

Lecture 1: Causal Identification

POL-GA 1251
Quantitative Political Analysis II
Prof. Cyrus Samii
NYU Politics

January 28, 2019

Plan for the week

Mon:

- ▶ Discuss overview of class and syllabus.
- ▶ Explain what “causal identification” means.
- ▶ Introduce the “potential outcomes” framework
- ▶ Relate it to linear regression model.

Wed:

- ▶ Randomized experiments.
- ▶ Explain *estimation* concepts (estimand, estimators, bias, consistency, efficiency).
- ▶ Explain *statistical inference* concepts (sampling distribution, randomization distribution, CLT, confidence intervals, p -value).
- ▶ First homework distributed.

Where this class fits in

Model of quantitative research process:

- ▶ Theory motivates causal hypothesis or target of inference:
 - ▶ *H: manipulating X results in (...) effect on Y.*
- ▶ Hypothesis, statistical theory, and substantive theory motivate a research design:
 - ▶ Operationalize X and Y .
 - ▶ Define ways to get optimal variation in X and Y given constraints.
- ▶ Research design and statistical theory motivate analysis plan:
 - ▶ Optimal estimation strategy, given constraints.
 - ▶ Optimal testing strategy, given constraints.

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In this class we focus on **causal identification**. This is a specific application of the general idea of “identification,” distinct from some other applications:

What do we mean by “identification”?

Alternative application of “identification” (I):

Suppose someone says...

...they prefer Harris over Sanders, and

...they prefer Warren over Harris.

Does this information (data) **identify** the person's preference ordering over these three candidates?

What do we mean by “identification”?

Alternative application of “identification” (II):

Suppose none of the coefficients below are equal to zero but the error terms (last ones) are iid mean zero draws. Which system of simultaneous equations **identifies** its coefficients?

$$x_t = \alpha_1^a + \alpha_2^a y_t + \nu_t^a$$

$$y_t = \beta_1^a + \beta_2^a x_t + \epsilon_t^a$$

$$x_t = \alpha_1^b + \alpha_2^b y_t + \alpha_3^b w_t + \alpha_4^b v_t + \nu_t^b$$

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Angrist and Krueger (1999):

*The combination of a clearly labeled source of identifying variation in a causal variable and the use of a particular econometric[/statistical] technique to exploit this information is what we call an **identification strategy**.*

Potential outcomes

The Road Not Taken

*Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;*

*Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,*

*And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.*

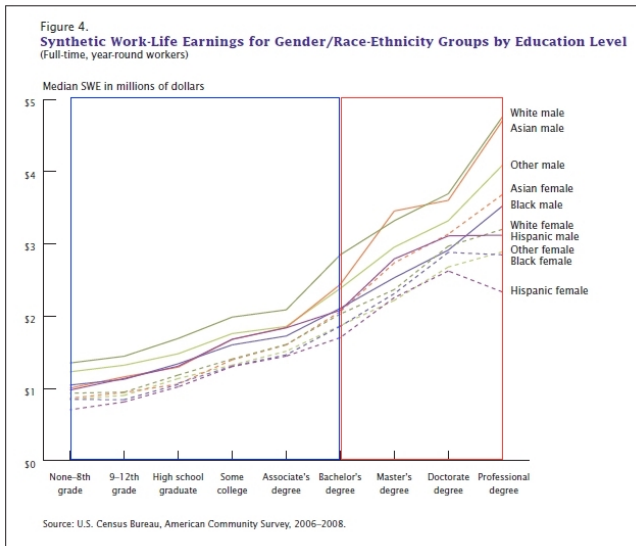
*I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.*

Modern frameworks for causal analysis:

- ▶ Potential outcomes (Neyman, 1923; Rubin, 1974, 1978).
- ▶ Causal graphs (Pearl, 2009).

Both rely on “counterfactual” logic.

Running Example: Effect of College on Earnings*



*SWE = expected total earnings over 25-64.

Potential outcomes

A causal effect can be defined as
a contrast between “potential outcomes.”

Potential outcomes

Experimental units in population P	Pretreatment values			Which treatment	Posttreatment values						Missing data indicator															
	X				W	Y						M														
						Y^1			...			Y^T			M^X			M^1			...			M^T		
	X_l	...	X_c			Y_l^1	...	Y_d^1		Y_l^T	...	Y_d^T	M_l^X	...	M_c^X	M_l^1	...	M_d^1		M_l^T	...	M_d^T				
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- Missing data indicators, $M_{i,j}^{(k)}$.

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- ▶ *Unit level causal effects* for members of \mathcal{P} are fixed a priori, and they compare y_{wi} to $y_{\tilde{w}i}$ for $w \neq \tilde{w}$ and $i \in \mathcal{P}$.
- ▶ *Population causal effects* for compare aggregates of unit level causal effects for members of \mathcal{P} .
- ▶ Effects are defined in an “agnostic” or “non-parametric” way.
- ▶ Potential outcomes and covariates are fixed, treatments and response indicators stochastic.
- ▶ Effects are defined by letting only treatments vary, holding units fixed.
- ▶ Thus, causal effects are clearly defined for units that can conceivably receive different treatment values.
- ▶ A test for the above is “manipulation” (Holland, 1986).

Potential outcomes, causal effects, and manipulability

Holland (1986) : “For causal inference, it is critical that each unit be potentially exposable to any one of the causes.”

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Issues arise when trying to interpret things like race or gender. See VanderWeele and Robinson (2014) for a formal treatment of ways to interpret “race effects.”

Potential outcomes and fundamental problem of causal inference

Recall, a unit level causal effect compares y_{wi} to $y_{\tilde{w}i}$ for $w \neq \tilde{w}$.

“Fundamental problem of causal inference” (Holland, 1986) : For each i potential outcomes for all w exist, but we only observe the potential outcome for the treatment value that i receives.

- ▶ “Scientific solution”: Use theory to determine when units are interchangeable.
- ▶ “Statistical solution”: Study features of conditional distributions, such as averages.

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- ▶ Consider a random draw, i , from \mathcal{P} , countable but large.
- ▶ Each draw is characterized by
 - ▶ a covariate vector, X_i ,
 - ▶ potential outcomes that under SUTVA are characterized as Y_{di} for all $d \in \mathcal{D}$, as well as
 - ▶ treatment assignments, $D_i \in \mathcal{D}$.

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$$\rho_i = Y_{1i} - Y_{0i},$$

for which,

$$E[\rho_i] = E[Y_{1i} - Y_{0i}] = \rho, \tag{1}$$

the average treatment effect (ATE).

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- ▶ For our running example, we have $D_i = 1$ if college, $D_i = 0$ if not. Outcome of interest is income. ρ is the average income benefit of college.

Causal identification under the potential outcomes model

- ▶ Consider simple difference mean college grad incomes vs mean no college incomes:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

- ▶ Now, suppose in the population $D_i = 1[f(X_i, U_i) > \eta_i]$.

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- ▶ For the full population,

$$\begin{aligned} E[Y_{1i}] &= E[Y_{1i}|f(X_i, U_i) > \eta_i] \Pr[f(X_i, U_i) > \eta_i] \\ &\quad + E[Y_{1i}|f(X_i, U_i) \leq \eta_i] \Pr[f(X_i, U_i) \leq \eta_i], \end{aligned}$$

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- ▶ Shows source of bias from selection into treatment.
- ▶ Similar could be shown for no-college outcomes.

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- ▶ E.g, consider a decomposition wrt ATT:

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- ▶ Could do similar wrt to ATC or effect heterogeneity (cf. CCI).

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(Why? E.g., if $A \perp\!\!\!\perp B$, what does this imply about $E[A|B]$?)

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As such,

$$\underbrace{E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]}_{\text{ATT}} = E[Y_{1i} - Y_{0i}],$$

so the simple difference, (2), equals ρ .

Causal identification under the potential outcomes model

Identifying assumption 2 (conditionally independent/unconfounded/strongly ignorable assignment):

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) | X_i \text{ and } 0 < Pr[D_i = 1 | X_i = x] < 1 \text{ for all } x \in \mathcal{X} \quad (4)$$

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Marginalization over \mathcal{X} , the support of X_i , yields,

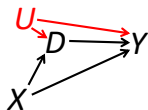
$$\int_{\mathcal{X}} \rho(x) dF(x) = \rho.$$

Causal graphs

- ▶ Alternative framework is causal graph framework (Pearl 2009).

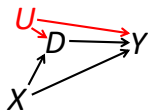
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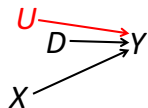
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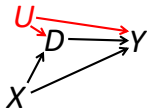
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equivalent to the post-intervention graph



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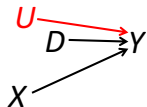
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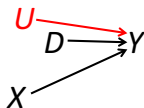
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- ▶ Difference between $E[Y|do(D_i = 1)] - E[Y|do(D_i = 0)]$ and $E[Y|D_i = 1] - E[Y|D_i = 0]$ is “backdoor paths” from D to Y .

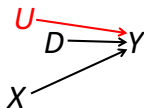
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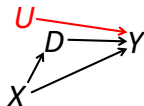


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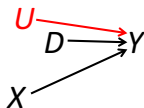


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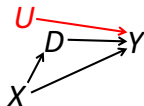


Causal graphs

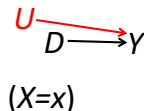
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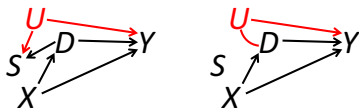
- ▶ Conditioning on X removes the other backdoor path:



- ▶ Marginalize over x to recover the intervention graph.

Causal graphs

- ▶ These are examples of “closing” backdoor paths.
- ▶ Other operations, e.g., “opening” backdoor paths by conditioning on “colliders”:



Looking forward to the rest of the class

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 - ▶ Plausibility is suspect, so need either other sources of identification or ways to check sensitivity to violations.

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- ▶ Time for questions.