Problem Set 1: Multiple Linear Regression

2 May 2023

For this problem set, we will study the dataset we will be studying is part of the **World Economics and Politics Dataverse** (WEP Dataverse).

The WEP Dataverse is a joint effort of the **Security and Political Economy** (SPEC) Lab at the **University of Southern California** and the **Niehaus Center for Globalization and Governance** at **Princeton University** to create a queryable data resource for comparative and international political economy.

The WEP Dataverse contains more than 800 variables from nearly 100 country-year and data-year datasets. You can visit https://ncgg.princeton.edu/wep/dataverse.html for more information, including the codebook.

Before you start, please download the dataset from Moodle and import it into `RStudio`.

If you would like to submit this problem set, please complete the questions at the end.

1 Variables

We will study the statistical association between corruption and economic development, one of the most classical questions in the political economy of development. Another way to rephrase the question is: **Does corruption hinder economic development?**

William Easterly, a Professor of Economics at New York University, has a brief discussion on this topic in his 2001 book *The Elusive Quest for Growth* published by the MIT Press (see Chapter 12). We will revisit this question again later this term when we cover observational or non-experimental causal inference.

We will need the following variables.

- `country`

- `year`
- `v2x_corr_VDEM` – the index of political corruption (continuous)
- `v2x_polyarchy_VDEM` – the index of electoral democracy (continuous)
- `democracy_DD` – the indicator of democracy (binary/dummy)
- `gdppc_WDI_PW` – gross domestic product, per capita (continuous)
- `lngdppc_WDI_PW` – logged gross domestic product, per capita (continuous)

The WEP Dataverse retrieved variables containing `_VDEM` in their names from from the **Varieties of Democracy** (V-Dem) Dataset, the largest cross-national datasets of democratic politics created by the University of Gothenburg, Sweden.

The binary indicator of democracy (i.e., it takes the value of 1 if the country is a democracy) is originally retrieved by the WEP Dataverse from the following article: **Cheibub, José Antonio, Jennifer Gandhi, and James Raymond Vreeland. 2010. "Democracy and Dictatorship Revisited."** ***Public Choice 143*****(1): 67-101.**

2 Preparation

Let us start by loading the following libraries.

```
library(car)
library(tidyverse)
library(stargazer)
```

Now make sure you have imported the dataset into `RStudio`. Since the dataset is a `.csv` file, we can use `read_csv()` from the `readr` package – this package has been included when you load `tidyverse`.

```
dta <- read_csv("05_01_23_0912pm_wep.csv")
```

We can use the `tidyverse` approach to process the original dataset to subset the dataset and select and recode the variables.

```
dta_sel <- dta |>
  dplyr::select(country, year,
  gdppc_WDI_PW, lngdppc_WDI_PW,
  democracy_DD,
  v2x_polyarchy_VDEM, v2x_corr_VDEM) |>
```

```
  mutate(v2x_polyarchy_VDEM_10 = v2x_polyarchy_VDEM*10,
         v2x_corr_VDEM_10 = v2x_corr_VDEM*10) |>
  filter(year == 2000) |>
  unique() |>
  drop_na()
```

We can use `head()` to take a quick a look at our dataset.

```
head(dta_sel)
```

Whenever you run into any new functions, it is always a good idea to use the question `?` to see what the function does. You can also consult the package manual, if necessary.

```
?head
```

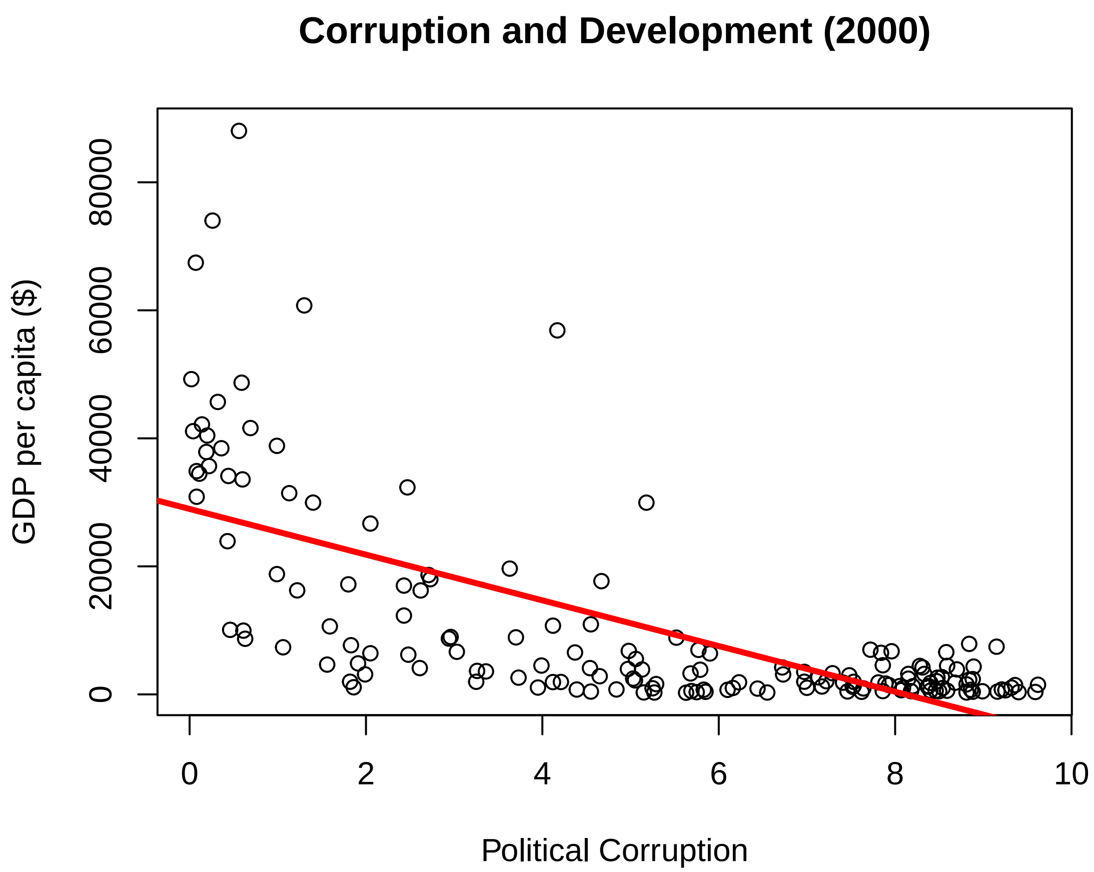
3 Task 1: Visualize the Data

Before we conduct any regression analysis, we can create a quick scatterplot to "see" the data. Since we are interested in the relationship between corruption and development, the variables of our particular interest now are `v2x_corr_VDEM_10` and `gdppc_WDI_PW`.

```
plot(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel,
     xlab = "Political Corruption",
     ylab = "GDP per capita ($)",
     main = "Corruption and Development (2000)")
```

Let's draw the regression line, using `abline`. The `abline` function has two arguments `col` and `lwd` we can use to choose the color and width of the line.

```
plot(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel,
     xlab = "Political Corruption",
     ylab = "GDP per capita ($)",
     main = "Corruption and Development (2000)")
abline(lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel),
       col = "red",
       lwd = 3)
```



What can we learn from this (two-dimensional) scatterplot? Anything we can do to make it better?

4 Task 2: Check Correlation without Using Regression

We can also check the pairwise correlation between `v2x_corr_VDEM_10` and `gdppc_WDI_PW`. One of the most common measures of pairwise correlation is the **Pearson correlation coefficient**. You can consult any statistical introductory textbook to see the formal definition or the formula.

To get the Pearson correlation coefficient between two variables, we can use `cor()`.

```
cor(dta_sel$v2x_corr_VDEM_10, dta_sel$gdppc_WDI_PW)
```

```
[1] -0.6780111
```

In the line above, we use the dollar sign `$` to call out a variable in a dataset.

5 Task 3: Bivariate Linear Regression

5.1 Model Specification

Now let us start the real business – linear regression. Again, we are interested in the correlation between political corruption and economic development. To put it more specifically, here political corruption is the **explanatory** variable with economic development as the **outcome** variable.

It is always a good idea to **specify** the model before we start any analysis.

$$Y = \alpha + \beta X + \epsilon,$$

where

- Y is GDP per capita (`gdppc_WDI_PW`)
- X is political corruption (`v2x_corr_VDEM_10`)

Our goal is to use linear regression to estimate α and β, the intercept and slope respectively. In other words, linear regression should generate the following fitted line:

$$Y = \hat{\alpha} + \hat{\beta} X.$$

5.2 Conduct the Analysis

The most canonical function to carry out linear regression in `R` is `lm()`. Below we will carry out the analysis and use `summary` to see the results.

```r
b_corruption <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel)
summary(b_corruption)
```

```
Call:
lm(formula = gdppc_WDI_PW ~ v2x_corr_VDEM_10, data = dta_sel)
```

```
Residuals:
   Min     1Q Median     3Q    Max
-21218  -7758   -397   4963  61059

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)         28966       1828   15.85   <2e-16 ***
v2x_corr_VDEM_10    -3570        304  -11.74   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11850 on 162 degrees of freedom
Multiple R-squared:  0.4597,    Adjusted R-squared:  0.4564
F-statistic: 137.8 on 1 and 162 DF,  p-value: < 2.2e-16
```

`R` also provides several useful tools for us to look into some more specific information of any linear regression analysis.

Function	Purpose
`coefficient()`	View the coefficients
`confint()`	View the confidence intervals
`resid()`	View the residuals
`fitted()`	View the predicted outcome

That being said, we can also use `ls()` to open (or "unearth") the output generated by the `lm()` function to obtain all the information above. We can similar use the dollar sign `$` to call them out.

```
ls(b_corruption)
```

```
 [1] "assign"        "call"          "coefficients"  "df.residual"
 [5] "effects"       "fitted.values" "model"         "qr"
 [9] "rank"          "residuals"     "terms"         "xlevels"
```

5.3 Statistical Inference

To begin with, **how should we interpret the estimated intercept and slope?** Are they statistically significant? Even if they are, does that mean corruption plays an important role in economic development?

Next, how about the **goodness-of-fit**? Let us take a look at the reported results again.

```r
b_corruption <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel)
summary(b_corruption)
```

```
Call:
lm(formula = gdppc_WDI_PW ~ v2x_corr_VDEM_10, data = dta_sel)

Residuals:
   Min     1Q Median     3Q    Max 
-21218  -7758   -397   4963  61059 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)         28966       1828   15.85   <2e-16 ***
v2x_corr_VDEM_10    -3570        304  -11.74   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11850 on 162 degrees of freedom
Multiple R-squared:  0.4597,    Adjusted R-squared:  0.4564 
F-statistic: 137.8 on 1 and 162 DF,  p-value: < 2.2e-16
```

R^2 is 0.46 while the adjusted R^2 is about the same. Again, R^2 is the fraction of the variance of the actual outcome explained by the variance of the predicted outcome. So we can verify the R^2 as follows.

```r
var_y_hat <- var(fitted(b_corruption))
var_y <- var(dta_sel$gdppc_WDI_PW)
var_y_hat/var_y
```

```
[1] 0.459699
```

The adjusted R^2 is a more advanced version to penalize the inclusion to too many variables (or the so-called "overfitting"). In other words, the formula of the adjusted R^2 will take the number of explanatory variables into account and the inclusion of more explanatory variables will suppress R^2 (i.e., adjust the original R^2 downwards).[1]

We can also try to calculate the sum of squared residuals as follows.

```
sum(resid(b_corruption)^2)
```

```
[1] 22756218447
```

Finally, you can try to run some **diagnostic tests to see how our model performs against the assumptions**.[2]

6 Task 4: Multiple Linear Regression

Now, let us consider another explanatory variable – democracy (`v2x_polyarchy_VDEM_10`). Political economists have also paid close attention to the relationship between democracy and development, with some arguing that democracy is more likely to induce long-term economic growth/development.

It is fairly straightforward to include more explanatory variables using `lm()`.

```
m_mod <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10 + v2x_polyarchy_VDEM_10,
data=dta_sel)
summary(m_mod)
```

```

Call:
lm(formula = gdppc_WDI_PW ~ v2x_corr_VDEM_10 + v2x_polyarchy_VDEM_10,
    data = dta_sel)

Residuals:
```

[1] See Section 4.2.6 in Imai (2018) for a more detailed discussion.
[2] See Section 1.6.1 in Roback and Leglar (2020) for a more detailed discussion.

```
   Min     1Q Median     3Q    Max
-22085  -7706   -266   4829  61175

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)    
(Intercept)            30112.6     4204.1   7.163 2.68e-11 ***
v2x_corr_VDEM_10       -3652.8      410.4  -8.900 1.11e-15 ***
v2x_polyarchy_VDEM_10   -139.0      458.8  -0.303    0.762    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11890 on 161 degrees of freedom
Multiple R-squared:   0.46, Adjusted R-squared:  0.4533 
F-statistic: 68.58 on 2 and 161 DF,  p-value: < 2.2e-16
```

And with one more predictor our model now becomes

$$\text{Development} = \alpha + \beta_1(\text{Corruption}) + \beta_2(\text{Democracy}) + \epsilon.$$

Instead of using X and Y, you can also use the abbreviation of the key variable names and explain the operationalization of each variable more carefully in the text. We will see good examples of quantitative research articles/notes in Week 5.

We can also use the `stargazer()` function to present several regression models.

Jack Russ has one of the most comprehensive guide on this function: https://www.jakeruss.com/cheatsheets/stargazer/. Here is a quick example.

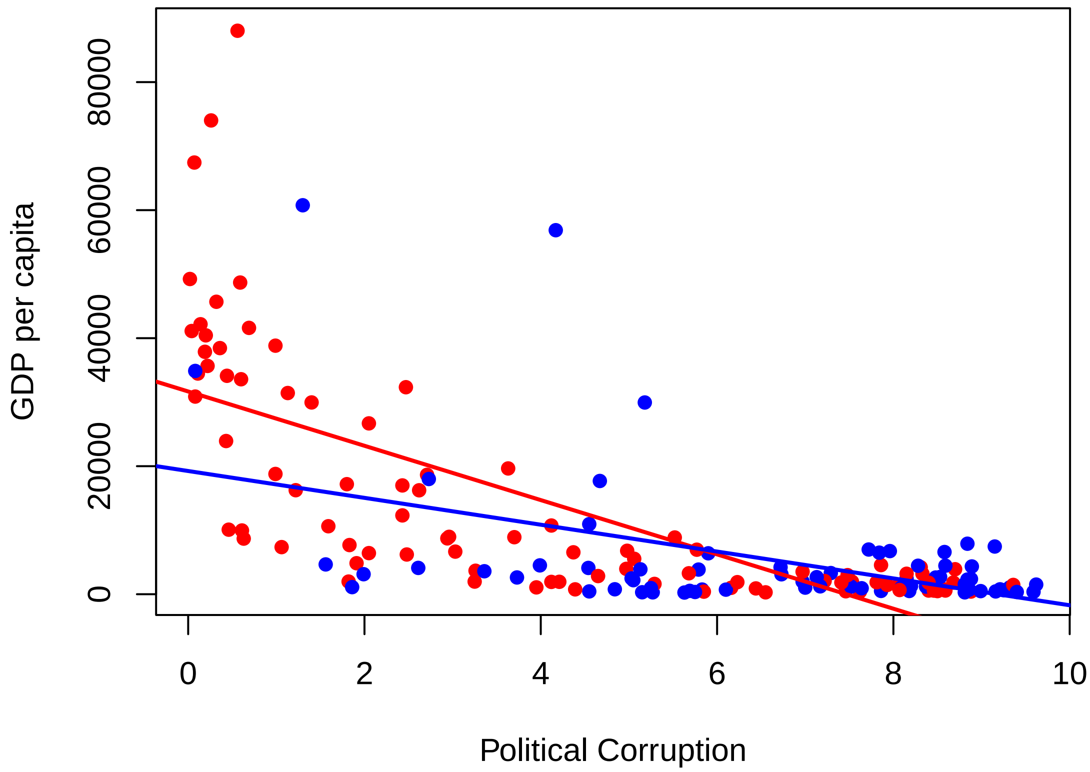
```r
stargazer(list(b_corruption, m_mod),
          omit.stat = c("f", "rsq", "ser"),
          dep.var.caption = "GDP per capita",
          column.labels = c("Model 1", "Model 2"),
          covariate.labels = c("Corruption", "Democracy"),
          type = "text",
          digits = 3,
          no.space = T,
          intercept.bottom = TRUE,
          star.cutoffs = c(0.05, 0.01, 0.001))
```

```
============================================
                      GDP per capita
             -------------------------------
                      gdppc_WDI_PW
                   Model 1         Model 2
                     (1)             (2)
--------------------------------------------
Corruption    -3,569.582***  -3,652.836***
                (304.047)       (410.420)
Democracy                       -139.042
                                (458.829)
Constant      28,966.030***  30,112.600***
               (1,827.460)     (4,204.064)
--------------------------------------------
Observations       164             164
Adjusted R2       0.456           0.453
============================================
Note:         *p<0.05; **p<0.01; ***p<0.001
```

7 Extra: Interaction (Using the Binary Predictor of Democracy)

7.1 Scatterplot with Regression Line by Group

```r
plot(gdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel,
     col = ifelse(dta_sel$democracy_DD == 1, "red", "blue"),
     pch = 16,
     xlab = "Political Corruption",
     ylab = "GDP per capita")
abline(lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10,
data=dta_sel[dta_sel$democracy_DD == 1,]),
col="red", lwd=2)
abline(lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10,
data=dta_sel[dta_sel$democracy_DD == 0,]),
col="blue", lwd=2)
```



7.2 Multiple Regression Analysis with Interaction Term

```
m_int_1 <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10 + democracy_DD,
data=dta_sel)
m_int_2 <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM_10 + democracy_DD +
v2x_corr_VDEM_10:democracy_DD, data=dta_sel)

stargazer(list(m_int_1, m_int_2),
          omit.stat = c("f", "rsq", "ser"),
          dep.var.caption = "GDP per capita",
          column.labels = c("Model 1", "Model 2"),
          covariate.labels = c("Corruption", "Democracy (=1)",
          "Corruption x Democracy (=1)"),
          type = "text",
          digits = 3,
          no.space = T,
          intercept.bottom = TRUE,
```

```r
          star.cutoffs = c(0.05, 0.01, 0.001))
```

```
==========================================================
                                GDP per capita
                            ------------------------------
                                  gdppc_WDI_PW
                               Model 1        Model 2
                                 (1)            (2)
----------------------------------------------------------
Corruption                  -3,573.812***  -2,097.128***
                              (335.680)      (586.749)
Democracy (=1)                -62.117       12,423.810**
                             (2,058.916)    (4,580.555)
Corruption x Democracy (=1)                -2,144.598**
                                             (707.101)
Constant                    29,022.800***  19,247.150***
                             (2,627.025)    (4,117.713)
----------------------------------------------------------
Observations                    164            164
Adjusted R2                    0.453          0.479
==========================================================
Note:                       *p<0.05; **p<0.01; ***p<0.001
```

8 Extra: Log Transformation

```r
m_log1 <- lm(lngdppc_WDI_PW ~ v2x_corr_VDEM_10, data=dta_sel)
m_log2 <- lm(lngdppc_WDI_PW ~ v2x_polyarchy_VDEM_10, data=dta_sel)
m_log3 <- lm(lngdppc_WDI_PW ~ v2x_corr_VDEM_10 + v2x_polyarchy_VDEM_10, data=dta_sel)

stargazer(list(m_log1, m_log2, m_log3),
          omit.stat = c("f", "rsq", "ser"),
          dep.var.caption = "GDP per capita (log)",
          column.labels = c("Model 1", "Model 2", "Model 3"),
          covariate.labels = c("Corruption", "Democracy (=1)"),
          type = "text",
```

```
          digits = 3,
          no.space = T,
          intercept.bottom = TRUE,
          star.cutoffs = c(0.05, 0.01, 0.001))
```

9 Extra: Panel Analysis (Use Country-Year as the Unit of Observation)

```
dta_sel_2 <- dta |>
  dplyr::select(country, year, gdppc_WDI_PW, lngdppc_WDI_PW, democracy_DD, v2x_polyarchy_V
  mutate(v2x_polyarchy_VDEM_10 = v2x_polyarchy_VDEM*10,
         v2x_corr_VDEM_10 = v2x_corr_VDEM*10) |>
  filter(year >= 2000 & year <= 2005) |>
  unique() |>
  drop_na()

m_long_1 <- lm(gdppc_WDI_PW ~ v2x_corr_VDEM + v2x_polyarchy_VDEM +
 as.factor(country) + as.factor(year),
data=dta_sel_2)
m_long_2 <- lm(lngdppc_WDI_PW ~ v2x_corr_VDEM + v2x_polyarchy_VDEM +
 as.factor(country) + as.factor(year),
data=dta_sel_2)

stargazer(list(m_long_1, m_long_2),
          omit = c("as.factor"),
          omit.stat = c("f", "rsq", "ser"),
          dep.var.caption = "GDP per capita",
          column.labels = c("Model 1", "Model 2"),
          covariate.labels = c("Corruption", "Democracy"),
          type = "text",
          digits = 3,
          no.space = T,
          intercept.bottom = TRUE,
          star.cutoffs = c(0.05, 0.01, 0.001))
```

10 Questions

Question 1 Visit the V-Dem Dataset's website, https://v-dem.net/data/the-v-dem-dataset/, and download their codebook to summarize the definitions of political corruption and electoral democracy. Do you agree with their definitions? Discuss.

Question 2

Repeat Task 3 but now use `v2x_polyarchy_VDEM_10` (the democracy score) as the explanatory variable. Name the model as `b_democracy` in the script.

- Interpret the estimated intercept and slope.
- Calculate the 95% confidence interval of the estimated slope ($\hat{\beta}$) and use it to explain whether $\hat{\beta}$ is statistically significance. If yes, also discuss its substantive significance.
- Use `stargazer` to show `b_democracy` and `m_mod` together. Look across all model specifications and discuss which explanatory variable may have a larger influence on economic development. Use at least one statistical tool to determine which model is the best fit.