The Butterfly Effect

MTG 6418

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24 April 2024

Could a butterfly flapping its wings cause a tornado on the other side of the world? This seems improbable. A concept commonly called the Butterfly Effect could make this appear more likely than a person might think. The Butterfly Effect demonstrates how sensitive chaotic dynamical systems are to initial conditions and shows how many conditions that seem unrelated are actually interconnected. This discovery was made while attempting to predict meteorological events on a computer. Weather forecasting is both an art and a science, relying on an understanding of atmospheric dynamics, it is not the only application of the Butterfly Effect. Although it can be extended in a similar vein to predict climate change, it can also be applied to diverse fields such as our legal system. At the epicenter of this exploration is Edward Lorenz, who laid the foundation we use today for complex dynamical systems. It is important to hear of his journey and contributions to understand where chaos was initiated from.

In his youth, Lorenz had an aptitude for mathematics and an interest in weather. After he graduated from Dartmouth in 1938, he was called up to fight in World War II, specifically as a weather forecaster for the Army Air Corps – a change in his mathematical journey perhaps, but not a dissatisfying one for him. While he did continue publishing his work in the field of mathematics afterwards, he also continued investigating meteorology. Meteorologists at this point in time, such as those at the Massachusetts Institute of Technology where Lorenz received his doctorate, did not consider forecasting to be real science. It was disorderly and did not produce useful solutions. These same meteorologists did not feel that computers were helping their cause. Computers were not for theoretical science, and "numerical weather modeling was something of a bastard problem"[1].

It had long been believed that the world was a deterministic environment. The same Newtonian laws that allowed the tracking of planets and tides should also apply to weather. Computers could do calculations incapable of being done by hand, therefore taking the guesswork out of short-term predictions. The complications of weather should not exclude it from the laws of nature. Even Laplace, a prominent mathematician and philosopher in the 18th century, was hopeful for the future of weather prediction. His optimism was kept in mind as computers were developed and soon put to use with the “ideal task”[1] of weather modeling. The issue at hand was the problem of approximation. It had been concluded previously that close approximations of data would lead to close approximations of solutions. This was the idea of convergence: that many small changes were not going to build up and change the entire outcome.

In the meantime, Lorenz had deconstructed everything weather was thought to be on his computer, the Royal McBee. It was large, it was noisy, and it was prone to break down. However, it was programmed essentially to run a mock environment. It would periodically print out the results of the that imaginary day’s weather. He noticed that the same weather pattern never repeated itself, although it did match up with his intuition about what the patterns should be doing. The eureka moment came from a shortcut. Lorenz wanted to scrutinize a sequence of results, so he re-ran the sequence starting partway through. To do that, he entered the starting data directly from a previous printout with one small, unnoticed change. When he saw the results, he thought his computer had broken again. The weather pattern he had attempted to reproduce had diverged so far from the original printout that it was unrecognizable. What was the change? Lorenz had rounded his numbers. Instead of entering up to six decimal places, he had only entered up to the third. It turns out the one one-thousandth really did matter.

This one-thousandth matters because even though small errors should seemingly cancel each other out somewhere, in Lorenz’s computer model they wreaked havoc. When he made this discovery, he decided that “long-range weather forecasting must be doomed”[1]. Then along came von Neumann, a charismatic man who excited people with the concept of controlling the weather, as he talked of geodomes and seeding clouds. His theory was that if he treated the weather as a dynamical system with specific unstable points, he could use those unstable points to make changes that would roll over into desired results. Lorenz contradicted this idea, suggesting that “it would be like giving an extra shuffle to an already well-shuffled pack of cards. You know it will change your luck, but you don’t know whether for better or worse”[1].

Lorenz took this opportunity to research and returned to his roots of being a mathematician. He felt that his weather model was not merely based on chance, but that there was a pattern to it. This would be a pattern of aperiodicity, just like other natural occurrences such as animal populations and epidemics. If the weather exactly repeated itself, we wouldn’t need to predict it. Instead, it comes close to repeating itself but instead becomes unpredictable. It was through this that he solidified his concept of the Butterfly Effect, or by its technical name, sensitive dependence on initial conditions. Small changes might not always cause drastic results, but at an unstable point in any system, a small change can change a system with startling rapidness[1].

It is important to understand the essence of Lorenz's Butterfly Effect, or rather what it has been narrowed down to: the Lorenz or Strange Attractor. Lorenz set his weather system aside and dove deeper into complex behavior, coming up with a system of fourteen nonlinear equations, which he soon narrowed down to twelve. With these, he went and consulted with Barry Saltzman, who was working on a system of six equations to model convection. This gave Lorenz the information necessary to adapt his model to be constructed only from three variables, described as "the development of a solvable problem, a transformation to alternative coordinate systems, and the approximation of solutions"[6].

He was able to narrow his system down because of inspiration from convection, a type of fluid motion. On a small scale, an example of convection is when the air is seen moving above hot pavement. On a large scale, the sun heats the surface of the earth, and therefore the air. The hot air rises and cools, and the newly cooled air circulates back down to the surface where it is reheated, and the cycle continues. This creates a rolling motion of the gas or liquid being heated. This can be tested by creating a "cell of fluid"[1] where the floor and ceiling are close in temperature. The fluid tends to remain at rest, and any small disturbances dissipate. Imagine now that there is a larger difference in temperature – the low-level fluid heats up, expands, loses density, and circulates to the top, while the reverse is happening to the upper level (Figure 1). At extreme temperature differences, the system begin to fall apart. This falling-apart is well illustrated by the construction of the waterwheel.

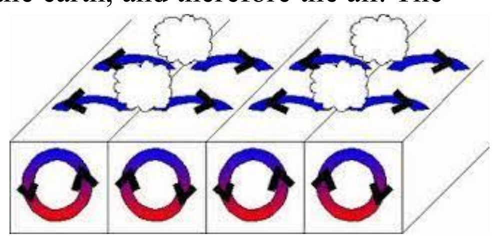


Figure 1

Visualize a waterwheel where water is being poured steadily through from directly above. The top bucket is being filled up, but not enough to cause motion. This is representative of when the difference in temperature between the top and bottom of the fluid cell negligible. When enough water fills the top bucket to set the wheel in motion it begins to turn, and the filled bucket pulls the wheel around, causing empty buckets to

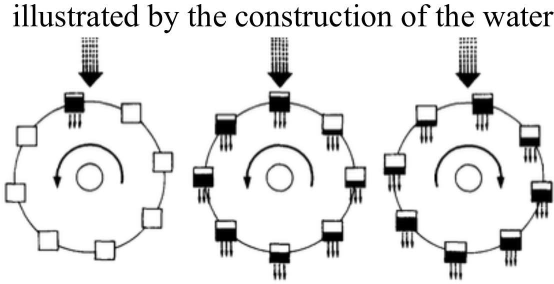


Figure 2

rise up to the top and get filled. As the buckets circle around, they are losing water which is how they are able to move back up on the other side. This is the same way air or fluid in the cell heats back up as it rotates back to the bottom of the cell, giving it the ability to rise once more. The chaos comes in when the wheel is spinning too quickly. The buckets do not fill enough at the top or empty enough on their journey around the wheel. The wheel starts moving erratically, spinning in different directions (Figure 2).

The Lorenz Attractor is a graphical model of these scenarios. A point travels in three dimensions, governed by the three equations Lorenz developed. It moves in a figure-eight pattern, except that it never goes through the same point twice. This can be seen with its trace. The trace of this points traces what can be described as the wings of a butterfly. Sometimes the point will circle around one wing of the butterfly more than another for a while, and then pass back over to the other side. A second point set in motion at the same time, with a slightly different starting location, will end up in a completely different spot on the wings than the original point after some iterations go by (Figure 3). The speed of the point is also not constant, as can be seen by the distances between the points in the trace.

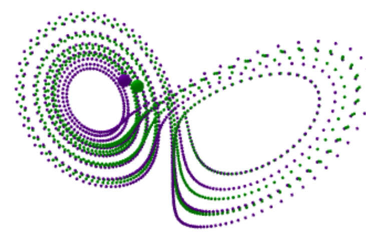
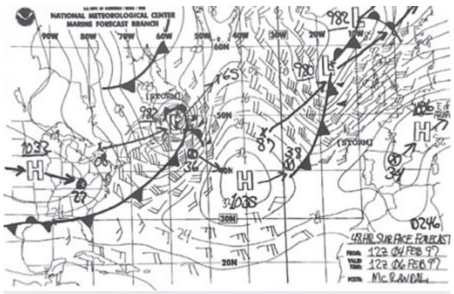


Figure 3

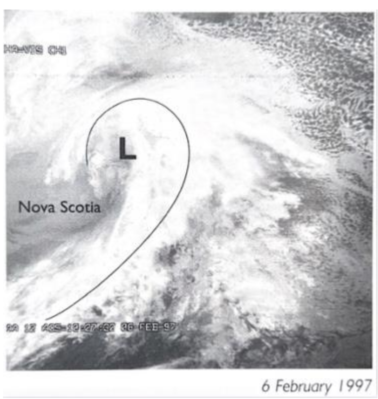
For a portion of my research, I interviewed Michael Carr, a seasoned mariner and author. Carr presented a thorough scenario in which chaotic dynamical systems would be applied in weather forecasting today, which he explains in more detail in *Weather Predicting Simplified*[2]. The challenge presented to us is to gather data, create a system of equations that weights the data based on importance or relevance, and run that system for a useful amount of time. A person might gather information such as height above sea level, barometric pressure, and ocean temperature. They then use that data to compute a local weather forecast. At the time the data was gathered, the system will output a relatively accurate output for a certain amount of time. Data is only accurate for the moment it was taken, and the longer the person looks into the future, the more unreliable the prediction becomes, because the real-time circumstances are changing.

Today is Saturday, and we may not be able to say with certainty that it will rain at 10:00 a.m. on this coming Friday. That would be a deterministic model of forecasting. A probabilistic forecast would tell us that there is an 80% chance of it raining on Friday morning, and now I can plan my day accordingly. The National Oceanographic and Atmospheric Administration, or NOAA, runs the National Weather Service. The National Weather Service operates satellites, weather buoys, and weather offices that measure heat, moisture, convection, and other parameters constantly and feeds them into NOAA's bank of computers. The computers take that data and insert it into weather models which then constantly update to give us the forecasts and visuals we have today. Trends also impact the probability of certain events occurring, such as moisture in the air or temperature increasing or decreasing over time, and then that amount of time being tracked has to be determined as well. Carr provided the formula $\triangle T = \triangle D = \triangle P =$ event, which means that a change in temperature leads to a change in density, which leads to a change in pressure, which leads to a weather event. These events are all products of the chaotic dynamical systems being used in forecasting and prediction.



A 48-hour surface forecast over the Atlantic Ocean for 6 Feb, 1997[2]

Carr stated that "weather forecasts can accurately predict up to 120 hours into the future. If we asked a computer for information five days in advance, it could give us results within a few hours." You may notice, as I did, that websites like weather.com will give 10 to 14-day forecasts. This is because of commercial weather companies "making gross assumptions and extrapolating them". Referring back to the more accurate five-day forecast, those five days are not static, but they do follow a trendline temporarily. It is also important to note that the quality of the data being input can impact the quality of predictions being output. A two-degree difference in accuracy one day can be the difference between a calm day and a stormy day next week.



An infrared image of the forecasted weather, as expected on 6 Feb, 1997[2]

Flash forward thirty-seven years from the publishing of Gleick's *CHAOS*, and Lorenz's strange attractor and sensitive dependence on initial conditions have become popular in many fields of mathematics and science. It is put into use in expected ways such as analyzing financial market patterns, but also in unexpected ways in literature and art[3]. One way chaotic dynamical systems are being applied now is in the predictions of climate change[4]. Climate change is not just one equation predicting that the atmosphere will heat up and the icebergs will all melt by date X, and then the polar bears will be extinct. It is a complex, detailed, and sensitive system of equations based on coupled equations for environmental models, covering every conceivable change. This includes the ocean, atmospheric, and land dynamical systems, together called the weather (short term) and the climate (long term)[4].

In order for the predictions to increase in accuracy, an increasing number of parameters and processes have to be included. More parameters and processes increase the complexity, and the whole system becomes unmanageable. Thus, it is often better to experiment with and study a system that is less complex and more easily understood, with the data included being specifically tailored to the problem being studied. This was illustrated in a comparison of computations on a data set completed on several computing models (ECHAM5, RegCM, and Eta-POM). Though they varied in levels of complexity, they produced comparable results[4].

An unexpected way that dynamical systems and chaos can be applied is in the law and legal science. An apt metaphor for chaotic systems is given in Thomas Burri's article[5]:

> Imagine that we are kneading dough...we use a rolling pin to roll out the biscuit dough until it is flat, then we fold the dough, placing one half on top of the other, and recommence rolling. This constitutes one kneading operation (one repetition). The idea is to repeat the same kneading operation *ad infinitum*. At any time while kneading some spices are placed on the dough and kneading continues. Then emergences a central property of chaos, which is called *sensitive dependence on initial conditions*...after extensive kneading it will end up at a completely different place in the dough. Small initial differences may thus produce large final differences[5].

The article continues on to explain mixing with a similar metaphor – that if you know where the spices are now in the dough, you cannot know from where in the dough they began. Chaos, and the iterations that lead to it, are actually quite common. Think computers, traffic lights, populations, and life cycles.

But what about the law? This may sound like a stretch, because what could mathematics and legal science possibly have to do with each other. Burri describes case law as a machine that "devours the facts of a new case and then produces a result"[5]. Now, however, the case law machinery involves individuals, meaning the client wanting their lawyer to reassure them of a positive outcome. Due to the sensitive dependence on initial conditions, or all the small decisions based on facts presented leading up to the final judgement, the lawyer cannot say with 100% certainty what the outcome will be. With chaos, it is possible to eventually reach any position from any previous starting points, which is known as mixing. The fixed points in law would be the "legal principles that resist change...sometimes it just takes a great number of iterations"[5]. Chaos theory produced a change in legal culture, for example whether an act by an individual caused an event has now shifted to the thought process of whether the act contributed to the event. The law has become less black and white; less binary.

Chaotic systems, or sensitive dependence on initial conditions, can bring about relationships between conditions that would never be suspected as being interconnected. Mixing, the reverse of chaos, also plays a role in this as it shows how present conditions could have been created by a multitude of beginning scenarios. Whether it is pure mathematical analysis of dynamical systems and bifurcation maps, the chaotic analysis of the data that allows us to forecast the weather within reasonable parameters, or the chaotic iterations of decisions in the legal systems, chaos is clearly everywhere.

Sources:

[1] Gleick, J. (1987). *CHAOS: Making a New Science*. Penguin Books.

[2] Carr, M. W. (1999). *Weather Predicting Simplified*. International Marine.

[3] Gleick, J. (2008). *CHAOS: Making a New Science.* Penguin Books.

[4] Mihailovic, D.T., Mimic, G., Arsenic, I. (2014). Climate Predictions: The Chaos and Complexity in Climate Models. *Advances in Meteorology*, vol. 2014, 14 pages. https://doi.org/10.1155/2014/878249.

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