1 Terms

=: Without any additional rewriting.
$\rightarrow_{flatten}$: Use rewrites to flatten body of term. Uses algebraic distribution property.
$\rightarrow_{match}$: Looks for common terms to cancel.
$\rightarrow_{refactor}$: Unflattens.Groups term with the same denominator.
$\rightarrow_{pull}$:. Makes the numerator in a division term 1.

2 PVO

field#4(3)[] F= $(\frac{V}{|V|}) \bullet (\frac{U}{|U|})$

Probe $\frac{(A \bullet C)}{<||A||*||C||>}$

Gradient .
$= ((A \bullet (((\frac{1}{|A|} * (((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) + ((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))))^T) + (\frac{1}{|A|} * \frac{1}{|C|} * (C \bullet \nabla \otimes A)))$

$\rightarrow_{flatten} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$

$\rightarrow_{match} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$

$\rightarrow_{refactor} (\frac{(((\nabla \otimes C)^T \bullet A)+(C \bullet \nabla \otimes A))}{(|A|*|C|)} - (\frac{((C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{((C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))}))$

$\rightarrow_{pull} ((((\nabla \otimes C)^T \bullet A) + (C \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|)} - (((C \bullet A) * (A \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|*(A \bullet A))} + ((C \bullet A) * (C \bullet \nabla \otimes C)) * \frac{1}{(|C|*|A|*(C \bullet C))}))$

Hessian .

$= ((A \bullet Trav(((\frac{1}{|A|} * (((\frac{(-(((|C|*(C \bullet C)*((C \bullet \nabla \otimes \nabla \otimes C)+((\nabla \otimes C)^T \bullet \nabla \otimes C))))-(-((((((2*|C|*(C \bullet \nabla \otimes C))+((C \bullet C)*\frac{(C \bullet \nabla \otimes C)}{|C|}))*(C$
$(|C|*(C \bullet C)*|C|*(C \bullet C))}$
$C)) + Trav(((\nabla \otimes C * \frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))}))) < 2, :, 0 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes C)) <$
$1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * \nabla \otimes C))) < 2, 0, 1 >)) + Trav(((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*(A \bullet A))} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T))))) < 1, 0, :> + ((\frac{(-(((|A|*|C|*(A \bullet A)*((A \bullet \nabla \otimes \nabla \otimes A)+((\nabla \otimes A)^T \bullet \nabla \otimes A))))-(-((((((|A|$
$(|A|*|C$
$C)) + Trav(((\nabla \otimes C * \frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))}))) < 2, :, 0 >)) < 1, 2, 0 >) + (((\frac{1}{|A|} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) + ((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))) \bullet \nabla \otimes A) +$
$(\frac{1}{|A|} * ((C \bullet Trav((Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes A)) < 1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} *$
$\nabla \otimes A))) < 2, 0, 1 >)) < 1, 2, 0 >) + (\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C)))) + (\frac{1}{|C|} * ((C \bullet$
$\nabla \otimes A) \otimes \frac{-(((A \bullet \nabla \otimes A)))}{(|A|*(A \bullet A))})))$

$$\rightarrow_{flatten} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+\frac{((C\bullet C)*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|C|*|A|*((C\bullet C))^2)}+$$
$$\frac{-(((|C|*(C\bullet C)*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+\frac{-(((|C|*(C\bullet C)*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+(\frac{-(((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+$$
$$\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)}+\frac{-((((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))}+(\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T+$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))}+\frac{(2*|C|*|A|*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}+\frac{(|A|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))}+$$
$$\frac{(|C|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{((|A|)^2*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))}+\frac{-(((|A|*|C|*(A\bullet A)*(C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}+\frac{-((|A|*|C|*(A\bullet A)*(C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}$$
$$(\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+(\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)}+$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)}+\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+$$
$$\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)}+\frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$

$$\rightarrow_{match} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)}+\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{-((|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}+$$
$$\frac{-((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}+(\frac{-((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)}+$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))}+(\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T+\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))}+$$
$$\frac{(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*((A\bullet A))^2)}+\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{((C\bullet C)*|A|*|C|*(A\bullet A))}+\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*((A\bullet A))^2)}+$$
$$\frac{-(((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))}+\frac{-(((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))}+(\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)}+(\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)}+$$
$$\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)}+\frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$

$$\rightarrow_{refactor} ((\frac{(((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))}{(|C|*|A|)}+$$
$$\frac{(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|A|*|C|*((A\bullet A))^2)}+\frac{((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C}{((C\bullet C)*|A|*|C|*(A\bullet A))}$$
$$\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)})-(\frac{((((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A}{(((((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))^T)+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+$$
$$\frac{((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}))$$

$$\rightarrow_{pull} (((((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))*\frac{1}{(|C|*|A|)}+(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|A|*|C|*((A\bullet A))^2)}+((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C))))*\frac{1}{((C\bullet C)*|A|*|C|*(A\bullet A))}+((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|C|*|A|*((C\bullet C))^2)}+(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*((C\bullet C))^2)})-(((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))+((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))+(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C)))+((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T)+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+(((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|C|*|A|*(A\bullet A))}+(((((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))*\frac{1}{(|A|*|C|*(C\bullet C))}+((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*(C\bullet C))}))$$

3 Moe

field#k(3)[3] G= ∇V
field#k(3)[] F= $(-\nabla|G|) \bullet (\frac{G}{|G|})$

Probe $\rightarrow_{none} ((-\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} \bullet \nabla A) * \frac{1}{|\nabla A|})$

$\rightarrow_{rewrite} \frac{(-1*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2)}$

Gradient .

$$= ((\frac{1}{|\nabla A|} * ((\nabla A \bullet (-(\frac{((|\nabla A| * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) - (((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T) +$$
$$(-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A))) + ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}))$$

$$\rightarrow_{flatten} (\frac{-((|\nabla A| * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{-((|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} +$$
$$\frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2)} + \frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))})$$

$$\rightarrow_{match} (\frac{-(((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} + \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} + \frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)} +$$
$$\frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)})$$

$$\rightarrow_{refactor} (\frac{-(((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) + ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} +$$
$$\frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)})$$

$$\rightarrow_{pull} (-(((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) +$$
$$((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))) * \frac{1}{(\nabla A \bullet \nabla A)} + (2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet$$
$$\nabla \otimes \nabla A)) * \frac{1}{(((\nabla A \bullet \nabla A))^2)})$$

Hessian .

$$= ((((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * ((\nabla A \bullet (-(\frac{((|\nabla A| * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) - (((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T$$
$$(-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A)))))^T + (\frac{1}{|\nabla A|} * ((-(\frac{((|\nabla A| * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) - (((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) * (\nabla A \bullet \nabla$$
$$(\nabla A \bullet \nabla A)}$$
$$\nabla \otimes \nabla A) + (\nabla A \bullet Trav(-(\frac{(((\nabla A \bullet \nabla A) * (((|\nabla A| * (Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) <:,2,1> + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) <1,2,0> + 1$$
$$1,2,0>) + ((\nabla \otimes \nabla A)^T \bullet (-(\frac{((|\nabla A| * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) - (((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T) +$$
$$(-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla \otimes \nabla A))) + (((((\nabla A \bullet (-(\frac{((|\nabla A| * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) - (((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) * (\nabla A$$
$$(\nabla A \bullet \nabla A)}$$
$$(-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A)) * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})))^T + ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) *$$
$$\frac{(-((|\nabla A| * (\nabla A \bullet \nabla A) * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A) + (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) - (-((((((\nabla A \bullet \nabla A) * \frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) + (2 * |\nabla A| * (\nabla A \bullet \nabla \otimes \nabla A))) * (\nabla A \bullet \nabla \otimes$$
$$(|\nabla A| * (\nabla A \bullet \nabla A) * |\nabla A| * (\nabla A \bullet \nabla A))$$

$$\rightarrow_{flatten} ((\frac{(|\nabla A| * ((\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))})^T + (\frac{(|\nabla A| * ((\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))})^T +$$
$$(\frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)})^T + (\frac{(((\nabla A \bullet \nabla \otimes \nabla A) * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))})^T +$$

$$\left(\frac{-((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}\right)^T+\left(\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}\right)^T+$$
$$\left(\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))}\right)^T+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+\left(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}\right)^T+$$
$$\left(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}\right)^T+\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+\left(\frac{-(((\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}\right)^T+$$
$$\frac{(2*|\nabla A|*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(2*|\nabla A|*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\left(\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}\right)^T+\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))}+\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes$$
$$\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+\frac{(((\nabla A\bullet\nabla\otimes\nabla A)\otimes((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))}+$$
$$\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{((|\nabla A|)^2*|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{((|\nabla A|)^2*|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*|\nabla A|*(\nabla A\bullet\nabla A))^2)}+\frac{(-2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A))}{((|\nabla A|)^2*|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}$$

$$\rightarrow_{match}\left(\left(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}\right)^T+\left(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}\right)^T+\right.$$
$$\left(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}\right)^T+\left(\frac{(2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}\right)^T+$$
$$\left(\frac{-(((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}\right)^T+\left(\frac{-(((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}\right)^T+\left(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A$$
$$\frac{-(((\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla$$
$$\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-((((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}+\frac{-((((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}$$
$$\left(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}\right)^T+\left(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}\right)^T+$$
$$\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}\Big)$$

$$\rightarrow_{refactor}\left(\frac{(((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))))^T+((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))))^T+((2*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla\right.\right.$$
$$\left.\left(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}\right)^T+\frac{-((((((((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))+(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(^T\right.\right.$$

$$\rightarrow_{pull}\left(((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))))^T+((((\nabla A\bullet\right.$$
$$\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))))^T+((2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet$$
$$(\nabla A\bullet\nabla\otimes\nabla A)))))^T+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+$$
$$(((((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+((2*((\nabla A\bullet(\nabla A\bullet$$
$$\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes$$
$$\nabla A))+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))+(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet$$
$$\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))-$$
$$(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))-(((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes$$

$$(\nabla A \bullet \nabla \otimes \nabla A))))) * \frac{1}{(((\nabla A \bullet \nabla A))^2)} + ((-8 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet \nabla \otimes \nabla A))) * \frac{1}{(((\nabla A \bullet \nabla A))^2 * (\nabla A \bullet \nabla A))})^T + -((((((((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) + (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 0, 1 > \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + ((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla A))) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 0, 2 >)) + (((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))^T) + (((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))^T)) * \frac{1}{(\nabla A \bullet \nabla A)})$$

4 Canny

Gradient $= ((\frac{1}{|\nabla A|} * (\nabla A \bullet (-(2 * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)))^T)) +$
$(-(2 * (\nabla A \bullet \nabla \otimes \nabla A)) \bullet ((((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T + ((\frac{(-1*(\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A))))^T))$

$\rightarrow_{flatten} (\frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} +$
$\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))})$

$\rightarrow_{match} (\frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} +$
$\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))})$

$\rightarrow_{refactor} (\frac{(((-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) + (-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{|\nabla A|} +$
$\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))})$

$\rightarrow_{pull} ((((-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes$
$\nabla A)))) + (-2 * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) * \frac{1}{|\nabla A|} + (2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes$
$\nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(|\nabla A| * (\nabla A \bullet \nabla A))})$

Hessian $= ((\frac{1}{|\nabla A|} * (-(2 * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) \bullet$
$\nabla \otimes \nabla A)) + (\nabla A \bullet Trav((Trav(((\frac{(-1*(\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * -(2 * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)$
$1, 0, :> + (\frac{1}{|\nabla A|} * -(2 * (Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) <:, 2, 1> + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1$
$1, 2, 0 >) + ((((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T + ((\frac{(-1*(\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A))) \bullet (-(2 * ((((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla$
$(-(2 * (\nabla A \bullet \nabla \otimes \nabla A)) \bullet Trav((Trav(((\frac{(-1*(\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla \otimes \nabla A))) < 2, 0, 1 >$
$+ Trav((\frac{1}{|\nabla A|} * \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\nabla \otimes \nabla A * \frac{(-1*(\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))}))) <$
$2, :, 0 > + ((\frac{((-1*|\nabla A| * (\nabla A \bullet \nabla A) * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A) + (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))) - ((-1*((((\nabla A \bullet \nabla A) * \frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) + (2*|\nabla A| * (\nabla A \bullet \nabla \otimes$
$\nabla A)))) < 1, 2, 0 >))$

$\rightarrow_{flatten} ((\frac{(-2*((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})^T + (\frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|})^T +$
$\frac{(2*((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(2*((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} +$
$\frac{(-2*(\nabla A \bullet Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 0, 2 >))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla ($
$\frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} +$
$(\frac{(2*((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T + (\frac{(2*((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T +$
$\frac{(2*(((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(-2*(Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 0, 1 > \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} +$
$(\frac{(2*(((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T + \frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} +$
$\frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(-2*(\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} +$
$\frac{(-4*|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)})$

$\rightarrow_{match} ((\frac{(-2*((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})^T + (\frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|})^T +$

$$\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+$$
$$\frac{(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla($$
$$\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+$$
$$(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+$$
$$\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\frac{(-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+$$
$$(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})$$

$$\rightarrow_{refactor}(\frac{((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))+(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla$$
$$\frac{((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$(((((((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A$$
$$\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})$$

$$\rightarrow_{pull}(((((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))+(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))+((-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))^T)+((-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))^T)*\frac{1}{|\nabla A|}+((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2)}+(((((((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T)+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T)+(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))*\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A))}+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))*\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})$$