1 PVO computation

image(3)[3] u = image("../data/inputfile0.nrrd"); image(3)[3] v = image("../data/inputfile1.nrrd");
kernel#4 k= c4hexic;
field#4(3)[3]U =k⊛ u; field#4(3)[3]V =k ⊛ v;
field#4(3)[] G= $(\frac{V}{|V|}) \bullet (\frac{U}{|U|})$

1.1 best version

Probe Diderot-Dev –exec –readEinLatex –readEin1 PVO.diderot
$\rightarrow_{flat} \frac{(v \bullet u)}{(|v|*|u|)}$

Gradient Diderot-Dev –exec –readEinLatex –readEin3 PVO.diderot

$\rightarrow_{refactor} ((\frac{(u \bullet \nabla \otimes v)}{(|v|*|u|)} + \frac{(v \bullet \nabla \otimes u)}{(|u|*|v|)}) - (\frac{((u \bullet v)*(v \bullet \nabla \otimes v))}{(|v|*|u|*(v \bullet v))} + \frac{((u \bullet v)*(u \bullet \nabla \otimes u))}{(|u|*|v|*(u \bullet u))}))$

Hessian Diderot-Dev –exec –readEinLatex –readEin2 PVO.diderot

$\rightarrow_{match} (\frac{(2*|u|*(u \bullet v)*((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u)))}{(|v|*(u \bullet u)*((u \bullet u))^2)} + \frac{((u \bullet v)*((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u)))}{(|u|*|v|*((u \bullet u))^2)} + \frac{-(((|u|*(u \bullet v)*(u \bullet \nabla \otimes \nabla \otimes u)))}{(|v|*(u \bullet u)*(u \bullet u))} + \frac{-(((|u|*(u \bullet v)*((\nabla \otimes u)^T \bullet \nabla \otimes u)))}{(|v|*(u \bullet u)*(u \bullet u))} + \frac{-((((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes u))))}{(|u|*|v|*(u \bullet u))} + \frac{(v \bullet \nabla \otimes \nabla \otimes u)}{(|u|*|v|)} + \frac{-((((v \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))))}{(|u|*|v|*(u \bullet u))} + \frac{((u \bullet v)*((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes v)))}{(|u|*|v|*(u \bullet u)*(v \bullet v))} + \frac{-((((v \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes v))))}{(|u|*|v|*(v \bullet v))} + \frac{(2*(u \bullet v)*((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))}{(|u|*|v|*((v \bullet v))^2)} + \frac{((u \bullet v)*((v \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes u)))}{((u \bullet u)*|v|*|u|*(v \bullet v))} + \frac{((u \bullet v)*((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))}{(|u|*|v|*((v \bullet v))^2)} + \frac{-(((u \bullet v)*(v \bullet \nabla \otimes \nabla \otimes v)))}{(|v|*|u|*(v \bullet v))} + \frac{-(((u \bullet v)*((\nabla \otimes v)^T \bullet \nabla \otimes v)))}{(|v|*|u|*(v \bullet v))} + \frac{-((((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes u))))}{(|v|*|u|*(v \bullet v))} + \frac{-((((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes v))))}{(|u|*|v|*(u \bullet u))} + \frac{((\nabla \otimes u)^T \bullet \nabla \otimes v)}{(|u|*|v|)} + \frac{-((((v \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes v))))}{(|v|*|u|*(v \bullet v))} + \frac{(u \bullet \nabla \otimes \nabla \otimes v)}{(|u|*|v|)} + \frac{-((((u \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes u))))}{(|v|*|u|*(u \bullet u))} + \frac{((\nabla \otimes v)^T \bullet \nabla \otimes u)}{(|u|*|v|)} + \frac{-((((u \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v))))}{(|u|*|v|*(v \bullet v))})$

Diderot-Dev –exec –readEinLatex –readEin4 PVO.diderot

$\rightarrow_{pull-division} (((((\nabla \otimes v)^T \bullet \nabla \otimes u) + (v \bullet \nabla \otimes \nabla \otimes u) + ((\nabla \otimes u)^T \bullet \nabla \otimes v) + (u \bullet \nabla \otimes \nabla \otimes v)) * \frac{1}{(|u|*|v|)} + (((u \bullet v) * ((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v))) + (2 * (u \bullet v) * ((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))) * \frac{1}{(|u|*|v|*((v \bullet v))^2)} + ((u \bullet v) * ((v \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{((u \bullet u)*|v|*|u|*(v \bullet v))} + ((u \bullet v) * ((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes v))) * \frac{1}{(|u|*|v|*(u \bullet u)*(v \bullet v))} + ((u \bullet v) * ((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|u|*|v|*((u \bullet u))^2)} + (2 * |u| * (u \bullet v) * ((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|v|*(u \bullet u)*((u \bullet u))^2)}) - (((((v \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes v))) + (((u \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))) * \frac{1}{(|u|*|v|*(v \bullet v))} + (((u \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|v|*|u|*(u \bullet u))} + ((((v \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes v))) + ((u \bullet v) * (v \bullet \nabla \otimes \nabla \otimes v)) + ((u \bullet v) * ((\nabla \otimes v)^T \bullet \nabla \otimes v)) + (((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes u)))) * \frac{1}{(|v|*|u|*(v \bullet v))} + ((((v \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) + (((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes u))) + (((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes v)))) * \frac{1}{(|u|*|v|*(u \bullet u))} + ((|u| * (u \bullet v) * (u \bullet \nabla \otimes \nabla \otimes u)) + (|u| * (u \bullet v) * ((\nabla \otimes u)^T \bullet \nabla \otimes u))) * \frac{1}{(|v|*(u \bullet u)*(u \bullet u))}))$

1.2 Multiple rewrites

Diderot-Dev –exec –readEinLatex –readEinRewrite PVO.diderot
Each computation goes through four steps of rewriting →. Here we list the output for all four steps. The most readable version varies with computation.
Probe
$\rightarrow_{flat} \frac{(v \bullet u)}{(|v|*|u|)}$

$\rightarrow_{match} \frac{(v \bullet u)}{(|v|*|u|)}$

$\rightarrow_{refactor} \frac{(v \bullet u)}{(|v|*|u|)}$

$\rightarrow_{pull-division} (v \bullet u) * \frac{1}{(|v|*|u|)}$

Gradient
$\rightarrow_{flat} (\frac{-(((u \bullet v)*(u \bullet \nabla \otimes u)))}{(|u|*|v|*(u \bullet u))} + \frac{(v \bullet \nabla \otimes u)}{(|u|*|v|)} + \frac{-(((u \bullet v)*(v \bullet \nabla \otimes v)))}{(|v|*|u|*(v \bullet v))} + \frac{(u \bullet \nabla \otimes v)}{(|v|*|u|)})$

$\rightarrow_{match} (\frac{-(((u \bullet v)*(u \bullet \nabla \otimes u)))}{(|u|*|v|*(u \bullet u))} + \frac{(v \bullet \nabla \otimes u)}{(|u|*|v|)} + \frac{-(((u \bullet v)*(v \bullet \nabla \otimes v)))}{(|v|*|u|*(v \bullet v))} + \frac{(u \bullet \nabla \otimes v)}{(|v|*|u|)})$

$$\rightarrow_{refactor} ((\frac{(u\bullet\nabla\otimes v)}{(|v|*|u|)} + \frac{(v\bullet\nabla\otimes u)}{(|u|*|v|)}) - (\frac{((u\bullet v)*(v\bullet\nabla\otimes v))}{(|v|*|u|*(v\bullet v))} + \frac{((u\bullet v)*(u\bullet\nabla\otimes u))}{(|u|*|v|*(u\bullet u))}))$$

$$\rightarrow_{pull-division} (((u \bullet \nabla \otimes v) * \frac{1}{(|v|*|u|)} + (v \bullet \nabla \otimes u) * \frac{1}{(|u|*|v|)}) - (((u \bullet v) * (v \bullet \nabla \otimes v)) * \frac{1}{(|v|*|u|*(v\bullet v))} + ((u \bullet v) * (u \bullet \nabla \otimes u)) * \frac{1}{(|u|*|v|*(u\bullet u))}))$$

Hessian

$$\rightarrow_{flat} (\frac{(2*|u|*(u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{((|u|)^2*|v|*(u\bullet u)*(u\bullet u))} + \frac{((u\bullet u)*(u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{((|u|)^2*|u|*|v|*((u\bullet u))^2)} + \frac{-(((|u|*(u\bullet u)*(u\bullet v)*(u\bullet\nabla\otimes\nabla\otimes u)))}{((|u|)^2*|v|*(u\bullet u)*(u\bullet u))} + \frac{-(((|u|*(u\bullet u)*(u\bullet v)*((\nabla\otimes u)^T$$
$$\frac{-((((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes u))))}{(|u|*|v|*(u\bullet u))} + \frac{(v\bullet\nabla\otimes\nabla\otimes u)}{(|u|*|v|)} + \frac{-((((v\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u))))}{(|u|*|v|*(u\bullet u))} + \frac{((u\bullet v)*((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v)))}{(|u|*|v|*(u\bullet u)*(v\bullet v))} + \frac{-((((v\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*(v\bullet v))} +$$
$$\frac{(2*|u|*|v|*(u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v)))}{(|v|*|u|*|v|*|u|*((v\bullet v))^2)} + \frac{(|v|*(v\bullet v)*(u\bullet v)*((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u)))}{(|u|*|v|*|u|*|v|*|u|*(v\bullet v)*(v\bullet v))} + \frac{(|u|*(v\bullet v)*(u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v)))}{(((|v|)^2*|u|*|v|*|u|*(v\bullet v)*(v\bullet v))} + \frac{-(((|v|*|u|*(v\bullet v)*(u\bullet v)*(v\bullet$$
$$\frac{-((|v|*|u|*(v\bullet v)*(u\bullet v)*((\nabla\otimes v)^T\bullet\nabla\otimes v)))}{(|v|*|u|*|v|*|u|*((v\bullet v))^2)} + \frac{-((((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes u))))}{(|v|*|u|*(v\bullet v))} + \frac{-((((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes v))))}{(|u|*|v|*(u\bullet u))} + \frac{((\nabla\otimes u)^T\bullet\nabla\otimes v)}{(|u|*|v|)} + \frac{-((((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes v))))}{(|v|*|u|*(v\bullet v))} +$$
$$\frac{(u\bullet\nabla\otimes\nabla\otimes v)}{(|u|*|v|)} + \frac{-((((u\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u))))}{(|v|*|u|*(u\bullet u))} + \frac{((\nabla\otimes v)^T\bullet\nabla\otimes u)}{(|u|*|v|)} + \frac{-((((u\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*(v\bullet v))})$$

$$\rightarrow_{match} (\frac{(2*|u|*(u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{(|v|*(u\bullet u)*((u\bullet u))^2)} + \frac{((u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{(|u|*|v|*((u\bullet u))^2)} + \frac{-(((|u|*(u\bullet v)*(u\bullet\nabla\otimes\nabla\otimes u)))}{(|v|*(u\bullet u)*(u\bullet u))} + \frac{-(((|u|*(u\bullet v)*((\nabla\otimes u)^T\bullet\nabla\otimes u)))}{(|v|*(u\bullet u)*(u\bullet u))} +$$
$$\frac{-((((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes u))))}{(|u|*|v|*(u\bullet u))} + \frac{(v\bullet\nabla\otimes\nabla\otimes u)}{(|u|*|v|)} + \frac{-((((v\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u))))}{(|u|*|v|*(u\bullet u))} + \frac{((u\bullet v)*((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v)))}{(|u|*|v|*(u\bullet u)*(v\bullet v))} + \frac{-((((v\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*(v\bullet v))} +$$
$$\frac{(2*(u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v)))}{(|u|*|v|*((v\bullet v))^2)} + \frac{((u\bullet v)*((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u)))}{((u\bullet u)*|v|*|u|*(v\bullet v))} + \frac{((u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v)))}{(|u|*|v|*((v\bullet v))^2)} + \frac{-(((u\bullet v)*(v\bullet\nabla\otimes\nabla\otimes v)))}{(|v|*|u|*(v\bullet v))} + \frac{-(((u\bullet v)*((\nabla\otimes v)^T\bullet\nabla\otimes v)))}{(|v|*|u|*(v\bullet v))} +$$
$$\frac{-((((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes u))))}{(|v|*|u|*(v\bullet v))} + \frac{-((((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes v))))}{(|u|*|v|*(u\bullet u))} + \frac{((\nabla\otimes u)^T\bullet\nabla\otimes v)}{(|u|*|v|)} + \frac{-((((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes v))))}{(|v|*|u|*(v\bullet v))} + \frac{(u\bullet\nabla\otimes\nabla\otimes v)}{(|u|*|v|)} + \frac{-((((u\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u))))}{(|v|*|u|*(u\bullet u))} +$$
$$\frac{((\nabla\otimes v)^T\bullet\nabla\otimes u)}{(|u|*|v|)} + \frac{-((((u\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*(v\bullet v))})$$

$$\rightarrow_{refactor} (((\frac{(((\nabla\otimes v)^T\bullet\nabla\otimes u)+(v\bullet\nabla\otimes\nabla\otimes u)+((\nabla\otimes u)^T\bullet\nabla\otimes v)+(u\bullet\nabla\otimes\nabla\otimes v))}{(|u|*|v|)} + \frac{(((u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v)))+(2*(u\bullet v)*((v\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*((v\bullet v))^2)} +$$
$$\frac{((u\bullet v)*((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u)))}{((u\bullet u)*|v|*|u|*(v\bullet v))} + \frac{((u\bullet v)*((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v)))}{(|u|*|v|*(u\bullet u)*(v\bullet v))} + \frac{((u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{(|u|*|v|*((u\bullet u))^2)} + \frac{(2*|u|*(u\bullet v)*((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))}{(|v|*(u\bullet u)*((u\bullet u))^2)}) -$$
$$(\frac{((((v\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes v)))+(((u\bullet\nabla\otimes v)\otimes(v\bullet\nabla\otimes v))))}{(|u|*|v|*(v\bullet v))} + \frac{(((u\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes u)))}{(|v|*|u|*(u\bullet u))} + \frac{((((v\bullet\nabla\otimes v)\otimes(u\bullet\nabla\otimes v)))+((u\bullet v)*(v\bullet\nabla\otimes\nabla\otimes v))+((u\bullet v)*((\nabla\otimes v)^T\bullet\nabla\otimes v))+($$
$$\frac{((((v\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes u)))+(((u\bullet\nabla\otimes u)\otimes(v\bullet\nabla\otimes u)))+(((u\bullet\nabla\otimes u)\otimes(u\bullet\nabla\otimes v))))}{(|u|*|v|*(u\bullet u))} + \frac{((|u|*(u\bullet v)*(u\bullet\nabla\otimes\nabla\otimes u))+(|u|*(u\bullet v)*((\nabla\otimes u)^T\bullet\nabla\otimes u)))}{(|v|*(u\bullet u)*(u\bullet u))}))$$

$$\rightarrow_{pull-division} (((((\nabla \otimes v)^T \bullet \nabla \otimes u) + (v \bullet \nabla \otimes \nabla \otimes u) + ((\nabla \otimes u)^T \bullet \nabla \otimes v) + (u \bullet \nabla \otimes \nabla \otimes v)) * \frac{1}{(|u|*|v|)} +$$
$$(((u \bullet v) * ((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v))) + (2 * (u \bullet v) * ((v \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))) * \frac{1}{(|u|*|v|*((v\bullet v))^2)} + ((u \bullet v) * ((v \bullet \nabla \otimes$$
$$v) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{((u\bullet u)*|v|*|u|*(v\bullet v))} + ((u \bullet v) * ((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes v))) * \frac{1}{(|u|*|v|*(u\bullet u)*(v\bullet v))} + ((u \bullet v) * ((u \bullet$$
$$\nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|u|*|v|*((u\bullet u))^2)} + (2 * |u| * (u \bullet v) * ((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|v|*(u\bullet u)*((u\bullet u))^2)}) - ((((((v \bullet$$
$$\nabla \otimes u) \otimes (v \bullet \nabla \otimes v))) + (((u \bullet \nabla \otimes v) \otimes (v \bullet \nabla \otimes v)))) * \frac{1}{(|u|*|v|*(v\bullet v))} + (((u \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes u))) * \frac{1}{(|v|*|u|*(u\bullet u))} +$$
$$((((v \bullet \nabla \otimes v) \otimes (u \bullet \nabla \otimes v))) + ((u \bullet v) * (v \bullet \nabla \otimes \nabla \otimes v)) + ((u \bullet v) * ((\nabla \otimes v)^T \bullet \nabla \otimes v)) + (((v \bullet \nabla \otimes v) \otimes (v \bullet$$
$$\nabla \otimes u)))) * \frac{1}{(|v|*|u|*(v\bullet v))} + ((((v \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes u))) + (((u \bullet \nabla \otimes u) \otimes (v \bullet \nabla \otimes u))) + (((u \bullet \nabla \otimes u) \otimes (u \bullet \nabla \otimes$$
$$v)))) * \frac{1}{(|u|*|v|*(u\bullet u))} + ((|u| * (u \bullet v) * (u \bullet \nabla \otimes \nabla \otimes u)) + (|u| * (u \bullet v) * ((\nabla \otimes u)^T \bullet \nabla \otimes u))) * \frac{1}{(|v|*(u\bullet u)*(u\bullet u))})$$