New language features

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1 Defining operators between fields

1.1 Abs, Max, and Min

Implementation lift max and min operators to the field level and represents the magnitude of scalar fields with absolute expressions. Extra syntax is created to support differentiation.

Design   Need to add new syntax to support differentiation.

$$
\begin{array}{rcll}
cond & = & e > e \mid e < e & \text{conditional} \\
e & = & \text{Max}(a,b) \mid \text{Min}(a,b) & \text{Binary EIN operators} \\
& \mid & \text{if}(cond, e, e) & \text{If wrapper returns tensor-valued expression} \\
& \mid & \text{Abs}(e) & \text{Absolute function} \\
& \mid & \text{Sgn}(e) & \text{Returns Sign (-1, 0, 1)}
\end{array}
$$

Differentiation rules   Differentiation of an absolute expression:

$$\frac{\partial}{\partial x_\alpha}\text{abs}(e) \rightarrow (\frac{\partial}{\partial x_\alpha}e) * (\text{Sgn}(e))$$

Differentiation creates an if wrapper expressions.

$$\frac{\partial}{\partial x_\alpha}\text{Max}(a,b) \rightarrow if(a > b, \frac{\partial}{\partial x_\alpha}a, \frac{\partial}{\partial x_\alpha}b)$$

$$\frac{\partial}{\partial x_\alpha}\text{Min}(a,b) \rightarrow if(a < b, \frac{\partial}{\partial x_\alpha}a, \frac{\partial}{\partial x_\alpha}b)$$

Differentiation of an If wrapper is pushed to leaves.

$$\frac{\partial}{\partial x_\alpha}\text{If}(cond, c, d) \rightarrow if(cond, \frac{\partial}{\partial x_\alpha}c, \frac{\partial}{\partial x_\alpha}d)$$

other rules   Otherwise, Max and Min are treated like other binary operators. The following pushes the probes to the leaves.

$$(\text{Max}(a,b))(x) \rightarrow \text{Max}(a(x), b(x))$$

2 Testing results

XT6 Using If Wrapper with other field operators

```
field #4(2)[]G = compose(minF((F0),(F1)),(F2*0.1));
```

Field operators need to be applied to the leaves in if wrapper. The composition is a field operator so it needed to be pushed to the leaves.

$$(\text{If}(c, e3, e4)) \circ es \rightarrow \text{If}(c, e3 \circ es, e4 \circ es)$$