

1 PVO

$$\text{field}\#4(3) \parallel F = (\frac{V}{|V|}) \bullet (\frac{U}{|U|})$$

$$\text{Probe } \frac{(A \bullet C)}{\langle ||A|| * ||C|| \rangle}$$

Gradient .

$$\rightarrow_{none} ((\frac{1}{|C|} * \frac{1}{|A|} * (C \bullet \nabla \otimes A)) + (A \bullet (((((C * \frac{(-1 * (A \bullet \nabla \otimes A))}{(|C| * |A| * (A \bullet A))})))^T + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes C))^T + (((C * \frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)})))^T))))^T)$$

$$\rightarrow_{rewriteM-H} (\frac{(A \bullet \nabla \otimes C)}{(|A| * |C|)} + \frac{(-1 * (A \bullet C) * (C \bullet \nabla \otimes C))}{(|A| * |C| * (C \bullet C))} + \frac{(-1 * (A \bullet C) * (A \bullet \nabla \otimes A))}{(|C| * |A| * (A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A| * |C|)})$$

Hessian .

$$\begin{aligned} &\rightarrow_{none} (((\frac{1}{|C|} * (\frac{(-1 * (A \bullet \nabla \otimes A))}{((A \bullet A) * |A|)} * (C \bullet \nabla \otimes A))))^T + (\frac{1}{|A|} * ((\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C)) + \\ &(C \bullet Trav((Trav(((\nabla \otimes A * \frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)}))) < 1, 2, 0 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes \\ &A)) < 1, 2, 0 >))) < 1, 2, 0 >))) + (((((C * \frac{(-1 * (A \bullet \nabla \otimes A))}{(|C| * |A| * (A \bullet A))})))^T + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes \\ &C))^T + (((C * \frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)})))^T))) \bullet \nabla \otimes A) + (A \bullet Trav((Trav(((\frac{(-1 * (A \bullet \nabla \otimes A))}{(|C| * |A| * (A \bullet A))} * \\ &\nabla \otimes C))) < :, 2, 1 > + Trav(((C * \frac{((-1 * |C| * |A| * (A \bullet A) * (((\nabla \otimes A)^T \bullet \nabla \otimes A) + (A \bullet \nabla \otimes \nabla \otimes A)) - (-1 * ((A \bullet \nabla \otimes A) \otimes (((A \bullet A) * |C| * \frac{(A \bullet \nabla \otimes A))}{|A|})))}{(|C| * |A| * |A| * (A \bullet A) * (A \bullet A) * |C|)}) \\ &1, 2, 0 > + Trav((((\frac{1}{|C|} * \nabla \otimes C))^T + (((C * \frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)})))^T * \frac{(-1 * (A \bullet \nabla \otimes A))}{((A \bullet A) * |A|)}))) < : \\ &, 2, 1 > + (\frac{1}{|A|} * (Trav(((\nabla \otimes C * \frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)}))) < 1, 2, 0 > + Trav((\frac{1}{|C|} * \nabla \otimes \\ &\nabla \otimes C)) < 1, 2, 0 > + Trav(((\frac{(-1 * (C \bullet \nabla \otimes C))}{((C \bullet C) * |C|)} * \nabla \otimes C))) < :, 2, 1 > + Trav(((C * \\ &\frac{((-1 * (C \bullet C) * |C| * (((\nabla \otimes C)^T \bullet \nabla \otimes C) + (C \bullet \nabla \otimes \nabla \otimes C))) - (-1 * ((C \bullet \nabla \otimes C) \otimes (((C \bullet C) * \frac{(C \bullet \nabla \otimes C))}{|C|}) + (|C| * 2 * (C \bullet \nabla \otimes C))))}{((C \bullet C) * |C| * (C \bullet C) * |C|)}))) < \\ &1, 2, 0 >))) < 1, 2, 0 >)) \end{aligned}$$

$$\begin{aligned} &\rightarrow_{rewriteM} (\frac{(|C| * -1 * (A \bullet C) * (C \bullet C) * (Trav(\nabla \otimes \nabla \otimes C) < 2, 0, 1 > \bullet C))}{((|C|)^2 * (C \bullet C) * |A| * (C \bullet C))} + (\frac{(|C| * -1 * (A \bullet C) * (C \bullet C) * ((\nabla \otimes C)^T \bullet \nabla \otimes C))}{((|C|)^2 * (C \bullet C) * |A| * (C \bullet C))})^T + \\ &\frac{(|C| * 2 * (A \bullet C) * (((\nabla \otimes C)^T \bullet C) \otimes ((\nabla \otimes C)^T \bullet C))}{((|C|)^2 * (C \bullet C) * |A| * (C \bullet C))} + (\frac{((C \bullet C) * (A \bullet C) * (((\nabla \otimes C)^T \bullet C) * ((\nabla \otimes C)^T \bullet C)))}{((|C|)^2 * |A| * |C| * ((C \bullet C))^2})^T + \\ &\frac{(|A| * -1 * (A \bullet A) * |C| * (A \bullet C) * (Trav(\nabla \otimes \nabla \otimes A) < 2, 0, 1 > \bullet A))}{(|A| * |C| * |A| * |C| * ((A \bullet A))^2)} + (\frac{(|A| * -1 * (A \bullet A) * |C| * (A \bullet C) * ((\nabla \otimes A)^T \bullet \nabla \otimes A))}{(|A| * |C| * |A| * |C| * ((A \bullet A))^2)})^T + \\ &(\frac{(|A| * 2 * |C| * (A \bullet C) * (((\nabla \otimes A)^T \bullet A) * ((\nabla \otimes A)^T \bullet A)))}{(|A| * |C| * |A| * |C| * ((A \bullet A))^2)})^T + \frac{((A \bullet C) * |C| * (A \bullet A) * (((\nabla \otimes A)^T \bullet A) \otimes ((\nabla \otimes A)^T \bullet A)))}{((|A|)^2 * |A| * |C| * (A \bullet A) * |C| * (A \bullet A))} + \\ &\frac{((A \bullet C) * |A| * (A \bullet A) * (((\nabla \otimes A)^T \bullet A) \otimes ((\nabla \otimes C)^T \bullet C)))}{(|A| * |C| * |A| * |C| * (A \bullet A) * |C| * (A \bullet A))} + \frac{(-1 * (((\nabla \otimes A)^T \bullet C) \otimes ((\nabla \otimes A)^T \bullet A)))}{(|A| * |C| * (A \bullet A))} + \frac{(-1 * (((\nabla \otimes A)^T \bullet A) \otimes ((\nabla \otimes C)^T \bullet A)))}{(|A| * |C| * (A \bullet A))} + \\ &\frac{(-1 * (((\nabla \otimes C)^T \bullet C) \otimes ((\nabla \otimes C)^T \bullet A)))}{(|C| * |A| * (C \bullet C))} + \frac{(Trav(\nabla \otimes \nabla \otimes C) < 2, 0, 1 > \bullet A)}{(|C| * |A|)} + (\frac{(-1 * (((\nabla \otimes C)^T \bullet C) * ((\nabla \otimes C)^T \bullet A)))}{(|C| * |A| * (C \bullet C))})^T + \\ &(\frac{((C \bullet A) * (((\nabla \otimes A)^T \bullet A) * ((\nabla \otimes C)^T \bullet C)))}{(|C| * |A| * (C \bullet C) * (A \bullet A))})^T + (\frac{(-1 * (((\nabla \otimes A)^T \bullet A) * ((\nabla \otimes C)^T \bullet A)))}{(|C| * |A| * (A \bullet A))})^T + \frac{(-1 * (((\nabla \otimes A)^T \bullet A) \otimes (C \bullet \nabla \otimes A)))}{(|A| * |C| * (A \bullet A))} + \\ &\frac{(-1 * (((\nabla \otimes C)^T \bullet C) \otimes (C \bullet \nabla \otimes A)))}{(|C| * |A| * (C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet \nabla \otimes A)}{(|C| * |A|)} + (\frac{((\nabla \otimes C)^T \bullet \nabla \otimes A)}{(|C| * |A|)})^T + \frac{(Trav(\nabla \otimes \nabla \otimes A) < 2, 0, 1 > \bullet C)}{(|C| * |A|)} + \\ &(\frac{(-1 * (((\nabla \otimes C)^T \bullet C) * ((\nabla \otimes A)^T \bullet C)))}{(|A| * |C| * (C \bullet C))})^T) \\ &\rightarrow_{rewriteH} ((\frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})^T + (\frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|})^T + \end{aligned}$$

$$\begin{aligned}
& \left(\frac{(2*((\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{((\nabla A \bullet \nabla A) * |\nabla A|)} \right)^T + \left(\frac{(2*((\nabla A \bullet \nabla \otimes \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{((\nabla A \bullet \nabla A) * |\nabla A|)} \right)^T + \\
& \frac{(-2 * (\nabla A \bullet Trav((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 0, 2 >))}{|\nabla A|} + \frac{(-2 * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2 * (\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla \otimes \nabla A)) < 2, 1, 0 >))}{|\nabla A|} \\
& \frac{(-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \left(\frac{(2*((\nabla A \bullet \nabla \otimes \nabla A) * ((\nabla \otimes \nabla A)^T \bullet \nabla A \bullet \nabla \otimes \nabla A)))}{((\nabla A \bullet \nabla A) * |\nabla A|)} \right)^T + \frac{(-2 * (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 0, 1 > \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) < 2, 0, 1 >))}{|\nabla A|} \\
& \frac{(2 * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((\nabla A \bullet \nabla A) * |\nabla A|)} + \frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(2 * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{((\nabla A \bullet \nabla A) * |\nabla A|)} \\
& \frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(2 * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{((\nabla A \bullet \nabla A) * |\nabla A|)} + \frac{(-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla A) * ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A) * |\nabla A| * (\nabla A \bullet \nabla A))} \\
& \frac{(|\nabla A| * -4 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(|\nabla A| * 2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla A) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \\
& \frac{(|\nabla A| * 2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)}
\end{aligned}$$

2 Canny

$$\begin{aligned}
& \text{field\#k(3)[3] } G = \nabla F \\
& \text{field\#k(3)[] } F = (-\nabla(G \bullet G)) \bullet (\frac{G}{|G|})
\end{aligned}$$

Probe .

$$\begin{aligned}
& \rightarrow_{none} .(((- (2 * (\nabla A \bullet \nabla \otimes \nabla A)) \bullet \nabla A) * \frac{1}{|\nabla A|})) \\
& \rightarrow_{rewriteM-H} \frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * -2)}{|\nabla A|}
\end{aligned}$$

Gradient .

$$\begin{aligned}
& \rightarrow_{none} (((\frac{1}{|\nabla A|} * (\nabla A \bullet (-2 * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T))))^T) + \\
& (-2 * (\nabla A \bullet \nabla \otimes \nabla A)) \bullet (((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T + (((\nabla A * \frac{(-1 * (\nabla A \bullet \nabla \otimes \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)}))^T))^T) \\
& \rightarrow_{rewriteM} (\frac{(2 * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)} + \frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla A)))}{|\nabla A|} + \\
& \frac{(-2 * (\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla A))}{|\nabla A|} + \frac{(-2 * (\nabla A \bullet (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))}{|\nabla A|}) \\
& \rightarrow_{rewriteH} (\frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)} + \frac{(-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \\
& \frac{(-2 * (\nabla A \bullet (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))}{|\nabla A|} + \frac{(-2 * (\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla A))}{|\nabla A|})
\end{aligned}$$

Hessian .

$$\begin{aligned}
& \rightarrow_{none} (((\frac{1}{|\nabla A|} * (-2 * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) \bullet \\
& \nabla \otimes \nabla A) + (\nabla A \bullet Trav((Trav(((- (2 * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T)) * \\
& \frac{(-1 * (\nabla A \bullet \nabla \otimes \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)})))) < :, 2, 1 > + (\frac{1}{|\nabla A|} * -2 * (Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < :, 2, 1 > + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 >)) + (((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T + (((\nabla A * \frac{(-1 * (\nabla A \bullet \nabla \otimes \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)}))^T)) \bullet (-2 * (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T + ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T) \\
& (-2 * (\nabla A \bullet \nabla \otimes \nabla A)) \bullet Trav((Trav(((\nabla \otimes \nabla A * \frac{(-1 * (\nabla A \bullet \nabla \otimes \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)})))) < 1, 2, 0 > \\
& + Trav((\frac{1}{|\nabla A|} * \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\frac{(-1 * (\nabla A \bullet \nabla \otimes \nabla A))}{((\nabla A \bullet \nabla A) * |\nabla A|)} * \nabla \otimes \nabla A))) < :
\end{aligned}$$

