Terms:
$\rightarrow_{none}$ No rewriting
$\rightarrow_{rewriteM}$ Does some rewriting to flatten terms.
$\rightarrow_{rewriteH}$ Does more aggressive rewriting.
$\rightarrow_{rewriteM-H}$ output is the same for both.

1 PVO

field#4(3)[] F= $(\frac{V}{|V|}) \bullet (\frac{U}{|U|})$

1.1 In one line

/Users/chariseechiw/diderot/chiw17/bin/diderotc –exec –readEinDebug PVO.diderot

Probe $= (\frac{1}{|A|} * \frac{1}{|C|} * (A \bullet C))$
$\rightarrow_{flatten} \frac{(A \bullet C)}{(|A|*|C|)}$
$\rightarrow_{match} \frac{(A \bullet C)}{(|A|*|C|)}$
$\rightarrow_{refactor} \frac{(A \bullet C)}{(|A|*|C|)}$
$\rightarrow_{pull} (A \bullet C) * \frac{1}{(|A|*|C|)}$

Gradient $. = ((A \bullet (((\frac{1}{|A|} * (((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) +$
$((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))))^T) + (\frac{1}{|A|} * \frac{1}{|C|} * (C \bullet \nabla \otimes A)))$
$\rightarrow_{flatten} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$
$\rightarrow_{match} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$
$\rightarrow_{refactor} (\frac{(((\nabla \otimes C)^T \bullet A)+(C \bullet \nabla \otimes A))}{(|A|*|C|)} - (\frac{((C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{((C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))}))$
$\rightarrow_{pull} ((((\nabla \otimes C)^T \bullet A) + (C \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|)} - (((C \bullet A) * (A \bullet \nabla \otimes A)) *$
$\frac{1}{(|A|*|C|*(A \bullet A))} + ((C \bullet A) * (C \bullet \nabla \otimes C)) * \frac{1}{(|C|*|A|*(C \bullet C))}))$

Hessian $. = ((A \bullet Trav(((\frac{1}{|A|} * (((\frac{(-(((|C|*(C \bullet C)*((C \bullet \nabla \otimes \nabla \otimes C)+((\nabla \otimes C)^T \bullet \nabla \otimes C))))) - (-((((((2*|C|*(C \bullet \nabla \otimes C))+((C \bullet C)*\frac{(C \bullet$
$(|C|*(C \bullet C))*|C|*(C \bullet C))}{|}$
$C)) + Trav(((\nabla \otimes C * \frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))}))) < 2, :, 0 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes C)) <$
$1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * \nabla \otimes C))) < 2, 0, 1 >)) + Trav(((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*(A \bullet A))} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)))) < 1, 0, :> + ((\frac{(-((|A|*|C|*(A \bullet A)*((A \bullet \nabla \otimes \nabla \otimes A)+((\nabla \otimes A)^T \bullet \nabla \otimes A))))) - (-((((((|A|$
$(|A|*|C$
$C)) + Trav(((\nabla \otimes C * \frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))}))) < 2, :, 0 >)) < 1, 2, 0 >) + (((\frac{1}{|A|} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) + ((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))) \bullet \nabla \otimes A) +$
$(\frac{1}{|A|} * ((C \bullet Trav((Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes A)) < 1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} *$
$\nabla \otimes A))) < 2, 0, 1 >)) < 1, 2, 0 >) + (\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C)))) + (\frac{1}{|C|} * ((C \bullet$

$$\nabla \otimes A) \otimes \frac{-(((A\bullet\nabla\otimes A)))}{(|A|*(A\bullet A))}))))$$
$$\rightarrow_{flatten} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))} + \frac{((C\bullet C)*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|C|*|A|*((C\bullet C))^2)} +$$
$$\frac{-((|C|*(C\bullet C)*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))} + \frac{-((|C|*(C\bullet C)*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))} + (\frac{-(((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T +$$
$$\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)} + \frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))} + (\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T +$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))} + \frac{(2*|C|*|A|*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)} + \frac{(|A|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))} +$$
$$\frac{(|C|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{((|A|)^2*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))} + \frac{-(((|A|*|C|*(A\bullet A)*(C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)} + \frac{-((|A|*|C|*|A|*|C|*(A\bullet A)*(C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}$$
$$(\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*|C|*(A\bullet A))})^T + (\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T + \frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)} +$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*|C|*(A\bullet A))})^T + \frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)} + \frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*|C|*(C\bullet C))} +$$
$$\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)} + \frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$
$$\rightarrow_{match} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)} + \frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)} + \frac{-((|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))} +$$
$$\frac{-((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))} + (\frac{-(((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T + \frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)} +$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))} + (\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T + \frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))} +$$
$$\frac{(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*((A\bullet A))^2)} + \frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{((C\bullet C)*|A|*|C|*(A\bullet A))} + \frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*((A\bullet A))^2)} +$$
$$\frac{-(((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))} + \frac{-(((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))} + (\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T +$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T + \frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)} + (\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T + \frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)} +$$
$$\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))} + \frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)} + \frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$
$$\rightarrow_{refactor} ((\frac{(((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))}{(|C|*|A|)} +$$
$$\frac{(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|A|*|C|*((A\bullet A))^2)} + \frac{((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C$$
$$\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)} + \frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)}) - (\frac{((((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A$$
$$\frac{(((((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))} +$$
$$\frac{((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}))$$
$$\rightarrow_{pull} (((((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet$$
$$\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))*\frac{1}{(|C|*|A|)}+(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes$$
$$(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|A|*|C|*((A\bullet A))^2)}+$$
$$((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C))))*$$
$$\frac{1}{((C\bullet C)*|A|*|C|*(A\bullet A))}+((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|C|*|A|*((C\bullet C))^2)}+$$
$$(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*((C\bullet C))^2)})-(((((((((\nabla\otimes$$
$$C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))+((C\bullet A)*((\nabla\otimes A)^T\bullet$$
$$\nabla\otimes A)))+(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))^T)+((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes$$
$$A))))^T)+(((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|C|*|A|*(A\bullet A))}+(((((((\nabla\otimes C)^T\bullet A)*$$
$$(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes$$
$$C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))*\frac{1}{(|A|*|C|*(C\bullet C))}+((|C|*(C\bullet A)*$$
$$((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*(C\bullet C))}))$$

2 Canny

field#k(3)[3] G= ∇V
field#k(3)[] F= $(-\nabla(G \bullet G)) \bullet (\frac{G}{|G|})$

Probe $. = (\frac{1}{|\nabla A|} * (-((2 * (\nabla A \bullet \nabla \otimes \nabla A))) \bullet \nabla A))$
$\rightarrow_{flatten} \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|}$
$\rightarrow_{match} \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|}$
$\rightarrow_{refactor} \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|}$
$\rightarrow_{pull} (-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A))) * \frac{1}{|\nabla A|}$

Gradient $= ((-((2 * (\nabla A \bullet \nabla \otimes \nabla A))) \bullet ((((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A)) + ((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T))^T) + (\frac{1}{|\nabla A|} * (\nabla A \bullet (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))))^T)))$
$\rightarrow_{flatten} (\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})$
$\rightarrow_{match} (\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})$
$\rightarrow_{refactor} (\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(((-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) + (-2*(\nabla A \bullet$
$\rightarrow_{pull} ((2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(|\nabla A| * (\nabla A \bullet \nabla A))} + (((-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) + (-2 * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) * \frac{1}{|\nabla A|})$

Hessian $= ((-((2*(\nabla A \bullet \nabla \otimes \nabla A))) \bullet Trav((((\frac{(-(((|\nabla A| * (\nabla A \bullet \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) - (-((((((2*|\nabla$
$(|\nabla A| * (\nabla A \bullet \nabla A)) * |\nabla A$
$\nabla A)) + Trav(((\nabla \otimes \nabla A * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}))) < 2, :, 0 > + Trav((\frac{1}{|\nabla A|} * \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla \otimes \nabla A))) < 2, 0, 1 >)) < 1, 2, 0 >) + ((((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A)) + ((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T) \bullet (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))))^T) + (\nabla A \bullet Trav(((\frac{1}{|\nabla A|} * -((2 * (Trav((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav((Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) <:, 2, 1 >)))) + Trav(((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * -((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)))))) < 1, 0, :>)) < 1, 2, 0 >) + (\frac{1}{|\nabla A|} * (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))) \bullet \nabla \otimes \nabla A)))$
$\rightarrow_{flatten} (\frac{(-4*|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(-2*(\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla$
$\frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} +$

$$
\begin{aligned}
&\left(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}\right)^T+\frac{(-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}+\left(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}\right)^T+\\
&\left(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}\right)^T+\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))<1,0,2>))}{|\nabla A|}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla
\end{aligned}
$$

$$
\begin{aligned}
&\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}+\left(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}\right)^T+\\
&\left(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}\right)^T)\\
&\rightarrow_{match}\left(\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)\right)\\
&\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\\
&\left(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}\right)^T+\frac{(-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\left(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}\right)^T+\\
&\left(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}\right)^T+\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\\
&\frac{(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))<1,0,2>))}{|\nabla A|}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla
\end{aligned}
$$

$$
\begin{aligned}
&\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla\otimes\nabla A))}+\left(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}\right)^T+\\
&\left(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}\right)^T)\\
&\rightarrow_{refactor}\left(\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A\right)\right.\\
&\frac{((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+\\
&\underline{((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))}\\
&\underline{((((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))+(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)}
\end{aligned}
$$

$$
\begin{aligned}
&\rightarrow_{pull}((-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))*\\
&\frac{1}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\\
&\nabla A)))*\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\\
&\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2)}+\\
&((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\\
&\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+\\
&((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\\
&\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))*\\
&\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A))}+((((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))+\\
&(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\\
&\nabla A))))+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<\\
&2,1,0>\bullet\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet\\
&Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))+((-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\\
&\nabla\otimes\nabla\otimes\nabla A))))^T)+((-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))^T)*\frac{1}{|\nabla A|})
\end{aligned}
$$

3 Moe

field#k(3)[3] G= ∇V
field#k(3)[] F= $(-\nabla|G|) \bullet (\frac{G}{|G|})$

Probe $. = ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{1}{|\nabla A|})$
$\rightarrow_{flatten} \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2)}$
$\rightarrow_{match} \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}$
$\rightarrow_{refactor} -(\frac{(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A))}{(\nabla A \bullet \nabla A)})$
$\rightarrow_{pull} -((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(\nabla A \bullet \nabla A)})$

Gradient $\dot{}$
$= (((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}) + (\frac{1}{|\nabla A|} * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes$
$\nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T))))$
$\rightarrow_{flatten} (\frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} + \frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2)} + \frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} +$
$\frac{-((|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{-((|\nabla A| * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))})$
$\rightarrow_{match} (\frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} + \frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)} + \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} +$
$\frac{-(((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)})$
$\rightarrow_{refactor} (\frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)} - \frac{((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes$
$\rightarrow_{pull} ((2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(((\nabla A \bullet \nabla A))^2)} - ((((\nabla \otimes$
$\nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet$
$\nabla \otimes \nabla A))) * \frac{1}{(\nabla A \bullet \nabla A)})$

Hessian $= (((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{(-((|\nabla A| * (\nabla A \bullet \nabla A) * ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) - (-((((((2 * |\nabla A| * (\nabla A \bullet$
$(((((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla$
$\frac{-(((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})))^T + (\frac{1}{|\nabla A|} * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet$
$(-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T) +$

$(\nabla A \bullet Trav(-(\frac{(((\nabla A \bullet \nabla A) * ((Trav(((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)))) < 1,0,: > + (|\nabla A| * (Trav((\nabla A \bullet \nabla$
$1,2,0>) + (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}) \bullet$
$\nabla \otimes \nabla A))) + (((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}) * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla$
$\rightarrow_{flatten} (\frac{(-2 * |\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} + \frac{-(((\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla$
$\frac{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} + \frac{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} +$
$\frac{(((\nabla A \bullet \nabla \otimes \nabla A) \otimes ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} + \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} +$

$$\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2)}+\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))}+$$
$$\frac{-((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\frac{-((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+(\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}$$
$$\frac{(2*|\nabla A|*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(2*|\nabla A|*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$(\frac{-(((\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T+\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)})^T+\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+(\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))})^T+$$
$$(\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+(\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+$$
$$(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))})^T+(\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))})^T+(\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))})^T)$$
$$\rightarrow_{match}(\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+$$
$$\frac{-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A}{(((\nabla A\bullet\nabla A))^2)}$$
$$\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$\frac{-((((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}+\frac{-((((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+$$
$$\frac{-(((\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(\nabla A\bullet\nabla A)}+(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{-((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)})^T+(\frac{-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)})^T+(\frac{(2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A}{(((\nabla A\bullet\nabla A))^2)}$$
$$(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T+(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)})^T)$$
$$\rightarrow_{refactor}((\frac{((((((((((((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+(2*(\nabla A\bullet(\nabla A$$
$$(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T)-\frac{((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))^T+((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A}{}$$
$$\rightarrow_{pull}((((((((((((((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+(2*((\nabla A\bullet$$
$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet$$
$$\nabla\otimes\nabla\otimes\nabla A)))+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))+((2*((\nabla A\bullet$$
$$(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T)+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*$$
$$(\nabla A\bullet\nabla\otimes\nabla A))))^T)-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))-(((\nabla A\bullet$$
$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*$$
$$(\nabla A\bullet\nabla\otimes\nabla A))))^T)+((2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A)))))^T)+$$
$$((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))))^T)+((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet$$
$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))))^T)*\frac{1}{(((\nabla A\bullet\nabla A))^2)}+((-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet$$

$$\nabla \otimes \nabla A) * (\nabla A \bullet \nabla \otimes \nabla A))) * \frac{1}{(((\nabla A \bullet \nabla A))^2 * (\nabla A \bullet \nabla A))})^T) - ((((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))^T + ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) + (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 0, 1 > \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + ((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)) + (\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) + (\nabla A \bullet Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 0, 2 >) + (((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))^T) * \frac{1}{(\nabla A \bullet \nabla A)})$$