1 New ways to define a field

1.1 Polynomial

It is natural to define a field with a polynomial expression.

$$F = x^3$$

In the surface language we added function poly(). The first argument is a variable and the second argument is a field definition.

```
vec2 x;
ofield #2(2)[2] V = poly(x, x);
ofield #2(2)[] S = poly(x, x²);
```

This allows the programmer to differentiate this type of field.

```
ofield #1(2)[2] GS = ∇S;
```

We illustrate the expected structure below:

$$V = \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} \qquad \nabla \otimes V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad S = V_0^2 + V_1^2 \qquad \nabla S = \begin{bmatrix} 2 * V_0 \\ 2 * V_1 \end{bmatrix}$$

Representation .
As an ongoing example: A field F is defined by taking the cube of the input variable.

```
real p;
ofield #1(2)[] F = poly(p, p³);
tensor[] out = F(pos);
```

$$\underset{init}{\Longrightarrow} \begin{array}{l} \text{F} = \lambda()\big\langle PolyWrap_p(p^3)\big\rangle() \\ \text{out} = \lambda(F,x)\langle F(x)\rangle\text{(F,x)} \end{array}$$

Substitution creates:
$\underset{subst}{\Longrightarrow}$ out=$\lambda(x)\big\langle PolyWrap_p(p^3)(x)\big\rangle$ (x)

Replace polynomial variable .
The variable p represents a vector of length 2, where p= [X,Y].
The term P_0 represents the 0th component of the vector, or X. In the following, we will use the terms X and Y, in place of P_0 and P_1.

The polynomial variable is instantiated with the position.
$\Rightarrow$ out=$\lambda(p)\langle e\rangle$(x) where e=$p * p * p$.
The EIN term (p) is replaced with an EIN term that represents the vector components. In the 2-d case there are two terms indexed with a constant index in $p = X\delta_{0i} + Y\delta_{1i}$

Occurrances for P are replaced inside the expression:
$\rightarrow (P_0\delta_{0i} + P_1\delta_{1i}) * (P_0\delta_{0i} + P_1\delta_{1i}) * (P_0\delta_{0i} + P_1\delta_{1i})$.
$= (X\delta_{0i} + Y\delta_{1i}) * (X\delta_{0i} + Y\delta_{1i}) * (X\delta_{0i} + Y\delta_{1i})$.

Normalization .
Similar terms are collected:

$$P_0 * P_0 \rightarrow {P_0}^2 \quad \text{or} \quad X * X \rightarrow X^2$$

The differentiation operator is distributed over the EIN term, as usual, and pushed to a polynomial term

$$\frac{\partial}{\partial x_i}({P_0}^2 + e) \rightarrow \frac{\partial}{\partial x_i}{P_0}^2 + \frac{\partial}{\partial x_i}e \quad \text{or} \quad \frac{\partial}{\partial x_i}(X^2 + e) \rightarrow \frac{\partial}{\partial x_i}X^2 + \frac{\partial}{\partial x_i}e$$

Evaluation .
During the evaluation the variable index in a differentiation operator is bound to a number. An EIN term such as $\frac{\partial}{\partial x_i} P_c{}^n$ is evaluated.
When i and c are both 0:

$$\frac{\partial}{\partial x_0} X \to 1 \qquad \frac{\partial}{\partial x_0} X^2 \to 2 * X \qquad \frac{\partial}{\partial x_0} X^3 \to 3 * X^2$$

When i and c are both 1:

$$\frac{\partial}{\partial x_1} Y \to 1 \qquad \frac{\partial}{\partial x_1} Y^2 \to 2 * Y \qquad \frac{\partial}{\partial x_1} Y^3 \to 3 * Y^2$$

When i and c are not the same

$$\frac{\partial}{\partial x_1} X^n \to 0 \qquad \frac{\partial}{\partial x_1} Y^n \to 0$$