1 Terms

=: Without any additional rewriting.
$\rightarrow_{flatten}$: Use rewrites to flatten body of term. Uses algebraic distribution property.
$\rightarrow_{match}$: Looks for common terms to cancel.
$\rightarrow_{refactor}$: Unflattens.Groups term with the same denominator.
$\rightarrow_{pull}$:. Makes the numerator in a division term 1.

2 PVO

field#4(3)[] F= $(\frac{V}{|V|}) \bullet (\frac{U}{|U|})$

Probe $\frac{(A \bullet C)}{<||A||*||C||>}$

Gradient .
$= ((A \bullet (((\frac{1}{|A|} * (((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) + ((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))))^T) + (\frac{1}{|A|} * \frac{1}{|C|} * (C \bullet \nabla \otimes A)))$

$\rightarrow_{flatten} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$

$\rightarrow_{match} (\frac{-(((C \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{-(((C \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} + \frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)})$

$\rightarrow_{refactor} (\frac{(((\nabla \otimes C)^T \bullet A)+(C \bullet \nabla \otimes A))}{(|A|*|C|)} - (\frac{((C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{((C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))}))$

$\rightarrow_{pull} ((((\nabla \otimes C)^T \bullet A) + (C \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|)} - (((C \bullet A) * (A \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|*(A \bullet A))} + ((C \bullet A) * (C \bullet \nabla \otimes C)) * \frac{1}{(|C|*|A|*(C \bullet C))}))$

Hessian .

$= ((A \bullet Trav(((\frac{1}{|A|} * (((\frac{(-(((|C|*(C \bullet C)*((C \bullet \nabla \otimes \nabla \otimes C)+((\nabla \otimes C)^T \bullet \nabla \otimes C))))-(-((((((2*|C|*(C \bullet \nabla \otimes C))+((C \bullet C)*\frac{(C \bullet \nabla \otimes C)}{|C|}))*(C$
$(|C|*(C \bullet C)*|C|*(C \bullet C))}$
$C)) + Trav(((\nabla \otimes C * \frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))}))) < 2, :, 0 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes C)) <$
$1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * \nabla \otimes C))) < 2, 0, 1 >)) + Trav(((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*(A \bullet A))} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T))))) < 1, 0, :> + ((\frac{(-(((|A|*|C|*(A \bullet A)*((A \bullet \nabla \otimes \nabla \otimes A)+((\nabla \otimes A)^T \bullet \nabla \otimes A))))-(-((((((|A|$
$(|A|*|C$
$C)) + Trav(((\nabla \otimes C * \frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))}))) < 2, :, 0 >)) < 1, 2, 0 >) + (((\frac{1}{|A|} *$
$(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} * C)) + ((\frac{1}{|C|} * \nabla \otimes C))^T)) + ((\frac{-(((A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} * C))) \bullet \nabla \otimes A) +$
$(\frac{1}{|A|} * ((C \bullet Trav((Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes A)) < 1, 2, 0 > + Trav(((\frac{-(((C \bullet \nabla \otimes C)))}{(|C|*(C \bullet C))} *$
$\nabla \otimes A))) < 2, 0, 1 >)) < 1, 2, 0 >) + (\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C)))) + (\frac{1}{|C|} * ((C \bullet$
$\nabla \otimes A) \otimes \frac{-(((A \bullet \nabla \otimes A)))}{(|A|*(A \bullet A))})))$

$$\rightarrow_{flatten} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+\frac{((C\bullet C)*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|C|*|A|*((C\bullet C))^2)}+$$
$$\frac{-(((|C|*(C\bullet C)*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+\frac{-(((|C|*(C\bullet C)*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))}+(\frac{-(((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+$$
$$\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)}+\frac{-((((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))}+(\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T+$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))}+\frac{(2*|C|*|A|*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}+\frac{(|A|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))}+$$
$$\frac{(|C|*(A\bullet A)*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{((|A|)^2*|C|*|A|*|C|*(A\bullet A)*(A\bullet A))}+\frac{-(((|A|*|C|*(A\bullet A)*(C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}+\frac{-((|A|*|C|*(A\bullet A)*(C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A}{(|A|*|C|*|A|*|C|*((A\bullet A))^2)}$$
$$(\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+(\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)}+$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)}+\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+$$
$$\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)}+\frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$

$$\rightarrow_{match} (\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)}+\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{-((|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}+$$
$$\frac{-((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}+(\frac{-((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)}+$$
$$\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))}+(\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T+\frac{-(((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))}+$$
$$\frac{(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*((A\bullet A))^2)}+\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{((C\bullet C)*|A|*|C|*(A\bullet A))}+\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*((A\bullet A))^2)}+$$
$$\frac{-(((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))}+\frac{-(((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))}+(\frac{-(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+$$
$$(\frac{-((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))}{(|C|*|A|*(C\bullet C))})^T+\frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)}+(\frac{-((((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A))))}{(|A|*|C|*(A\bullet A))})^T+\frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)}+$$
$$\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)}+\frac{-((((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|C|*|A|*(A\bullet A))})$$

$$\rightarrow_{refactor} ((\frac{(((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))}{(|C|*|A|)}+$$
$$\frac{(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|A|*|C|*((A\bullet A))^2)}+\frac{((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C}{((C\bullet C)*|A|*|C|*(A\bullet A))}$$
$$\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)})-(\frac{((((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A}{(((((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))^T)+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))}{(|A|*|C|*(C\bullet C))}+$$
$$\frac{((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}))$$

$$\rightarrow_{pull} (((((\nabla\otimes A)^T\bullet\nabla\otimes C)+(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))*\frac{1}{(|C|*|A|)}+(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|A|*|C|*((A\bullet A))^2)}+((((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C))))*\frac{1}{((C\bullet C)*|A|*|C|*(A\bullet A))}+((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|C|*|A|*((C\bullet C))^2)}+(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*((C\bullet C))^2)})-(((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))+((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))+(((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C)))+((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T)+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+(((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|C|*|A|*(A\bullet A))}+(((((((((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C))))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)+((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C))))*\frac{1}{(|A|*|C|*(C\bullet C))}+((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*(C\bullet C))}))$$

3 Moe

field#k(3)[3] G= ∇V
field#k(3)[] F= $(-\nabla|G|) \bullet (\frac{G}{|G|})$

Probe $\rightarrow_{none} ((-\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} \bullet \nabla A) * \frac{1}{|\nabla A|})$

$\rightarrow_{rewrite} \frac{(-1*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2)}$

Gradient .
$= (((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}) + (\frac{1}{|\nabla A|} * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T))))$

$\rightarrow_{flatten} (\frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} + \frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2)} + \frac{((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} + \frac{-((|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{-((|\nabla A| * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(|\nabla A| * (\nabla A \bullet \nabla A))})$

$\rightarrow_{match} (\frac{-((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} + \frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)} + \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)} + \frac{-(((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)})$

$\rightarrow_{refactor} (\frac{(2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(((\nabla A \bullet \nabla A))^2)} - \frac{((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes$
$(\nabla A \bullet \nabla A)}$

$\rightarrow_{pull} ((2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(((\nabla A \bullet \nabla A))^2)} - ((((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) + (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))) + (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))) * \frac{1}{(\nabla A \bullet \nabla A)})$

Hessian .
$= (((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla A) * \frac{(-((|\nabla A| * (\nabla A \bullet \nabla A) * ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) - (-((((((2 * |\nabla A| * (\nabla A \bullet \nabla \otimes \nabla A)) + ((\nabla$
$(|\nabla A| * (\nabla A \bullet \nabla A) * |\nabla A| * (\nabla A \bullet \nabla A))}$
$(((((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla$
$(\nabla A \bullet \nabla A)}$
$\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})))^T + (\frac{1}{|\nabla A|} * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet$
$(-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}))^T) +$

$(\nabla A \bullet Trav(-(\frac{(((\nabla A \bullet \nabla A) * ((Trav(((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)))) < 1,0,: > + (|\nabla A| * (Trav((\nabla A \bullet \nabla$
$1,2,0 >) + (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)) - ((\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} * (\nabla A \bullet \nabla \otimes \nabla A))))}{(\nabla A \bullet \nabla A)}) \bullet$
$\nabla \otimes \nabla A))) + (((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * ((-(\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|}) \bullet \nabla \otimes \nabla A) + (\nabla A \bullet (-(\frac{((|\nabla A| * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla$
$(\nabla A \bullet \nabla A)}$

$\rightarrow_{flatten} (\frac{(-2 * |\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} + \frac{-(((\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla$
$\frac{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} + \frac{(|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))} +$
$\frac{(((\nabla A \bullet \nabla \otimes \nabla A) \otimes ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * (\nabla A \bullet \nabla A))} + \frac{-(((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A))))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} +$

$$\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2)}+\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))}+$$
$$\frac{-((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+\frac{-((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+(\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}$$
$$\frac{(2*|\nabla A|*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{(2*|\nabla A|*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$(\frac{-(((\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T+\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(|\nabla A|*(\nabla A\bullet\nabla A)*((\nabla A\bullet\nabla A))^2)}+(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)})^T+\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-(((\nabla A\bullet\nabla A)*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)}+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+$$
$$\frac{-(((|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+(\frac{((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))})^T+$$
$$(\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+(\frac{-(((|\nabla A|*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+$$
$$(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*(\nabla A\bullet\nabla A))})^T+(\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A))))}{((|\nabla A|)^2*((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))})^T+(\frac{(|\nabla A|*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))})^T)$$

$$\rightarrow_{match}(\frac{-(((Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+$$
$$\frac{-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A}{(((\nabla A\bullet\nabla A))^2)}$$
$$\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$\frac{-((((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}+\frac{-((((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+\frac{-(((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)}+$$
$$\frac{-(((\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))}{(\nabla A\bullet\nabla A)}+(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{-((((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)})^T+(\frac{-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(\nabla A\bullet\nabla A)})^T+(\frac{(2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A)}{(((\nabla A\bullet\nabla A))^2)}$$
$$(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T+(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)})^T+$$
$$(\frac{(((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))}{(((\nabla A\bullet\nabla A))^2)})^T)$$

$$\rightarrow_{refactor}((\frac{((((((((((((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+(2*(\nabla A\bullet(\nabla A$$
$$(\frac{(-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T)-\frac{((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))^T+((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A$$

$$\rightarrow_{pull}((((((((((((((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+(2*((\nabla A\bullet$$
$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet$$
$$\nabla\otimes\nabla\otimes\nabla A)))+(2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))+((2*((\nabla A\bullet$$
$$(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))^T)+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*$$
$$(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))^T)-(((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))-(((\nabla A\bullet$$
$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*$$
$$(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))^T)+((2*((\nabla A\bullet\nabla\otimes\nabla A)*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))))^T)+$$
$$((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))))^T)+((((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet$$

$$((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))))^T)*\frac{1}{(((\nabla A\bullet\nabla A))^2)}+((-8*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)*(\nabla A\bullet\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))})^T)-((((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))^T+((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))+(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A))+((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))+(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))+(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))+(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))+(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)+(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))^T)*\frac{1}{(\nabla A\bullet\nabla A)})$$

4 Canny

Gradient $= ((-((2 * (\nabla A \bullet \nabla \otimes \nabla A))) \bullet ((((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A)) + (((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T))^T) + (\frac{1}{|\nabla A|} * (\nabla A \bullet (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))))^T)))$

$\rightarrow_{flatten}$ $(\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})$

$\rightarrow_{match}$ $(\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|})$

$\rightarrow_{refactor}$ $(\frac{(2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + \frac{(((-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) + (-2*(\nabla A \bullet$
$|\nabla A|}$

$\rightarrow_{pull}$ $((2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)) * \frac{1}{(|\nabla A| * (\nabla A \bullet \nabla A))} + (((-2 * ((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A))) + (-2 * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))) + (-2 * (\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) * \frac{1}{|\nabla A|})$

Hessian $= ((-((2*(\nabla A \bullet \nabla \otimes \nabla A))) \bullet Trav((((\frac{(-(((|\nabla A| * (\nabla A \bullet \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A) + ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))) - (-(((((2*|\nabla$
$(|\nabla A| * (\nabla A \bullet \nabla A)) * |\nabla A$
$\nabla A)) + Trav(((\nabla \otimes \nabla A * \frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))}))) < 2, :, 0 > + Trav((\frac{1}{|\nabla A|} * \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla \otimes \nabla A))) < 2, 0, 1 >)) < 1, 2, 0 >) + ((((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * \nabla A)) + ((\frac{1}{|\nabla A|} * \nabla \otimes \nabla A))^T) \bullet (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))))^T) + (\nabla A \bullet Trav(((\frac{1}{|\nabla A|} * -((2 * (Trav((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav((Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 2, 0 > + Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) <:, 2, 1 >)))) + Trav(((\frac{-(((\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} * -((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T)))))) < 1, 0, :>)) < 1, 2, 0 >) + (\frac{1}{|\nabla A|} * (-((2 * (((\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A))^T + (((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))^T))) \bullet \nabla \otimes \nabla A)))$

$\rightarrow_{flatten}$ $(\frac{(-4*|\nabla A| * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(-2*(\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla A \bullet \nabla \otimes \nabla A) \otimes (\nabla$
$((|\nabla A|)^2 * |\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet \nabla A))$
$\frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} + \frac{(2*|\nabla A| * (\nabla A \bullet \nabla A) * (\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A))}{((|\nabla A|)^2 * ((\nabla A \bullet \nabla A))^2)} +$
$(\frac{(2*(((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T + \frac{(-2*(Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 0, 1 > \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{|\nabla A|} +$
$\frac{(2*(((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + (\frac{(2*((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T +$
$(\frac{(2*((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) * (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))})^T + \frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} +$
$\frac{(-2*((\nabla \otimes \nabla A)^T \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet (Trav(\nabla \otimes \nabla \otimes \nabla A) < 2, 1, 0 > \bullet \nabla \otimes \nabla A)))}{|\nabla A|} +$
$\frac{(-2*(\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|} + \frac{(-2*(\nabla A \bullet Trav(((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla \otimes \nabla A)) < 1, 0, 2 >))}{|\nabla A|} + \frac{(2*((\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla$
$(|\nabla A| * (\nabla A \bullet \nabla A))$
$\frac{(2*((\nabla A \bullet ((\nabla \otimes \nabla A)^T \bullet \nabla \otimes \nabla A)) \otimes (\nabla A \bullet \nabla \otimes \nabla A)))}{(|\nabla A| * (\nabla A \bullet \nabla A))} + (\frac{(-2*((\nabla \otimes \nabla A)^T \bullet (\nabla A \bullet \nabla \otimes \nabla \otimes \nabla A)))}{|\nabla A|})^T +$

$$(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{|\nabla A|})^T)$$

$$\rightarrow_{match} (\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}$$
$$\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+\frac{(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$(\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+\frac{(-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+$$
$$\frac{(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+(\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+$$
$$(\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))})^T+\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+$$
$$\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+$$
$$\frac{(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{|\nabla A|}+\frac{(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>))}{|\nabla A|}+\frac{(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla}{(|\nabla A|*(\nabla A\bullet\nabla A))}$$
$$\frac{(2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(|\nabla A|*(\nabla A\bullet\nabla A))}+(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A)))}{|\nabla A|})^T+$$
$$(\frac{(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{|\nabla A|})^T)$$

$$\rightarrow_{refactor} (\frac{(-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+\frac{(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}$$
$$\frac{((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))}{(((\nabla A\bullet\nabla A))^2)}+$$
$$\frac{((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))}{(((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))+(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)}$$

$$\rightarrow_{pull} ((-4*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2*(\nabla A\bullet\nabla A))}+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla A\bullet\nabla\otimes\nabla A)\otimes(\nabla A\bullet\nabla\otimes\nabla A)))*\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A)*(\nabla A\bullet\nabla A))}+((2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))+(2*|\nabla A|*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla A))*((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))*\frac{1}{(((\nabla A\bullet\nabla A))^2)}+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+((2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*(((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A)))+((2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+((2*((\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))*(\nabla A\bullet\nabla\otimes\nabla A))))^T+(2*((\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))\otimes(\nabla A\bullet\nabla\otimes\nabla A))))*\frac{1}{(|\nabla A|*(\nabla A\bullet\nabla A))}+((((((((((-2*(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,0,1>\bullet(\nabla A\bullet\nabla\otimes\nabla A)))+(-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A)))+(-2*(\nabla A\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet(Trav(\nabla\otimes\nabla\otimes\nabla A)<2,1,0>\bullet\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))))+(-2*(\nabla A\bullet Trav(((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla\otimes\nabla A))<1,0,2>)))+((-2*((\nabla\otimes\nabla A)^T\bullet(\nabla A\bullet\nabla\otimes\nabla\otimes\nabla\otimes\nabla A))))^T)+((-2*((\nabla\otimes\nabla A)^T\bullet((\nabla\otimes\nabla A)^T\bullet\nabla\otimes\nabla A))))^T)*\frac{1}{|\nabla A|})$$

5 tmp

$$=((A\bullet\nabla\otimes\nabla\otimes C)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(C\bullet\nabla\otimes\nabla\otimes A)+((\nabla\otimes A)^T\bullet\nabla\otimes C))$$

$$\rightarrow_{flatten} ((A\bullet\nabla\otimes\nabla\otimes C)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(C\bullet\nabla\otimes\nabla\otimes A)+((\nabla\otimes A)^T\bullet\nabla\otimes C))$$

$$\rightarrow_{match} ((A\bullet\nabla\otimes\nabla\otimes C)+((\nabla\otimes C)^T\bullet\nabla\otimes A)+(C\bullet\nabla\otimes\nabla\otimes A)+((\nabla\otimes A)^T\bullet\nabla\otimes C))$$

$\rightarrow_{refactor} (((\nabla \otimes A)^T \bullet \nabla \otimes C) + (C \bullet \nabla \otimes \nabla \otimes A) + ((\nabla \otimes C)^T \bullet \nabla \otimes A) + (A \bullet \nabla \otimes \nabla \otimes C))$

$\rightarrow_{pull} (((\nabla \otimes A)^T \bullet \nabla \otimes C) + (C \bullet \nabla \otimes \nabla \otimes A) + ((\nabla \otimes C)^T \bullet \nabla \otimes A) + (A \bullet \nabla \otimes \nabla \otimes C))$