1 PVO

field#4(3)[] F= $(\frac{V}{|V|}) \bullet (\frac{U}{|U|})$

Probe $\quad \frac{(A \bullet C)}{<||A|| * ||C||>}$

Gradient .
$= ((\frac{1}{|A|} * \frac{1}{|C|} * (C \bullet \nabla \otimes A)) + (A \bullet ((((\frac{(-1*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} * C)) + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes C))^T + ((\frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))} * C)))))))^T))$

$\rightarrow_{flatten} (\frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)} + \frac{(-1*(C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{(-1*(C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))})$

$\rightarrow_{match} (\frac{(C \bullet \nabla \otimes A)}{(|A|*|C|)} + \frac{(-1*(C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|C|*|A|)} + \frac{(-1*(C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))})$

$\rightarrow_{refactor} (\frac{(((\nabla \otimes C)^T \bullet A)+(C \bullet \nabla \otimes A))}{(|A|*|C|)} + \frac{-((C \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))} + \frac{-((C \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))})$

$\rightarrow_{pull} (((((\nabla \otimes C)^T \bullet A) + (C \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|)} + -((C \bullet A)*(C \bullet \nabla \otimes C)) * \frac{1}{(|C|*|A|*(C \bullet C))} + -((C \bullet A)*(A \bullet \nabla \otimes A)) * \frac{1}{(|A|*|C|*(A \bullet A))})$

Hessian .

$= ((\frac{1}{|C|} * ((C \bullet \nabla \otimes A) \otimes \frac{(-1*(A \bullet \nabla \otimes A))}{(|A|*(A \bullet A))})) + (\frac{1}{|A|} * ((\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C)) + (C \bullet Trav((Trav(((\frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))} * \nabla \otimes A))) < 2,0,1 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes A)) < 1,2,0 >)) < 1,2,0 >))) + ((((\frac{(-1*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} * C)) + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes C))^T + ((\frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))} * C))))) \bullet \nabla \otimes A) + (A \bullet Trav((Trav(((\nabla \otimes C * \frac{(-1*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))})))$ <
$2,:,0 > + ((\frac{((-1*|A|*|C|*(A \bullet A)*(((\nabla \otimes A)^T \bullet \nabla \otimes A)+(A \bullet \nabla \otimes \nabla \otimes A)))-((-1*(((|C|*(A \bullet A)* \frac{(A \bullet \nabla \otimes A)}{|A|})+(|A|*((A \bullet A)* \frac{(C \bullet \nabla \otimes C)}{|C|})+(2*|}{(|A|*|C|*(A \bullet A)*|A|*|C|*(A \bullet A))}$
$C)) + Trav(((\frac{(-1*(A \bullet \nabla \otimes A))}{(|A|*(A \bullet A))} * (((\frac{1}{|C|} * \nabla \otimes C))^T + ((\frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))} * C)))))) < 1,0,:>$
$+ (\frac{1}{|A|} * (Trav(((\frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))} * \nabla \otimes C))) < 2,0,1 > + Trav((\frac{1}{|C|} * \nabla \otimes \nabla \otimes C)) <$
$1,2,0 > + Trav(((\nabla \otimes C * \frac{(-1*(C \bullet \nabla \otimes C))}{(|C|*(C \bullet C))}))) < 2,:,0 > + ((\frac{((-1*|C|*(C \bullet C)*(((\nabla \otimes C)^T \bullet \nabla \otimes C)+(C \bullet \nabla \otimes \nabla \otimes C)))-((-1*((((C}{(|C|*(C \bullet C)*|C|*(C}$
$C)))))) < 1,2,0 >))$

$\rightarrow_{flatten} (\frac{(-1*((C \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{(|C|*|A|*(A \bullet A))} + \frac{((\nabla \otimes A)^T \bullet \nabla \otimes C)}{(|C|*|A|)} + \frac{(-1*(((\nabla \otimes A)^T \bullet C) \otimes (C \bullet \nabla \otimes C)))}{(|A|*|C|*(C \bullet C))} +$
$\frac{(Trav(\nabla \otimes \nabla \otimes A)<2,0,1> \bullet C)}{(|C|*|A|)} + (\frac{(-1*((C \bullet \nabla \otimes A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))})^T + \frac{((\nabla \otimes C)^T \bullet \nabla \otimes A)}{(|C|*|A|)} + (\frac{(-1*((C \bullet \nabla \otimes A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))})^T +$
$(\frac{(-1*(((\nabla \otimes C)^T \bullet A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))})^T + \frac{-1*|A|*|C|*(A \bullet A)*(C \bullet A)*((\nabla \otimes A)^T \bullet \nabla \otimes A))}{(|A|*|C|*|A|*|C|*((A \bullet A))^2)} + \frac{-1*|A|*|C|*(A \bullet A)*(C \bullet A)*(A \bullet \nabla \otimes \nabla \otimes A))}{(|A|*|C|*|A|*|C|*((A \bullet A))^2)} +$
$\frac{|C|*(A \bullet A)*(C \bullet A)*((A \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{((|A|)^2*|C|*|A|*|C|*(A \bullet A)*(A \bullet A))} + \frac{|A|*(A \bullet A)*(C \bullet A)*((A \bullet \nabla \otimes A) \otimes (C \bullet \nabla \otimes C)))}{(|C|*|A|*|C|*|A|*|C|*(A \bullet A)*(A \bullet A))} +$
$\frac{(2*|C|*|A|*(C \bullet A)*((A \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{(|A|*|C|*|A|*|C|*((A \bullet A))^2)} + \frac{(-1*(((\nabla \otimes C)^T \bullet A) \otimes (A \bullet \nabla \otimes A)))}{(|C|*|A|*(A \bullet A))} + (\frac{((C \bullet A)*((A \bullet \nabla \otimes A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C)*(A \bullet A))})^T +$
$\frac{(-1*(((\nabla \otimes C)^T \bullet A) \otimes (C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} + \frac{(Trav(\nabla \otimes \nabla \otimes C)<2,0,1> \bullet A)}{(|C|*|A|)} + (\frac{(-1*(((\nabla \otimes C)^T \bullet A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))})^T +$
$\frac{(-1*|C|*(C \bullet C)*(C \bullet A)*((\nabla \otimes C)^T \bullet \nabla \otimes C))}{((|C|)^2*|A|*(C \bullet C)*(C \bullet C))} + \frac{(-1*|C|*(C \bullet C)*(C \bullet A)*(C \bullet \nabla \otimes \nabla \otimes C))}{((|C|)^2*|A|*(C \bullet C)*(C \bullet C))} + \frac{((C \bullet C)*(C \bullet A)*((C \bullet \nabla \otimes C) \otimes (C \bullet \nabla \otimes C)))}{((|C|)^2*|C|*|A|*((C \bullet C))^2)} +$

$$\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{((|C|)^2*|A|*(C\bullet C)*(C\bullet C))})$$

$\rightarrow_{match}$ $(\frac{(-1*((C\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*(A\bullet A))}+\frac{((\nabla\otimes A)^T\bullet\nabla\otimes C)}{(|C|*|A|)}+\frac{(-1*(((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*|C|*(C\bullet C))}+$
$\frac{(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C)}{(|C|*|A|)}+(\frac{(-1*((C\bullet\nabla\otimes A)*(A\bullet\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))})^T+\frac{((\nabla\otimes C)^T\bullet\nabla\otimes A)}{(|C|*|A|)}+(\frac{(-1*((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C))})^T+$
$(\frac{(-1*(((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A)))}{(|A|*|C|*(A\bullet A))})^T+\frac{(-1*(C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A))}{(|A|*|C|*(A\bullet A))}+\frac{(-1*(C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A))}{(|A|*|C|*(A\bullet A))}+$
$\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|A|*|C|*((A\bullet A))^2)}+\frac{((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))}{((C\bullet C)*|A|*|C|*(A\bullet A))}+\frac{(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*((A\bullet A))^2)}+$
$\frac{(-1*(((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))}{(|C|*|A|*(A\bullet A))}+(\frac{((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C)*(A\bullet A))})^T+\frac{(-1*(((\nabla\otimes C)^T\bullet A)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C))}+$
$\frac{(Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)}{(|C|*|A|)}+(\frac{(-1*(((\nabla\otimes C)^T\bullet A)*(C\bullet\nabla\otimes C)))}{(|C|*|A|*(C\bullet C))})^T+\frac{(-1*|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))}{(|A|*(C\bullet C)*(C\bullet C))}+$
$\frac{(-1*|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C))}{(|A|*(C\bullet C)*(C\bullet C))}+\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)})$

$\rightarrow_{refactor}$ $(\frac{-((((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))+((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes A)))+(((((\nabla\otimes C)^T\bullet A)*(A\bullet\nabla\otimes A)))^T}{(|C|*|A|*(A\bullet A))}$
$\frac{((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|C|*|A|*((C\bullet C))^2)}+\frac{(((((Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A))+(Trav(\nabla\otimes\nabla\otimes A)<2,0,1>\bullet C))+(\nabla\otimes A)^T\bullet\nabla\otimes C}{(|C|*|A|)}$
$\frac{-((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes\nabla\otimes C)))}{(|A|*(C\bullet C)*(C\bullet C))}+\frac{-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C)))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)}{(|A|*|C|*(C\bullet C)}$
$\frac{(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))}{(|A|*(C\bullet C)*((C\bullet C))^2)}+\frac{(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))+(((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T)}{((C\bullet C)*|A|*|C|*(A\bullet A))}+$
$\frac{(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))}{(|A|*|C|*((A\bullet A))^2)})$

$\rightarrow_{pull}$ $(-(((((((((\nabla\otimes C)^T\bullet A)\otimes(A\bullet\nabla\otimes A)))+((C\bullet A)*(A\bullet\nabla\otimes\nabla\otimes A)))+((C\bullet A)*((\nabla\otimes A)^T\bullet\nabla\otimes$
$\frac{1}{(|C|*|A|*(A\bullet A))}+((C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|C|*|A|*((C\bullet C))^2)}+$
$(((((Trav(\nabla\otimes\nabla\otimes C)<2,0,1>\bullet A)+((\nabla\otimes C)^T\bullet\nabla\otimes A))+(Trav(\nabla\otimes\nabla\otimes A)<$
$2,0,1>\bullet C))+((\nabla\otimes A)^T\bullet\nabla\otimes C))*\frac{1}{(|C|*|A|)}+-((|C|*(C\bullet A)*((\nabla\otimes C)^T\bullet\nabla\otimes C))+(|C|*(C\bullet A)*(C\bullet\nabla\otimes$
$\frac{1}{(|A|*(C\bullet C)*(C\bullet C))}+-(((((\nabla\otimes A)^T\bullet C)\otimes(C\bullet\nabla\otimes C)))+((((C\bullet\nabla\otimes A)*(C\bullet\nabla\otimes C))))^T+((((\nabla\otimes C)^T\bullet A$
$\frac{1}{(|A|*|C|*(C\bullet C))}+(2*|C|*(C\bullet A)*((C\bullet\nabla\otimes C)\otimes(C\bullet\nabla\otimes C)))*\frac{1}{(|A|*(C\bullet C)*((C\bullet C))^2)}+$
$(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(C\bullet\nabla\otimes C)))+(((C\bullet A)*((A\bullet\nabla\otimes A)*(C\bullet\nabla\otimes$
$C))))^T)*\frac{1}{((C\bullet C)*|A|*|C|*(A\bullet A))}+(((C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A)))+(2*$
$(C\bullet A)*((A\bullet\nabla\otimes A)\otimes(A\bullet\nabla\otimes A))))*\frac{1}{(|A|*|C|*((A\bullet A))^2)})$

2 Moe

field#k(3)[3] G= ∇V
field#k(3)[] F= $(-\nabla |G|) \bullet (\frac{G}{|G|})$

Probe $\rightarrow_{none}$ $((-\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} \bullet \nabla A) * \frac{1}{|\nabla A|})$

$\rightarrow_{rewrite}$ $\frac{(-1*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2)}$

Gradient .

Hessian .