

1 PVO

$$\text{field}\#4(3) \parallel F = \left(\frac{V}{|V|}\right) \bullet \left(\frac{U}{|U|}\right)$$

$$\text{Probe} \quad \frac{(A \bullet C)}{<||A||*||C||>}$$

Gradient .

$$\rightarrow_{\text{none}} \left(\left(\frac{1}{|C|} * \frac{1}{|A|} * (C \bullet \nabla \otimes A) \right) + (A \bullet (((((C * \frac{(-1*(A \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))}))^T + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes C))^T + (((C * \frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)}))^T))))^T) \right)$$

$$\rightarrow_{\text{rewrite}} \left(\frac{(C \bullet \nabla \otimes A)}{(|C|*|A|)} + \frac{(-1*(C \bullet A)*(A \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))} + \frac{((\nabla \otimes C)^T \bullet A)}{(|A|*|C|)} + \frac{(-1*(C \bullet A)*(C \bullet \nabla \otimes C))}{(|A|*|C|*(C \bullet C))} \right)$$

Hessian .

$$\begin{aligned} &\rightarrow_{\text{none}} \left(\left(\frac{1}{|C|} * \frac{(-1*(A \bullet \nabla \otimes A))}{((A \bullet A)*|A|)} * (C \bullet \nabla \otimes A) \right) \right)^T + \left(\frac{1}{|A|} * \left(\left(\frac{1}{|C|} * ((\nabla \otimes A)^T \bullet \nabla \otimes C) \right) + \right. \right. \\ &(C \bullet \text{Trav}((\text{Trav}(((\nabla \otimes A * \frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)}))) < 1, 2, 0 > + \text{Trav}((\frac{1}{|C|} * \nabla \otimes \nabla \otimes \\ &A)) < 1, 2, 0 >))) + (((((C * \frac{(-1*(A \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))}))^T + (\frac{1}{|A|} * (((\frac{1}{|C|} * \nabla \otimes \\ &C))^T + (((C * \frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)}))^T))) \bullet \nabla \otimes A) + (A \bullet \text{Trav}((\text{Trav}(((\frac{(-1*(A \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))} * \\ &\nabla \otimes C))) < :, 2, 1 > + \text{Trav}(((C * \frac{((-1*|C|*|A|*(A \bullet A)*((\nabla \otimes A)^T \bullet \nabla \otimes A) + (A \bullet \nabla \otimes \nabla \otimes A)) - (-1*((A \bullet \nabla \otimes A) \otimes (((A \bullet A)*|C| * \frac{(A \bullet \nabla \otimes A)}{|A|} \\ &1, 2, 0 > + \text{Trav}((((((\frac{1}{|C|} * \nabla \otimes C))^T + (((C * \frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)}))^T * \frac{(-1*(A \bullet \nabla \otimes A))}{((A \bullet A)*|A|)}))) < : \\ &, 2, 1 > + (\frac{1}{|A|} * (\text{Trav}(((\nabla \otimes C * \frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)}))) < 1, 2, 0 > + \text{Trav}((\frac{1}{|C|} * \nabla \otimes \\ &\nabla \otimes C)) < 1, 2, 0 > + \text{Trav}(((\frac{(-1*(C \bullet \nabla \otimes C))}{((C \bullet C)*|C|)} * \nabla \otimes C))) < :, 2, 1 > + \text{Trav}(((C * \\ &\frac{((-1*(C \bullet C)*|C|*((\nabla \otimes C)^T \bullet \nabla \otimes C) + (C \bullet \nabla \otimes \nabla \otimes C)) - (-1*((C \bullet \nabla \otimes C) \otimes (((C \bullet C)* \frac{(C \bullet \nabla \otimes C)}{|C|} + (|C|*2*(C \bullet \nabla \otimes C)))))) < \\ &\frac{((C \bullet C)*|C|*(C \bullet C)*|C|)}{((C \bullet C)*|C|*(C \bullet C)*|C|)} >))) < 1, 2, 0 >)) \end{aligned}$$

$$\begin{aligned} &\rightarrow_{\text{rewriteM}} \left(\frac{(-1*((A \bullet \nabla \otimes A)*(C \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} \right)^T + \frac{((\nabla \otimes A)^T \bullet \nabla \otimes C)}{(|A|*|C|)} + \frac{(-1*((C \bullet \nabla \otimes C)*((\nabla \otimes A)^T \bullet C))}{(|C|*|A|*(C \bullet C))} \right)^T + \\ &\frac{(\text{Trav}(\nabla \otimes \nabla \otimes A) < 2, 0, 1 > \bullet C)}{(|A|*|C|)} + \frac{(-1*((A \bullet \nabla \otimes A) \otimes (C \bullet \nabla \otimes A)))}{(|C|*|A|*(A \bullet A))} + \frac{((\nabla \otimes C)^T \bullet \nabla \otimes A)}{(|A|*|C|)} + \frac{(-1*((C \bullet \nabla \otimes C) \otimes (C \bullet \nabla \otimes A)))}{(|A|*|C|*(C \bullet C))} + \\ &\frac{(-1*((A \bullet \nabla \otimes A) \otimes ((\nabla \otimes C)^T \bullet A))}{(|C|*|A|*(A \bullet A))} + \frac{(-1*(C \bullet A)*((\nabla \otimes A)^T \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))} + \frac{(-1*(C \bullet A)*(A \bullet \nabla \otimes \nabla \otimes A))}{(|C|*|A|*(A \bullet A))} + \\ &\left(\frac{((C \bullet A)*((A \bullet \nabla \otimes A)*(A \bullet \nabla \otimes A)))}{((|A|)^2*|C|*|A|*(A \bullet A))} \right)^T + \left(\frac{((C \bullet A)*((C \bullet \nabla \otimes C)*(A \bullet \nabla \otimes A)))}{((|C|)^2*|C|*|A|*(A \bullet A))} \right)^T + \left(\frac{(2*(C \bullet A)*((A \bullet \nabla \otimes A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*((A \bullet A))^2)} \right)^T + \\ &\left(\frac{(-1*((A \bullet \nabla \otimes A)*((\nabla \otimes C)^T \bullet A))}{(|A|*|C|*(A \bullet A))} \right)^T + \frac{((C \bullet A)*((C \bullet \nabla \otimes C) \otimes (A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A)*(C \bullet C))} + \left(\frac{(-1*((C \bullet \nabla \otimes C)*((\nabla \otimes C)^T \bullet A))}{(|A|*|C|*(C \bullet C))} \right)^T + \\ &\frac{(\text{Trav}(\nabla \otimes \nabla \otimes C) < 2, 0, 1 > \bullet A)}{(|A|*|C|)} + \frac{(-1*((C \bullet \nabla \otimes C) \otimes ((\nabla \otimes C)^T \bullet A))}{(|A|*|C|*(C \bullet C))} + \frac{(-1*(C \bullet A)*((\nabla \otimes C)^T \bullet \nabla \otimes C))}{(|A|*|C|*(C \bullet C))} + \\ &\frac{(-1*(C \bullet A)*(C \bullet \nabla \otimes \nabla \otimes C))}{(|A|*|C|*(C \bullet C))} + \left(\frac{((C \bullet A)*((C \bullet \nabla \otimes C)*(C \bullet \nabla \otimes C)))}{((|C|)^2*|A|*|C|*(C \bullet C))} \right)^T + \left(\frac{(2*(C \bullet A)*((C \bullet \nabla \otimes C)*(C \bullet \nabla \otimes C)))}{(|A|*|C|*((C \bullet C))^2)} \right)^T) \\ &\rightarrow_{\text{rewrite}} \left(\frac{(-1*((C \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{(|C|*|A|*(A \bullet A))} + \frac{((\nabla \otimes A)^T \bullet \nabla \otimes C)}{(|C|*|A|)} + \frac{(-1*((\nabla \otimes A)^T \bullet C) \otimes (C \bullet \nabla \otimes C))}{(|A|*|C|*(C \bullet C))} \right) + \\ &\frac{(\text{Trav}(\nabla \otimes \nabla \otimes A) < 2, 0, 1 > \bullet C)}{(|C|*|A|)} + \left(\frac{(-1*((C \bullet \nabla \otimes A)*(A \bullet \nabla \otimes A)))}{(|A|*|C|*(A \bullet A))} \right)^T + \frac{((\nabla \otimes C)^T \bullet \nabla \otimes A)}{(|C|*|A|)} + \left(\frac{(-1*((C \bullet \nabla \otimes A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C))} \right)^T + \\ &\left(\frac{(-1*((\nabla \otimes C)^T \bullet A)*(A \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} \right)^T + \frac{(-1*(C \bullet A)*((\nabla \otimes A)^T \bullet \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \frac{(-1*(C \bullet A)*(A \bullet \nabla \otimes \nabla \otimes A))}{(|A|*|C|*(A \bullet A))} + \\ &\frac{((C \bullet A)*((A \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{((|A|)^2*|A|*|C|*(A \bullet A))} + \frac{((C \bullet A)*((A \bullet \nabla \otimes A) \otimes (C \bullet \nabla \otimes C)))}{((|C|)^2*|A|*|C|*(A \bullet A))} + \frac{(2*(C \bullet A)*((A \bullet \nabla \otimes A) \otimes (A \bullet \nabla \otimes A)))}{(|C|*|A|*((A \bullet A))^2)} + \end{aligned}$$

$$\begin{aligned}
& \frac{(-1*((\nabla \otimes C)^T \bullet A) \otimes (A \bullet \nabla \otimes A))}{(|C|*|A|*(A \bullet A))} + \left(\frac{((C \bullet A)*((A \bullet \nabla \otimes A)*(C \bullet \nabla \otimes C)))}{(|C|*|A|*(C \bullet C)*(A \bullet A))} \right)^T + \frac{(-1*((\nabla \otimes C)^T \bullet A) \otimes (C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))} + \\
& \frac{(Trav(\nabla \otimes \nabla \otimes C) < 2, 0, 1 > \bullet A)}{(|C|*|A|)} + \left(\frac{(-1*((\nabla \otimes C)^T \bullet A)*(C \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))} \right)^T + \frac{(-1*(C \bullet A)*((\nabla \otimes C)^T \bullet \nabla \otimes C))}{(|C|*|A|*(C \bullet C))} + \\
& \frac{(-1*(C \bullet A)*(C \bullet \nabla \otimes \nabla \otimes C))}{(|C|*|A|*(C \bullet C))} + \frac{((C \bullet A)*((C \bullet \nabla \otimes C) \otimes (C \bullet \nabla \otimes C)))}{((|C|)^2*|C|*|A|*(C \bullet C))} + \frac{(2*(C \bullet A)*((C \bullet \nabla \otimes C) \otimes (C \bullet \nabla \otimes C)))}{(|C|*|A|*((C \bullet C))^2)}
\end{aligned}$$

2 Moe

$$\rightarrow_{none} \left(\left(-\frac{(\nabla A \bullet \nabla \otimes \nabla A)}{|\nabla A|} \bullet \nabla A \right) * \frac{1}{|\nabla A|} \right)$$

$$\rightarrow_{rewrite} \frac{(-1*(\nabla A \bullet (\nabla A \bullet \nabla \otimes \nabla A)))}{((|\nabla A|)^2)}$$