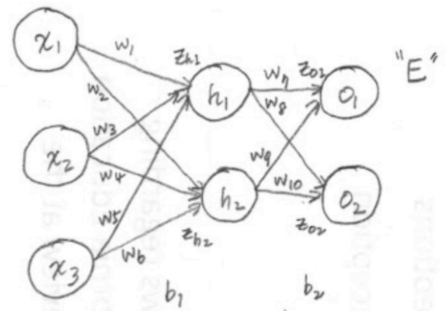

① initialize weights

② forward propagate to get output

③ define error/cost function

④ backpropagate

⑤ update weights

$$\begin{bmatrix} x_1 = 1 \\ x_2 = 4 \\ x_3 = 5 \end{bmatrix} \quad \begin{bmatrix} t_1 = 0.1 \\ t_2 = 0.05 \end{bmatrix}$$

input   target

② forward propagate

$$w_1 \cdot x_1 + w_3 x_2 + w_5 x_3 + b_1 = z_{h_1}$$

$$w_2 \cdot x_1 + w_4 x_2 + w_6 x_3 + b_1 = z_{h_2}$$

$$h_1 = \sigma(z_{h_1})$$

$$h_2 = \sigma(z_{h_2})$$

$$w_7 h_1 + w_9 \cdot h_2 + b_2 = z_{o_1}$$

$$w_8 h_1 + w_{10} h_2 + b_2 = z_{o_2}$$

$$o_1 = \sigma(z_{o_1})$$

$$o_2 = \sigma(z_{o_2})$$

③ error function

$$E = \frac{1}{2}\left[(o_1 - t_1)^2 + (o_2 - t_2)^2\right]$$

$$\frac{\partial E}{\partial o_1} = o_1 - t_1$$

$$\frac{\partial E}{\partial o_2} = o_2 - t_2$$

⑤ update weights

$$w_1 = w_1 - \alpha \cdot \frac{\partial E}{\partial w_1}$$

$$\vdots \qquad \vdots$$

$$w_{10} = w_{10} - \alpha \cdot \frac{\partial E}{\partial w_{10}}$$

$$b_1 = b_1 - \alpha \cdot \frac{\partial E}{\partial b_1}$$

$$b_2 = b_2 - \alpha \cdot \frac{\partial E}{\partial b_2}$$

④ backpropagate

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\frac{d\sigma}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \sigma(x)(1 - \sigma(x))$$

$$\because \frac{e^{-x}}{1 + e^{-x}} = 1 - \frac{1}{1 + e^{-x}}$$

error derivatives w.r.t. w_7, w_8, w_9, w_{10}, b_2

$$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial w_7} = (o_1 - t_1) \cdot (o_1(1 - o_1)) \cdot h_1$$

$$\frac{\partial E}{\partial w_8} = \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial w_8}$$

$$\frac{\partial E}{\partial w_9} = \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial w_9}$$

$$\frac{\partial E}{\partial w_{10}} = \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial w_{10}}$$

$$\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial b_2} + \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial b_2}$$

error derivatives w.r.t. $w_1, w_2, \cdots, w_6$, b_1

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_{h_1}} \cdot \frac{\partial z_{h_1}}{\partial w_1}$$

$$\frac{\partial E}{\partial h_1} \rightarrow \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial h_1} + \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial h_1}$$

$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_{h_1}} \cdot \frac{\partial z_{h_1}}{\partial w_3}$$

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_{h_1}} \cdot \frac{\partial z_{h_1}}{\partial w_5}$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial h_2} \cdot \frac{\partial h_2}{\partial z_{h_2}} \cdot \frac{\partial z_{h_2}}{\partial w_2}$$

$$\frac{\partial E}{\partial h_2} \rightarrow \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial h_2} + \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial h_2}$$

$$\frac{\partial E}{\partial w_4} = \frac{\partial E}{\partial h_2} \cdot \frac{\partial h_2}{\partial z_{h_2}} \cdot \frac{\partial z_{h_2}}{\partial w_4}$$

$$\frac{\partial E}{\partial w_6} = \frac{\partial E}{\partial h_2} \cdot \frac{\partial h_2}{\partial z_{h_2}} \cdot \frac{\partial z_{h_2}}{\partial w_6}$$

$$\frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial o_1} \cdot \frac{\partial o_1}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_{h_1}} \cdot \frac{\partial z_{h_1}}{\partial b_1} + \frac{\partial E}{\partial o_2} \cdot \frac{\partial o_2}{\partial z_{o_2}} \cdot \frac{\partial z_{o_2}}{\partial h_2} \cdot \frac{\partial h_2}{\partial z_{h_2}} \cdot \frac{\partial z_{h_2}}{\partial b_1}$$