Quantum Analogues of Probability Distributions

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Binomial Distribution

Bernoulli Trials With Coins

$$x = heads$$
$$y = tails$$
$$(x + y)(x + y) = 2\ coin\ flips$$

Equation

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

$$\binom{n}{k} p^k (1-p)^{n-k}.$$

Expected Value and Variance

$$x\frac{\partial}{\partial x}(x+y)^n = \sum_{k=0}^{n} k\binom{n}{k}x^k y^{n-k}.$$

$$x\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}(x+y)^n\right) = \sum_{k=0}^{n} k^2\binom{n}{k}x^k y^{n-k}$$

02

Quantum Analogues With Binomial Distributions

The Order is Not Preserved

xyy

yxy

yyx

$$xyy,\ yxy,\ yyx \longrightarrow 3xy^2$$

Quantum Analogue

$$yx = qxy$$

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{\prod_{i=n-k+1}^{n}(q^i - 1)}{\prod_{i=1}^{k}(q^i - 1)}.$$

Quantom Binomial Distribution

$$(x+y)^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q x^k y^{n-k}.$$

Inferences

Expected value	$\binom{n}{2}p(1-p)$
Variance	$\binom{n}{2}p(1-p)\left(\frac{2n-1}{3}-p(1-p)(2n-3)\right)$
Expected value given k successes	$\frac{k(n-k)}{2}$
Variance given k successes	$\frac{k(n-k)(n+1)}{12}$

03

Box Plots and Partitions

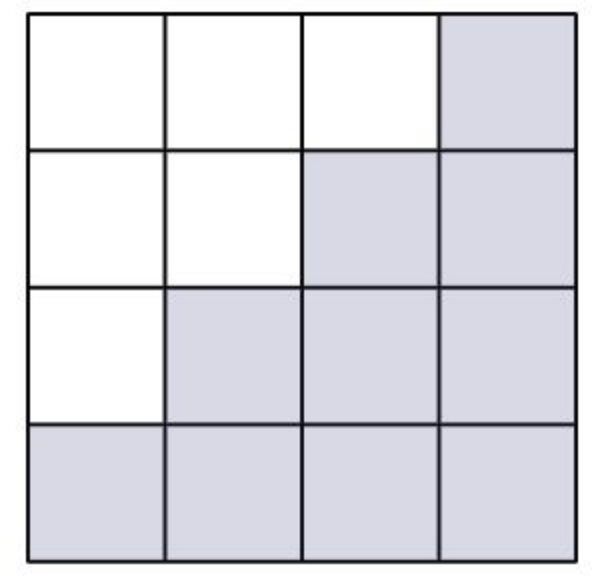
Partitions

A set of Positive Integers:

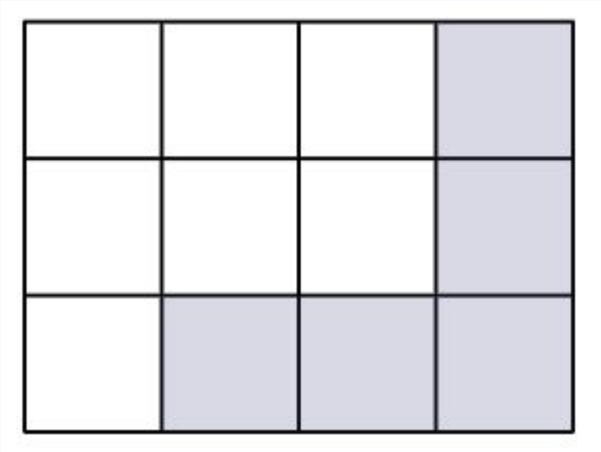
[1,2,3,4]

[0,1,1,3]

Box Plots

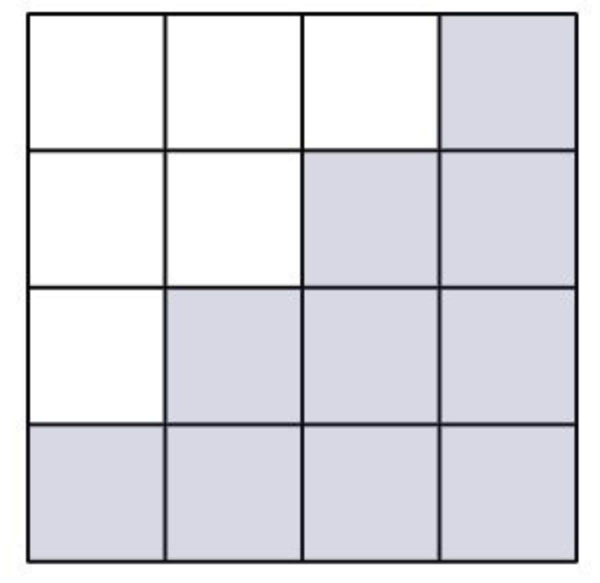


Partition: [1,2,3,4]
Binomial: xyxyxyxy



Partition: [0,1,1,3]
Binomial: yxyyxxy

Inversions



Equations

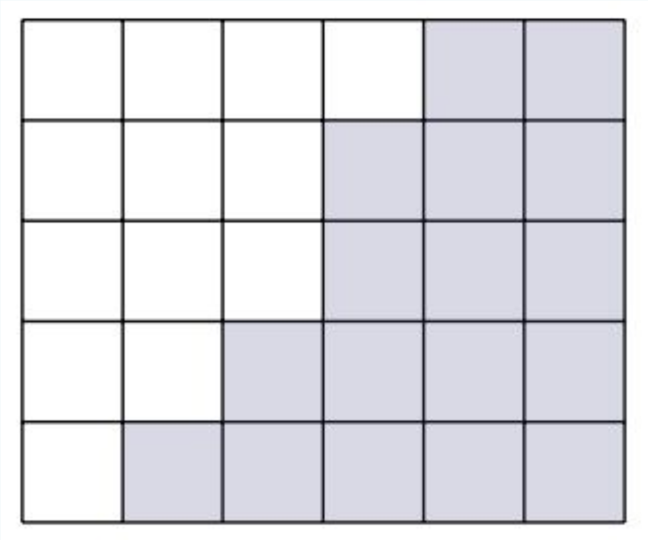
Additions of all possible partitions by area underneath path for an mxn box.

$$F_q(m,n) = \begin{bmatrix} m+n \\ n \end{bmatrix}_q$$

04

Fibonacci Distribution and K-bonacci Distribution

Fibonacci Distribution



A Way to Solve This

$$f_1(m,n) + f_2(m,n) = g(m,n)$$

$$f_1(m,n) = F(m-n+1, n)$$

$$f_2(m,n) = (0, 1, 2, \ldots, n-1)$$

$$(a_1, a_2 + 1, a_3 + 2, \ldots, a_n + n - 1)$$

K–Bonacci Distribution

Imagine a scenario where we are allowed to pick up a total of **k** number of fruits from **n** variety fruits given that we pick up each fruit a set number of times.

Pick up how many number of fruit 1 up to a given number s.

Then, pick up less than s number of fruit 2’s.

And so on until n fruits!

K-Bonacci Distribution

$$[z^k]\sum_{n\geq 0} A_n z^n = A_k$$

K-Bonacci Distribution

$$[z^k]\Pi_{j=1}^{n}(1 + zq^j + z^2q^{zj} + z^3q^{3j} + \ldots + z^sq^{sj})$$

In Conclusion

$$g_s(m,n) = \Sigma_{i=0}^{n}(-1)^i q^{\binom{i+1}{2}(s+1)} \begin{bmatrix} n \\ i \end{bmatrix}_{q^{s+1}} \begin{bmatrix} n+k-1-(s+1)i \\ n-1 \end{bmatrix} q^{k-(s+1)i}$$

So far was the joint work with Professor Matthew Just.

References

[1] G. E. Andrews, The Theory of Partitions, Cambridge University Press, 1988.
[2] G. H. Hardy and S. Ramanujan, Asymptotic formulae in combinatory anal- ysis, Proceedings of the London Mathematical Society, Second Series, 17 (1918), 75–115.
[3] M. Lothaire, Combinatorics on Words, Cambridge University Press, 1997.

Thank You!