Homework 4 Solutions

Gabriel J. Young

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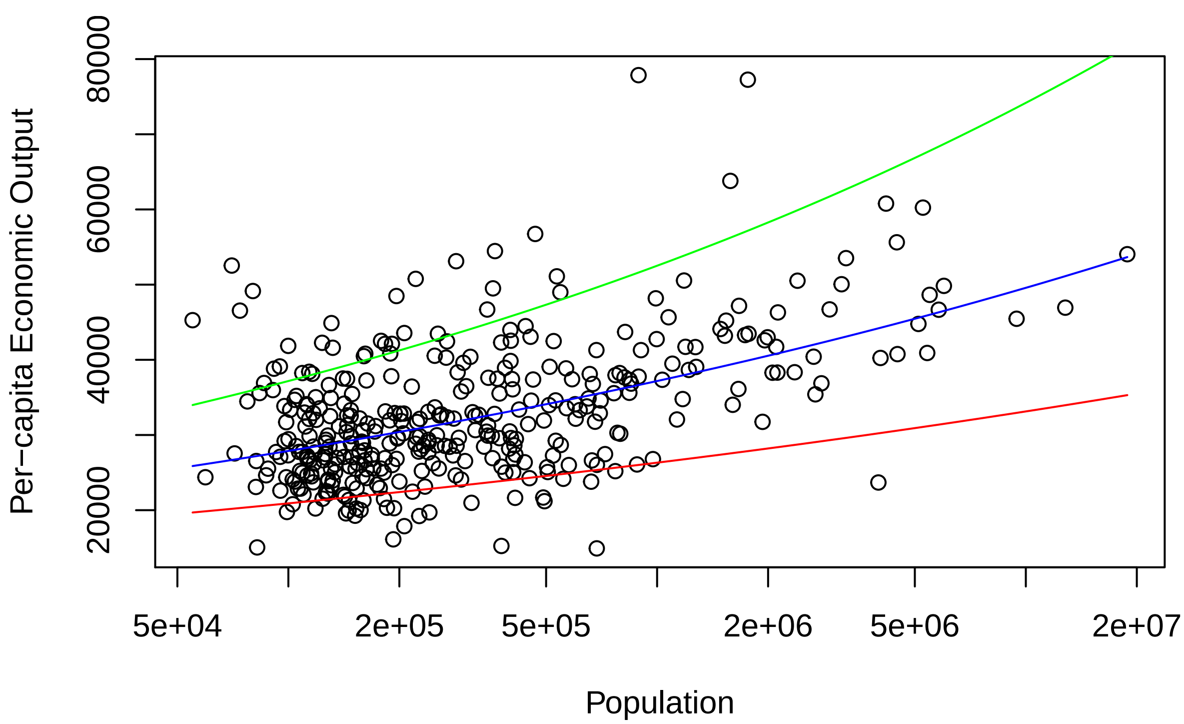
```
set.seed(1)
```

```
gmp     <- read.table("gmp.txt", header = TRUE)
gmp$pop <- round(gmp$gmp/gmp$pcgmp)
```

i.

```
plot(gmp$pop, gmp$pcgmp, log = "x", xlab = "Population",
     ylab = "Per-capita Economic Output")
curve(6611*x^{1/8}, add = TRUE, col = "blue")
curve(6611*x^{0.1}, add = TRUE, col = "red")
curve(6611*x^{0.15}, add = TRUE, col = "green")
```



ii.

```
mse <- function(beta, pop = gmp$pop, pcgmp = gmp$pcgmp) {
  return(mean((pcgmp - beta[1]*pop^beta[2])^2))
}
mse(c(6611, 0.15))
```

```
## [1] 207057513
```

```r
mse(c(5000, 0.10))
```

```
## [1] 298459914
```

iii.

```r
nlm(mse, c(beta0 = 6611, beta1 = 0.125))
```

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6611.0000000    0.1263177
##
## $gradient
## [1] 50.048639 -9.983778
##
## $code
## [1] 2
##
## $iterations
## [1] 3
```

```r
nlm(mse, c(beta0 = 6611, beta1 = 0.15))
```

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6610.9999997    0.1263182
##
## $gradient
## [1]   51.76354 -210.18948
##
## $code
## [1] 2
##
## $iterations
## [1] 7
```

```r
nlm(mse, c(beta0 = 5000, beta1 = 0.10))
```

```
## $minimum
## [1] 62521484
##
## $estimate
## [1] 5000.0000008    0.1475913
##
## $gradient
## [1] -1028.22544    11.38762
##
```

```
## $code
## [1] 2
##
## $iterations
## [1] 5
```

iv.

```
plm <- function(b0, b1, pop = gmp$pop, pcgmp = gmp$pcgmp) {
  hatb0 <- nlm(mse, c(beta0 = b0, beta1 = b1), pop, pcgmp)$estimate[1]
  hatb1 <- nlm(mse, c(beta0 = b0, beta1 = b1), pop, pcgmp)$estimate[2]
  MSE   <- nlm(mse, c(beta0 = b0, beta1 = b1), pop, pcgmp)$minimum
  return(list(hatb0 = hatb0, hatb1 = hatb1, MSE = MSE))
}
plm(b0 = 6611, b1 = 0.15)
```

```
## $hatb0
## [1] 6611
##
## $hatb1
## [1] 0.1263182
##
## $MSE
## [1] 61857060
```

```
plm(b0 = 5000, b1 = 0.10)
```

```
## $hatb0
## [1] 5000
##
## $hatb1
## [1] 0.1475913
##
## $MSE
## [1] 62521484
```

v.

(a)

```
pcgmp_mean <- mean(gmp$pcgmp)
pcgmp_sd   <- sd(gmp$pcgmp)
n          <- length(gmp$pcgmp)
pcgmp_se   <- pcgmp_sd/sqrt(n)
```

(b)

```
sample.mean <- function(indices) {
  return(mean(gmp$pcgmp[indices]))
}
```

(c)

```
B <- 100
bootstrap.means <- rep(NA, B)
for (i in 1:B) {
  indices <- sample(1:n, n, replace = TRUE)
  bootstrap.means[i] <- sample.mean(indices)
}
```

(d)

```
sd(bootstrap.means)
```

```
## [1] 435.2417
```

```
pcgmp_se
```

```
## [1] 481.9195
```

vi.

```
plm.bootstrap <- function(b0, b1, pop = gmp$pop, pcgmp = gmp$pcgmp, B = 100) {
  bootstrap.b0 <- rep(NA, B)
  bootstrap.b1 <- rep(NA, B)
  n <- length(pop)
  for (i in 1:B) {
    indices <- sample(1:n, n, replace = TRUE)
    bootstrap.b0[i] <- plm(b0, b1, pop = pop[indices], pcgmp = pcgmp[indices])$hatb0
    bootstrap.b1[i] <- plm(b0, b1, pop = pop[indices], pcgmp = pcgmp[indices])$hatb1
  }
  return(c(se.hatb0 = sd(bootstrap.b0), se.hatb1 = sd(bootstrap.b1)))
}
plm.bootstrap(b0 = 6611, b1 = 0.15)
```

```
##     se.hatb0     se.hatb1
## 1.322035e-08 1.054700e-03
```

```
plm.bootstrap(b0 = 5000, b1 = 0.10)
```

```
##     se.hatb0     se.hatb1
## 1.548400e-08 9.525965e-04
```

vii.

```
gmp2013     <- read.table("gmp-2013.txt", header = TRUE)
gmp2013$pop <- round(gmp2013$gmp/gmp2013$pcgmp)

nlm(mse, c(beta0 = 6611, beta1 = 0.125), pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
## $minimum
## [1] 135210524
##
## $estimate
## [1] 6611.0000002    0.1433688
##
## $gradient
## [1] -1493.88166     1.66893
##
## $code
## [1] 2
##
## $iterations
## [1] 7
```

```r
nlm(mse, c(beta0 = 6611, beta1 = 0.15), pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
## $minimum
## [1] 135210524
##
## $estimate
## [1] 6610.9999999    0.1433688
##
## $gradient
## [1] -1493.881707    -2.324581
##
## $code
## [1] 2
##
## $iterations
## [1] 5
```

```r
nlm(mse, c(beta0 = 5000, beta1 = 0.10), pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
## $minimum
## [1] 139208731
##
## $estimate
## [1] 5000.000001    0.164427
##
## $gradient
## [1] -3764.45251    72.85103
##
## $code
## [1] 2
##
## $iterations
## [1] 7
```

```r
plm(b0 = 6611, b1 = 0.15, pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
## $hatb0
```

```
## [1] 6611
##
## $hatb1
## [1] 0.1433688
##
## $MSE
## [1] 135210524
```

```r
plm(b0 = 5000, b1 = 0.10, pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
## $hatb0
## [1] 5000
##
## $hatb1
## [1] 0.164427
##
## $MSE
## [1] 139208731
```

```r
plm.bootstrap(b0 = 6611, b1 = 0.15, pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
##     se.hatb0     se.hatb1
## 1.395700e-08 1.039769e-03
```

```r
plm.bootstrap(b0 = 5000, b1 = 0.10, pop = gmp2013$pop, pcgmp = gmp2013$pcgmp)
```

```
##     se.hatb0     se.hatb1
## 6.107205e-08 1.160188e-03
```