MoM of Beta(α,β)

$$p(x|\alpha,\beta) = B(\alpha,\beta)\, x^{\alpha-1}(1-x)^{\beta-1},\ 0<x<1$$

$$Ex = \int_0^1 x\, B(\alpha,\beta)\, x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \ = \frac{\alpha}{\alpha+\beta}$$

$$\text{Var } x = E[x^2] - (Ex)^2$$

$$= \int_0^1 x^2\, p(x|\alpha,\beta)\,dx - \left(\frac{\alpha}{\alpha+\beta}\right)^2$$

$$= \frac{\alpha\beta}{(\alpha+\beta)^2 + (\alpha+\beta+1)}$$

MoM

2 parameters $\Rightarrow$ use $\bar{x}$ and s^2

Let $m = \bar{x}$, $v = s^2$

$$\underbrace{m = \frac{\alpha}{\alpha+\beta}}_{①},\quad \underbrace{v = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}_{②}$$

① $m(\alpha+\beta) = \alpha$, $m\alpha + m\beta = \alpha$, $m\beta = \alpha(1-m)$

$\alpha = \frac{\beta m}{1-m}$ (*)

plug * into 2

$$v = \frac{\frac{\beta m}{1-m} * \beta}{\left(\frac{\beta}{1-m}\right)^2\left(\frac{\beta+1-m}{1-m}\right)}$$

$$\alpha+\beta = \frac{\beta m}{1-m} + \beta = \frac{\beta m + \beta - m\beta}{1-m}$$

$$= \frac{\beta}{1-m}$$

$$V = \frac{\frac{\beta^2 m}{1-m}}{\frac{\beta^2}{(1-m)^3} * \beta+1-m} = \frac{m(1-m)^2}{\beta+1-m}$$

$\rightarrow \beta v + v - mv = m(1-m)$

$$\beta = \frac{m(1-m)^2 - v + mv}{v}$$ MoM est of β $\rightarrow$ plug into α

$$\alpha = \frac{m}{(1-m)}\left[\frac{m(1-m)^2 - v + mv}{v}\right]$$

For # 6 in lab

$$p(x|\overset{\text{params}}{\widehat{\alpha,\beta}}) = B(\alpha,\beta)\,x^{\alpha-1}(1-x)^{\beta-1},\ 0<x<1$$

$$L(\alpha,\beta) = \left[B(\alpha,\beta)\right]^n \left(\prod_{i=1}^n X_i\right)^{\alpha-1} \cdot \left(\prod_{i=1}^n (1-X_i)\right)^{\beta-1}$$

$$\ell(\alpha,\beta) = n\log B(\alpha,\beta) + (\alpha-1)\sum_{i=1}^n \log X_i + (\beta-1)\sum_{i=1}^n \log(1-X_i)$$