Implication of Time Varying Stock-Bond Correlation for Portfolio Diversification

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Abstract

There are divided views on existence and extent of time variation within stock-bond correlation. Diversification benefit of the portfolio depends on the stock-bond correlation. Numerous studies have been conducted to examine the factors responsible for movement in stock-bond correlation. Surprisingly, the implications of time varying stock bond correlation on portfolio diversification and risk are not well developed. Through our empirical findings we demonstrate that the stock-bond correlation vary with time. The correlations are estimated using rolling window and DCC GARCH(1,1) model. Our findings indicate that portfolio diversification and portfolio performance show a linear as well as quadratic relationship. We further analyse the relationship with a vector autoregressive (VAR) model and report the impulse response of diversification benefit to shocks in correlation.

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List of abbreviations

RW - Rolling Window

MV - Minimum Variance Portfolio

MSR - Maximum Sharpe Ratio Portfolio

EW - Equally Weighted Portfolio

MV 36 - Minimum Variance Portfolio with 36 month estimation windows

MSR 36 - Maximum Sharpe Ratio Portfolio with 36 month estimation windows

EW 36 - Equally Weighted Portfolio with 36 month estimation windows

MV 60 - Minimum Variance Portfolio with 60 month estimation windows

MSR 60 - Maximum Sharpe Ratio Portfolio with 60 month estimation windows

EW 60 - Equally Weighted Portfolio with 60 month estimation windows

VAR - Vector Autoregressive model

1 Introduction

Correlation between different assets is an important feature which is used while portfolio construction for generating diversified portfolios. **?** Modern Portfolio Theory being a single period model does not take into consideration the impact of changes in the correlation between assets. The existing research has a divided view on this assumption. Some of the research finds the stock - bond correlation to be constant over the time whereas others find the correlation to be time-varying. The time variation between the stock-bond correlation could possess a problem for the diversification characteristics when the portfolios are constructed using mean variance model.

The first objective of this research thesis is to determine whether there is a time variation between the stock-bond correlation. We estimate the correlation for the return time series of stock and bond over various time periods. We employ two methods to estimate correlation namely rolling window approach and conditional correlation using DCC GARCH(1,1) model. A careful examination of the correlation time series thus obtain clearly indicate an existence of time variation. This finding is in sync with various studies conducted previously.

The second objective is to construct portfolios using the selected stock & bond using the mean variance framework. We construct portfolios using minimum variance strategy and maximum Sharpe ratio strategy within this framework. In order to provide a comparison we also construct the portfolio using a naive equally weighted strategy. We use this naive strategy as a benchmark for determining the performance of the portfolios constructed using mean variance framework.

The third objective is to examine the relationship between the diversification characteristic and time varying stock-bond correlation. We examine this with help of a linear and a quadratic model. Empirical findings suggest existence of both a linear and quadratic relationship. Further using similar models we examine the relationship between the portfolio performance measured using Sharpe ratio and the time varying correlation.

To investigate the response of diversification benefits of portfolio to shocks of correlation we use a Vector Autoregressive model with impulse response function. We observe that a positive shock in correlation affects the diversification benefit in a negative direction for minimum variance and equally weighted portfolios.

Finally, we examine the performance of the portfolios constructed using above mentioned strategies using measures such as max draw-down, tracking error and information ratio. We find that the naive equally weighted strategy performs better than the other portfolios.

This study contributes to the existing literature, by examining the relationship between the time variation in correlation between assets and the portfolio diversification & portfolio performance. The study conducted can be used to effectively factor in the time variation between assets while constructing mean variance portfolios. A further evolution of this study would be to construct framework for a tactical asset allocation strategy which can take advantage of the time variation in correlation.

We conduct this study within the financial markets of the United States. The sheer motivation to use financial markets from the United States is the availability of data for longest period of time. S&P 500 index has been used as an indicator of stock returns and US Government 10 year bond has been used as an indicator of bond returns for this study. We have used monthly data ranging from December 1974 to December 2016, however the data from December 1974 to December 1979 is used only for the purpose of estimation. The study, therefore, is for the time period from 1980 to 2016.

The rest of the paper is organised as follows. Section 2 deals with review of the literature on time variant correlation and mean variance framework for portfolio construction.

In section 3 we describe the methodology used to calculate the correlation, to construct the mean variance portfolios, and to establish the relationship between the correlation and portfolio diversification, and portfolio performance. In section 4 we present the empirical results of the study followed by a brief conclusion in section 5.

2 Literature Review

2.1 Correlation between assets

? demonstrated the importance of correlations between the assets of a portfolio for diversification. According to Markowitz Modern Portfolio Theory, maximum diversification is possible when assets in a portfolio follow a negative correlation. These findings were further supported by ?. Correlation is one of the most powerful tools that can be used for reducing the volatility of the portfolio.

In the current scenario, there are many asset classes that can be used for the purpose of portfolio diversification. ? in their research demonstrated that gold would provide a perfect hedge against the volatility of the stock returns particularly in extreme market conditions. Although many such asset classes like derivatives, real estate and others have made the selection process more difficult, bonds and stocks still remain to be major components of any diversified portfolio.

Critical analysis of the correlation between the assets within a portfolio is required in order to construct an optimal diversified portfolio in a mean-variance framework. There have been many studies conducted in order to analyse the relationship between the stocks and bonds in a portfolio. ? were amongst the first to study the relationship between stock and bond returns. The existing research provides a divided view of how to look at the correlation between stocks and bonds. ? observed stock-bond correlation to be negative due to changes in common interest rate factor, the correlation being time invariant. ? also imply that the correlation between the stock and bond is invariant in the time.

In contrast, ?, ? have shown that the correlation between the stocks and bond show a time variation. ? observed that the stock-bond correlation varies according to the expectations of inflation. ? implies that the uncertainty with respect to inflation is responsible for a stronger negative stock-bond correlation whereas **?** show that inflation volatility weakens the correlation.

? demonstrated that the stock-bond correlation shifts during stock market crashes. **?** show that equity crashes alter the long run stock-bond correlation and crash leads to capital preserving behaviour as opposed to the strategic reallocation of government securities. They also describe unique stock-bond correlation adjustments in US markets post 9/11 and the flight to safety reaction related to it. The flight to safety (quality) phenomenon was further supported by **?** findings that the phenomenon leads to diversification effect in the times when it is needed the most. **?** used rolling window approach in conjunction with conditional correlation using DCC GARCH(1,1) model to estimate the time variation in stock-bod correlation. **?** proposed to use AG-DCC GARCH model to study the stock-bond correlation and found that both bond and equities exhibit asymmetries in conditional correlation.

? analysed the correlations after bisecting the data into an emerging market and bear market data series. They provide strong evidence of variation of US stock-bond correlations due to flight to quality. The time variation in stock-bond correlation largely affects the diversifying properties in the portfolio. It was found that during the global financial crises of 2007, the portfolio with 50% stock / 50% bond performed better than the diversified portfolio made out of eight equally weighted assets.

Existence of time varying stock-bond correlation cannot be overlooked while constructing an efficient optimal portfolio. Diversification benefit of the portfolio constructed using mean-variance model stays true until the correlation is constant, therefore the model needs to be revisited in light of the time varying stock-bond correlations. I intend to study how time variation between stock-bond correlation would affect the diversification characteristic of a portfolio constructed using mean variance model.

2.2 Mean Variance Framework

? introduced the mean variance framework for portfolio optimization and extended it further in his book Portfolio Selection: Efficient Diversification in 1959. The mean variance framework provides a way to construct the portfolios based on the expected returns and the risk appetite of the investor.

? states that the Markowitz mean variance framework provides a quantitative measure for the concept of diversification in form of correlation between the assets. The basic premise of the framework is to estimate the expected returns, volatility and correlation for all the securities and then optimize the solution for the required risk return profile. An efficient frontier can then be constructed for varying risk return profiles. Minimum variance portfolio is the portfolio at the left most tip of this efficient frontier. Figure 1 provides an overview of the process for constructing an efficient frontier.

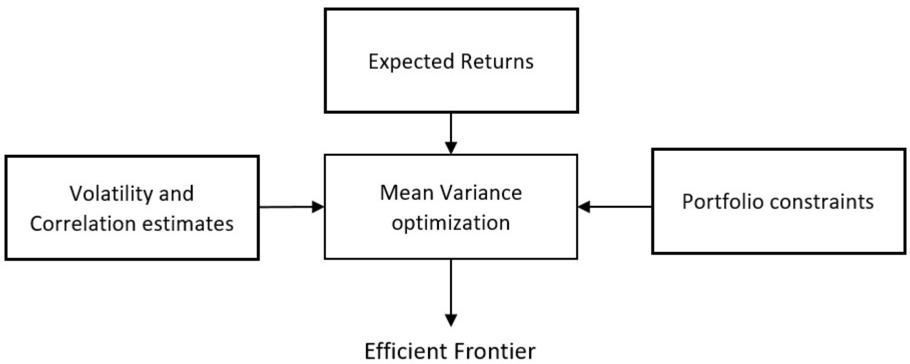

Figure 1: Efficient frontier framework.Adapted from Fabozzi et al.(2002)

Works of **?**,**?** and **?** saw introduction of a risk free asset to the set of risky assets in mean variance framework. The tangent line drawn from the risk free asset to the efficient frontier (Capital Market Line) would yield a super-efficient portfolio at the point of tangency. This would be the one which maximizes the excess returns per unit of risk and hence named as a Maximum Sharpe Ratio portfolio. Figure 2 exhibits a description of efficient frontier, minimum variance portfolio and the maximum sharpe ratio portfolio.

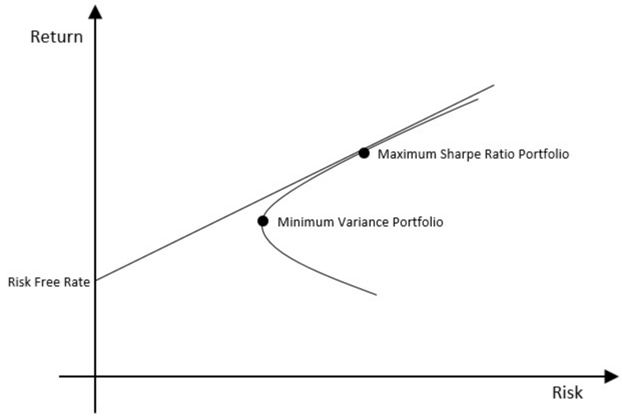

Figure 2: Representation of an efficient framework and CML

A major problem us mean variance framework is the errors associated with forecasting the expected returns. Capital Asset Pricing Model (CAPM) was used to estimate the expected returns, however **?** invalidated use of CAPM for any applications. Minimum variance strategy however,does not require expected return forecast and hence will be used for this study.

Another source of error in a mean variance framework is the covariance matrix. **?** has enumerated various problems associated with estimation of covariance matrix. A Bayesian shrinkage procedure can be applied to reduce the errors. A new shrinkage procedure was proposed by **?** which aims at minimizing the estimation errors.

Although marred with crticism, mean variance framework combined with the asset pricing models are widely accepted by practitioners [**?**] and hence I have selected this framework to construct the portfolios and observe the effects of time varying correlation.

A naive approach of using equal weights for the assets in the portfolio has been used during the study. This approach is often considered as a special case in mean variance optimal portfolio. **?**, in their study, find that the out of sample performance of 14 mean-variance model portfolios is not consistently better than the naive approach measured in terms of Sharpe ratio.

2.3 Implication of time varying correlation

Regression analysis is the simplest model that can be employed to determine the relationship between two or more variables. **?** in his study used regression analysis to determine the impact of stock-bond correlation on asset allocation. **?** used linear regression to determine which macroeconomic factors affect the stock-bond correlation over time. **?**, in their Harvard Business Review paper used regression to determine the relationship between the managerial ownership and risk taking of a bank.**?** employed regression analysis to determine the extent to which the Fama-French factors are responsible for variation in excess returns of minimum variance portfolios.

Vector autoregressive (VAR) model is the easy to use and widely used multivariate time series analysis model. In VAR model each variable is estimated as a function of lag of its own value and other variables in the system. **?** used the VAR model for the first time in economics and since then several studies have used VAR model to determine the relationship between multivariate time series. **?** used VAR model to determine whether monetary policies of a country lead to recession. **?** determined the relationship between the Foreign Direct Investments in a country and its macroeconomic factors using VAR model. VAR model was used by **?** to determine the serial dependence of stock returns and therefore the out-of-sample performance of the portfolios.

VAR models can also be used to forecast the variables withing the system. **?** used VAR model to determine the predictive power of long term interest rates for estimating future inflation. Using impulse response functions we can determine response of one variable in the system to impulse in another variable.**?** determined the impact of currency exchange rates on monetary policies in countries of France, Germany and Italy. In a working paper series of Bank of International Settlements, **?** used VAR model and impulse response function to study the effects of monetary policy shocks in east Asian economies. **?** used impulse response function with VAR model framework to measure effects of shocks in monetary policy in Nigeria. **?** used impulse response function to gauge the response of FDI to shocks in economic factors.

Due to its wide presence in literature I have selected regression model, VAR model and impulse response function to study the relationship between the stock-bond correlation and portfolio diversification.

3 Research Methodology

3.1 Sample Data

We have used S&P 500 index and US Government 10 year treasury bond as proxies for stock and bond asset class respectively. S&P 500 is a cap-weighted index comprising of 500 large companies listed on New York Stock Exchange (NYSE) and NASDAQ. S&P 500 is a major stock index in the United States and closely resemble the US economy making it a good choice to be considered as a proxy for asset class of stock. Previous studies like **?** and **?** have used US Government 10 year treasury as they represent a long term view on economy and are liquid. 30 day US treasury bills have been used as a representative of risk free rate for calculation of Sharpe ratio.

Monthly data ranging from December 1974 to December 2016 derived from the CRSP database has been used in the study, however the data from Decemeber 1974 to Decemeber 1979 has been used only for the purpose of calculation of parameters fo rolling window as described in next section. The actual study of stock-bond correlation and its implications ranges from January 1980 to December 2016.

The nominal log returns of the stock and bond time series are calculated using the formula $r_t = \log \frac{P_t}{P_{t-1}}$ where P_t is the price of the asset at time t.

3.2 Correlation

The first preliminary objective of the research is to study the variation in correlation between the stocks and bonds with respect to time. I have used a simple rolling window approach to calculate the sample correlation between the stocks and bond. Two sampling windows of 36 months and 60 months respectively have been used in this study. The purpose of using two different time frames for sampling is to substantiate the finding over different periods. The correlation in a rolling window approach is calculated using the following formula

$$\widehat{\rho} = \frac{\sum_{t=1}^{n}(r_{s,t} - \overline{r_s})(r_{b,t} - \overline{r_b})}{\sqrt{\sum_{i=1}^{n}(r_{s,t} - \overline{r_s})^2 \sum_{i=1}^{n}(r_{b,t} - \overline{r_b})^2}}$$

where $r_{s,t}$ and $r_{b,t}$ are returns on stock and bond respectively at time t and $\overline{r_s}$ and $\overline{r_b}$ are average returns of stock and bond respectively over the entire estimation window, and n is the length of rolling windows.

The second method used to estimate the stock-bond correlation is the DCC GARCH(1,1) model proposed by **?**. DCC GARCH model estimates the conditional correlation in two steps. First, we estimate variance equation of the time series using the mean equation in a univariate GARCH framework. Second, we then estimate the covariance matrix and hence the correlation between stock and bonds. The idea in DCC GARCH model is that we can decompose the covariance matrix into a conditional standard deviation matrix and a correlation matrix. The general form of DCC GARCH model used in this study is as follows

$$r_t = \mu_t + a_t$$

$$a_t = \mathbf{H}_t^{1/2} z_t$$

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t$$

where r_t is a vector of log returns, a_t is a vector of mean corrected returns, μ_t is aa vector of expected value of conditional returns,$\mathbf{H}_t$ is conditional covariance matrix, $\mathbf{D}_t$ is diagonal matrix of standard deviations, $\mathbf{R}_t$ is conditional correlation matrix, and z_t is vector of errors terms.

We report the correlation obtained using both approaches in this study. The rolling window approach captures the changes in correlation with respect to time in a most simple way. We further check for presence of auto correlation within the time series of returns of stock and bond. Visual inspection for the presence of auto correlation on conducted by plotting a correlogram for both the time series. To verify the finding from correlogram we implement Ljung-Box Q test [?] which examines the null hypothesis that the time series of residuals exhibit no auto correlation.

We then check for presence of stationarity in the time series of estimated correlation. We apply two unit root test namely Phillips-Perron (PP) test and Augmented Dickey-Fuller (ADF) test, similar to study conducted by (?). Both test assess the null hypothesis of presence of unit root for the time series implying the time series to be non stationary.

3.3 Portfolio Construction

I adopt the methodology used by **?** for construction of portfolios using the mean variance framework in a rolling window paradigm and testing the out of sample performance. At the beginning of each month starting from January 1980, I estimate the covariance matrix of stock and bond using the historical data of 36 months. The use of historical data is necessary to keep the study model free. The estimated covariance matrix is then fed into an optimizer process in Matlab to generate the weights of stock and bond for a minimum variance portfolio. A long only constraint is specified for optimization with no limit set for weight of an asset. Returns are calculated for an investment horizon of one month using the weights thus obtained. This process is repeated for every month for the period under the study. The similar set of steps are then performed for a 60 month window of historical data used for estimation.

A naive strategy similar to used by **?** is also implemented over the same period to facilitate a comparison with minimum variance and maximum Sharpe ratio portfolios.

Sharpe ratio [?] is used as a performance measure for the portfolios constructed using above methodology. Sharpe ratio is calculated in two ways, first for the entire period under the study, and second for sub periods of 60 months starting from January 1980 to December 1984 and advancing it by one month thereafter, thus the final period for calculation is from January 2010 to December 2012. Sharpe ratio is defined as the mean excess returns of the portfolio over the sample volatility and is represented by the following equation. Sharpe ratio thus calculated is compared with the correlation during the same sub periods.

$$SR_t = \frac{\widehat{\mu_t}}{\widehat{\sigma_t}}$$

3.4 Implications of time varying correlation

Robust Regression

After observing the time variation in stock-bond correlation and the performance of optimal and naive portfolios over the same time frame we now try to establish a relationship between the observed correlation and diversification characteristic of the portfolio. Similar models are then examined for performance of portfolios measured by Sharpe Ratio. The models under the study are as follows

$Model\ 1 : w_{s,t} = \beta_0 + \beta_1 \widehat{\rho_t}$

$Model\ 2 : w_{s,t} = \beta_0 + \beta_1 \widehat{\rho_t} + \beta_2 \widehat{\rho_t}^2$

where $w_{s,t}$ is the weight of stock at time t and $\widehat{\rho_t}$ is the estimated correlation.

$Model\ 3 : SR_t = \beta_0 + \beta_1 \widehat{\rho_t}$

$Model\ 4 : SR_t = \beta_0 + \beta_1 \widehat{\rho_t} + \beta_2 \widehat{\rho_t}^2$

where SR_t is the Sharpe ratio at time t and $\widehat{\rho_t}$ is the estimated correlation.

From the empirical results mentioned in the next section we observe that the correlation time series are non - stationary. The least squares method for estimating regression coefficients would therefore not be suitable for the above mentioned model. Robust regressions [?] provides an alternative for estimating the parameters for a non stationary time series. We use a bi-square estimator for all our model estimates. The bi-square estimate minimizes the weighted sum of squares for the data set. The data points near the estimated line get full weight whereas the data points far would get reduced weight. We use Matlab econometrics toolbox for all our estimations.

Impulse Response Function

Our objective of the study is to find the effects of correlation on the diversification of the portfolio. We employ a impulse function based on vector autoregressive model (VAR) to find the effects of correlation shocks on the diversification benefit of portfolios measured using Sharpe ratio. The VAR model specification used is as follows

$$y_t = G_0 + G_1 y_{t-1} + G_2 y_{t-2} + \ ... \ + G_p y_{t-p} + e_t$$

where y_t is the matrix of variables, G_0 is the vector of constants, G is the matrix of coefficients, e_t is the matrix of innovations, and p is the number of lags used for modelling.

We estimate the above mentioned model for all the portfolios generated using 36 month and 60 month estimation windows. Before estimating the model we need to to establish the number of lags to be used for VAR modelling. According to study conducted by **?** the Akaike Information Criteria (AIC) provides the most accurate lag value for a monthly VAR model. Therefore, for each of the portfolios under study, we first determine the number of lags to be used based upon the AIC.

After determining the number of lags to be used, we estimate the VAR model. We then estimate the impulse response of Sharpe ratio of portfolios to the shocks in correlation. The shocks in correlation are generated using widely used Cholskey decomposition method. We apply one standard deviation shocks and report the results for all portfolios.

4 Empirical Results

4.1 Sample Statistics

Figure 3 is the plot of monthly return time series for the stock and bond over the entire period under study. Descriptive statistics of the stock & bond return time series are reported in Table 1.

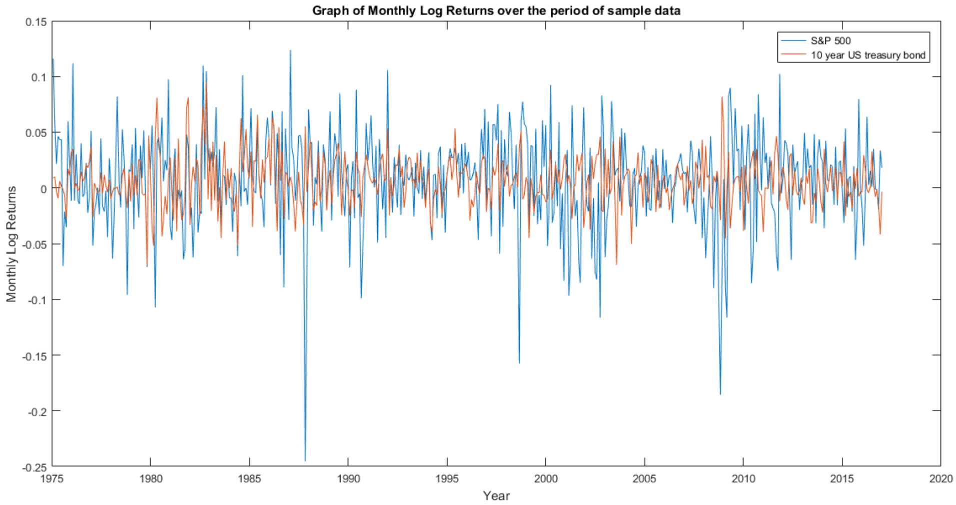

Figure 3: Log returns time series for Stock and Bond

Summary Statistic	S&P 500	10 yr US Treasury Bond
No. of observations	504	504
Mean Returns	0.0069	0.0061
Std. Deviation	0.0433	0.0227
Skewness	-0.7999	0.2087
Kurtosis	5.9903	4.1546
Jarque-Bera	241.5287	31.6528
p - value JB Statistic	0	0

Table 1: Summary Statistics of Sample Data (all values are monthly)

We have 504 observations for stock and bond return time series. It is observed that the stock return time series has a negative skewness and the bond return time series has a positive skewness. Further Jarque - Bera (JB) statistic [?] is calculated for the time series. JB statistic is used to examine the normality in a time series. It examines the skewness and kurtosis of the time series and determines the normality. The skewness, kurtosis, and JB statistic value for the stock and bond time series suggest that both the time series dose not follow a normal distribution.

4.2 Correlation

Table 4.2 enlists the summary statistics of observed correlation estimated using different methods. The correlation between stock and bond for the entire period under observation is found to be 0.0893. When a rolling window approach is applied to the time series, the correlation is found to be varying between -0.73038 and 0.63964 for a 36 month rolling window, whereas it is between -0.55917 and 0.60367 for a 60 month rolling window. For conditional correlation estimated using DCC GARCH(1,1) model the average correlation is 0.1320 and varies between -0.5647 and 0.6332. All three methods used have very similar results.

	36m RW	**60m RW**	**Conditional**
No. of observations	444	444	480
Mean	0.0783	0.0820	0.1320
Minimum	-0.73038	-0.55917	-0.5647
Maximum	0.63964	0.60367	0.6332
Std. Dev.	0.3950	0.3566	0.2891
Skewness	-0.2261	-0.2416	-0.3219
Kurtosis	1.8601	1.6659	2.0379
Jarque-Bera	27.8230	37.2291	26.7989

Table 2: Summary statistics of correlations

Although the stock-bond correlation over the entire period is observed to be positive, from Figure 4 it is clearly evident that the correlation is rather unstable and varies with time. Period of sustained positive and negative correlation are clearly visible. Also the time variation in the correlation is substantiated by observing similar patters for 36 month and 60 month rolling windows. The correlation is observed to have a varying but positive till 2002 followed by a period of negative correlation till the end of observed data set. The results are similar to one obtained by **?**.

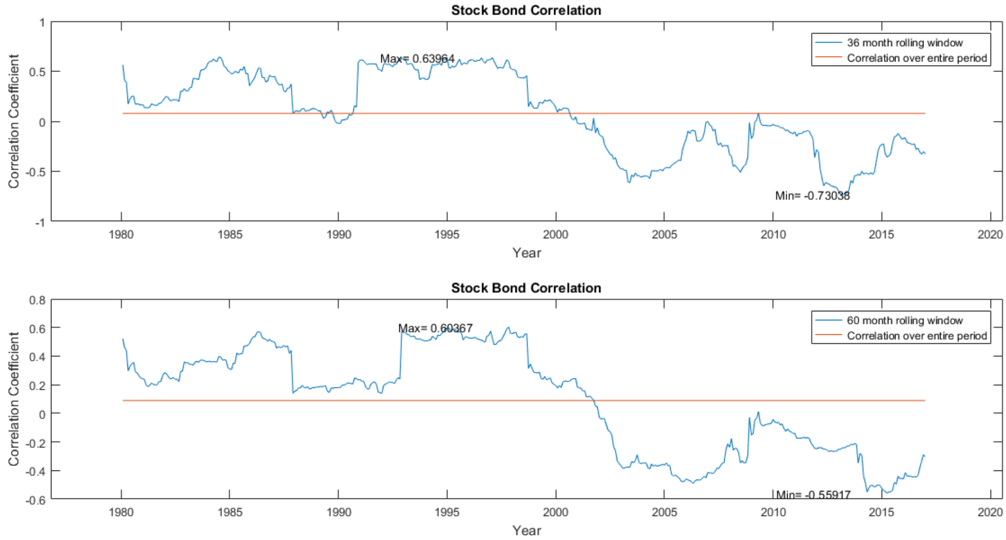

Figure 4: Time variation in stock-bond correlation

Conditional correlation estimated using DCC GARCH(1,1) model along with the rolling window correlation computed earlier are compared in Figure 5. We can observe that the correlation derived from two approaches are very similar to each other. For conditional correlation the average value is positive but a clear variation with respect to time is apparent. The results are in-line with the one obtained by **?**.

Figure 5: Comparison of rolling window and conditional correlation

We plot the histogram for correlations in Figure 6. It is evident that the correlations do not follow any distribution.

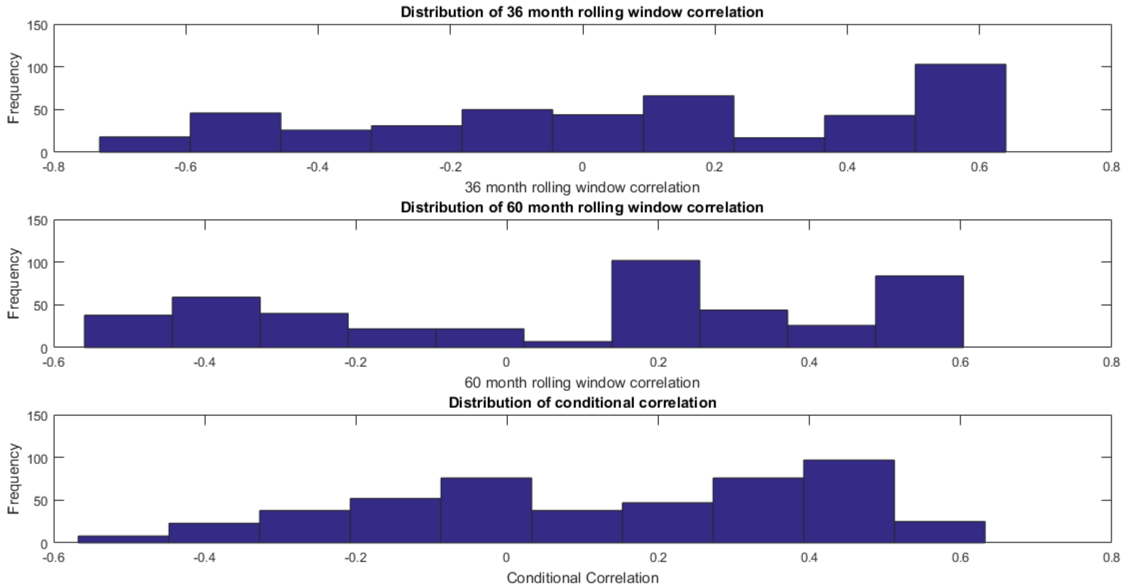

Figure 6: Distribution of correlation

To check the existence on auto correlation within the return time series, a correlogram is plotted for simple and squared returns of the stock and bond return. Figure 7 and Figure 8 represent the correlogram for the stock and bond return time series respectively. No significant auto correlation effect is observed for simple and squared stock returns, whereas a significant autocorrelation is found for squared returns of the bond returns. Further to verify the observations from correlogram Ljung-Box Q Test is conducted on the time series. We report the results of the test in Table 4.2. The results suggest that there is a significant autocorrelation present in the bond return time series whereas no significant autocorrelation in the stock return time series. The results from the correlogram and the Q test are in unison.

Estimates	S&P 500	10 yr US Bond
Test statistic	30.7554	133.6861
p - value	0.0585	0

Table 3: Test statistics for Ljung-Box Q test

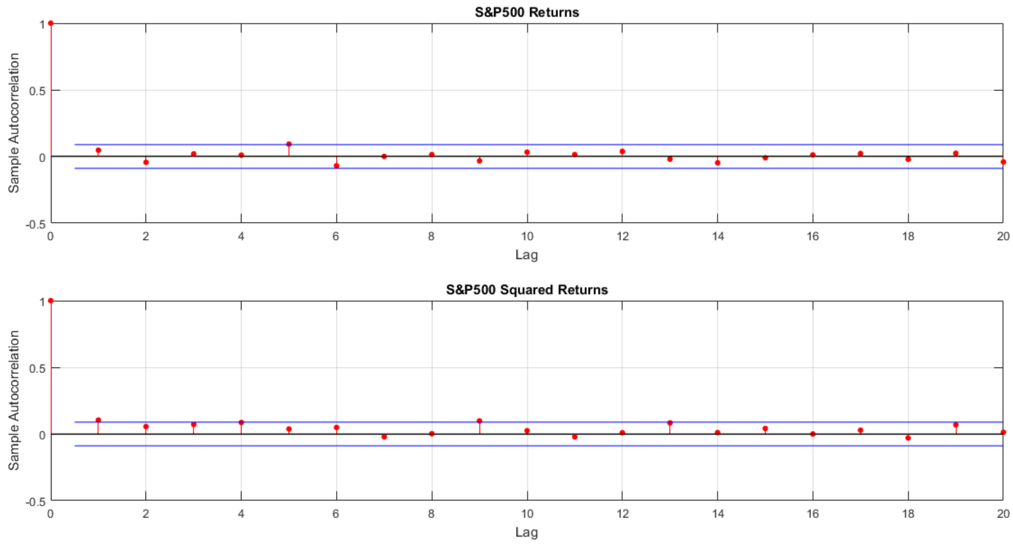

Figure 7: Corrleogram for stock returns

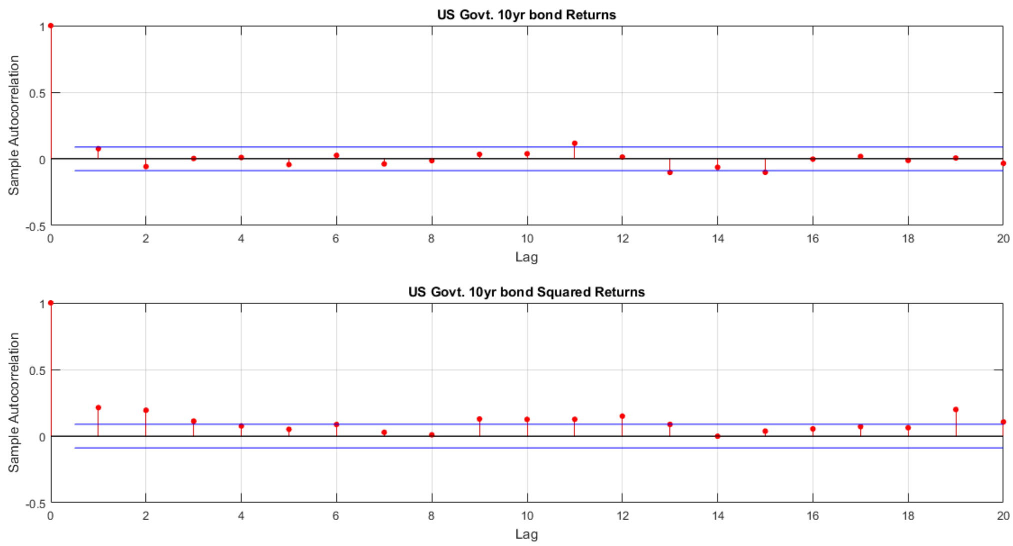

Figure 8: Corrleogram for Bond returns

We further apply tests to determine whether the time series of estimated correlation are stationary or not. We apply unit root stationary test in form of Phillips-Perron (PP) test and Augmented Dickey-Fuller (ADF) test. The results of the test are enumerated in Table 4. The 36 month and 60 month estimated correlation time series are found to be non - stationery according to the tests conducted. These findings reinforce the previous finding of time variation within the stock-bond correlation.

Test for Stationarity	**PP - stat**	**p - value**	**ADF - stat**	**p - value**
36 month correlation	-1.6406	0.0952	-2.0540	0.5658
60 month correlation	-1.4006	0.1501	-1.7444	0.7185

Table 4: Test of stationarity for estimated correlation

4.3 Portfolio Performance

In this section we check the performance of the portfolios formed using minimum variance, maximum Sharpe ratio and equally weighted strategies.

Table 4.3 represents the out of sample performance of portfolios,constructed using 36 month and 60 month window of estimation over the entire period. Minimum variance portfolio gives best out of sample performance measured in terms of Sharpe ratio for both 36 month and 60 month estimation windows. The naive strategy of equally weighing the assets performs better than the maximum Sharpe ratio strategy for both scenarios.

Measure	36m RW			60m RW		
	Return	Volatility	Sharpe Ratio	Return	Volatility	Sharpe Ratio
Minimum Variance	0.0064	0.0211	0.1312	0.0063	0.0215	0.1247
Maximum Sharpe	0.0061	0.0297	0.0821	0.0053	0.0364	0.0562
Equally Weighted	0.0066	0.0253	0.1188	0.0066	0.0253	0.1188

Table 5: Portfolio performance over entire period (all observations are monthly)

To examine the performance of portfolios in light of time varying correlation, we plot the evolution of Sharpe ratios for all portfolios with respect to the time varying stock-bond correlation. From Figure 9 it is evident that there is a direct relationship between the Sharpe ratio and the stock-bond correlation till the year 1997 and the relationship is inverse thereafter. We examine this relationship in detail in next section. A similar comparison can be seen in Figure 10 for the portfolios constructed using 60 month estimation window.

Figure 9: Correlation v/s Sharpe Ratio : 36 month estimation window

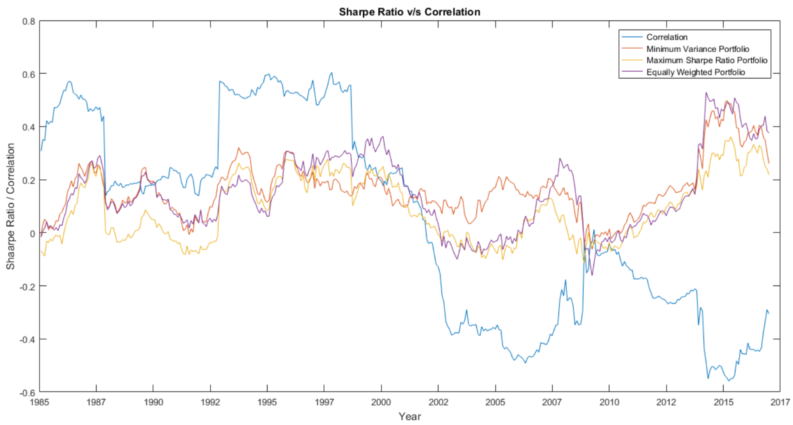

Figure 10: Correlation v/s Sharpe Ratio : 60 month estimation window

Further we conduct parametric tests to determine the significance of the difference between the evolution of Sharpe ratios for all portfolios. We report the results of these tests in Table 4.3. We find that the Sharpe ratios are significantly different for all the portfolios except for the pair of MSR36 & EW36.

	36m estimation window		**60m estimation window**	
	t - statistic	**p - value**	**t - statistic**	**p - value**
MV & MSR	8.6365	0	20.7994	0
MV & EW	6.8777	0	3.9634	0
MSR & EW	-1.7109	0.0879	-15.8368	0

Table 6: Test of significance for Sharpe Ratio time series

4.4 Implications of correlation on Portfolio Diversification

In this section we model the relationship between the time varying correlation and portfolio diversification. Weightage of stocks in a portfolio is considered as a measure of portfolio diversification. The models under study are as follows

$Model\ 1 : w_{s,t} = \beta_0 + \beta_1 \widehat{\rho_t}$

$Model\ 2 : w_{s,t} = \beta_0 + \beta_1 \widehat{\rho_t} + \beta_2 \widehat{\rho_t}^2$

where $w_{s,t}$ is the weight of stock at time t and $\widehat{\rho_t}$ is the estimated correlation.

The regression results for the $Model\ 1$ are reported in Table 7. The obtained results suggest that that there is a negative relationship between the weight of stocks for the portfolios constructed using minimum variance strategy and the stock bond correlation. We obtain similar results 36 month and 60 month estimation window. The results are not decisive for the portfolios constructed using the Max Sharpe ratio strategy. The results are also enumerated in scatter plots represented in Figure 11 and Figure 12.It can be observed that there is no particular pattern visible for the MSR portfolios.

Portfolios	β_0	β_1	**Adjusted** R^2
MV 36	0.2146*	-0.1344*	0.06339
MV 60	0.1943*	-0.1711*	0.2691
MSR 36	0.3562*	0.0764*	-
MSR 60	0.2773*	0.2212*	-
* - Significant at 95% confidence level			

Table 7: Parameter estimates for *Model* 1

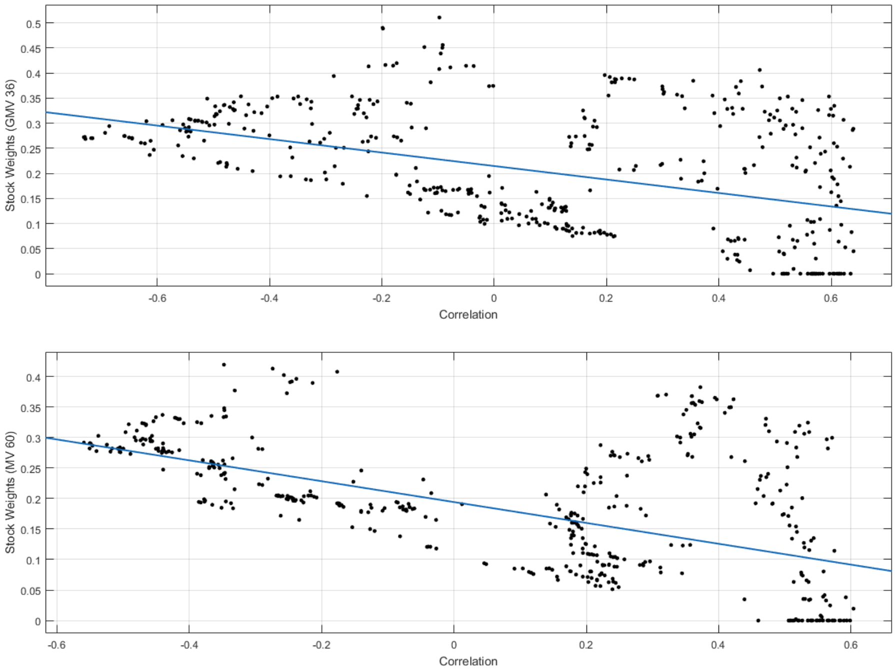

Figure 11: Scatter plot for Stock Weights (MV) with correlation (*Model* 1)

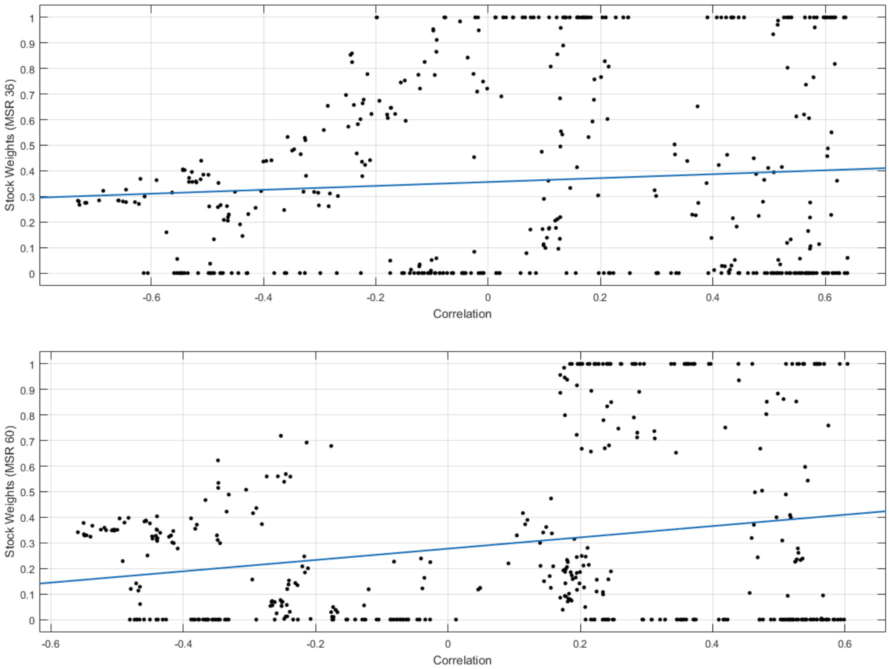

Figure 12: Scatter plot for Stock Weights (MSR) with correlation ($Model$ 1)

The regression results for the $Model$ 2 are reported in Table 8. The negative relationship between the weight of stock in minimum variance strategy and stock-bond correlation is significant, reinforce the results obtained from linear models. Similar to the results obtained in linear model the MSR portfolio do not imply any relationship with the correlation.

Portfolios	β_0	β_1	β_2	**Adjusted** R^2
MV 36	0.2132*	-0.1347*	0.0089 *	0.0618
MV 60	0.11846*	-0.1759*	0.07152*	0.2782
MSR 36	0.4530*	0.1011*	-0.6043*	-
MSR 60	0.3069*	0.2410*	-0.2003*	-
* - Significant at 95% confidence level				

Table 8: Parameter estimates for $Model$ 2

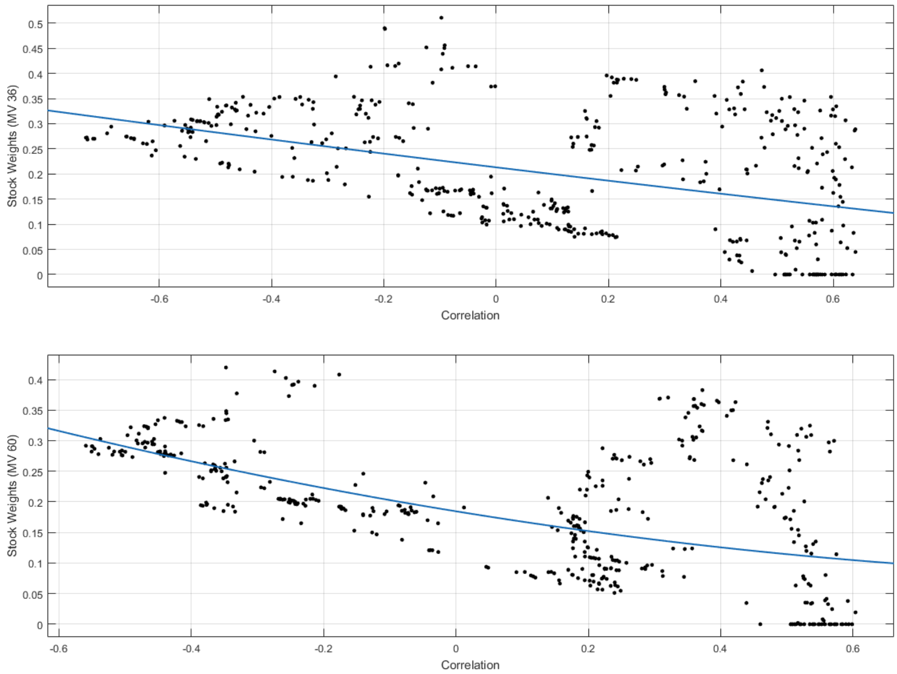

Figure 13: Scatter plot for Stock Weights (MV) with correlation ($Model$ 2)

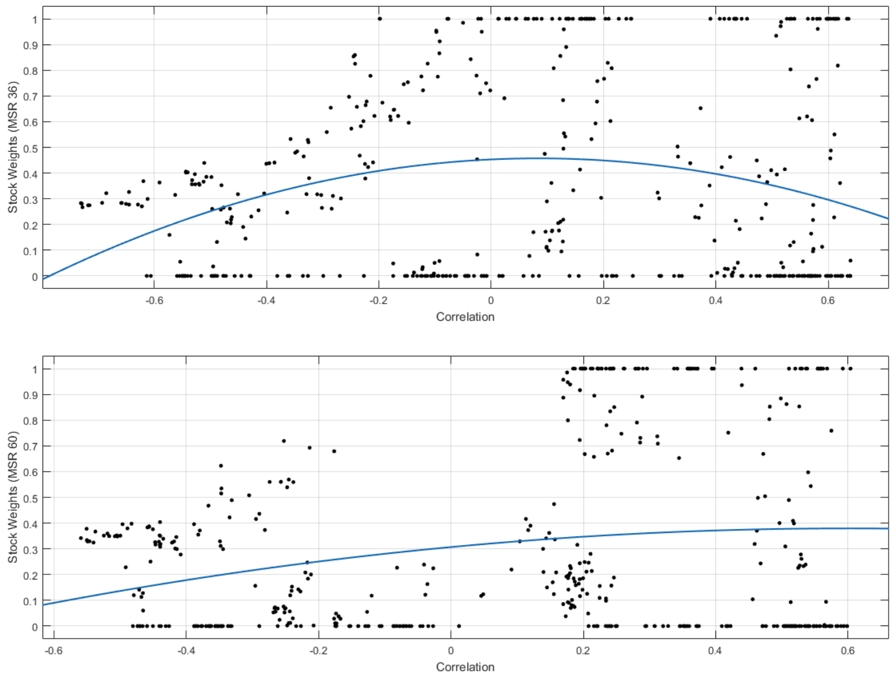

Figure 14: Scatter plot for Stock Weights (MSR) with correlation ($Model$ 2)

4.5 Implication of correlation on Portfolio Performance

After studying the effect of time varying correlation on portfolio diversification, in this section we focus our attention on performance of portfolios. We use Sharpe ratio as the measure of performance of the portfolios. The models under the study are as follows

$Model\ 3 : SR_t = \beta_0 + \beta_1 \widehat{\rho_t}$

$Model\ 4 : SR_t = \beta_0 + \beta_1 \widehat{\rho_t} + \beta_2 \widehat{\rho_t}^2$

where SR_t is the Sharpe ratio of the portfolio at time t and $\widehat{\rho_t}$ is the estimated correlation.

The regression results for $Model$ 3 are reported in Table 9. A significant positive relationship is observed between the Sharpe Ratio for MV portfolios and the correlation. Analogous to results obtained in previous section we do not observe any significant relationship for MSR portfolios. The Sharpe Ratio of EW portfolios also exhibit a positive relationship with the correlation but with lower values of adjusted R^2. The relationship are also presented in a scatter plot in Figure 15, Figure 16, and Figure 17 for MV, MSR and EW portfolios respectively.

Portfolios	β_0	β_1	**Adjusted** R^2
MV 36	0.1568*	0.0169*	0.2402
MSR 36	0.1353*	0.0778*	-
EW 36	0.1365*	0.06246	0.0017
MV 60	0.1431*	0.05028*	0.3089
MSR 60	0.0828*	0.0852*	-
EW 60	0.1297*	0.0887*	0.07854
* - Significant at 95% confidence level			

Table 9: Parameter estimates for $Model$ 3

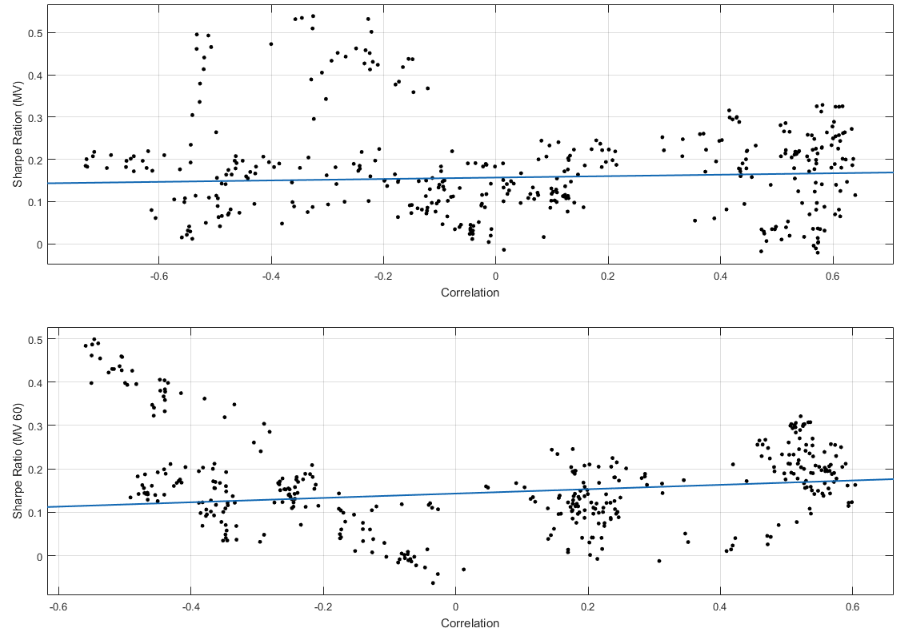

Figure 15: Scatter plot for Sharpe Ratio (MV) with correlation ($Model$ 3)

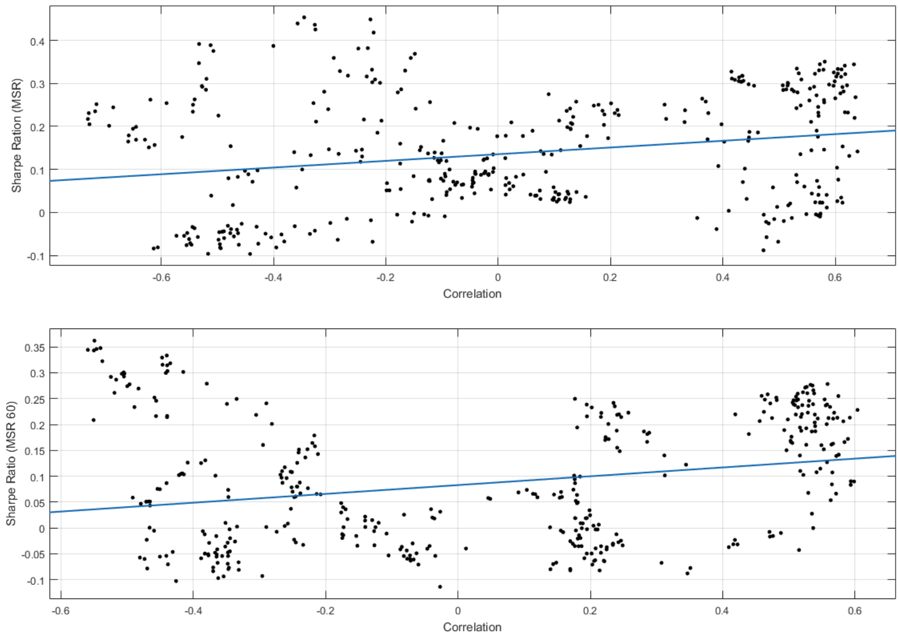

Figure 16: Scatter plot for Sharpe Ratio (MSR) with correlation ($Model$ 3)

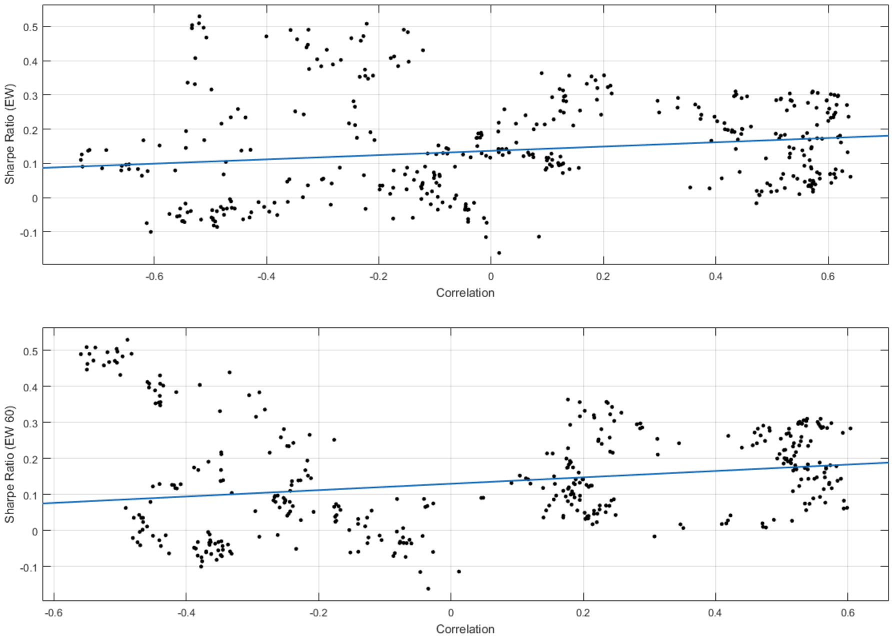

Figure 17: Scatter plot for Sharpe Ratio (EW) with correlation (*Model* 3)

The regression results for *Model* 4 are reported in Table 10. There are conflicting results for MV portfolio for 36 month and 60 month estimation window. Sharpe ratio of MV portfolios display a positive relationship with the correlation. For MSR 36 portfolio the model is insignificant, whereas for MSR 60 model is significant with high adjusted R^2 value. Similarly Sharpe ratio of EW 36 have a very weak relationship with correlation whereas EW 60 display a more stronger relationship.

Portfolios	β_0	β_1	β_2	**Adjusted** R^2
MV 36	0.1371*	0.0201*	0.1050*	0.3254
MSR 36	0.1190*	0.0731*	0.1024*	-
EW 36	0.1493*	0.0674*	-0.0769*	0.0049
MV 60	0.0814*	-0.0801*	0.6152*	0.3142
MSR 60	-0.0001*	0.01*	0.6461*	0.2092
EW 60	0.0750*	0.0146*	0.4679*	0.0353
* - Significant at 95% confidence level				

Table 10: Parameter Estimates for *Model* 4

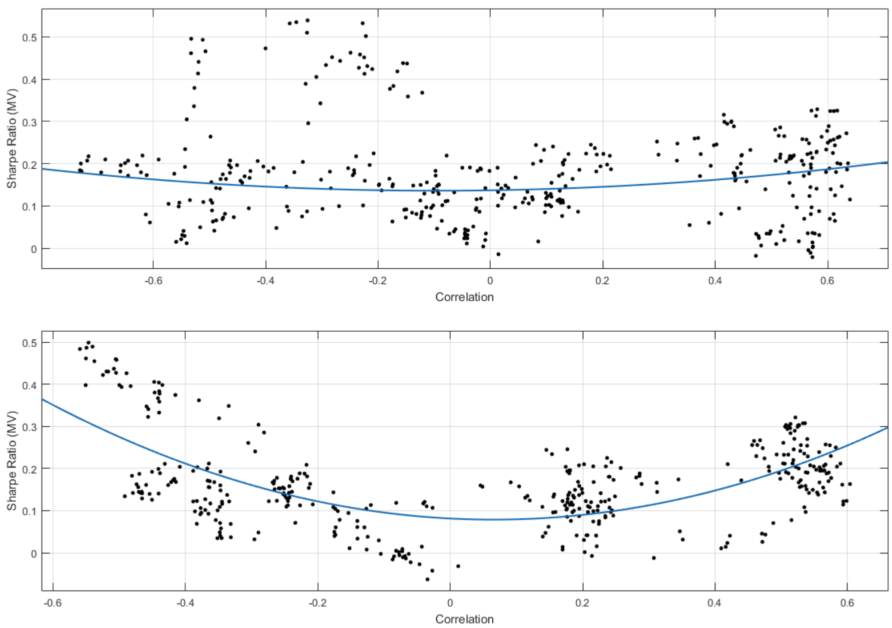

Figure 18: Scatter plot for Sharpe Ratio (MV) with correlation (*Model* 4)

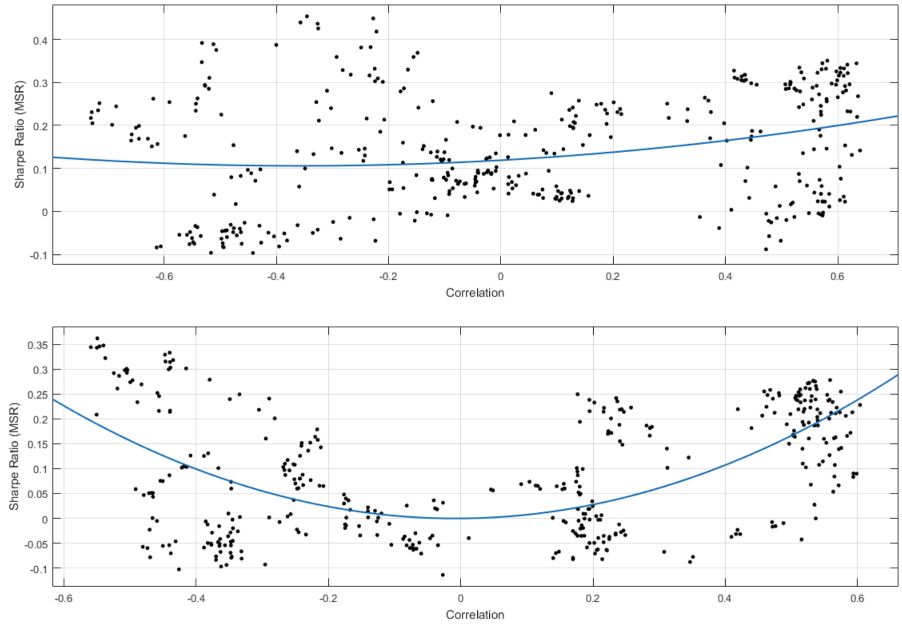

Figure 19: Scatter plot for Sharpe Ratio (MSR) with correlation ($Model$ 4)

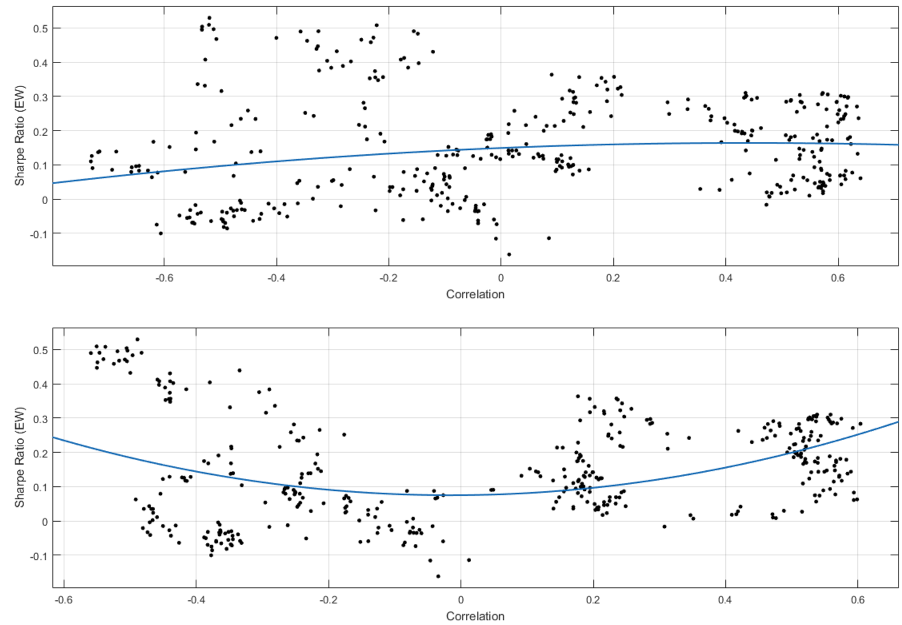

Figure 20: Scatter plot for Sharpe Ratio (EW) with correlation ($Model$ 4)

4.6 Impulse Response

In this section we report the impulse responses of benefits from diversification to shocks in correlation. Estimation of lags and the coefficients for all the VAR models are enumerated in Appendix 1.

Figure 21 shows the response of Sharpe ratios of MV36, MSR36 and EW36 portfolios to a positive shock in correlation. For MV36 portfolio is can be observed that as the correlation increases there is a fall in the Sharpe ratio and it takes about 50 months to recover to its original level. This observation is in-line with the theoretical construct that the diversification benefit decreases with increase in correlation. Similar results can be observed for EW36 portfolios with a recovery time of 25 months. However, MSR36 portfolio entail a positive response to the shocks in correlation.

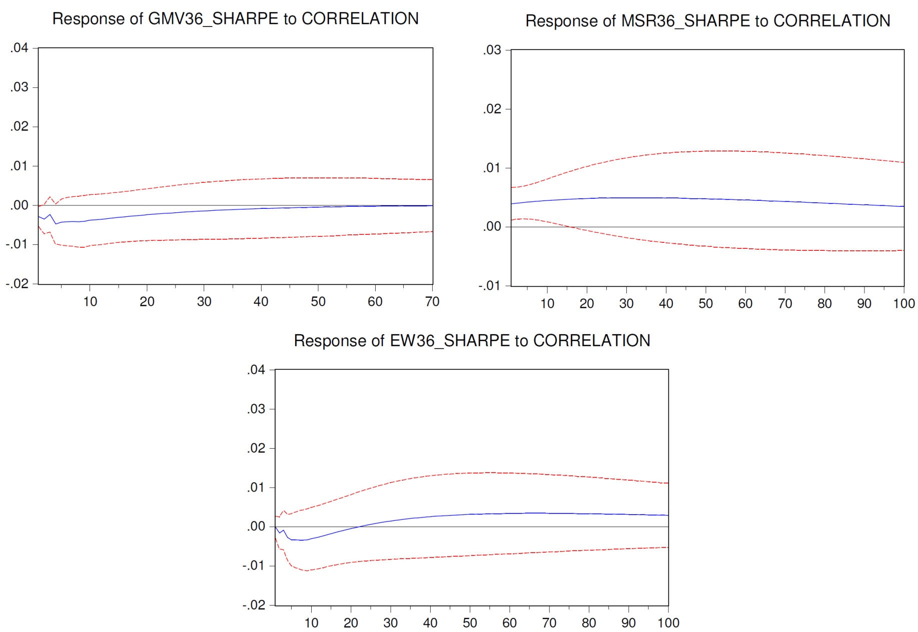

Figure 21: Response of Sharpe ratio to shocks in correlation

Figure 22 shows the response of Sharpe ratios of MV60, MSR60 and EW60 portfolios to a positive shock in correlation. The nature of response to Sharpe ratios of all portfolios is similar to the one obtained earlier.

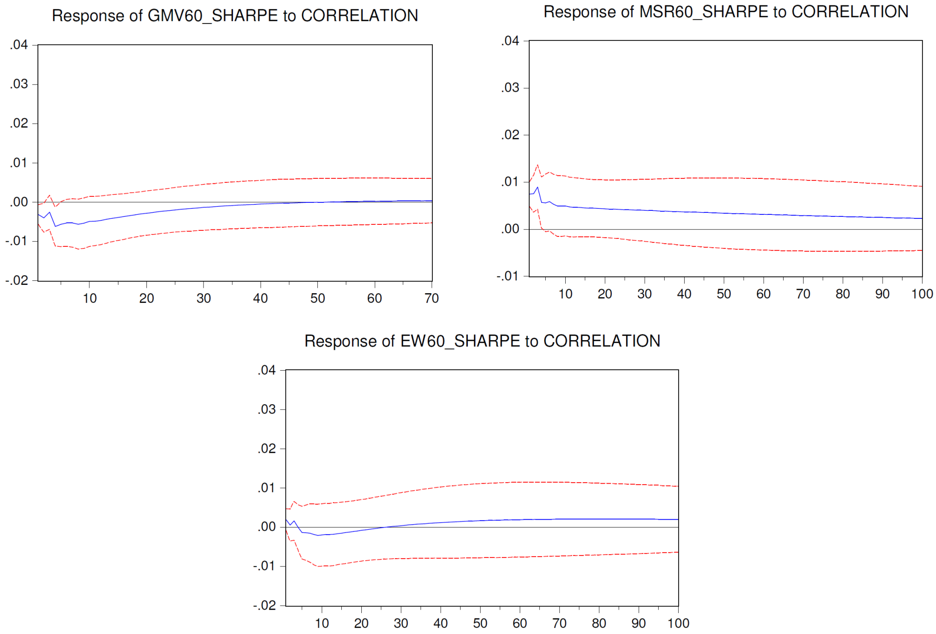

Figure 22: Response of Sharpe ratio to shocks in correlation

However, if we compare the response to correlation shocks with the average values of Sharpe ratio as reported in Table 4.3, the responses seems to be very insignificant. We can infer from our findings that there is no significant response of diversification benefit to shocks in correlation.

We now examine whether the past correlation can be used to forecast the diversification benefits of a portfolio. We apply Granger causality test with a null hypothesis that lagged value of stock-bond correlation does not affect the value of Sharpe ratio of the portfolio We report the results of this test in Table 11. from the results we can observe that the null hypothesis can be rejected only for MV60 portfolio at 5% significance and for MSR60 portfolio at 10% significance. We do not observe any significant Granger causality in all other portfolios. Due to conflicting results from our empirical results we can say that past correlation between the assets cannot be used to forecast the future portfolio diversification benefits.

Portfolios	chi - square	p - value
MV36	5.1253	0.4008
MV60	11.6254	0.0403*
MSR36	0.9135	0.3392
MSR60	10.5231	0.0617**
EW36	4.1281	0.5311
EW60	5.1729	0.3951
* - significant at 5%		
** - significant at 10%		

Table 11: VAR Granger Casulaity

4.7 Other measures of portfolio performance

In this section we take a look at some other portfolio performance measures which would lead to a deeper insight to examine the effect of correlation on performance of portfolios.We calculate benchmark relative performance measures of tracking error and information ratio, the results are reported in Table 12. Similar to the strategy adopted by **?** we have considered the naive equally weighted portfolio as our benchmark for calculation of these performance measures. The results indicate similar performance for 36 month and 60 month estimation periods. Also, the MV portfolio performs better than the MSR portfolio as it has a lower tracking error. The information ratio is negative for all the portfolio which imply that the benchmark (EW) outperform all other portfolios.

We then calculate the Maximum Draw-down of all the portfolios, the values of which are in Table 13. According to this measures the MV portfolios outperform the MSR and EW in both the time periods of estimation. This implies that in case of a period of crisis the MV portfolio would have less downside risk than the MSR and EW portfolios.

Portfolios	**Tracking Error**	**Information Ratio**
MV 36	0.0156	-0.0145
MSR 36	0.0204	-0.0274
MV 60	0.0158	-0.0204
MSR 60	0.0205	-0.0629

Table 12: Tracking Error and Information Ratio with EW portfolios as benchmark

Portfolios	**Max Draw Down**
MV 36	0.1044
MSR 36	0.2653
EW 36	0.2557
MV 60	0.1257
MSR 60	0.2457
EW 60	0.2557

Table 13: Max Draw Down of portfolios

5 Conclusion

The first objective of this study was to examine whether their is a time variation within the stock-bond correlation. We estimate the correlation using the rolling window method for estimation window of 36 months and 60 months, and using conditional correlation.The empirical findings suggest an existence of time variation between the stock-bond correlation. The finding is reinforced by the fact that similar patters of time variation are obtained for 36 month,60 month estimation window,and conditional correlations.

The second objective of the study was to construct the portfolios using mean variance framework and a naive equally weighted strategy.We report the out of sample performance of these portfolios. We find that the portfolio constructed using the minimum variance strategy outperform, in terms of risk adjusted returns, the other portfolios in both 36 month and 60 month estimation periods. The equally weighted strategy outperforms the portfolios constructed using the maximum Sharpe ratio strategy.

The third objective of the study was to establish the implications of time variation in stock-bond correlation on portfolio diversification & portfolio performance. The weight of stocks is regarded as a measure of portfolio diversification for this study. The results reported indicate that there is a negative linear relationship between the weight of stocks in minimum variance strategy portfolios and the stock-bond correlation. As the weight of stock increases in a portfolio the diversification characteristic decrease, hence from the results we can imply that as the correlation increases the diversification characteristic of the portfolio decrease. A quadratic relationship is also significant for minimum variance strategy portfolios. We do not observe any significant relationship for maximum Sharpe ratio strategy portfolios.

Sharpe ratio of the portfolio is considered as a measure of portfolio performance to examine the relationship with correlation. We find significant quadratic relation for minimum variance and equally weighted portfolios, whereas we observe conflicting results for maximum Sharpe ratio portfolios of 36 months and 60 months estimation window. From the findings we can imply that the Sharpe ratio has a minimum value near zero correlation whereas Sharpe ratio increase for higher values of positive as well as negative correlation.

Further, we study the impulse response of diversification benefits to shocks in correlation using VAR model. We observe that positive shocks in correlation affect the diversification benefit in negative way, but these responses are not found to be significant. This observation is further strengthened by using Granger causality test.

We extend the study by reporting portfolio performance measures of max draw-down, tracking error, and information ratio. We find that the minimum variance portfolio would limit the downside in times of market fall in comparison to maximum Sharpe ratio and equally weighted portfolios. Whereas over the entire period the naive strategy of equally weighted assets outperform the minimum variance and maximum Sharpe ratio strategies.

The findings of this thesis can be used by the individuals who use mean variance framework for portfolio construction. Eventually, it would propel the research on constructing a tactical asset allocation strategy which takes advantage of the time variation within stock-bond correlation to generate excess returns over the market.

Finally, a limitation of this study is that, it uses basic asset classes like stock index and bond for determining the time variation in correlation. Further, the sample data used ranges for a period of 37 years, a more wide sample could be used to substantiate the findings. A findings can be reinforced by replicating the study for financial markets from Europe and Asia.

Appendix

A1: Lag estimate and VAR Model estimates for MV36, MSR36 and EW36 portfolios

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION GMV36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:05
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	98.83090	NA	0.002043	-0.517812	-0.496827	-0.509480
1	1399.728	2580.924	1.99e-06	-7.453091	-7.390135*	-7.428095*
2	1401.321	3.142733	2.01e-06	-7.440217	-7.335291	-7.398557
3	1406.797	10.74749	2.00e-06	-7.448112	-7.301215	-7.389787
4	1412.672	11.46733	1.98e-06	-7.458139	-7.269271	-7.383149
5	1419.132	12.54042*	1.95e-06*	-7.471295*	-7.240456	-7.379641
6	1421.856	5.257381	1.96e-06	-7.464468	-7.191659	-7.356150
7	1424.353	4.794997	1.98e-06	-7.456434	-7.141654	-7.331452
8	1426.304	3.723988	2.00e-06	-7.445475	-7.088725	-7.303829
9	1427.031	1.381012	2.04e-06	-7.427975	-7.029254	-7.269664
10	1430.222	6.023292	2.05e-06	-7.423648	-6.982956	-7.248673

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION MSR36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:07
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	54.79638	NA	0.002585	-0.282334	-0.261348	-0.274001
1	1375.926	2621.064*	2.26e-06*	-7.325806*	-7.262850*	-7.300810*
2	1376.587	1.305248	2.30e-06	-7.307953	-7.203027	-7.266293
3	1380.124	6.941418	2.30e-06	-7.305477	-7.158580	-7.247152
4	1384.011	7.585869	2.30e-06	-7.304870	-7.116002	-7.229880
5	1385.699	3.276963	2.33e-06	-7.292507	-7.061668	-7.200853
6	1387.881	4.212207	2.36e-06	-7.282785	-7.009975	-7.174467
7	1388.187	0.588774	2.40e-06	-7.263034	-6.948254	-7.138052
8	1391.788	6.874773	2.41e-06	-7.260901	-6.904150	-7.119255
9	1393.792	3.803711	2.43e-06	-7.250225	-6.851504	-7.091915
10	1396.513	5.135665	2.45e-06	-7.243384	-6.802692	-7.068409

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION EW36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:12
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	3.946620	NA	0.003393	-0.010410	0.010576	-0.002078
1	1364.123	2698.532	2.40e-06	-7.262692	-7.199736*	-7.237695*
2	1366.973	5.622223	2.42e-06	-7.256538	-7.151611	-7.214877
3	1370.094	6.125227	2.43e-06	-7.251837	-7.104940	-7.193512
4	1373.706	7.051195	2.44e-06	-7.249765	-7.060897	-7.174776
5	1382.064	16.22345*	2.38e-06*	-7.273067*	-7.042229	-7.181414
6	1385.908	7.421259	2.38e-06	-7.272235	-6.999425	-7.163917
7	1388.251	4.497383	2.40e-06	-7.263372	-6.948592	-7.138390
8	1390.375	4.055666	2.43e-06	-7.253342	-6.896591	-7.111695
9	1390.975	1.139085	2.47e-06	-7.235160	-6.836439	-7.076849
10	1395.829	9.162609	2.46e-06	-7.239726	-6.799034	-7.064751

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

A2: Lag estimate and VAR Model estimates for MV60, MSR60 and EW60 portfolios

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION GMV36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:05
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	98.83090	NA	0.002043	-0.517812	-0.496827	-0.509480
1	1399.728	2580.924	1.99e-06	-7.453091	-7.390135*	-7.428095*
2	1401.321	3.142733	2.01e-06	-7.440217	-7.335291	-7.398557
3	1406.797	10.74749	2.00e-06	-7.448112	-7.301215	-7.389787
4	1412.672	11.46733	1.98e-06	-7.458139	-7.269271	-7.383149
5	1419.132	12.54042*	1.95e-06*	-7.471295*	-7.240456	-7.379641
6	1421.856	5.257381	1.96e-06	-7.464468	-7.191659	-7.356150
7	1424.353	4.794997	1.98e-06	-7.456434	-7.141654	-7.331452
8	1426.304	3.723988	2.00e-06	-7.445475	-7.088725	-7.303829
9	1427.031	1.381012	2.04e-06	-7.427975	-7.029254	-7.269664
10	1430.222	6.023292	2.05e-06	-7.423648	-6.982956	-7.248673

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION MSR36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:07
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	54.79638	NA	0.002585	-0.282334	-0.261348	-0.274001
1	1375.926	2621.064*	2.26e-06*	-7.325806*	-7.262850*	-7.300810*
2	1376.587	1.305248	2.30e-06	-7.307953	-7.203027	-7.266293
3	1380.124	6.941418	2.30e-06	-7.305477	-7.158580	-7.247152
4	1384.011	7.585869	2.30e-06	-7.304870	-7.116002	-7.229880
5	1385.699	3.276963	2.33e-06	-7.292507	-7.061668	-7.200853
6	1387.881	4.212207	2.36e-06	-7.282785	-7.009975	-7.174467
7	1388.187	0.588774	2.40e-06	-7.263034	-6.948254	-7.138052
8	1391.788	6.874773	2.41e-06	-7.260901	-6.904150	-7.119255
9	1393.792	3.803711	2.43e-06	-7.250225	-6.851504	-7.091915
10	1396.513	5.135665	2.45e-06	-7.243384	-6.802692	-7.068409

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Order Selection Criteria
Endogenous variables: CORRELATION EW36_SHARPE
Exogenous variables: C
Date: 06/07/17 Time: 10:12
Sample: 1 384
Included observations: 374

Lag	LogL	LR	FPE	AIC	SC	HQ
0	3.946620	NA	0.003393	-0.010410	0.010576	-0.002078
1	1364.123	2698.532	2.40e-06	-7.262692	-7.199736*	-7.237695*
2	1366.973	5.622223	2.42e-06	-7.256538	-7.151611	-7.214877
3	1370.094	6.125227	2.43e-06	-7.251837	-7.104940	-7.193512
4	1373.706	7.051195	2.44e-06	-7.249765	-7.060897	-7.174776
5	1382.064	16.22345*	2.38e-06*	-7.273067*	-7.042229	-7.181414
6	1385.908	7.421259	2.38e-06	-7.272235	-6.999425	-7.163917
7	1388.251	4.497383	2.40e-06	-7.263372	-6.948592	-7.138390
8	1390.375	4.055666	2.43e-06	-7.253342	-6.896591	-7.111695
9	1390.975	1.139085	2.47e-06	-7.235160	-6.836439	-7.076849
10	1395.829	9.162609	2.46e-06	-7.239726	-6.799034	-7.064751

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

B1: VAR model estimates for MV36 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:04
Sample (adjusted): 6 384
Included observations: 379 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	GMV36_SHA
CORRELATION(-1)	0.956609	-0.008212
	(0.05229)	(0.02384)
	[18.2930]	[-0.34448]
CORRELATION(-2)	0.162783	0.024076
	(0.07179)	(0.03273)
	[2.26745]	[0.73567]
CORRELATION(-3)	-0.049112	-0.059823
	(0.07225)	(0.03294)
	[-0.67976]	[-1.81636]
CORRELATION(-4)	-0.131905	0.061471
	(0.07217)	(0.03290)
	[-1.82771]	[1.86843]
CORRELATION(-5)	0.051249	-0.017579
	(0.05249)	(0.02393)
	[0.97636]	[-0.73464]
GMV36_SHARPE(-1)	-0.037423	1.064471
	(0.11400)	(0.05197)
	[-0.32827]	[20.4832]
GMV36_SHARPE(-2)	0.112060	-0.186769
	(0.16618)	(0.07576)
	[0.67431]	[-2.46537]
GMV36_SHARPE(-3)	0.117596	0.162226
	(0.16539)	(0.07539)
	[0.71103]	[2.15171]
GMV36_SHARPE(-4)	-0.382527	0.085103
	(0.16488)	(0.07517)
	[-2.31996]	[1.13221]
GMV36_SHARPE(-5)	0.205188	-0.157642
	(0.11445)	(0.05217)
	[1.79285]	[-3.02152]
C	-0.004064	0.006219
	(0.00533)	(0.00243)
	[-0.76228]	[2.55891]
R-squared	0.982281	0.952547
Adj. R-squared	0.981799	0.951258
Sum sq. resids	1.095386	0.227635
S.E. equation	0.054558	0.024871
F-statistic	2040.035	738.7102
Log likelihood	570.1207	867.8478
Akaike AIC	-2.950505	-4.521624
Schwarz SC	-2.836223	-4.407342
Mean dependent	0.031554	0.177228
S.D. dependent	0.404403	0.112653
Determinant resid covariance (dof adj.)		1.82E-06
Determinant resid covariance		1.71E-06
Log likelihood		1440.484
Akaike information criterion		-7.485406
Schwarz criterion		-7.256842

B2: VAR model estimates for MSR36 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:07
Sample (adjusted): 2 384
Included observations: 383 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	MSR36_SHA
CORRELATION(-1)	0.991038	0.003356
	(0.00708)	(0.00351)
	[139.983]	[0.95575]
MSR36_SHARPE(-1)	-0.009299	0.972744
	(0.02176)	(0.01079)
	[-0.42735]	[90.1416]
C	-0.000431	0.004476
	(0.00411)	(0.00204)
	[-0.10485]	[2.19734]
R-squared	0.981801	0.957453
Adj. R-squared	0.981705	0.957229
Sum sq. resids	1.140003	0.280362
S.E. equation	0.054772	0.027162
F-statistic	10250.18	4275.639
Log likelihood	570.5028	839.1210
Akaike AIC	-2.963461	-4.366167
Schwarz SC	-2.932537	-4.335243
Mean dependent	0.036313	0.141075
S.D. dependent	0.404947	0.131339
Determinant resid covariance (dof adj.)		2.17E-06
Determinant resid covariance		2.13E-06
Log likelihood		1413.704
Akaike information criterion		-7.350936
Schwarz criterion		-7.289087

B3: VAR model estimates for EW36 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:13
Sample (adjusted): 6 384
Included observations: 379 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	EW36_SHA
CORRELATION(-1)	0.956148	-0.026716
	(0.05169)	(0.02582)
	[18.4980]	[-1.03460]
CORRELATION(-2)	0.156978	0.040967
	(0.07130)	(0.03562)
	[2.20167]	[1.15013]
CORRELATION(-3)	-0.049858	-0.047424
	(0.07186)	(0.03590)
	[-0.69381]	[-1.32099]
CORRELATION(-4)	-0.108677	0.025142
	(0.07164)	(0.03579)
	[-1.51703]	[0.70252]
CORRELATION(-5)	0.034543	0.010169
	(0.05192)	(0.02594)
	[0.66531]	[0.39207]
EW36_SHARPE(-1)	0.078408	1.069476
	(0.10292)	(0.05142)
	[0.76180]	[20.7993]
EW36_SHARPE(-2)	-0.063124	-0.139608
	(0.15167)	(0.07577)
	[-0.41620]	[-1.84255]
EW36_SHARPE(-3)	0.029120	0.127646
	(0.15258)	(0.07623)
	[0.19085]	[1.67458]
EW36_SHARPE(-4)	-0.295971	0.072936
	(0.15227)	(0.07607)
	[-1.94372]	[0.95879]
EW36_SHARPE(-5)	0.254398	-0.154244
	(0.10338)	(0.05165)
	[2.46083]	[-2.98656]
C	-0.001736	0.003983
	(0.00404)	(0.00202)
	[-0.42951]	[1.97247]
R-squared	0.982260	0.965787
Adj. R-squared	0.981778	0.964857
Sum sq. resids	1.096654	0.273702
S.E. equation	0.054590	0.027272
F-statistic	2037.634	1038.818
Log likelihood	569.9015	832.9235
Akaike AIC	-2.949348	-4.337327
Schwarz SC	-2.835066	-4.223045
Mean dependent	0.031554	0.149181
S.D. dependent	0.404403	0.145478
Determinant resid covariance (dof adj.)		2.22E-06
Determinant resid covariance		2.09E-06
Log likelihood		1402.827
Akaike information criterion		-7.286687
Schwarz criterion		-7.058123

B4: VAR model estimates for MV60 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:16
Sample (adjusted): 6 384
Included observations: 379 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	GMV60_SHA
CORRELATION(-1)	0.934747	-0.015548
	(0.05202)	(0.03274)
	[17.9694]	[-0.47491]
CORRELATION(-2)	0.154563	0.041490
	(0.07062)	(0.04445)
	[2.18856]	[0.93347]
CORRELATION(-3)	0.010568	-0.120907
	(0.07105)	(0.04472)
	[0.14873]	[-2.70390]
CORRELATION(-4)	-0.117152	0.128117
	(0.07135)	(0.04490)
	[-1.64197]	[2.85316]
CORRELATION(-5)	0.011083	-0.032821
	(0.05246)	(0.03302)
	[0.21126]	[-0.99406]
GMV60_SHARPE(-1)	0.026246	1.076935
	(0.08249)	(0.05192)
	[0.31816]	[20.7439]
GMV60_SHARPE(-2)	0.121504	-0.220658
	(0.12098)	(0.07614)
	[1.00436]	[-2.89815]
GMV60_SHARPE(-3)	-0.062274	0.173900
	(0.12004)	(0.07555)
	[-0.51877]	[2.30183]
GMV60_SHARPE(-4)	-0.347784	0.111710
	(0.11897)	(0.07487)
	[-2.92332]	[1.49199]
GMV60_SHARPE(-5)	0.267409	-0.178471
	(0.08202)	(0.05162)
	[3.26039]	[-3.45752]
C	-0.002121	0.006357
	(0.00380)	(0.00239)
	[-0.55828]	[2.65910]
R-squared	0.989458	0.948273
Adj. R-squared	0.989171	0.946867
Sum sq. resids	0.548383	0.217208
S.E. equation	0.038603	0.024295
F-statistic	3453.862	674.6209
Log likelihood	701.2336	876.7332
Akaike AIC	-3.642394	-4.568513
Schwarz SC	-3.528112	-4.454231
Mean dependent	0.042119	0.166353
S.D. dependent	0.370959	0.105398
Determinant resid covariance (dof adj.)		8.65E-07
Determinant resid covariance		8.15E-07
Log likelihood		1581.151
Akaike information criterion		-8.227707
Schwarz criterion		-7.999143

B5: VAR model estimates for MSR60 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:17
Sample (adjusted): 6 384
Included observations: 379 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	MSR60_SHA
CORRELATION(-1)	0.934444	-0.009141
	(0.05360)	(0.03658)
	[17.4325]	[-0.24990]
CORRELATION(-2)	0.113560	0.069201
	(0.07337)	(0.05007)
	[1.54787]	[1.38222]
CORRELATION(-3)	0.030860	-0.154589
	(0.07303)	(0.04984)
	[0.42257]	[-3.10195]
CORRELATION(-4)	-0.031055	0.086928
	(0.07369)	(0.05029)
	[-0.42140]	[1.72856]
CORRELATION(-5)	-0.053622	0.009594
	(0.05438)	(0.03711)
	[-0.98602]	[0.25851]
MSR60_SHARPE(-1)	-0.015790	1.058531
	(0.07925)	(0.05408)
	[-0.19924]	[19.5737]
MSR60_SHARPE(-2)	0.108088	-0.182655
	(0.11512)	(0.07856)
	[0.93888]	[-2.32497]
MSR60_SHARPE(-3)	-0.077052	0.188308
	(0.11376)	(0.07763)
	[-0.67731]	[2.42565]
MSR60_SHARPE(-4)	-0.241365	-0.037461
	(0.11390)	(0.07773)
	[-2.11904]	[-0.48195]
MSR60_SHARPE(-5)	0.217073	-0.055357
	(0.07929)	(0.05411)
	[2.73779]	[-1.02311]
C	-0.000430	0.002982
	(0.00252)	(0.00172)
	[-0.17059]	[1.73501]
R-squared	0.989310	0.953852
Adj. R-squared	0.989020	0.952598
Sum sq. resids	0.556053	0.258945
S.E. equation	0.038872	0.026526
F-statistic	3405.711	760.6346
Log likelihood	698.6014	843.4266
Akaike AIC	-3.628504	-4.392753
Schwarz SC	-3.514221	-4.278471
Mean dependent	0.042119	0.092809
S.D. dependent	0.370959	0.121838

Determinant resid covariance (dof adj.)	9.79E-07
Determinant resid covariance	9.23E-07
Log likelihood	1557.649
Akaike information criterion	-8.103690
Schwarz criterion	-7.875126

B6: VAR model estimates for EW60 portfolio

Vector Autoregression Estimates
Date: 06/07/17 Time: 10:19
Sample (adjusted): 6 384
Included observations: 379 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	CORRELATI	EW60_SHA
CORRELATION(-1)	0.930525	-0.039635
	(0.05115)	(0.03615)
	[18.1937]	[-1.09632]
CORRELATION(-2)	0.136810	0.071261
	(0.06983)	(0.04936)
	[1.95923]	[1.44374]
CORRELATION(-3)	0.019914	-0.076949
	(0.07001)	(0.04949)
	[0.28445]	[-1.55495]
CORRELATION(-4)	-0.065154	0.008720
	(0.06988)	(0.04940)
	[-0.93237]	[0.17654]
CORRELATION(-5)	-0.028466	0.037873
	(0.05142)	(0.03635)
	[-0.55362]	[1.04202]
EW60_SHARPE(-1)	0.029863	1.074764
	(0.07282)	(0.05147)
	[0.41009]	[20.8803]
EW60_SHARPE(-2)	0.067853	-0.145089
	(0.10777)	(0.07618)
	[0.62961]	[-1.90458]
EW60_SHARPE(-3)	-0.057736	0.135520
	(0.10850)	(0.07669)
	[-0.53212]	[1.76701]
EW60_SHARPE(-4)	-0.324476	0.062294
	(0.10830)	(0.07656)
	[-2.99597]	[0.81371]
EW60_SHARPE(-5)	0.285260	-0.151084
	(0.07346)	(0.05192)
	[3.88333]	[-2.90971]
C	-0.001327	0.003902
	(0.00285)	(0.00202)
	[-0.46551]	[1.93606]
R-squared	0.989498	0.965883
Adj. R-squared	0.989213	0.964956
Sum sq. resids	0.546256	0.272936
S.E. equation	0.038528	0.027234
F-statistic	3467.452	1041.838
Log likelihood	701.9700	833.4548
Akaike AIC	-3.646280	-4.340131
Schwarz SC	-3.531998	-4.225849
Mean dependent	0.042119	0.149181
S.D. dependent	0.370959	0.145478

Determinant resid covariance (dof adj.)	1.10E-06
Determinant resid covariance	1.03E-06
Log likelihood	1536.386
Akaike information criterion	-7.991482
Schwarz criterion	-7.762918