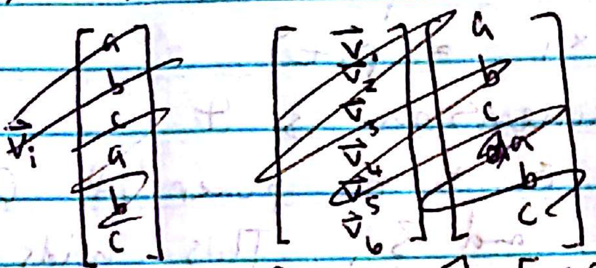
1. a) It can recover from only one erasure. It can't handle patterns where both a's are lost or both b's, etc.

b)



$$\begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \alpha_3 & \beta_3 & \gamma_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \alpha_4 & \beta_4 & \gamma_4 \\ \alpha_5 & \beta_5 & \gamma_5 & \alpha_5 & \beta_5 & \gamma_5 \\ \alpha_6 & \beta_6 & \gamma_6 & \alpha_6 & \beta_6 & \gamma_6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ a \\ b \\ c \end{bmatrix}$$

c) [1, 0, 0], [0 1 0], [0 0 1], [1 0 0], [0 1 0], [0 0 1]

d) When the letters in the place of the ones at that time i is present. For example, you wouldn't be able to figure out "a" if "a" was missing at $i=1$ and "a" and "b" was missing at $i=4$ and etc.

e)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ ? \\ ? \\ 2 \\ 3 \\ ? \\ ? \end{bmatrix}$$

$a = 6$
$b = -4$
$c = -3$

f) She should choose the strategy in part a because at this point he knows neither a, b, nor c, and it's hard to solve for combinations of summations of a, b, c if ~~he~~ he doesn't know any of them.

2. a) $p(t)$ is closed under both addition and multiplication

$p(t) + 0 = p(t)$

$p(t) \times 1 = p(t)$

the dimension is 4

one is to the power of 0, and 1, and 2, and 3. This adds up to 4 different dimensions

b) $p(t)$ is literally a linear combination of the canonical polynomials.

$$[c_0 \; c_1 \; c_2 \; c_3] \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$$

$= c_0 + c_1 t + c_2 t^2 + c_3 t^3$

Does this look familiar?

It's the exact same as $p(t)$ but with c's instead of p's

c) the c vector consists of the constants that we are multiplying the elements of $\varphi(t)$ by

d) True; $\varphi(t)$ is definitely linearly independent, as any combination of t^2 can't equal t and etc., and linear combinations of the elements of $\varphi(t)$ can form any ~~poly~~ cubic polynomial with real values

2 e) $(1-t)^3 = (t^2 - 2t + 1)(1-t) = t^2 - 2t + 1 - t^3 + 2t^2 - t$

$= -t^3 + 3t^2 - 3t + 1$

$3t(1-t)^2 = 3t^3 - 6t^2 + 3t$

$3t^2(1-t) = 3t^2 - 3t^3$

$$\vec{B}(t) = \begin{bmatrix} -t^3 + 3t^2 - 3t + 1 \\ 3t^3 - 6t^2 + 3t \\ -3t^3 + 3t^2 \\ t^3 \end{bmatrix}$$

calculator

$$R = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

because there is a pivot in each row, R is linearly independent and invertible

$$R^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad R^{-1}\vec{B}(t) = \vec{Q}(t)$$

$\cancel{p_0}\,B_0(t) + \frac{4}{3}\cancel{p_1}\,B_1(t) + 2\cancel{p_2}\,B_2(t) + 4\cancel{p_3}\,B_3(t)$

$= \cancel{p_0}t + \cancel{p_1}t + \cancel{p_2}t^2 + \cancel{p_3}t^3$

~~3~~

3. a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

c) no; $x_2[1]$ always equals zero so that gives us no information, and $x_1[1]$ is just $x_1[0] + x_2[0]$ and here we're trying to solve two variables with ~~the~~ one equation, which doesn't work.

d) It shows that ~~As~~ the form $A\vec{x} = \vec{b}$ doesn't have a unique solution and matrix A therefore isn't invertible. As a result, we can't do $A^{-1}\vec{b} = \vec{x}$ to find out the initial water levels

e) matrix A is therefore ~~linearly inde~~ invertible and we can recover the initial state, as when we put it in the form $A\vec{x} = \vec{b}$,

$$A^{-1}A\vec{x} = \cancel{A^{-1}\vec{b}}\ A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

and we can find $\vec{x}$, the initial state.

This means that the experiment is reproducible

4. a) $\vec{x}[1] = A\vec{x}[0] + \vec{b}u[0]$

b) $\vec{x}[2] = A(A\vec{x}[0] + \vec{b}u[0]) + \vec{b}u[1]$

$= A^2\vec{x}[0] + A\vec{b}u[0] + \vec{b}u[1]$

c) $\vec{x}[3] = A^3\vec{x}[0] + A^2\vec{b}u[0] + A\vec{b}u[1] + \vec{b}u[2]$

d) $\vec{x}[4] = A^4\vec{x}[0] + A^3\vec{b}u[0] + A^2\vec{b}u[1] + A\vec{b}u[2] + \vec{b}u[3]$

e) $\vec{x}[N] = A^N\vec{x}[0] + \vec{b}(A^{N-1}u[0] + A^{N-2}u[1] + \dots + A^0u[N-1])$

f) $\vec{x}[2] = A^2\vec{x}[0] + A\vec{b}u[0] + \vec{b}u[1]$

$\vec{0} - A^2\vec{x}[0] = A\vec{b}u[0] + \vec{b}u[1]$

$$\begin{bmatrix} A\vec{b} \\ \vec{b} \end{bmatrix} \begin{bmatrix} u[0] \\ u[1] \end{bmatrix} = -A^2\vec{x}[0]$$

no; the columns of the coefficent matrix aren't linearly independent

g) $\vec{x}[3] - A^3\vec{x}[0] = A^2\vec{b}u[0] + A\vec{b}u[1] + \vec{b}u[2]$

$$\begin{bmatrix} A^2\vec{b} \\ A\vec{b} \\ \vec{b} \end{bmatrix} \begin{bmatrix} u[0] \\ u[1] \\ u[2] \end{bmatrix} = -A^3\vec{x}[0]$$

no; there isn't a pivot in every column so the coefficient matrix can't be solved with a unique solution

h) $$\begin{bmatrix} A^3\vec{b} \\ A^2\vec{b} \\ A\vec{b} \\ \vec{b} \end{bmatrix} \begin{bmatrix} u[0] \\ u[1] \\ u[3] \\ u[4] \end{bmatrix} = -A^4\vec{x}[0]$$

yes; ~~it row red~~ the coefficient matrix now reduces ~~to~~ the identity matrix, so the columns are linearly independent and there is a unique ~~so~~ solution

j) $\vec{0} - A^N x[0]$ must be in the span of
$\{\vec{b}, A\vec{b}, A^2\vec{b}, \ldots, A^{N-1}\vec{b}\}$

k) $\vec{x}[N] - A^N x[0]$ needs to be in
span $\{\vec{b}, A\vec{b}, A^2\vec{b}, \ldots, A^{N-1}\vec{b}\}$

yes this would be cool!

5. By myself