1. a)

$$\begin{array}{ccc} 1 & 2 & 3 \\ \begin{bmatrix} 0 \\ b \\ c \end{bmatrix} & \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \qquad \begin{array}{ccc} 1 & 2 & 3 \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} b \\ a \\ 0 \end{bmatrix} & \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} \end{array}$$

b) the total number of the people in the system stays the same

c) $\begin{bmatrix} 0 & 0 & 0 \\ 0.5 & 0.4 & 0.2 \\ 0 & 0.6 & 0.65 \end{bmatrix}$ People are leaving the system

d) $$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_1x_1 + a_2x_2 + a_3x_3 \\ b_1x_1 + b_2x_2 + b_3x_3 \\ c_1x_1 + c_2x_2 + c_3x_3 \end{bmatrix}$$

$a_1 + a_2 + a_3 = 1$

$b_1 + b_2 + b_3 = 1$

$c_1 + c_2 + c_3 = 1$

$x_1 = x_2 = x_3$

$$= \begin{bmatrix} (a_1 + a_2 + a_3)x_1 \\ (b_1 + b_2 + b_3)x_2 \\ (c_1 + c_2 + c_3)x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2. if v_n is dependent, then

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$$

for some constants $a_1, \dots, a_n$

if we multiply v_n by A, we get

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n \end{bmatrix}$$

~~and since we know that~~

$$\cancel{c_1x_1 + c_2x_2 + \dots + c_nx_n = 0}$$

~~if we multiply each element~~

the linear combs of this vector can be written as

$$(a_{11} + \dots + a_{n1})x_1 + (a_{12} + \dots + a_{n2})x_2 + \dots (a_{1n} + \dots + a_{nn})x_n$$

and since we know that $c_1x_1 + c_2x_2 + \dots c_nx_n = 0$ from the original vector,

$$\frac{c_1(a_{11} + \dots + a_{n1})}{(a_{11} + \dots + a_{n1})}x_1 + \frac{c_2(a_{12} + \dots + a_{n2})}{(a_{12} + \dots + a_{n2})}x_2 + \dots \frac{c_n(a_{1n} + \dots + a_{nn})}{(a_{1n} + \dots + a_{nn})}x_n = 0$$

therefore, v_n multiplied by any matrix A will be linearly dependent

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3. a) $P_1 = \frac{1}{2}T_1 + \frac{1}{2}T_2$

$P_2 = \frac{1}{2}T_2 + \frac{1}{2}T_3$

$P_3 = \frac{1}{2}T_3 + \frac{1}{2}T_4$

$P_4 = \frac{1}{2}T_4 + \frac{1}{2}T_5$

$P_5 = \frac{1}{2}T_5 + \frac{1}{2}T_6$

$P_6 = \frac{1}{2}T_6 + \frac{1}{2}T_1$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix}$$

$$\sim \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

we have a row of zeros, so this system doesn't have a unique solution

$T_1 = 2$, $T_2 = 4$, $T_3 = 4$, $T_4 = 4$, $T_5 = 4$, $T_6 = 4$

$T_1 = 4$, $T_2 = 2$, $T_3 = 6$, $T_4 = 2$, $T_5 = 6$, $T_6 = 2$

b) $$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

we can, because when we row reduce the coefficient matrix we get an identity matrix, meaning that $A\vec{x} = \vec{b}$ has a unique solution $\vec{x}$ ~~and that~~ since A is linearly independent

c) when n is odd, you can figure out the top

4. a) $q_x = R_{xx} P_x + R_{xy} P_y + T_x$

$q_y = R_{yx} P_x + R_{yy} P_y + T_y$

there are six unknowns

you need six equations

you need three pairs of common points

b) $q_{1x} = R_{1xx} P_{1x} + R_{1xy} P_{1y} + T_{1x}$

$q_{1y} = R_{1yx} P_{1x} + R_{1yy} P_{1y} + T_{1y}$

$q_{2x} = R_{2xx} P_{2x} + R_{2xy} P_{2y} + T_{2x}$

$q_{2y} = R_{2yx} P_{2x} + R_{2yy} P_{2y} + T_{2y}$

$q_{3x} = R_{3xx} P_{3x} + R_{3xy} P_{3y} + T_{3x}$

$q_{3y} = R_{3yx} P_{3x} + R_{3yy} P_{3y} + T_{3y}$

c)
$$\begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} R_{xx} \\ R_{xy} \\ R_{yx} \\ R_{yy} \\ T_x \\ T_y \end{bmatrix}$$

d) If $\vec{p}_1$, $\vec{p}_2$, p_3 is colinear, then the ~~system~~ ~~the~~ columns of the matrix will be linearly dependent and there won't be a unique solution. It makes sense geometrically, because you can't define a ~~3d space~~ 2d plane with less than 3 unique points that lie on diffent lines.