1. $\text{proj}_{\vec{v}_1}\vec{y} = \frac{\vec{v}_1 \cdot \vec{y}}{|\vec{v}_1|^2}\vec{v}_1$ $\quad$ $\text{proj}_{\vec{v}_2}\vec{y} = \frac{\vec{v}_2 \cdot \vec{y}}{|\vec{v}_2|^2}\vec{v}_2$

a)

b) $\cos^{-1}\frac{\vec{y}\cdot\vec{y}_i}{|\vec{y}|\,|\vec{y}_i|} = \cos^{-1}\frac{\vec{y}\cdot\frac{\vec{v}_i\cdot\vec{y}}{|\vec{v}_i|^2}\vec{v}_i}{|\vec{y}|\,\left|\frac{\vec{v}_i\cdot\vec{y}}{|\vec{v}_i|^2}\vec{v}_i\right|}$

c) $(\vec{y}-\vec{y}_i)\cdot(\vec{y}-\vec{y}_i) = \left(\vec{y}-\frac{\vec{v}_i\cdot\vec{y}}{|\vec{v}_1|^2}\vec{v}_1\right)^2$

d) $\left(\vec{y}-\frac{\vec{v}_1\cdot\vec{y}}{|\vec{v}_1|^2}\vec{v}_1\right)^2 < \left(\vec{y}-\frac{\vec{v}_2\cdot\vec{y}}{|\vec{v}_2|^2}\vec{v}_2\right)^2$

2. a) $\text{Span}\left\{\begin{bmatrix}1\\3\\1\end{bmatrix}, \begin{bmatrix}-1\\-3\\-1\end{bmatrix}, \begin{bmatrix}-3\\-5\\-1\end{bmatrix}, \begin{bmatrix}4\\8\\2\end{bmatrix}\right\}$

dimension = 4

c) $\text{span}\{(1\ -1\ -3\ 4), (3\ -3\ -5\ 8), (1\ -1\ -1\ 2)\}$

dimension = 3

b) $\left[\begin{array}{cccc|c}1 & -1 & -3 & 4 & 0\\ 3 & -3 & -5 & 8 & 0\\ 1 & -1 & -1 & 2 & 0\end{array}\right] \sim \left[\quad\right]$

2. $\begin{bmatrix} 1 & -1 & -3 & 4 \\ 3 & -3 & -5 & 8 \\ 1 & -1 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a) $\text{span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ -1 \end{bmatrix} \right\}$ $\dim = 2$

c ~~a~~) $\text{span}\left\{ (1 \;\; -1 \;\; -3 \;\; 4), (3 \;\; -3 \;\; -5 \;\; 8) \right\}$ $\dim = 2$

b ~~a~~) $x_1 = x_2 - x_4$

$x_3 = x_4$

$$\begin{bmatrix} x_2 - x_4 \\ x_2 \\ x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} x_4$$

$\text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ $\dim = 2$

d) $A^T = \begin{bmatrix} 1 & 3 & 1 \\ -1 & -3 & -1 \\ -3 & -5 & -1 \\ 4 & 8 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $x_1 = -x_2$

$x_3 = -2x_2$ $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

$\text{span}\left\{ \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \right\}$ $\dim = 1$

3. a) $$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_i \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \\ \vdots & & \ddots & \\ 0 & & & \lambda_i \end{bmatrix} [\vec{v}_1, \vec{v}_2 \cdots \vec{v}_i]^T$$

b) 400 eigenvectors are required

c) ipython

d) 15

4. a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) 1 one-hop
0 two-hop
1 three-hop

c) both are $\frac{1}{2}$

d) $\begin{bmatrix} 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & 0 \end{bmatrix}$

e) 1 two-hop path
1 three-hop path

f) 0.33, 0.17, 0.12, 0.38

g) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \end{bmatrix}$

h) 0 paths

i) 0.2, 0.2, 0.4, 0.1, 0.1

In graph c some nodes don't relate to each other at all

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