

1.

這一個建議作業與程式作業相關

在任課老師的程式作業答案中，使用到 (not) 指令。這一個指令上課時間沒有講到。

這一組簡短範例示範如何使用 (not) 指令

- (not) 指令限制在 facts 裡面沒有特定 facts 時，rules 的前提才算得到滿足
- 這跟我們上課所講的因為匹配而滿足是互補的觀念
 - "因為匹配而滿足"是 AIMA 第九章所講的 unification
- 更多參考資料
 - CLIPS documentation
 - https://sourceforge.net/projects/clipsrules/files/CLIPS/6.31/clips_documentation_631.zip/download
 - https://sourceforge.net/projects/clipsrules/files/CLIPS/6.31/clips_documentation_631.zip/download
 - basic programming guide (3.1)
 - Section 5.4.5

操作程序

- 進入 CLIPS 環境，依序輸入以下定義和指令

```

CLIPS> (defrule stop_then_go
  (numlist $? 4 $?)
  (stop_sign)
=>
  (printout t "hello world" crlf)
)

CLIPS> (defrule no_stop_then_go
  (numlist $? 4 $?)
  (not (stop_sign))
=>
  (printout t "hello world not" crlf)
)

CLIPS> (assert (numlist 1 2 3 4))
CLIPS> (agenda)
CLIPS> (assert (stop_sign))
CLIPS> (agenda)
CLIPS> (retract 2) ;; [備註：這裡用固定的 2 是有危險的，必須是你剛剛真的是剛剛進入CLIPS]
CLIPS> (agenda)

```

觀察 agenda 裡面兩條規則在甚麼狀況之下被啟動



按上述程式碼，執行結果如下：

```

CLIPS> (defrule stop_then_go
  (numlist $? 4 $?)
  (stop_sign)
=>
  (printout t "hello world" crlf))
CLIPS> (defrule no_stop_then_go
  (numlist $? 4 $?)
  (not (stop_sign)))
=>
  (printout t "hello word not" crlf))
CLIPS> (assert (numlist 1 2 3 4))
<Fact-1>
CLIPS> (agenda)
0      no_stop_then_go: f-1,*
For a total of 1 activation.
CLIPS> (assert (stop_sign))
<Fact-2>
CLIPS> (agenda)
0      stop_then_go: f-1,f-2
For a total of 1 activation.
CLIPS> (retract 2)
CLIPS> (agenda)
0      no_stop_then_go: f-1,*
For a total of 1 activation.

```

觀察如下：

1. 第一時間 assert 一筆 fact 時，此時尚無 stop_sign 之 fact，因此第一次的 agenda 會發現程式會觸發 no_stop_then_go 的 rule
2. 當 assert (stop_sign) 進去之後，第二次的 agenda 會發現程式會觸發 stop_then_go 的 rule
3. 而當 retract 2 之後，當然，這步可以寫一個 function 或 rule 來取代直接 retract 特定 fact address 的方式，stop_sign 被移除了
4. 第三次執行 agenda 時，便發現最後又回到了 no_stop_then_go 的 rule，因為此時 stop_sign 不存在，使得 (not (stop_sign)) 的條件成立

2.

依據 joint.distribution.4.random.variables.csv 的數據

- 利用 AIMA (13.6)、(13.8) 或者 Sec. 13.5 所解釋的貝氏定理計算以下機率
 - $\Pr(A=a1)$
 - $\Pr(D=d2)$
 - $\Pr(A=a1|D=d2)$
 - $\Pr(D=d2|A=a1)$
 - $\Pr(D=d2|A=a2, B=b2)$
 - $\Pr(c1, d1 | a1, b1)$

In [2]:

```
import pandas as pd

joint_csv = pd.read_csv('joint.distribution.4.random.variables.csv')

print('csv content:')
print(joint_csv)

print('sum: ', joint_csv['Pr(A, B, C, D)'].sum())
```

csv content:

	A	B	C	D	Pr(A, B, C, D)
0	a1	b1	c1	d1	0.003
1	a1	b1	c1	d2	0.047
2	a1	b1	c2	d1	0.085
3	a1	b1	c2	d2	0.036
4	a1	b2	c1	d1	0.041
5	a1	b2	c1	d2	0.111
6	a1	b2	c2	d1	0.098
7	a1	b2	c2	d2	0.093
8	a2	b1	c1	d1	0.034
9	a2	b1	c1	d2	0.034
10	a2	b1	c2	d1	0.106
11	a2	b1	c2	d2	0.054
12	a2	b2	c1	d1	0.083
13	a2	b2	c1	d2	0.028
14	a2	b2	c2	d1	0.023
15	a2	b2	c2	d2	0.122

sum: 0.998

In [18]:

```
import pandas as pd

joint_csv = pd.read_csv('joint.distribution.4.random.variables.csv')

#  $Pr(A=a1)$ 
pr1 = joint_csv[joint_csv.A == 'a1']
print('==> 1.  $Pr(A=a1)$  集合內容:\n{}'.format(pr1))
print('答案為 {}'.format(pr1['Pr(A, B, C, D)'].sum()))

#  $Pr(D=d2)$ 
pr2 = joint_csv[joint_csv.D == 'd2']
print('==> 2.  $Pr(D=d2)$  集合內容:\n{}'.format(pr2))
print('答案為 {}'.format(pr2['Pr(A, B, C, D)'].sum()))

#  $Pr(A=a1|D=d2)$ 
pr3 = pr2[pr2.A == 'a1']
print('==> 3.  $Pr(A=a1, D=d2)$  集合內容:\n{}'.format(pr3))
print('sum: {}'.format(pr3['Pr(A, B, C, D)'].sum()))

print('Pr(A=a1|D=d2) = Pr(A=a1, D=d2) / Pr(D=d2)')
print('答案為 {}'.format(0.287 / 0.525))

#  $Pr(D=d2|A=a1)$ 
print('==> 4.  $Pr(D=d2|A=a1) = Pr(D=d2, A=a1) / Pr(A=a1)$ ')
print('答案為 {}'.format(0.287 / 0.514))

#  $Pr(D=d2|A=a2, B=b2)$ 
pr4 = joint_csv[(joint_csv.A=='a2') & (joint_csv.B=='b2')]
print('==> 5.  $Pr(A=a2, B=b2)$  集合內容:\n{}'.format(pr4))
print('sum: {}'.format(pr4['Pr(A, B, C, D)'].sum()))

pr5 = pr4[pr4.D=='d2']
print('Pr(A=a2, B=b2, D=d2) 集合內容:\n{}'.format(pr5))
print('sum: {}'.format(pr5['Pr(A, B, C, D)'].sum()))

print('Pr(D=d2|A=a2, B=b2) = Pr(D=d2, A=a2, B=b2) / Pr(A=a2, B=b2)')
print('答案為 {}'.format(0.15 / 0.256))

#  $Pr(c1, d1 | a1, b1)$ 
pr6 = joint_csv[(joint_csv.A == 'a1') & (joint_csv.B == 'b1')]
print('==> 6.  $Pr(A=a1, B=b1)$  集合內容:\n{}'.format(pr6))
print('sum: {}'.format(pr6['Pr(A, B, C, D)'].sum()))

pr7 = pr6[(pr6.C == 'c1') & (pr6.D == 'd1')]
print('Pr(C=c1, D=d1, A=a1, B=b1) 集合內容:\n{}'.format(pr7))
print('sum: {}'.format(pr7['Pr(A, B, C, D)'].sum()))

print('Pr(c1, d1 | a1, b1) = Pr(c1, d1, a1, b1) / Pr(a1, b1)')
print('答案為 {}'.format(0.003 / 0.171))
```

==> 1. $\Pr(A=a1)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
0	a1	b1	c1	d1	0.003
1	a1	b1	c1	d2	0.047
2	a1	b1	c2	d1	0.085
3	a1	b1	c2	d2	0.036
4	a1	b2	c1	d1	0.041
5	a1	b2	c1	d2	0.111
6	a1	b2	c2	d1	0.098
7	a1	b2	c2	d2	0.093

答案為 0.514

==> 2. $\Pr(D=d2)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
1	a1	b1	c1	d2	0.047
3	a1	b1	c2	d2	0.036
5	a1	b2	c1	d2	0.111
7	a1	b2	c2	d2	0.093
9	a2	b1	c1	d2	0.034
11	a2	b1	c2	d2	0.054
13	a2	b2	c1	d2	0.028
15	a2	b2	c2	d2	0.122

答案為 0.525

==> 3. $\Pr(A=a1, D=d2)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
1	a1	b1	c1	d2	0.047
3	a1	b1	c2	d2	0.036
5	a1	b2	c1	d2	0.111
7	a1	b2	c2	d2	0.093

sum: 0.28700000000000003

$\Pr(A=a1|D=d2) = \Pr(A=a1, D=d2) / \Pr(D=d2)$

答案為 0.5466666666666666

==> 4. $\Pr(D=d2|A=a1) = \Pr(D=d2, A=a1) / \Pr(A=a1)$

答案為 0.5583657587548637

==> 5. $\Pr(A=a2, B=b2)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
12	a2	b2	c1	d1	0.083
13	a2	b2	c1	d2	0.028
14	a2	b2	c2	d1	0.023
15	a2	b2	c2	d2	0.122

sum: 0.256

$\Pr(A=a2, B=b2, D=d2)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
13	a2	b2	c1	d2	0.028
15	a2	b2	c2	d2	0.122

sum: 0.15

$\Pr(D=d2|A=a2, B=b2) = \Pr(D=d2, A=a2, B=b2) / \Pr(A=a2, B=b2)$

答案為 0.5859375

==> 6. $\Pr(A=a1, B=b1)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
0	a1	b1	c1	d1	0.003
1	a1	b1	c1	d2	0.047
2	a1	b1	c2	d1	0.085
3	a1	b1	c2	d2	0.036

sum: 0.171

$\Pr(C=c1, D=d1, A=a1, B=b1)$ 集合內容:

	A	B	C	D	$\Pr(A, B, C, D)$
0	a1	b1	c1	d1	0.003

sum: 0.003

$\Pr(c1, d1 | a1, b1) = \Pr(c1, d1, a1, b1) / \Pr(a1, b1)$

答案為 0.017543859649122806

3.

確知如何檢驗機率變數的獨立和條件獨立關係，以下 joint probability 中的 A 和 B 是否獨立？

[備註：機率表的讀法： $\Pr(A=a_1, B=b_1)=0.1$; $\Pr(A=a_2, B=b_2)=0.2$]

[備註：參考 AIMA Sec. 13.4]

A	B	$\Pr(A, B)$
a1	b1	0.1
a1	b2	0.1
a2	b1	0.1
a2	b2	0.2
a3	b1	0.3
a3	b2	0.2

當我們說兩事件(或 random variables) A、B 獨立時，有以下特性：

1. $\Pr(A|B) = \Pr(A)$
2. $\Pr(B|A) = \Pr(B)$
3. $\Pr(A, B) = \Pr(A) \times \Pr(B)$

In [22]:

```

import pandas as pd

a_set = ['a1', 'a2', 'a3']
b_set = ['b1', 'b2']
key_pr = 'Pr(A,B)'

df = pd.DataFrame({'A': ['a1', 'a1', 'a2', 'a2', 'a3', 'a3'],
                    'B': ['b1', 'b2', 'b1', 'b2', 'b1', 'b2'],
                    key_pr: [0.1, 0.1, 0.1, 0.2, 0.3, 0.2]})

print('==> DataFrame content:\n{}'.format(df))
print('==> sum: {}'.format(df['Pr(A,B)'].sum()))

PrA = {a: round(df[df.A==a][key_pr].sum(), 1) for a in a_set}
PrB = {b: round(df[df.B==b][key_pr].sum(), 1) for b in b_set}

PrAandB = {(a, b): df[(df.A==a) & (df.B==b)][key_pr].values[0]
            for a in a_set
            for b in b_set}

PrAgvnB = {(a, b): round(PrAandB[(a, b)] / PrB[b], 3)
            for a in a_set
            for b in b_set}

PrBgvnA = {(b, a): round(PrAandB[(a, b)] / PrA[a], 3)
            for b in b_set
            for a in a_set}

print('Pr(A)', PrA)
print('Pr(B)', PrB)
print('Pr(A,B)', PrAandB)
print('Pr(A|B)', PrAgvnB)
print('Pr(B|A)', PrBgvnA)

try:
    print('檢查 Pr(A,B) = Pr(A) x Pr(B)')
    for key in PrAandB:
        key_a = key[0]
        key_b = key[1]
        print('check Pr({}, {}) == Pr({})Pr({})'.format(key_a, key_b, key_a, key_b))
        assert (PrAandB[key] == PrA[key_a] * PrB[key_b]), "{} 與 {} 並非獨立".format(key_a, key_b)

    print('檢查 Pr(A|B) = Pr(A)')
    for key in PrAgvnB:
        key_a = key[0]
        key_b = key[1]
        print('check: Pr({}|{}) == Pr({})'.format(key_a, key_b, key_a))
        assert (PrAgvnB[key] == PrA[key_a]), "{} 與 {} 並非獨立".format(key_a, key_b)

    print('檢查 Pr(B|A) = Pr(B)')
    for key in PrBgvnA:
        key_a = key[1]
        key_b = key[0]
        assert (PrBgvnA[key] == PrB[key_b]), "{} 與 {} 並非獨立".format(key_b, key_a)

```

```

except AssertionError as err:
    print(str(err))
    print("故 A 與 B 並非獨立")
else:
    print("A 與 B 互相獨立")

==> DataFrame content:
   A     B   Pr(A,B)
0  a1   b1      0.1
1  a1   b2      0.1
2  a2   b1      0.1
3  a2   b2      0.2
4  a3   b1      0.3
5  a3   b2      0.2
==> sum: 1.0
Pr(A) {'a1': 0.2, 'a2': 0.3, 'a3': 0.5}
Pr(B) {'b1': 0.5, 'b2': 0.5}
Pr(A,B) {('a1', 'b1'): 0.1, ('a1', 'b2'): 0.1, ('a2', 'b1'): 0.1,
          ('a2', 'b2'): 0.2, ('a3', 'b1'): 0.3, ('a3', 'b2'): 0.2}
Pr(A|B) {('a1', 'b1'): 0.2, ('a1', 'b2'): 0.2, ('a2', 'b1'): 0.2,
          ('a2', 'b2'): 0.4, ('a3', 'b1'): 0.6, ('a3', 'b2'): 0.4}
Pr(B|A) {('b1', 'a1'): 0.5, ('b1', 'a2'): 0.333, ('b1', 'a3'): 0.6,
          ('b2', 'a1'): 0.5, ('b2', 'a2'): 0.667, ('b2', 'a3'): 0.4}
檢查 Pr(A,B) = Pr(A) x Pr(B)
check Pr(a1,b1) == Pr(a1)Pr(b1)
check Pr(a1,b2) == Pr(a1)Pr(b2)
check Pr(a2,b1) == Pr(a2)Pr(b1)
a2 與 b1 並非獨立
故 A 與 B 並非獨立

```

4.

依據以下的 joint probability distribution回答問題

- B 和 C 是否獨立？[備註：參考 AIMA Sec. 13.4]
- 若給定機率變數A，B和C是否條件獨立？

[備註：參考 AIMA p. 498]

A	B	C	Pr(A,B,C)
a1	b1	c1	0.216
a1	b1	c2	0.024
a1	b2	c1	0.324
a1	b2	c2	0.036
a2	b1	c1	0.056
a2	b1	c2	0.224
a2	b2	c1	0.024
a2	b2	c2	0.096

當我們說隨機變數 B 與 C 在給定 A 時條件獨立，其結果如下：

1. $Pr(B, C|A) = Pr(B|A) \times Pr(C|A)$
2. $Pr(B|C, A) = Pr(B|A)$
3. $Pr(C|B, A) = Pr(C|A)$

In [9]:

```

import pandas as pd

a_set = ['a1', 'a2']
b_set = ['b1', 'b2']
c_set = ['c1', 'c2']
key_pr = 'Pr(A,B,C)'

df = pd.DataFrame({'A': ['a1', 'a1', 'a1', 'a1', 'a2', 'a2', 'a2', 'a2'],
                   'B': ['b1', 'b1', 'b2', 'b2', 'b1', 'b1', 'b2', 'b2'],
                   'C': ['c1', 'c2', 'c1', 'c2', 'c1', 'c2', 'c1', 'c2'],
                   'Pr(A,B,C)': [0.216, 0.024, 0.324, 0.036, 0.056, 0.224, 0.024,
                   , 0.096]})

print('==> DataFrame content:\n{}'.format(df))
print('==> sum:{}\n'.format(df[key_pr].sum()))

## 單論 B、C 的獨立性

print('==> Pr(b1):{}\n'.format(df[df.B=='b1'][key_pr].sum()))
print('==> Pr(c1):{}\n'.format(df[df.C=='c1'][key_pr].sum()))
print('==> Pr(b1,c1):{}\n'.format(df[(df.B=='b1') & (df.C=='c1')][key_pr].values[0]))
print('==> Pr(b1|c1): {}\n'.format(df[(df.B=='b1') & (df.C=='c1')][key_pr].values[0] / df[df.C=='c1'][key_pr].sum()))
print('因 Pr(b1|c1) != Pr(b1) 故 B & C 並非獨立的兩個 random variable.\n')

## 加上 A 的因素後

print('==> Pr(a1):{}\n'.format(df[df.A=='a1'][key_pr].sum()))
print('==> Pr(b1,c1|a1) = Pr(b1,c1,a1) / Pr(a1) = {}\n'.format(df[(df.B=='b1') & (df.C=='c1') & (df.A=='a1')][key_pr].values[0] / df[df.A=='a1'][key_pr].sum()))
print('==> Pr(B|A) = Pr(B,A) / Pr(A) = {}\n'.format(df[(df.B=='b1') & (df.A=='a1')][key_pr].sum() / df[df.A=='a1'][key_pr].sum()))
print('==> Pr(C|A) = Pr(C,A) / Pr(A) = {}\n'.format(df[(df.C=='c1') & (df.A=='a1')][key_pr].sum() / df[df.A=='a1'][key_pr].sum()))
print('==> Pr(B|A) x Pr(C|A) = {}\n'.format(0.39999999*0.89999999))

print('因加上 a1 後，b1, c1 符合條件隨機定義，故我們繼續用程式的方式來解題')

PrA = {a: round(df[df.A==a][key_pr].sum(), 3) for a in a_set}

PrBCandA = {(a,b,c): round(df[(df.A==a) & (df.B==b) & (df.C==c)][key_pr].values[0], 3)
             for a in a_set
             for b in b_set
             for c in c_set}

PrBgvnA = {(b,a): round(df[(df.B==b) & (df.A==a)][key_pr].sum() / PrA[a], 3)
            for b in b_set
            for a in a_set}

PrCgvnA = {(c,a): round(df[(df.C==c) & (df.A==a)][key_pr].sum() / PrA[a], 3)
            for c in c_set
            for a in a_set}

diff = 0.01

try:
    for a in a_set:
        for b in b_set:

```

```
for c in c_set:
    print('check {}, {} by given {}'.format(b, c, a))

Pr_abc = df[(df.A==a) & (df.B==b) & (df.C==c)][key_pr].values[0]
Pr_a = PrA[a]
Pr_b_gvn_a = PrBgvnA[(b,a)]
Pr_c_gvn_a = PrCgvnA[(c,a)]
Pr_bc_gvn_a = Pr_abc / Pr_a

print('Pr({},{})|{} = {}'.format(b, c, a, Pr_bc_gvn_a))
print('Pr({})|{} x Pr({})|{} = {}'.format(b, a, c, a, (Pr_b_gvn_a * Pr_c_gvn_a)))

assert diff > (Pr_bc_gvn_a - (Pr_b_gvn_a * Pr_c_gvn_a)), '在給定
{} 的條件下，{}、{} 不是獨立的'.format(a, b, c)
except AssertionError as err:
    print(str(err))
    print("故 B 與 C 在給定 A 的條件下並非獨立")
else:
    print("B 與 C 在給定 A 的條件下，互相獨立")
```

```

==> DataFrame content:
    A   B   C   Pr(A,B,C)
 0  a1  b1  c1      0.216
 1  a1  b1  c2      0.024
 2  a1  b2  c1      0.324
 3  a1  b2  c2      0.036
 4  a2  b1  c1      0.056
 5  a2  b1  c2      0.224
 6  a2  b2  c1      0.024
 7  a2  b2  c2      0.096
==> sum:1.0
==> Pr(b1):0.52
==> Pr(c1):0.6200000000000001
==> Pr(b1,c1):0.216
==> Pr(b1|c1): 0.3483870967741935
因 Pr(b1|c1) != Pr(b1) 故 B & C 並非獨立的兩個 random variable.
==> Pr(a1):0.6000000000000001
==> Pr(b1,c1|a1) = Pr(b1,c1,a1) / Pr(a1) = 0.3599999999999993
==> Pr(B|A) = Pr(B,A) / Pr(A) = 0.3999999999999999
==> Pr(C|A) = Pr(C,A) / Pr(A) = 0.8999999999999999
==> Pr(B|A) x Pr(C|A) = 0.3599998700000013
因加上 a1 後, b1, c1 符合條件隨機定義, 故我們繼續用程式的方式來解題
check b1, c1 by given a1
Pr(b1,c1|a1) = 0.36
Pr(b1|a1) x Pr(c1|a1) = 0.3600000000000004
check b1, c2 by given a1
Pr(b1,c2|a1) = 0.04
Pr(b1|a1) x Pr(c2|a1) = 0.0400000000000001
check b2, c1 by given a1
Pr(b2,c1|a1) = 0.54
Pr(b2|a1) x Pr(c1|a1) = 0.54
check b2, c2 by given a1
Pr(b2,c2|a1) = 0.06
Pr(b2|a1) x Pr(c2|a1) = 0.06
check b1, c1 by given a2
Pr(b1,c1|a2) = 0.1399999999999999
Pr(b1|a2) x Pr(c1|a2) = 0.1399999999999999
check b1, c2 by given a2
Pr(b1,c2|a2) = 0.5599999999999999
Pr(b1|a2) x Pr(c2|a2) = 0.5599999999999999
check b2, c1 by given a2
Pr(b2,c1|a2) = 0.06
Pr(b2|a2) x Pr(c1|a2) = 0.06
check b2, c2 by given a2
Pr(b2,c2|a2) = 0.24
Pr(b2|a2) x Pr(c2|a2) = 0.24
B 與 C 在給定 A 的條件下, 互相獨立

```

5.

貝氏網路預備工作

- 安裝 Hugin Lite : <https://www.hugin.com/> (<https://www.hugin.com/>)
- 下載 ChestClinic.net
- 嘗試開啟 ChestClinic.net