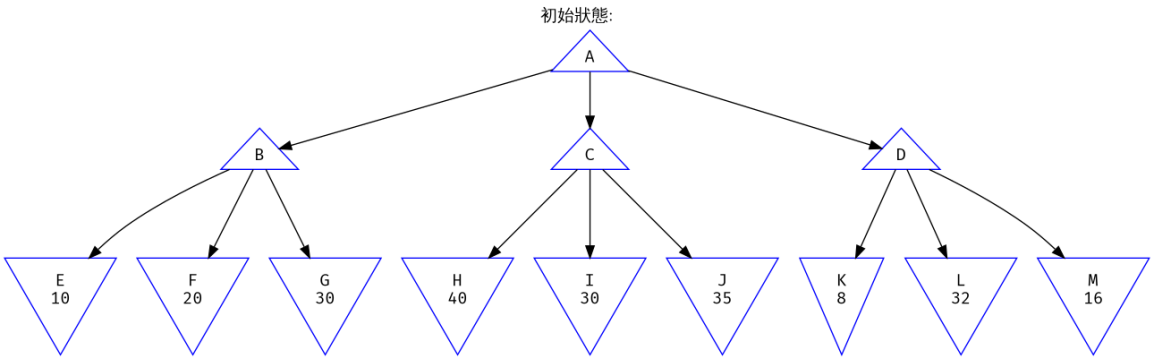
- 練習 minimax with alpha-beta pruning (AIMA Sec. 5.3) 考慮以下 game tree
  -  game tree
  - 注意：這一棵 game tree 和 AIMA 圖5.2雖然類似，但是有不少差異
  - 提示：注意 AIMA Sec. 5.3.1 所講的 move ordering 問題
  - 假設我們在執行 AIMA 圖 5.7 演算法中 for each a in Action(state) do 的時候，都是從上面 game tree 左邊的節點開始、逐次往右搜尋的話，則上述哪一些 subtree 可以被忽略？說明原因。
  - 假設我們在執行 AIMA 圖 5.7 演算法中 for each a in Action(state) do 的時候，都是從上面 game tree 右邊的節點開始、逐次往左搜尋的話，則上述哪一些 subtree 可以被忽略？說明原因

第一小題，從左邊節點往右

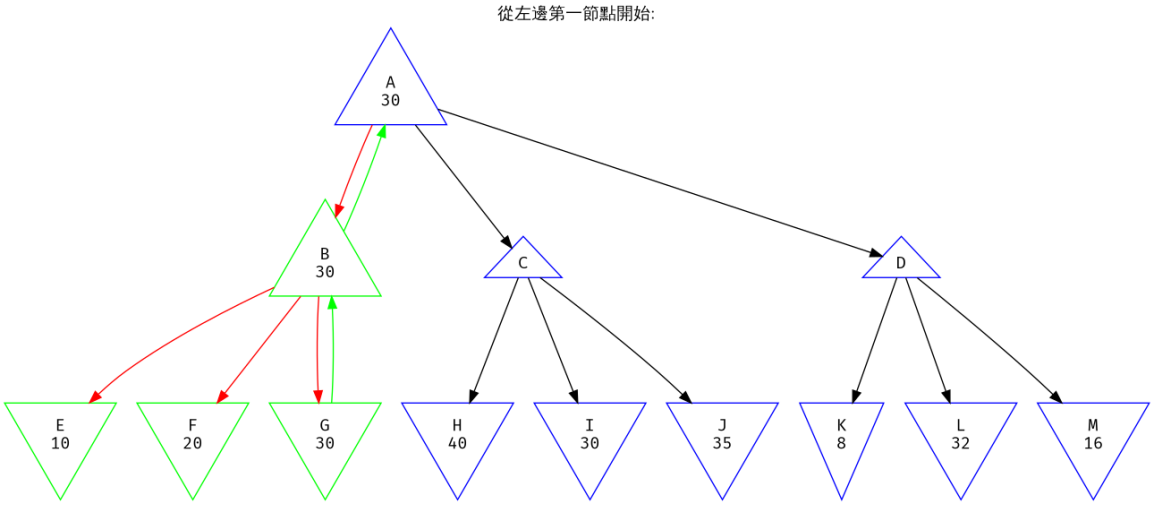
In [1]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "初始狀態:"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A", color=blue]

    B[shape=triangle, label="B", color=blue]
    C[shape=triangle, label="C", color=blue]
    D[shape=triangle, label="D", color=blue]

    A->B
    A->C
    A->D

    E[shape=invtriangle, label="E\n10", color=blue]
    F[shape=invtriangle, label="F\n20", color=blue]
    G[shape=invtriangle, label="G\n30", color=blue]
    H[shape=invtriangle, label="H\n40", color=blue]
    I[shape=invtriangle, label="I\n30", color=blue]
    J[shape=invtriangle, label="J\n35", color=blue]
    K[shape=invtriangle, label="K\n8", color=blue]
    L[shape=invtriangle, label="L\n32", color=blue]
    M[shape=invtriangle, label="M\n16", color=blue]

    B->E
    B->F
    B->G
    C->H
    C->I
    C->J
    D->K
    D->L
    D->M
}
```



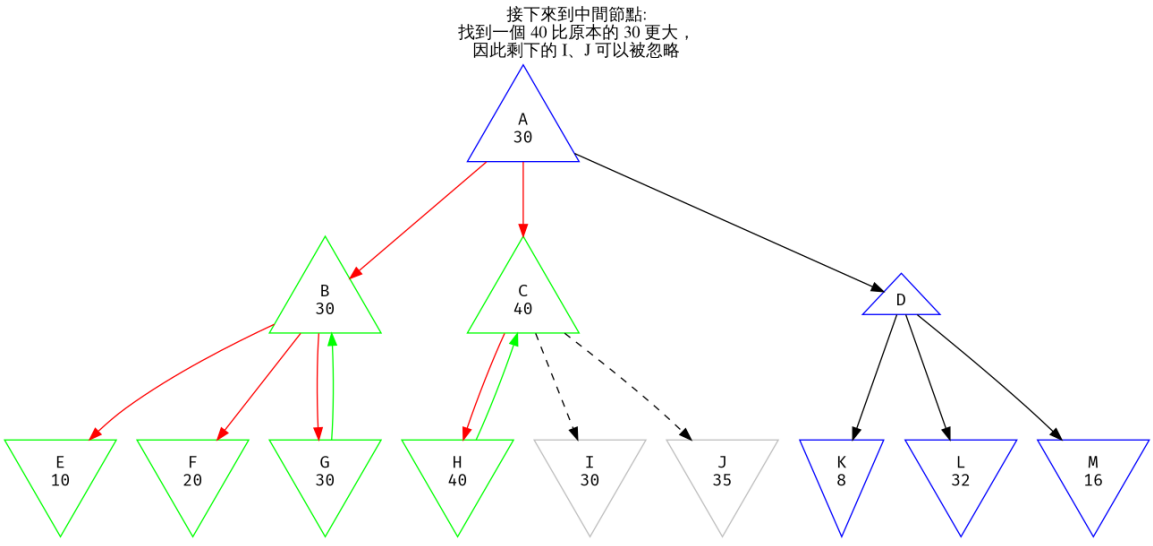
In [2]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "從左邊第一節點開始:"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n30", color=blue]

    B[shape=triangle, label="B\n30", color=green]
    C[shape=triangle, label="C", color=blue]
    D[shape=triangle, label="D", color=blue]

    A->B[color=red]
    B->A[color=green]
    A->C
    A->D

    E[shape=invtriangle, label="E\n10", color=green]
    F[shape=invtriangle, label="F\n20", color=green]
    G[shape=invtriangle, label="G\n30", color=green]
    H[shape=invtriangle, label="H\n40", color=blue]
    I[shape=invtriangle, label="I\n30", color=blue]
    J[shape=invtriangle, label="J\n35", color=blue]
    K[shape=invtriangle, label="K\n8", color=blue]
    L[shape=invtriangle, label="L\n32", color=blue]
    M[shape=invtriangle, label="M\n16", color=blue]

    B->E[color=red]
    B->F[color=red]
    B->G[color=red]
    G->B[color=green]
    C->H
    C->I
    C->J
    D->K
    D->L
    D->M
}
```



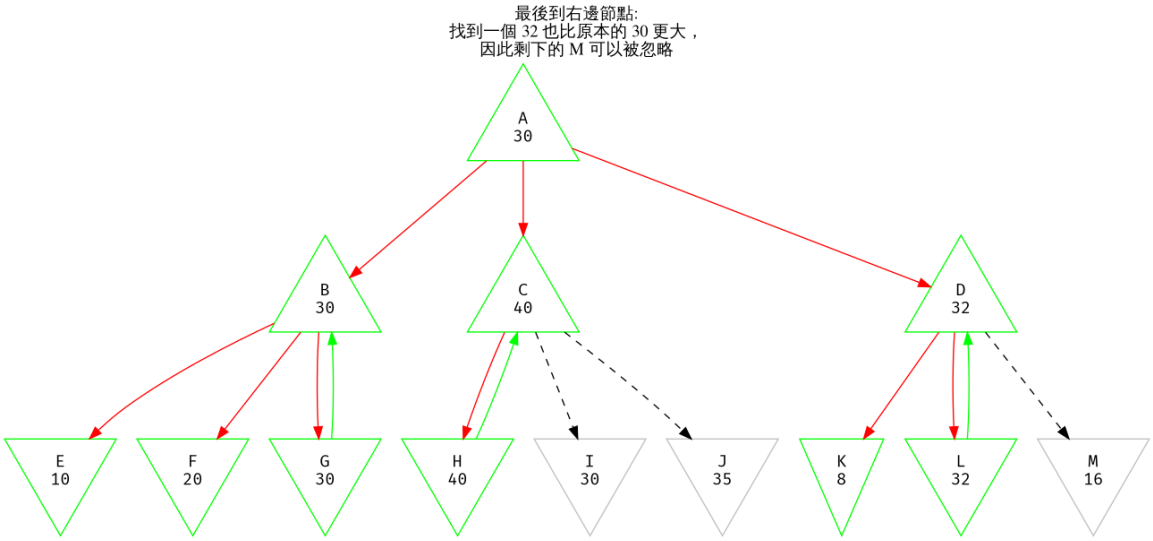
In [3]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "接下來到中間節點:\n找到一個 40 比原本的 30 更大，\n因此剩下的 I、J 可以被忽略"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n30", color=blue]

    B[shape=triangle, label="B\n30", color=green]
    C[shape=triangle, label="C\n40", color=green]
    D[shape=triangle, label="D", color=blue]

    A->B[color=red]
    A->C[color=red]
    A->D

    E[shape=invtriangle, label="E\n10", color=green]
    F[shape=invtriangle, label="F\n20", color=green]
    G[shape=invtriangle, label="G\n30", color=green]
    H[shape=invtriangle, label="H\n40", color=green]
    I[shape=invtriangle, label="I\n30", color=gray]
    J[shape=invtriangle, label="J\n35", color=gray]
    K[shape=invtriangle, label="K\n8", color=blue]
    L[shape=invtriangle, label="L\n32", color=blue]
    M[shape=invtriangle, label="M\n16", color=blue]

    B->E[color=red]
    B->F[color=red]
    B->G[color=red]
    G->B[color=green]
    C->H[color=red]
    H->C[color=green]
    C->I[style=dashed]
    C->J[style=dashed]
    D->K
    D->L
    D->M
}
```



In [4]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "最後到右邊節點:\n找到一個 32 也比原本的 30 更大，\n因此剩下的 M 可以被忽略"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n30", color=green]

    B[shape=triangle, label="B\n30", color=green]
    C[shape=triangle, label="C\n40", color=green]
    D[shape=triangle, label="D\n32", color=green]

    A->B[color=red]
    A->C[color=red]
    A->D[color=red]

    E[shape=invtriangle, label="E\n10", color=green]
    F[shape=invtriangle, label="F\n20", color=green]
    G[shape=invtriangle, label="G\n30", color=green]
    H[shape=invtriangle, label="H\n40", color=green]
    I[shape=invtriangle, label="I\n30", color=gray]
    J[shape=invtriangle, label="J\n35", color=gray]
    K[shape=invtriangle, label="K\n8", color=green]
    L[shape=invtriangle, label="L\n32", color=green]
    M[shape=invtriangle, label="M\n16", color=gray]

    B->E[color=red]
    B->F[color=red]
    B->G[color=red]
    G->B[color=green]
    C->H[color=red]
    H->C[color=green]
    C->I[style=dashed]
    C->J[style=dashed]
    D->K[color=red]
    D->L[color=red]
    L->D[color=green]
    D->M[style=dashed]
}
```

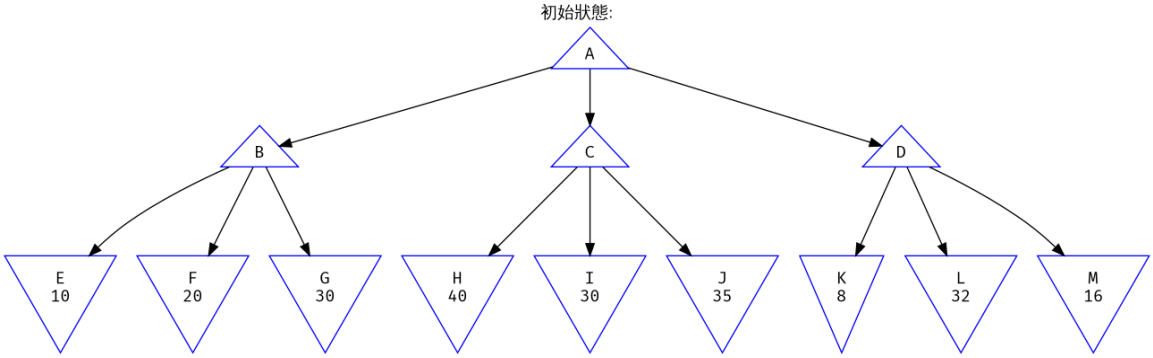


得出若從左邊開始，會找到 B，值為 30 的結果

第二小題，從右至左

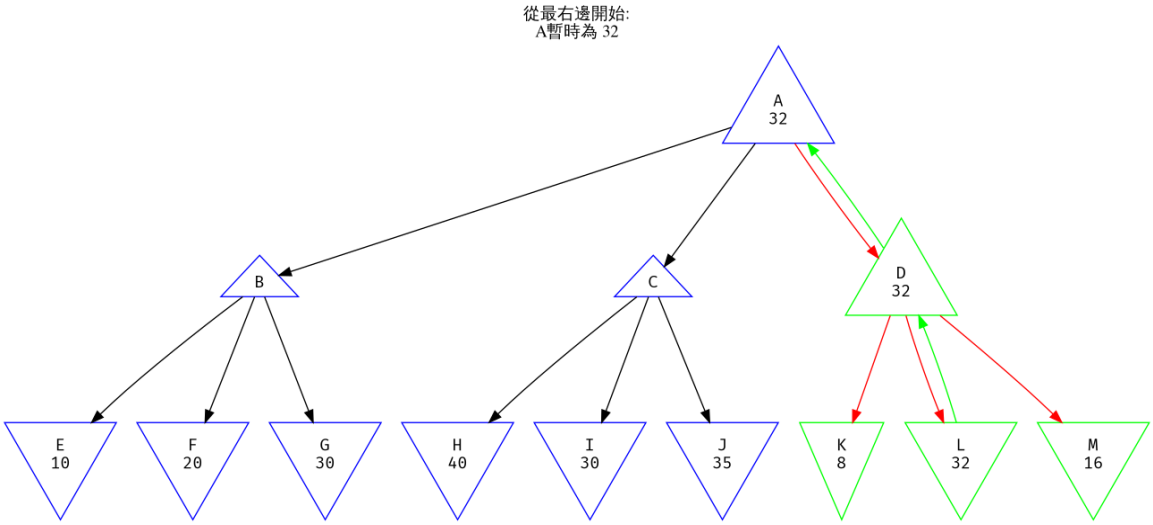
In [5]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "初始狀態:"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A", color=blue]

    B[shape=triangle, label="B", color=blue]
    C[shape=triangle, label="C", color=blue]
    D[shape=triangle, label="D", color=blue]

    A->B
    A->C
    A->D

    E[shape=invtriangle, label="E\n10", color=blue]
    F[shape=invtriangle, label="F\n20", color=blue]
    G[shape=invtriangle, label="G\n30", color=blue]
    H[shape=invtriangle, label="H\n40", color=blue]
    I[shape=invtriangle, label="I\n30", color=blue]
    J[shape=invtriangle, label="J\n35", color=blue]
    K[shape=invtriangle, label="K\n8", color=blue]
    L[shape=invtriangle, label="L\n32", color=blue]
    M[shape=invtriangle, label="M\n16", color=blue]

    B->E
    B->F
    B->G
    C->H
    C->I
    C->J
    D->K
    D->L
    D->M
}
```



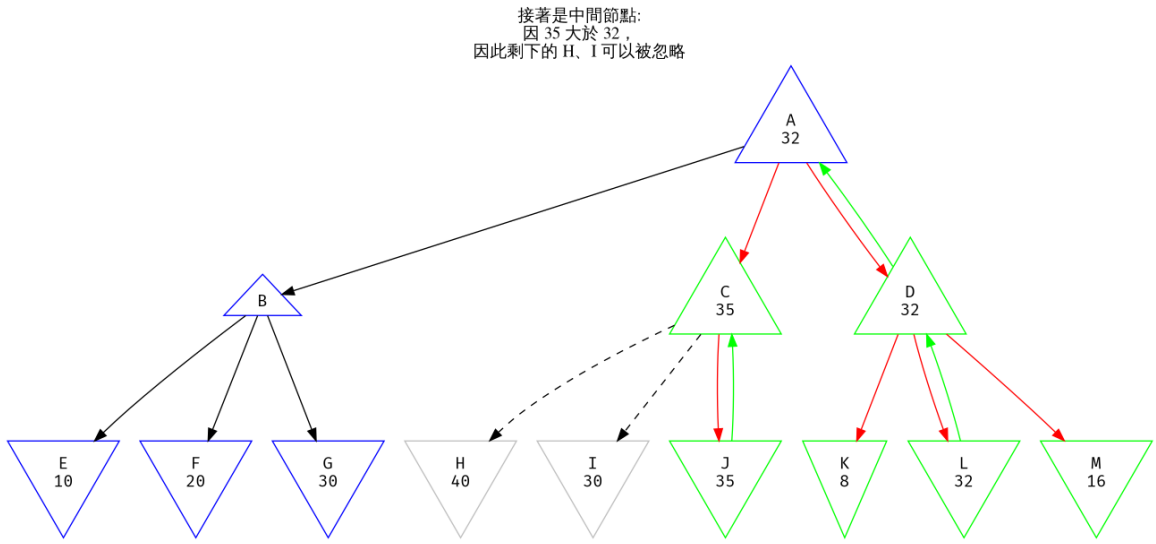
In [6]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "從最右邊開始:\nA暫時為 32"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n32", color=blue]

    B[shape=triangle, label="B", color=blue]
    C[shape=triangle, label="C", color=blue]
    D[shape=triangle, label="D\n32", color=green]

    A->B
    A->C
    A->D[color=red]
    D->A[color=green]

    E[shape=invtriangle, label="E\n10", color=blue]
    F[shape=invtriangle, label="F\n20", color=blue]
    G[shape=invtriangle, label="G\n30", color=blue]
    H[shape=invtriangle, label="H\n40", color=blue]
    I[shape=invtriangle, label="I\n30", color=blue]
    J[shape=invtriangle, label="J\n35", color=blue]
    K[shape=invtriangle, label="K\n8", color=green]
    L[shape=invtriangle, label="L\n32", color=green]
    M[shape=invtriangle, label="M\n16", color=green]

    B->E
    B->F
    B->G
    C->H
    C->I
    C->J
    D->K[color=red]
    D->L[color=red]
    L->D[color=green]
    D->M[color=red]
}
```



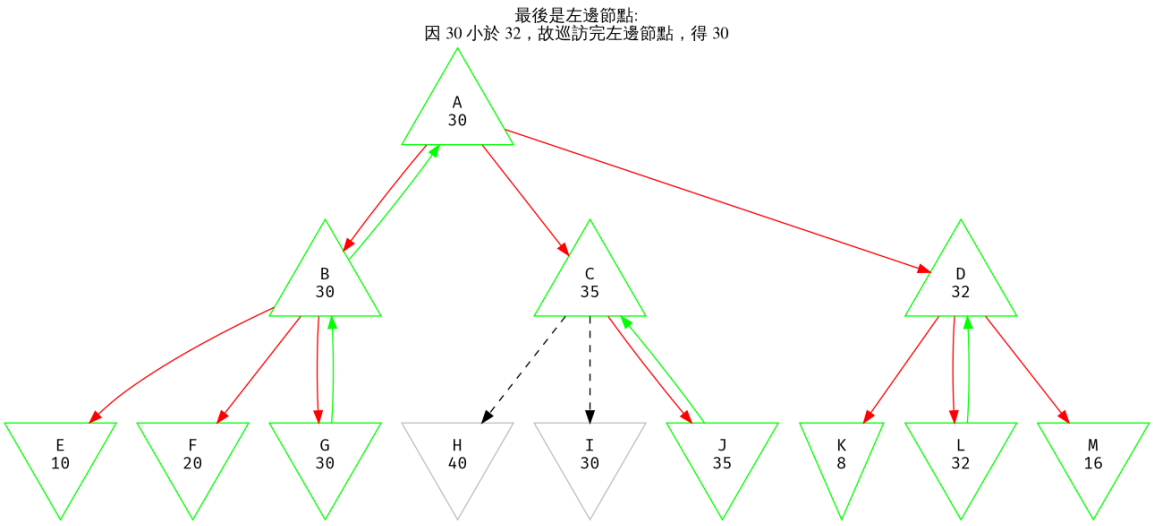
In [7]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "接著是中間節點:\n因 35 大於 32, \n因此剩下的 H、I 可以被忽略"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n32", color=blue]

    B[shape=triangle, label="B", color=blue]
    C[shape=triangle, label="C\n35", color=green]
    D[shape=triangle, label="D\n32", color=green]

    A->B
    A->C[color=red]
    A->D[color=red]
    D->A[color=green]

    E[shape=invtriangle, label="E\n10", color=blue]
    F[shape=invtriangle, label="F\n20", color=blue]
    G[shape=invtriangle, label="G\n30", color=blue]
    H[shape=invtriangle, label="H\n40", color=gray]
    I[shape=invtriangle, label="I\n30", color=gray]
    J[shape=invtriangle, label="J\n35", color=green]
    K[shape=invtriangle, label="K\n8", color=green]
    L[shape=invtriangle, label="L\n32", color=green]
    M[shape=invtriangle, label="M\n16", color=green]

    B->E
    B->F
    B->G
    C->H[style=dashed]
    C->I[style=dashed]
    C->J[color=red]
    J->C[color=green]
    D->K[color=red]
    D->L[color=red]
    L->D[color=green]
    D->M[color=red]
}
```



In [8]:

```
digraph {
    labelloc = "t"
    labelfontname = "文泉驛微米黑"
    label = "最後是左邊節點:\n因 30 小於 32，故巡訪完左邊節點，得 30"

    node [fontname="Fira Mono"]

    A[shape=triangle, label="A\n30", color=green]

    B[shape=triangle, label="B\n30", color=green]
    C[shape=triangle, label="C\n35", color=green]
    D[shape=triangle, label="D\n32", color=green]

    A->B[color=red]
    B->A[color=green]
    A->C[color=red]
    A->D[color=red]

    E[shape=invtriangle, label="E\n10", color=green]
    F[shape=invtriangle, label="F\n20", color=green]
    G[shape=invtriangle, label="G\n30", color=green]
    H[shape=invtriangle, label="H\n40", color=gray]
    I[shape=invtriangle, label="I\n30", color=gray]
    J[shape=invtriangle, label="J\n35", color=green]
    K[shape=invtriangle, label="K\n8", color=green]
    L[shape=invtriangle, label="L\n32", color=green]
    M[shape=invtriangle, label="M\n16", color=green]

    B->E[color=red]
    B->F[color=red]
    B->G[color=red]
    G->B[color=green]
    C->H[style=dashed]
    C->I[style=dashed]
    C->J[color=red]
    J->C[color=green]
    D->K[color=red]
    D->L[color=red]
    L->D[color=green]
    D->M[color=red]
}
```



- 模仿上課時以真值表證明 $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$ 的方式 (在白板上寫的證明)，證明以下關係
  - $P \Rightarrow Q \equiv \neg P \vee Q$

證明

依據 imply 之邏輯屬性，我們得出以下之 truth table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

且：

$\neg P$	Q	$\neg P \vee Q$
F	T	T
F	F	F
T	T	T
T	F	T

依兩 truth tables 結果之比較，可得： $P \Rightarrow Q \equiv \neg P \vee Q$

- 模仿 simple.proof.pdf 證明 $P_{2,2}$ 不成立$(P_{2,2} = False)$ 的程序，利用以下的 logic sentences，證明 $W_{2,2}$ 也不成立。
  - 參考 AI.logical.agents.pdf 第18頁上的說明，或者 AIMA Sec. 7.4.3 (p. 247) 的內容，我們使用 W 和 S 的符號
    1. R1: $W_{1,3} \Rightarrow S_{1,2} \wedge S_{2,3} \wedge S_{1,4}$
    2. R2: $S_{1,2} \Rightarrow W_{1,1} \vee W_{2,2} \vee W_{1,3}$
    3. R3: $W_{2,2} \Rightarrow S_{1,2} \wedge S_{3,2} \wedge S_{2,1} \wedge S_{2,3}$
    4. R4: $S_{2,1} \Rightarrow W_{1,1} \vee W_{2,2} \vee W_{3,1}$
    5. F1: $\neg S_{1,1}$
    6. F2: $S_{1,2}$
    7. F3: $\neg S_{2,1}$
    8. F4: $\neg W_{1,1}$

1. 用 implication elimination 推導 R3，得：

- R5: $\neg W_{2,2} \vee (S_{1,2} \wedge S_{3,2} \wedge S_{2,1} \wedge S_{2,3})$

2. 用 distributivity 展開 R5 得：

- R6: $(\neg W_{2,2} \vee S_{1,2}) \wedge (\neg W_{2,2} \vee S_{3,2}) \wedge (\neg W_{2,2} \vee S_{2,1}) \wedge (\neg W_{2,2} \vee S_{2,3})$

3. 我們從 F3 可得：

- F5: $\neg W_{2,2} \vee S_{1,2}$
- F6: $\neg W_{2,2} \vee S_{3,2}$
- F7: $\neg W_{2,2} \vee S_{2,1}$
- F8: $\neg W_{2,2} \vee S_{2,3}$

4. 根據 F3，得：

- F9: $\neg W_{2,2}$

5. 故得證：$W_{2,2}$ 為 $false$

In [ ]: