SeqSeies

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Question 1

0/10 pts 3 19

Consider the series $\sum_{n=1}^{\infty} \frac{4n^4}{5n^3+5}$

Based on the Divergence Test:

Theorem. If $\lim_{n \to \infty} a_n$ is not convergent to 0, $\sum_{n=1}^{\infty} a_n$ is divergent.

does this series Diverge?

- Inconclusive
- Diverges

Diverges

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Question 2

0/10 pts 3 19

Which statement(s) is(are) right?

- [ ] Sum of an infinite convergent sequence is also convergent.
- [ ] Harmonic series is convergent.
- [ ] Alternating harmonic series is convergent.
- [ ] Suppose that a_n is the n-th term of a convergent series. Then $\{a_n\}_n$ is convergent.
- [ ] Suppose that a_n is the n-th term of a divergent series. Then $\{a_n\}_n$ is always divergent.

Alternating harmonic series is convergent.
Suppose that a_n is the n-th term of a convergent series. Then $\{a_n\}_n$ is convergent.

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Question 3

0/10 pts 3 19

Suppose that
$a_0 = 0.6,$
$a_{n+1} = 2a_n$, if $a_n < 0.5$
or
$a_{n+1} = 2a_n - 1$ if $a_n > 0.5$

Which statement(s) is(are) right?

- ☐ $\{a_n\}_{n=0}^{\infty}$ is convergent.
- ☐ $\{a_n\}_{n=0}^{\infty}$ is divergent.
- ☐ $\{a_{4n}\}_{n=0}^{\infty}$ is convergent.
- ☐ $\{a_{2n}\}_{n=0}^{\infty}$ is convergent.

$\{a_n\}_{n=0}^{\infty}$ is divergent.
$\{a_{4n}\}_{n=0}^{\infty}$ is convergent.

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Question 4 0/10 pts 3 19

Consider the resursive series, $\sum_{n=0}^{\infty} a_n$, where a_b is defined as follows:

$$a_0 = \frac{1}{4}, a_n = \left(1 + \frac{1}{n}\right) a_{n-1}, n = 1, 2, 3, \cdots,$$

Answer the following about $\{a_n\}$ in explicit form but not in recursive formula:

1. $a_7 =$

2. a_n=

3, series is

- Divergent
- Convergent

Divergent

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Question 5

0/10 pts 3 19

Which one is(are) convergent?

- [ ] $$\sum_{n=1}^{\infty} \frac{n^3 - n + 1}{n^1 \cdot ((\ln(n^2+1)))^2}$$
- [ ] $$\sum_{n=1}^{\infty} \frac{n^1}{(n^3 - n + 1) \cdot (\ln(n^2+1))}$$
- [ ] $$\sum_{n=1}^{\infty} \frac{\left(\ln\left(n^2+1\right)\right)^2}{n^1 \cdot (n^3 - n + 1)}$$
- [ ] $$\sum_{n=1}^{\infty} \frac{n^1 \cdot \left(\ln\left(n^2+1\right)\right)}{n^3 - n + 1}$$
- [ ] $$\sum_{n=1}^{\infty} \frac{n^3 - n + 1}{n^1 \cdot (\ln(n^2+1))}$$

$$\sum_{n=1}^{\infty} \frac{n^1 \cdot \left(\ln\left(n^2+1\right)\right)}{n^3 - n + 1}$$
$$\sum_{n=1}^{\infty} \frac{n^1}{(n^3 - n + 1) \cdot (\ln(n^2+1))}$$
$$\sum_{n=1}^{\infty} \frac{\left(\ln\left(n^2+1\right)\right)^2}{n^1 \cdot (n^3 - n + 1)}$$

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Question 6 0/10 pts 3 19

Suppose that $\sum_{n=1}^{\infty} \frac{x^n (n+1)^{3n}}{(3n+1)!}$

It is convergent if $x \in$ [ ] $\left[-\frac{3^3}{\exp(3)}, \frac{3^3}{\exp(3)}\right]$

Note:

1. Euler number, input by e, for example: e^2 = e^2 input by exp(2),
2. Open interval, $\{x \in \mathbb{R} \mid a < x < b\}$, input by (a,b),
3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],

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Question 7 0/10 pts 3 19

Find all the Truths:

- [ ] If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$;
- [ ] If $\sum_{n=1}^{\infty} a_n$ diverges, $\lim_{n \to \infty} a_n \neq 0$;
- [ ] The series, $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$, converges;
- [ ] $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n \to \infty} a_n \neq 0$;
- [ ] The geometric series, $a + ar + ar^2 + \cdots + ar^n + \cdots$, is convergent only for $|r| < 1$ and for any $a \in \mathbb{R}$;

- [ ] $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n \to \infty} a_n = 0$;
- [ ] If $a_{n+2} = a_{n+1} + a_n$ where $a_1 = a_1 = 2$, then $\sum_{n=1}^{\infty} \frac{1}{a_{n+1}a_{n+2}}$ is convergent and equal to 1,
- [ ] If $\{a_n\}, n = 1, 2, \cdots,$ is bounded and monotonic, $\sum_{n=1}^{\infty} a_n$ converges
- [ ] The series, $1 - 1 + 1 - 1 + 1 - 1 \cdots$, converges,
- [ ] The series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$, converges;

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$;
$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n \to \infty} a_n \neq 0$;
The series, $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$, converges;

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Question 8 0/10 pts 3 19

Find out all the series which are convergent:

- [ ] $\sum_{n=2}^{\infty} \frac{\log n}{n}$
- [ ] $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$
- [ ] $\sum_{n=1}^{\infty} \frac{n}{3^n + 2}$
- [ ] $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$
- [ ] $\sum_{n=1}^{\infty} a_n^2$ where $\sum_{n=1}^{\infty} a_n$ is convergent;
- [ ] $\sum_{n=1}^{\infty} \left(1 - \frac{2}{n}\right)^{\frac{1}{n}}$
- [ ] $\sum_{n=1}^{\infty} a_n$ where $\sum_{n=1}^{\infty} a_n^2$ is convergent;
- [ ] $\sum_{n=2}^{\infty} \log\left(\frac{n+1}{n}\right)$
- [ ] $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 2n + 1}}$
- [ ] $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

$$\sum_{n=1}^{\infty} \frac{n}{3^n + 2}$$
$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$$
$$\sum_{n=2}^{\infty} \frac{\log n}{n^2}$$

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Question 9

0/10 pts 3 19

Finall all the Truth(s):

- [ ] $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$ is convergent;
- [ ] If $0 \leq a_n \leq b_n$ for $n = 1, 2, 3, \cdots$, $\sum_{n=1}^{\infty} a_n$ is divergent if $\sum_{n=1}^{\infty} b_n$ is divergent;
- [ ] If $0 \leq a_n \leq b_n + c_n$ for $n = 1, 2, 3, \cdots$, $\sum_{n=1}^{\infty} a_n$ is divergent if both $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} c_n$ are divergent;
- [ ] If $0 \leq a_n + b_n \leq c_n$ for $n = 1, 2, 3, \cdots$, both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent if $\sum_{n=1}^{\infty} c_n$ is convergent;
- [ ] $\sum_{n=1}^{\infty} \frac{\log(n)}{n^p}$ is divergent if $p \leq 1$;

☐ Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive series and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

then $\sum_{n=1}^{\infty} a_n$ is divergent if $\sum_{n=1}^{\infty} b_n$ is divergent;

☐ Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive series and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$

then $\sum_{n=1}^{\infty} a_n$ is divergent if $\sum_{n=1}^{\infty} b_n$ is divergent;

☐ If $0 \le a_n \le b_n$ for $n = 1, 2, 3, \cdots,$ $\sum_{n=1}^{\infty} b_n$ is divergent if $\sum_{n=1}^{\infty} a_n$ is divergent;

☐ If $0 < a_{n+7} \le b_n$ for $n = 1, 2, 3, \cdots,$ $\sum_{n=1}^{\infty} a_n$ is convergent if $\sum_{n=1}^{\infty} b_n$ is convergent;

☐ $\sum_{n=1}^{\infty} \frac{x_i}{10^n}$ is convergent where x_i is one of the numbers, $0, 1, 2, \cdots, 9;$

$\sum_{n=1}^{\infty} \frac{x_i}{10^n}$ is convergent where x_i is one of the numbers, $0, 1, 2, \cdots, 9$;

If $0 < a_{n+7} \le b_n$ for $n = 1, 2, 3, \cdots$, $\sum_{n=1}^{\infty} a_n$ is convergent if $\sum_{n=1}^{\infty} b_n$ is convergent;

$\sum_{n=1}^{\infty} \frac{\log(n)}{n^p}$ is divergent if $p \le 1$;

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive series and

$\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$

then $\sum_{n=1}^{\infty} a_n$ is divergent if $\sum_{n=1}^{\infty} b_n$ is divergent;

If $0 \le a_n \le b_n$ for $n = 1, 2, 3, \cdots$, $\sum_{n=1}^{\infty} b_n$ is divergent if $\sum_{n=1}^{\infty} a_n$ is divergent;

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Question 10

0/10 pts 3 19

Find all the Truth(s):

- [ ] $\sum_{n=1}^{\infty} \frac{n!}{xn!}$ is convergent if x is positive integer greater than 1;
- [ ] Suppose that $a_1 = \frac{1}{2}, a_{n+1} = \left(1 + \frac{1}{n}\right) a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent;

- [ ] $\sum_{n=1}^{\infty} \frac{n}{9^n}$ is divergent;
- [ ] Suppose that $a_1 = \frac{1}{9}, a_{n+1} = \frac{3n-1}{4n+1} a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent;
- [ ] Suppose that $a_1 = \frac{1}{9}, a_{n+1} = \frac{\sin(n)}{n} a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent;
- [ ] $\sum_{n=0}^{\infty} (-1)^n \left(\frac{(-1)^n}{\sqrt{2n+1}} + \frac{(-1)^{n+1}}{(2n)^2} \right)$ is convergent;
- [ ] $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n} \right)$ is convergent and equal to $\frac{1}{2}$;
- [ ] $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$ is convergent;
- [ ] If $\sum_{n=1}^{\infty} \frac{3^n}{4^n + 2}$ is convergent;
- [ ] Suppose that $a_1 = 1, a_2 = \frac{1 \cdot 2}{1 \cdot 3}, \cdots, a_{n+1} = \frac{n+1}{2n+1} a_n$, $\sum_{n=1}^{\infty} a_n$ is divergent;

$\sum_{n=1}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)$ is convergent and equal to $\frac{1}{2}$;

Suppose that $a_1=\frac{1}{9}$, $a_{n+1}=\frac{3n-1}{4n+1}a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent;

Suppose that $a_1=\frac{1}{9}$, $a_{n+1}=\frac{\sin(n)}{n}a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent;

$\sum_{n=2}^{\infty}\frac{1}{(\log n)^n}$ is convergent;

$\sum_{n=1}^{\infty}\frac{n}{9^n}$ is divergent;

$\sum_{n=1}^{\infty}\frac{n!}{xn!}$ is convergent if x is positive integer greater than 1;

If $\sum_{n=1}^{\infty}\frac{3^n}{4^n+2}$ is convergent;

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Question 11

0/10 pts 3 19

1. Suppose that $e^{2x}=\sum_{n=0}^{\infty} a_n x^n$.
   - coefficient, a_n, is $\frac{2^n}{n!}$;
   - the series is convergent if x in interval, I, and $I=$

$(-\infty, \infty)$.

2. Suppose that $e^x = \sum_{n=0}^{\infty} a_n (x+4)^n$.

   - coefficient, a_n, is $e^{-4} \cdot \frac{(-1)^n}{n!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

3. Suppose that $x^2 e^x = \sum_{n=2}^{\infty} a_n x^n$.

   - coefficient, a_n, is $\frac{1}{(n-2)!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

4. Suppose that $(x-4)e^x = \sum_{n=0}^{\infty} a_n x^n$.

   - coefficient, a_n where $n \geq 1$, is $\frac{n-4}{n!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

5. Suppose that $e^{-\frac{x^2}{2}} = \sum_{n=0}^{\infty} a_n x^{2n}$.

   - coefficient, a_n, is $\frac{(-1)^n}{(2)^n n!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

Note:

1. Euler number, e, input by e, for example: e^2 = e^2 = exp(2),
2. Open interval, $\{x \in mathb\{R\} \mid a < x < b\}$, input by (a,b),
3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],
4. $n!$, factorial of n, input by n!.
5. ∞, positive infinty, input by oo.

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Question 12

0/10 pts 3 19

1. Suppose that $\sin x = \sum_{n=0}^{\infty} a_n x^{2n+1}$.
   - coefficient, a_n, is $\frac{(-1)^n}{(2n+1)!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.
2. Suppose that $\cos x = \sum_{n=0}^{\infty} a_n x^{2n}$.
   - coefficient, a_n, is $\frac{(-1)^n}{(2n)!}$;
   - the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

3. Suppose that $\cos(\sqrt{x}) = \sum_{n=0}^{\infty} a_n x^n.$

- coefficient, a_n, is $\frac{(-1)^n}{(2n)!}$;
- the series is convergent if x in interval, I, and $I =$ $[0, \infty)$

.

4. Consider the taylor series of $\sin(x)$ is expanded at $x = \frac{\pi}{2}$; the series could be represented as follows

$$\sin(x) = \sum_{n=0}^{\infty} a_n \left(x - \frac{\pi}{2}\right)^{2n}.$$

- coefficient, a_n where $n \geq 1$, is $\frac{(-1)^n}{(2n)!}$;
- the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

5. Consider the taylor series of $\sin(x) - \cos$ is expanded at $x = \frac{\pi}{4}$; the series could be represented as follows

$$\sin x - \cos x = \sum_{n=0}^{\infty} a_n \left(x - \frac{\pi}{4}\right)^{2n}.$$

- coefficient, a_n, is $\sqrt{2} \cdot \frac{(-1)^n}{(2n+1)!}$;
- the series is convergent if x in interval, I, and $I =$ $(-\infty, \infty)$.

Note:

1. Euler number, e, input by e, for example: e^2 = e^2 = exp(2),
2. Open interval, $\{x \in mathbb\{R\} \mid a < x < b\}$, input by (a,b),

3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],
4. $n!$, factorial of n, input by n!.
5. ∞, positive infinty, input by oo.

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Question 13

0/10 pts 3 19

As well known result:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1.$$

Consider the following questions:

1. Consider $\frac{3}{1+x}$, expandeded at $x = 0$, is $\sum_{n=0}^{\infty} a_n x^n$.
   - coefficient, a_n, is [ ] $3 \cdot \frac{(-1)^n}{1^{n+1}}$;
   - the series is convergent if x in interval, I, and $I =$ [ ] $(-1, 1)$.
2. Consider $\frac{1}{x}$, expandeded at $x = 5$, is $\sum_{n=0}^{\infty} a_n (x-5)^n$.
   - coefficient, a_n, is [ ] $\frac{(-1)^n}{5^{n+1}}$;
   - the series is convergent if x in interval, I, and $I =$ [ ] $(0, 2 \cdot 5)$.

3. Consider $\dfrac{1}{(3+x)^2}$, expandeded at $x=0$, is $\sum_{n=0}^{\infty} a_n x^n$.
   - coefficient, a_n, is [ ] $(-1)^n \cdot \dfrac{n+1}{3^{n+2}}$;
   - the series is convergent if x in interval, I, and $I =$ [ ] $(-3, 3)$.
4. Infinite series, $(1+x)^p$ where $p<0$, is called binary series,
   - the binary series is convergent if x in interval, I, and $I =$ [ ] $(-1, 1)$.
   - For $p = \dfrac{1}{2}$, and the series is expandand at $x=0$ is $\sum_{n=0}^{\infty} (-1)^n \dfrac{a_n}{2^{2n-1}(n!)^2} x^n$ where a_n is [ ] $n(2 \cdot n - 2)!$.
   - For $p = -\dfrac{1}{2}$, and the series is expandand at $x=0$ is $\sum_{n=0}^{\infty} (-1)^n \dfrac{a_n}{2^{2n}(n!)^2} x^n$ where a_n is [ ] $(2n)!$.
   - Consider the $f(x) = \dfrac{1}{(6+x)^{\frac{1}{3}}}$. Its binaray series is convergent for $x \in I$, i.e. I = [ ] $(-6, 6)$.

Note:

1. Euler number, e, input by e, for example: e^2 = e^2 = exp(2),

2. Open interval, $\{x \in \mathbb{R} \mid a < x < b\}$, input by (a,b),
3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],
4. $n!$, factorial of n, input by n!.
5. ∞, positive infinty, input by oo.

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Question 14

0/10 pts 3 19

As well known result:
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1.$$
Consider the following questions:

the infinite series, $(1+x)^p$ where $p < 0$, is called binary series,

1. The binary series, for any $p < 0$, is convergent if x in interval, I, and $I =$ $(-1,1)$.
2. For $p = \frac{1}{2}$, and the series is expandand at $x = 0$ is
$$\sum_{n=0}^{\infty} (-1)^n \frac{a_n}{2^{2n-1}(n!)^2} x^n \text{ where}$$
a_n is $n(2n-2)!$.
3. For $p = -\frac{1}{2}$, and the series is expandand at $x = 0$ is
$$\sum_{n=0}^{\infty} (-1)^n \frac{a_n}{2^{2n}(n!)^2} x^n \text{ where}$$
a_n is $(2n)!$.
4. Suppose that

$$f(x) = \frac{1}{(6+x)^{\frac{1}{3}}}.$$

This function is also a binaray series and is convergent for $x \in I$, i.e. I = [ ] $(-6, 6)$.

5. The convergent radius of binary series is [ ] 6 .

Note:

1. Euler number, e, input by e, for example: e^2 = e^2 input by exp(2),
2. Open interval, $\{x \in \mathbb{R} \mid a < x < b\}$, input by (a,b),
3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],
4. $n!$, factorial of n, input by n!.
5. ∞, positive infinty, input by oo.

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Question 15

0/10 pts 3 19

Follows the following steps to determine convergence of the series: $S = \sum_{n=1}^{\infty} \left(5^{\frac{1}{n}} - 4^{\frac{1}{n}}\right)^2$

1. Suppose that $f(x) = x^{\frac{1}{n}}$ for $x \in [4, 5]$. By Mean Value Theorem, we have
$$5^{\frac{1}{n}} - 4^{\frac{1}{n}} = A \cdot c^{\frac{1}{n}-1} \cdot (5-4) = \frac{1}{\frac{1}{A} \cdot c^{1-\frac{1}{n}}},$$
where $A =$ [ ] $\frac{1}{n}$, and $c \in$ [ ]

$(4, 5)$.

2. For $n \neq 1$,

   $$5^{\frac{1}{n}} - 4^{\frac{1}{n}} \leq A \cdot \frac{1}{4^{1-\frac{1}{1}}} = \frac{1}{n^p}$$

   where $p =$ [ ] 1

3. To determinae S whether is convergent or not, S is compared to the following p-series:

   $$\sum_{n=1}^{\infty} \left(\frac{1}{n^C} \right)$$

   where $C =$ [ ] 2

4. Therefore, the series, S, is

   - [ ] Divergent
   - [ ] Convergent

Note:

1. Euler number, e, input by e, for example: e^x = e^x input by exp(x),
2. Open interval, $\{x \in \mathbb{R} \mid a < x < b\}$, input by (a,b),
3. closed interval, $\{x \in \mathbb{R} \mid a \leq x \leq b\}$, input by [a,b],
4. $n!$, factorial of n, input by n!.
5. ∞, positive infinty, input by oo.

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Question 16 0/10 pts 3 19

As well-known result, the sum of alternating harmonic series is:

$$\sum_{i=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots = \log 2.$$

Then

1. $\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} \cdots =$ $\frac{\log(2)}{2}$
2. $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots =$ $3\frac{\log(2)}{2}$

Note

1 Natural logarithm inputs by $\log(x)$.

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Question 17

0/10 pts 3 19

Although only real series are considered in our course, but also could be applied in complex field. The Euler formula is the most famous one

$e^{i\theta} = \cos\theta + i\sin\theta,$

where $i = \sqrt{-1}$ is the pure unitary imaginary number.

1. The Mclaurin's series of e^x is

   $$e^x = \sum_{n=0}^{\infty} \frac{A_n}{n!},$$

   where $A_n =$ x^n
2. By Euler formula, $e^{in} = B + C \cdot i,$

   where $B =$ $\cos(n)$

and $C =$ [ ] $\sin(n)$

3. Consider

$$e^{e^i} = \sum_{n=0}^{\infty} \frac{D_n}{n!} = \sum_{n=0}^{\infty} \frac{E + F \cdot i}{n!}$$

$D_n =$ [ ] $\exp(in)$,

$E =$ [ ] $\cos(n)$,

$F =$ [ ] $\sin(n)$,

4. On the other hand,

$e^i = \cos(1) + G \cdot i$, where $G =$ [ ] $\sin(1)$,

implies

$e^{e^i} = e^{\cos(1)} \cdot (H + i \cdot I)$ where

$H =$ [ ] $\cos(\sin(1))$, and $I =$ [ ] $\sin(\sin(1))$,

5. Finally, we get the result:

$$\sum_{n=0}^{\infty} \frac{\sin(n)}{n!} =$$ [ ] $\exp(\cos(1))\sin(\sin(1))$

Note

1 The unitary imaginary number, $\sqrt{-1}$ inputs by i or sqrt(-1).

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Question 18 0/10 pts 3 19

Geometric series is covergent as follows:

[I], $\dfrac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$, for $|x| < 1$.

1. $\dfrac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n A$, for $|x| < B$, where

   $A =$ [ ] x^{2n} , $B =$ [ ] 1

2. Integrating both sides of [I] above from $x = 0$ to $x = 1^-$ gets

   a. $\displaystyle\int_0^1 \frac{dx}{1+x^2} =$ [ ] $\dfrac{\pi}{4}$

   b. $\displaystyle\int_0^1 \left(\sum_{n=0}^{\infty} (-1)^n A\right) dx = \sum_{n=0}^{\infty} (-1)^n \int_0^1 A dx = \sum_{n=0}^{\infty} (-1)^n C,$

   where $C =$ [ ] $\dfrac{1}{2n+1}$,

   And concludes the following:

   $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots =$ [ ] $\dfrac{\pi}{4}$

Now consider to evaluate the sum of convergent series sum:

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)(2n+1)}$$

1. $S = D\displaystyle\sum_{n=1}^{\infty} \left(\frac{(-1)^n}{2n-1} - E\right),$

   D = [ ] 2 , E = [ ] $\dfrac{(-1)^n}{2n+1}$

2. From above result, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} =$ [ ] $-\dfrac{\pi}{4}$

3. and $\sum_{n=1}^{\infty} E =$ [ ] -0.21460183660255

Finally,

$S =$ [ ] -0.28539816339745

Note

1 Natural logarithm inputs by $\log(x)$.

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Question 19 0/10 pts 3 19

Consider the following series:

$$S = \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+5)}$$

1. Find all the true statements about S?
   - ☐ convergent
   - ☐ absolute convergent
   - ☐ divergent
   - ☐ convergent by limit comparison test and p-series test
   - ☐ conditional convergent
   - ☐ divergent by partal sum test
   - ☐ convergent by p-series test
   - ☐ convergent by comparison test
   - ☐ divergent by n-term test
   - ☐ convergent by integral test

   convergent
   absolute convergent
   conditional convergent
   convergent by limit comparison test and p-series test
2. $S =$ ______ 0.25555555555556

Note
If S is divergent, input DNE.

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Question 20 ☑ 0/10 pts ⟲ 3 ⇄ 19

Suppose that $P = \{2, 3, 5, 7, 11, \cdots\}$ is the set of ordered prime number, and p_n is th n-th prime number in P; for instance: $p_4 = 7$. Suppose that

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

Multipling with $\left(-\frac{1}{2^s}\right)$ on both sides gets:

$$\left(1 - \frac{1}{2^s}\right)\zeta(s) = \left(1 - \frac{1}{2^s}\right)\left(\frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \cdots\right)$$

$$= \left(\frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \cdots\right) - \left(\frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + A + B \cdots\right)$$

$$= \frac{1}{1^s} + \frac{1}{3^s} + \frac{1}{5^s} + C + D + \cdots$$

Next, multipling with $\left(-\frac{1}{3^s}\right)$ on both sides above gets:

$$\left(1 - \frac{1}{2^s}\right)\left(1 - \frac{1}{3^s}\right)\zeta(s) = \left(1 - \frac{1}{3^s}\right)\left(\frac{1}{1^s} + \frac{1}{3^s} + \frac{1}{5^s} + C + D + \cdots\right)$$

$$= \left(\frac{1}{1^s} + \frac{1}{3^s} + \frac{1}{5^s} + C + D + \cdots\right) - \left(\frac{1}{3^s} + \frac{1}{9^s} + \frac{1}{15^s} + \cdots\right)$$

$$= \frac{1}{1^s} + \frac{1}{5^s} + E + F + \cdots$$

Next multipling with $\left(-\frac{1}{5^s}\right)$, at which 5 is the third prime in P, and proceed similar steps for other primes in P, and so on; if the series is convergent, we can conclude the following:

$$\zeta(s) \prod_{p_n \in P} (1 - p_n^s) = G$$

After all, we have the result

$$\zeta(s) = \frac{G}{\prod_{p_n \in P} (1 - p_n^s)}$$

where

A = $\frac{1}{8^s}$

B = $\frac{1}{(10)^s}$

C = $\frac{1}{7^s}$

D = $\frac{1}{9^s}$

E = $\frac{1}{7^s}$

F = $\frac{1}{11^s}$

G = 1

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Question 21

0/10 pts 3 19

Binary is defined as followd: $(1+x)^m = 1 + \sum_{n=1}^{\infty} C_n^m x^n$

where m is a real but not nonpositive integer,

$C_n^m = \dfrac{m(m-1)\cdots(m-n+1)}{n!}$, $(n!) = 1 \cdot 2 \cdot 3 \cdot \cdots n$

and is convergent for $|x| < 1$. Consider the following binany series, $m = -\dfrac{1}{2}$:

$$(1+x)^{-\frac{1}{2}} = 1 + \sum_{n=1}^{\infty} C_n^{-\frac{1}{2}} x^n$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ \cdot \ \cdot A}{n!} B^n$$

Thus let $x = -\dfrac{1}{2}$, which its abosulte value is smaller than 1, we have:

$$1 + \frac{1}{4} + \frac{1 \times 3}{4 \times 8} + \frac{1 \times 3 \times 5}{4 \times 8 \times 12} + \cdots = D$$

where

A = $2n-1$

B = $-\dfrac{x}{2}$

D = $\sqrt{2}$

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Question 22 0/10 pts 3 19

Suppose the Fibonicci sequences is defined as follows:

$a_1 = a_2 = 1,$

$a_{n+2} = a_{n+1} + a_n$ where $n \geq 1$.
Then

1. the 6-term of this sequence is [ ] 8 .
2. Following steps below to represent a_n as closed form, i.e. a_n is in the form ar^n where a, r are constants to be determined as follows:
   $a_{n+2} = a_{n+1} + a_n$
   $\Rightarrow ar^{n+2} = ar^{n+1} + ar^n$
   $\Rightarrow r^2 + A - 1 = 0$
   $\Rightarrow r = r_1$ or r_2, where $r_2 < 0 < r_1$
   a. $A =$ [ ] $-r$
   b. r_1= [ ] $\frac{1+\sqrt{5}}{2}$, r_2= [ ] $\frac{1-\sqrt{5}}{2}$
   c. Thus we can represent the closed form of a_n as follows:
      $a_n = Cr_1^n + Dr_2^n$ for $n \geq 1$
      where
      C, D = [ ] $\frac{1}{\sqrt{5}}$, [ ] $-\frac{1}{\sqrt{5}}$
   d. Evaluate the following limit:
      $$\lim_{n\to\infty} \frac{a_{n+2}}{2^n} = \lim_{n\to\infty} \frac{1}{\sqrt{5}}\left(E\left(\frac{r_1}{2}\right)^n + F\left(\frac{r_2}{2}\right)^n\right) = G$$
      where
      E= [ ] $\frac{3+\sqrt{5}}{2}$
      F= [ ] $-\frac{3-\sqrt{5}}{2}$
      G= [ ] 0

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Question 23

0/10 pts 3 19

Given the sequence, $\{a_n\}$ where $a_n = 1 - (-1)^n$, determine at which statement(s) is(are) right?

- [ ] Sequence is bounded above,
- [ ] Sequence is bounded below,
- [ ] Sequence is divergent,
- [ ] Sequence is monotonic,
- [ ] Sequence is convergent and the limit of sequence is 0,
- [ ] Sequence is convergent since sequence is bounded and monotonic,

Sequence is bounded above,
Sequence is bounded below,
Sequence is divergent,

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Question 24

0/10 pts 3 19

Given the sequence, $\{a_n\}$ where $a_n = \left(1+\frac{2}{n}\right)^n$, determine at which statement(s) is(are) right?

- [ ] Sequence is bounded below,
- [ ] Sequence is divergent,
- [ ] Sequence is bounded above,
- [ ] Sequence is monotonic and decreasing,
- [ ] Sequence is monotonic and increasing,
- [ ] Sequence is convergent and the limit of sequence is e^{-2},

Sequence is monotonic and increasing,
Sequence is bounded above,
Sequence is bounded below,

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Question 25

0/10 pts 3 19

Find out all the statement(s) at which is(are) TRUE:

- [ ] Sequences diverge that approaches either ∞ or $-\infty$,
- [ ] If $a_{n+2} = a_{n+1} + a_n$ where $a_0 = a_1 = 1$, define $\{b_n\} = \frac{a_{n+1}}{a_n}$ for $n \geq 1$; $\{b_n\}$ is convergent to $\frac{1+\sqrt{5}}{2}$,
- [ ] If $\{a_n\}$ converges, then $\lim\limits_{n\to\infty} \frac{a_n}{n} = 0$,

- [ ] The limit of sequences of $\left\{\sqrt{4}, \sqrt{4+\sqrt{4}},\ +\sqrt{4+\sqrt{4+\sqrt{4}}}, \cdots\right\}$ exists and is irrational,
- [ ] The limit of sequences of irrational numbers is irrational if exists,
- [ ] The limit of sequences of rational numbers can not be irrational if exists,
- [ ] If $a_{n+2} = a_{n+1} + a_n$ where $a_0 = a_1 = 1$, define $\{b_n\} = \dfrac{a_{n+1}}{a_n}$ for $n \geq 1$; $\{b_n\}$ is convergent to 1,
- [ ] If $\{a_n\}$ converges and $\{b_n\}$ diverges, then $\{a_n - b_n\}$ converges,
- [ ] If $a_{n+2} = a_{n+1} + a_n$ where $a_0 = a_1 = 1$, define $\{b_n\} = \dfrac{a_{n+1}}{a_n}$ for $n \geq 1$; $\{b_n\}$ is divergent,
- [ ] If $\{a_n\}$ converges, then $\lim\limits_{n\to\infty} (a_n - a_{n+1}) = 0$,

If $\{a_n\}$ converges, then $\lim\limits_{n\to\infty} (a_n - a_{n+1}) = 0$,

If $\{a_n\}$ converges, then $\lim\limits_{n\to\infty} \dfrac{a_n}{n} = 0$,

If $a_{n+2} = a_{n+1} + a_n$ where $a_0 = a_1 = 1$, define $\{b_n\} = \dfrac{a_{n+1}}{a_n}$ for $n \geq 1$; $\{b_n\}$ is convergent to $\dfrac{1+\sqrt{5}}{2}$,

The limit of sequences of $\left\{\sqrt{4}, \sqrt{4+\sqrt{4}},\ +\sqrt{4+\sqrt{4+\sqrt{4}}}, \cdots\right\}$ exists and is irrational,

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Question 26 0/10 pts 3 19

Define the following recursive sequences, $\{a_n\}$, as follows:

$a_0 = \sqrt{x}$

$a_1 = \sqrt{x + 2\sqrt{a_0}}$

$\dots$

$a_{n+1} = \sqrt{x + 2\sqrt{a_n}}$, where $n \geq 1$

and this sequences is convergent for $|x| < 1$ by The Completed Axiom.

In other words, suppose that y is the limit of sequences:

$$y = \lim_{n \to \infty} a_n = \sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{\dots}}}}$$

1. For $|x| < 1$, find the closed form of y, i.e. $y = f(x)$ where
   $f(x)$ = $1 + \sqrt{x+1}$.
2. Evaluate the following definite integration:
   $\int_0^1 y dx$ = 2.2189514164975 .

Note

1. $\sqrt{x}$, square root of x, input by sqrt(x).
2. x^r, r-power of x, input y x^r

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