Lab 4: Velocity Kinematics

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○ Method

This is how I set up the coordinate frames according to DH convention.

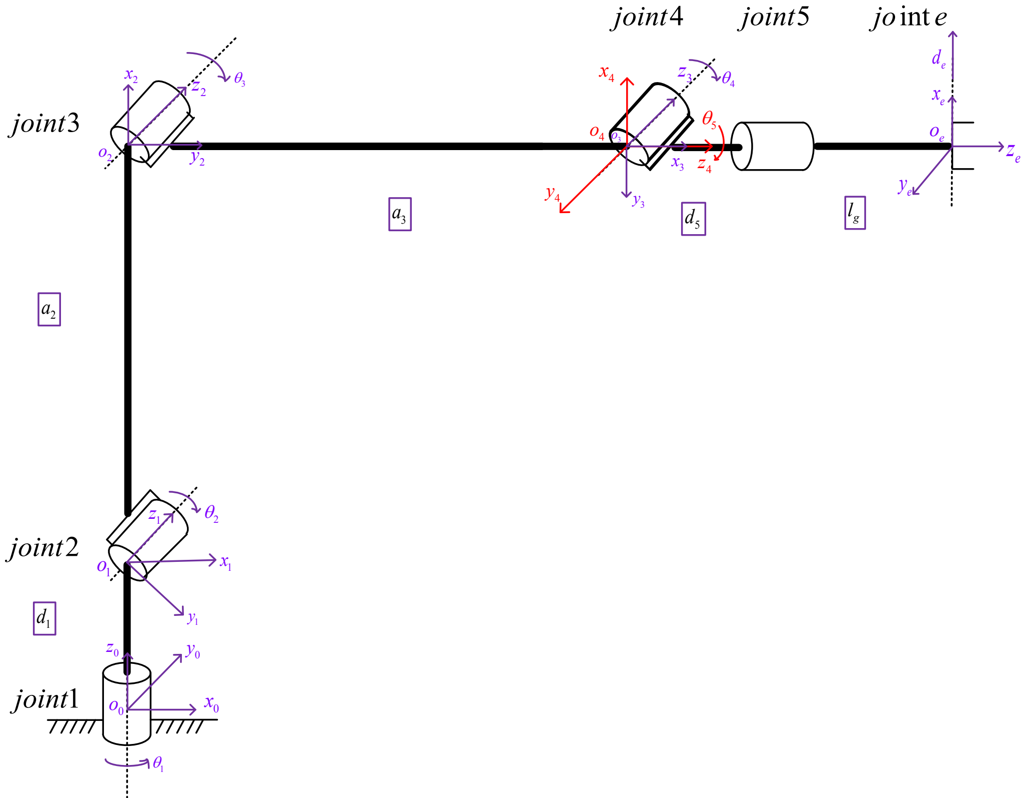


Figure 1 lynx coordinate frames

Link	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	d_1	θ_1^*
2	0	a_2	0	$\theta_2^* - \pi/2$
3	0	a_3	0	$\theta_3^* + \pi/2$
4	$-\pi/2$	0	0	$\theta_4 - \pi/2$
5	0	0	$d_5 + l_g$	θ_5

Table 1 DH parameter

We use two methods to compute forward Jacobian matrices including direct partial differential method and geometrical method. These two methods only differ in the way to compute linear velocity Jacobian matrix.

1. Direct Partial Differential Method using MATLAB symbolic toolbox (method 1)

$$T_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\times\cos(\alpha_i) & \sin(\theta_i)\times\sin(\alpha_i) & a_i\times\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\times\cos(\alpha_i) & -\cos(\theta_i)\times\sin(\alpha_i) & a_i\times\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = H_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = H_2^1 = \begin{bmatrix} \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ -\cos\theta_2 & \sin\theta_2 & 0 & -a_2\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = H_3^2 = \begin{bmatrix} -\sin\theta_3 & -\cos\theta_3 & 0 & -a_3\sin\theta_3 \\ \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4 = H_4^3 = \begin{bmatrix} \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ -\cos\theta_4 & 0 & \sin\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = H_5^4 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 + l_g \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_e^0 = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$$

(1) Linear velocity

In total, I use direct partial derivative to compute linear velocity. The position of end effector is saved in the 4th column of total transformation matrix $position = \begin{bmatrix} x & y & z \end{bmatrix}^T = T_e^0(1:3,4)$.

According to definition, the linear velocity Jacobian equals

$$\vec{V} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial t} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{6} \frac{\partial x}{\partial q_i}\frac{dq_i}{dt} \\ \sum_{i=1}^{6} \frac{\partial y}{\partial q_i}\frac{dq_i}{dt} \\ \sum_{i=1}^{6} \frac{\partial z}{\partial q_i}\frac{dq_i}{dt} \end{bmatrix} = J_v(\vec{q}) \cdot \dot{\vec{q}} = J_v \cdot \dot{\vec{q}} = J_v \cdot \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}^T$$

$$J_v(\vec{q}) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} & \frac{\partial x}{\partial q_4} & \frac{\partial x}{\partial q_5} & \frac{\partial x}{\partial q_6} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} & \frac{\partial y}{\partial q_4} & \frac{\partial y}{\partial q_5} & \frac{\partial y}{\partial q_6} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} & \frac{\partial z}{\partial q_4} & \frac{\partial z}{\partial q_5} & \frac{\partial z}{\partial q_6} \end{bmatrix}_{3\times 6}$$

$$J_v(\vec{q}) = \begin{bmatrix} J_{v\_11} & a_2\cos(q_1)\cos(q_2) - \#9 - \#5 - \#4 & -\#9 - \#5 - \#4 & -\#9 & 0 & 0 \\ J_{v\_21} & a_2\sin(q_1)\cos(q_2) - \#8 - \#3 - \#2 & -\#8 - \#3 - \#2 & -\#8 & 0 & 0 \\ 0 & \#6 - a_2\sin(q_2) - \#7 - \#1 & \#6 - \#7 - \#1 & -\#1 & 0 & 0 \end{bmatrix}$$

where

$$J_{v\_11} = \frac{\partial x}{\partial q_1} = \left(d_5 + l_g\right)\left(\cos(q_4)\#10 + \sin(q_4)\#11\right) - a_2\sin(q_1)\sin(q_2) - a_3\sin(q_1)\cos(q_2)\cos(q_3) + a_3\sin(q_1)\sin(q_2)\sin(q_3)$$

$$J_{v\_21} = \frac{\partial y}{\partial q_1} = \left(d_5 + l_g\right)\left(\cos(q_4)\#12 - \sin(q_4)\#13\right) + a_2\cos(q_1)\sin(q_2) + a_3\cos(q_1)\cos(q_2)\cos(q_3) - a_3\cos(q_1)\sin(q_2)\sin(q_3)$$

$$\#1 = \left(d_5 + l_g\right)\left(\cos(q_4)(\cos(q_2)\cos(q_3) - \sin(q_2)\sin(q_3)) - \sin(q_4)(\cos(q_2)\sin(q_3) + \sin(q_2)\cos(q_3))\right)$$

$$\#2 = a_3\sin(q_1)\sin(q_2)\cos(q_3)$$

$$\#3 = a_3\sin(q_1)\cos(q_2)\sin(q_3)$$

$$\#4 = a_3\cos(q_1)\sin(q_2)\cos(q_3)$$

$$\#5 = a_3\cos(q_1)\cos(q_2)\sin(q_3)$$

$$\#6 = a_3\sin(q_2)\sin(q_3)$$

$$\#7 = a_3\cos(q_2)\cos(q_3)$$

$$\#8 = \left(d_5 + l_g\right)\left(\cos(q_4)\#11 - \sin(q_4)\#10\right)$$

$$\#9 = \left(d_5 + l_g\right)\left(\cos(q_4)\#13 + \sin(q_4)\#12\right)$$

$$\#10 = \sin(q_1)\sin(q_2)\sin(q_3) - \sin(q_1)\cos(q_2)\cos(q_3)$$

$$\#11 = \sin(q_1)\cos(q_2)\sin(q_3) + \sin(q_1)\sin(q_2)\cos(q_3)$$

$$\#12 = \cos(q_1)\cos(q_2)\cos(q_3) - \cos(q_1)\sin(q_2)\sin(q_3)$$

$$\#13 = \cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\sin(q_2)\cos(q_3)$$

(2) Angular velocity

According to definition, the angular velocity Jacobian matrix is that

$$\vec{w} = \begin{bmatrix} w_x & w_y & w_z \end{bmatrix}^T = J_w \cdot \dot{\vec{q}} = J_w \cdot \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}^T$$

$$J_w(\vec{q}) = \sum_{i=1}^{6}\left(R_{i-1}^0 \cdot \hat{z}\right) = \begin{bmatrix} R_0^0 \cdot \hat{z} & R_1^0 \cdot \hat{z} & R_2^0 \cdot \hat{z} & R_3^0 \cdot \hat{z} & R_4^0 \cdot \hat{z} & R_5^0 \cdot \hat{z} \end{bmatrix}_{3\times 6}$$ Where $\hat{z} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

Actually, $\left(R_{i-1}^0 \cdot \hat{z}\right)$ represents coordinates of axis which q_i rotate around with respect to base frame (for revolute joint).

In this way, for revolute joint 1,2,3,4 and 5,

$R_0^0 \cdot \hat{z} = \left[\vec{z}_0\right]_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ $\quad R_1^0 \cdot \hat{z} = \left[\vec{z}_1\right]_0 = R_1^0 \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T = \left[T_1^0\right](1:3,3)$

$R_2^0 \cdot \hat{z} = \left[\vec{z}_2\right]_0 = R_1^0 \cdot R_2^1 \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T = \left[T_1^0 \cdot T_2^1\right](1:3,3)$ $\quad R_3^0 \cdot \hat{z} = \left[\vec{z}_3\right]_0 = R_1^0 \cdot R_2^1 \cdot R_3^2 \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T = \left[T_1^0 \cdot T_2^1 \cdot T_3^2\right](1:3,3)$

$R_4^0 \cdot \hat{z} = \left[\vec{z}_4\right]_0 = R_1^0 \cdot R_2^1 \cdot R_3^2 \cdot R_4^3 \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T = \left[T_1^0 \cdot T_2^1 \cdot T_3^2 \cdot T_4^3\right](1:3,3)$

For translational joint 6, it has nothing to do with angular velocity so $R_5^0 \cdot \hat{z} = [0 \quad 0 \quad 0]^T$.

$$J_w(\vec{q}) = \begin{bmatrix} 0 & -\sin(q_1) & -\sin(q_1) & -\sin(q_1) & J_{w\_15} & 0 \\ 0 & \cos(q_1) & \cos(q_1) & \cos(q_1) & J_{w\_25} & 0 \\ 1 & 0 & 0 & 0 & J_{w\_35} & 0 \end{bmatrix}$$

$$\begin{bmatrix} J_{w\_15} & J_{w\_25} & J_{w\_35} \end{bmatrix}^T = R_4^0 \cdot \hat{z}$$

$$J_{w\_15} = \cos(q_4)(\cos(q_1)\cos(q_2)\cos(q_3) - \cos(q_1)\sin(q_2)\sin(q_3)) - \sin(q_4)(\cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\sin(q_2)\cos(q_3))$$

$$J_{w\_25} = -\cos(q_4)(\sin(q_1)\sin(q_2)\sin(q_3) - \sin(q_1)\cos(q_2)\cos(q_3)) - \sin(q_4)(\sin(q_1)\cos(q_2)\sin(q_3) + \sin(q_1)\sin(q_2)\cos(q_3))$$

$$J_{w\_35} = -\cos(q_4)(\cos(q_2)\sin(q_3) + \sin(q_2)\cos(q_3)) - \sin(q_4)(\cos(q_2)\cos(q_3) - \sin(q_2)\sin(q_3))$$

Obviously, using some trigonometry knowledge, we can simplify that

$J_{w\_15} = \cos(q_1)\cos(q_2 + q_3 + q_4)$, $J_{w\_25} = \sin(q_1)\cos(q_2 + q_3 + q_4)$, $J_{w\_35} = -\sin(q_2 + q_3 + q_4)$

In the end, I can get that

$$J_w(\vec{q}) = \begin{bmatrix} 0 & -\sin(q_1) & -\sin(q_1) & -\sin(q_1) & \cos(q_1)\cos(q_2 + q_3 + q_4) & 0 \\ 0 & \cos(q_1) & \cos(q_1) & \cos(q_1) & \sin(q_1)\cos(q_2 + q_3 + q_4) & 0 \\ 1 & 0 & 0 & 0 & -\sin(q_2 + q_3 + q_4) & 0 \end{bmatrix}$$

(3) Summary

Finally, we get that

$$\begin{bmatrix} \vec{V} \\ \hline \vec{w} \end{bmatrix} = \begin{bmatrix} V_x & V_y & V_z & w_x & w_y & w_z \end{bmatrix}^T = J(\vec{q}) \cdot \dot{\vec{q}} = \begin{bmatrix} J_v(\vec{q}) \\ \hline J_w(\vec{q}) \end{bmatrix} \cdot \dot{\vec{q}} = \begin{bmatrix} J_v(\vec{q}) \\ \hline J_w(\vec{q}) \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}^T$$

Although this method is clear to understand and easy to execute, the running speed is really slow. We decide to use the second method during the whole experiment.

2. Geometrical Method using cross product (method 2)

(1) Linear velocity

(a) For prismatic joint i

$$T_n^0 = \begin{bmatrix} R_n^0 & o_n^0 \\ 0 & 1 \end{bmatrix} = T_{i-1}^0 \bullet T_i^{i-1} \bullet T_i^0 = \begin{bmatrix} R_{i-1}^0 & o_{i-1}^0 \\ 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} R_i^{i-1} & o_i^{i-1} \\ 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} R_i^0 & o_i^0 \\ 0 & 1 \end{bmatrix}$$

Or

$$T_n^0 = \begin{bmatrix} R_n^0 & R_i^0 o_n^i + R_{i-1}^0 o_i^{i-1} + o_{i-1}^0 \\ 0 & 1 \end{bmatrix}$$

Which gives that

$$o_n^0 = R_i^0 o_n^i + R_{i-1}^0 o_i^{i-1} + o_{i-1}^0$$

Obviously, if we only move joint i o_n^i o_{i-1}^0 and R_{i-1}^0 are constant

Differentiating the equation above, we get that

$$\frac{\partial o_n^0}{\partial q_i} = \frac{\partial}{\partial q_i} R_{i-1}^0 o_i^{i-1} = R_{i-1}^0 \frac{\partial}{\partial d_i} \begin{bmatrix} 0 \\ 0 \\ d_i \end{bmatrix} = \dot{d}_i R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \dot{d}_i \hat{z}_{i-1}^0$$

So we can know that, for prismatic joint i, $J_{vi} = \hat{z}_{i-1}^0$

(b) For revolute joint i

$q_i = \theta_i$ At that time, only o_{i-1}^0 is constant

$$\frac{\partial o_n^0}{\partial q_i} = \frac{\partial}{\partial \theta_i}\left[R_i^0 o_n^i + R_{i-1}^0 o_i^{i-1}\right] = \frac{\partial}{\partial \theta_i} R_i^0 o_n^i + R_{i-1}^0 \bullet \frac{\partial}{\partial \theta_i} o_i^{i-1} = \dot{\theta}_i S\left(z_{i-1}^0\right) R_i^0 o_n^i + \dot{\theta}_i S\left(z_{i-1}^0\right) R_{i-1}^0 o_i^{i-1} = \dot{\theta}_i S\left(z_{i-1}^0\right)\left[R_i^0 o_n^i + R_{i-1}^0 o_i^{i-1}\right]$$

Or $$\frac{\partial o_n^0}{\partial \theta_i} = \dot{\theta}_i S\left(z_{i-1}^0\right)\left(o_n^0 - o_{i-1}^0\right) = \dot{\theta}_i z_{i-1}^0 \times \left(o_n^0 - o_{i-1}^0\right)$$

In this way, $J_{vi} = \hat{z}_{i-1}^0 \times \left(o_n^0 - o_{i-1}^0\right)$

(2) Angular velocity

The method to find the angular velocity is totally the same as before.

(3) Forward kinematics

The upper part for Jacobian (linear velocity part) is given by

$$J_v = \left[J_{v_1} \mid J_{v_2} \mid \ldots \mid J_{v_n}\right] = \left[J_{v_1} \mid J_{v_2} \mid J_{v_3} \mid J_{v_4} \mid J_{v_5} \mid J_{v_6}\right]_{3\times 6}$$

Where for ith column of J_v, is that

$$J_{v_i} = \begin{cases} \hat{z}_{i-1}^0 \times \left(o_n^0 - o_{i-1}^0\right) & for\_revolute\_joint\_i \\ \hat{z}_{i-1}^0 & for\_prismatic\_joint\_i \end{cases}$$

The lower part for Jacobian (angular velocity part) is given by

$$J_w = \left[J_{w_1} \mid J_{w_2} \mid \ldots \mid J_{w_n}\right] = \left[J_{w_1} \mid J_{w_2} \mid J_{w_3} \mid J_{w_4} \mid J_{w_5} \mid J_{w_6}\right]_{3\times 6}$$

Where for ith column of J_w, is that

$$J_{w_i} = \begin{cases} \hat{z}_{i-1}^0 & for\_revolute\_joint\_i \\ 0 & for\_prismatic\_joint\_i \end{cases}$$

In a word, for revolute joint i

$$J_i = \begin{bmatrix} J_{vi} \\ J_{w_i} \end{bmatrix} = \begin{bmatrix} \hat{z}_{i-1}^0 \times \left(o_n^0 - o_{i-1}^0\right) \\ \hat{z}_{i-1}^0 \end{bmatrix}$$

For prismatic joint i

$$J_i = \begin{bmatrix} J_{vi} \\ J_{w_i} \end{bmatrix} = \begin{bmatrix} \hat{z}_{i-1}^0 \\ 0 \end{bmatrix}$$

Although this method seems to be difficult. It has nothing to do with symbolic representation so it runs really fast. We decide to use this method during the whole experiment.

Finally, the forward kinematic equation can be expressed as following. The coding work is shown in function e_vel = FK_velocity(q, qdot).

$$\begin{bmatrix} \vec{V} \\ \hline \vec{w} \end{bmatrix} = \begin{bmatrix} V_x & V_y & V_z & w_x & w_y & w_z \end{bmatrix}^T = J(\vec{q}) \cdot \dot{\vec{q}} = \begin{bmatrix} J_v(\vec{q}) \\ \hline J_w(\vec{q}) \end{bmatrix} \cdot \dot{\vec{q}}$$

So $\zeta = \begin{bmatrix} \vec{V} \\ \hline \vec{w} \end{bmatrix} = \left[J_1 \mid J_2 \mid J_3 \mid J_4 \mid J_5 \mid J_6\right] \cdot \left[\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad \dot{q}_4 \quad \dot{q}_5 \quad \dot{q}_6\right]^T$

Or $\zeta = \left[J_1 \mid J_2 \mid J_3 \mid J_4 \mid J_5 \mid 0_{6\times1}\right] \cdot \left[\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad \dot{q}_4 \quad \dot{q}_5 \quad 0\right]^T$

(4) Inverse kinematics

Since actually J is 6*5 matrix, we use pseudoinverse to solve inverse kinematic problems of velocity.

$$\zeta_{6\times1} = J_{6\times5}\dot{q}_{5\times1}$$

$\left(J^T J\right)_{5\times5}$ is 5×5 and $\left(J^T J\right)^{-1}$ exists.

$I = \left(J^T J\right)^{-1} J^T J = J^+ J$ where $J^+ = \left(J^T J\right)^{-1} J^T$

If a solution $\dot{q}$ exists, then $\dot{q}' = J^+\zeta + (I - J^+ J)b = J^+\zeta$ is a solution.

A solution to equation $\zeta_{6\times1} = J_{6\times5}\dot{q}$ is given by $\dot{q} = J^+\zeta$. This can be simply demonstrate that

$$J \cdot \dot{q} = J \cdot J^+ \cdot \zeta = J \cdot \left(J^T \cdot J\right)^{-1} \cdot J^T \cdot \zeta$$

$$= J \cdot \left(J^{-1} \cdot \left(J^T\right)^{-1}\right) \cdot J^T \cdot \zeta = J \cdot J^{-1} \cdot \left(J^T\right)^{-1} \cdot J^T \cdot \zeta = \zeta$$

Totally, we construct a pseudoinverse Jacobian matrix J^+ which is $J^+ = \left(J^T J\right)^{-1} J^T$. The inverse kinematic can be expressed as $\dot{q} = J^+\zeta$. The coding work is shown in function qdot = IK_velocity(q, e_vel).

○ Evaluation

1. Different kinds of end-effector motion

(1) Only One joint moving motion

During this experiment, we set one joint move at constant speed while other joint keep fixed. We start the experiment in zero configuration which $q=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$.

$$\begin{cases} \dot{q}_k = cons \\ \dot{q}_j = 0, j \neq k \end{cases} \quad k=1,2,3,4,5 \text{ , } j=1,2,3,4,5$$

(a) Only joint 1 move $\dot{q}_1 = 0.01 rad/s$

When only joint 1 moves, end effector can only have linear velocity in x, y direction and angular velocity in z direction where $w_z = c = 0.01 rad/s$.

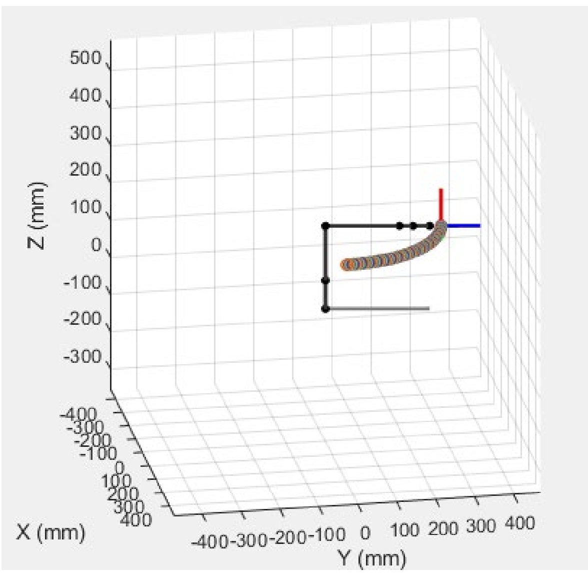


Figure 2 only joint 1 moving motion

There are some data about end effector velocity to prove it.

$e\_evl = \begin{bmatrix} v_x & v_y & v_z & w_x & w_y & w_z \end{bmatrix}$

v_x	v_y	v_z	w_x	w_y	w_z
0	2.92	0	0	0	0.01
-0.03	2.92	0	0	0	0.01
-0.06	2.92	0	0	0	0.01
-0.09	2.92	0	0	0	0.01
-0.12	2.92	0	0	0	0.01
-0.15	2.92	0	0	0	0.01
-0.18	2.92	0	0	0	0.01
-0.2	2.91	0	0	0	0.01
-0.23	2.91	0	0	0	0.01
-0.26	2.91	0	0	0	0.01
-0.29	2.91	0	0	0	0.01
-0.32	2.9	0	0	0	0.01
-0.35	2.9	0	0	0	0.01
-0.38	2.9	0	0	0	0.01

Table 2 joint one motion

(b) Only joint 2 move $\dot{q}_2 = 0.01 rad/s$

When only joint 2 moves, end effector can only have linear velocity in x, z direction and angular velocity in y direction where $w_y = c = 0.01 rad/s$ .There are some data about end effector velocity to prove it.

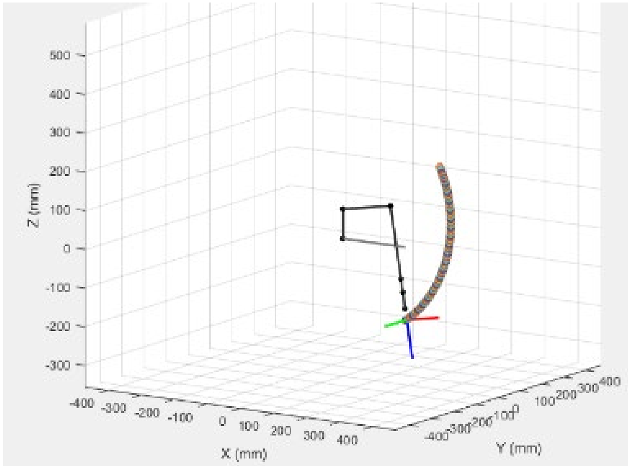


Figure 3 only joint 2 moving motion

$e\_evl = \begin{bmatrix} v_x & v_y & v_z & w_x & w_y & w_z \end{bmatrix}$

v_x	v_y	v_z	w_x	w_y	w_z
1.46	0	-2.92	0	0.01	0
1.43	0	-2.94	0	0.01	0
1.4	0	-2.95	0	0.01	0
1.37	0	-2.96	0	0.01	0
1.34	0	-2.98	0	0.01	0
1.31	0	-2.99	0	0.01	0
1.28	0	-3	0	0.01	0
1.25	0	-3.02	0	0.01	0
1.22	0	-3.03	0	0.01	0
1.19	0	-3.04	0	0.01	0
1.16	0	-3.05	0	0.01	0
1.13	0	-3.06	0	0.01	0
1.1	0	-3.07	0	0.01	0
1.07	0	-3.09	0	0.01	0

Table 3 joint two motion

(c) Only joint 3 move $\dot{q}_3 = 0.01 rad / s$

When only joint 3 moves, end effector can only have linear velocity in x, z direction and angular velocity in y direction where $w_y = c = 0.01 rad / s$.

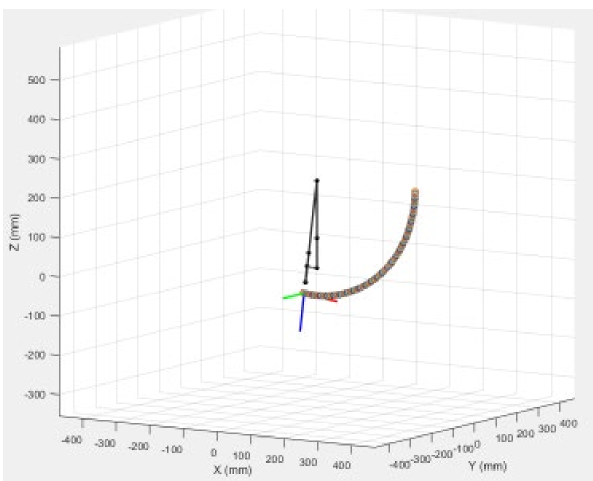


Figure 4 only joint 3 moving motion

There are some data about end effector velocity to prove it. $e\_evl = \begin{bmatrix} v_x & v_y & v_z & w_x & w_y & w_z \end{bmatrix}$

v_x	v_y	v_z	w_x	w_y	w_z
0	0	-2.92	0	0.01	0
-0.03	0	-2.92	0	0.01	0
-0.06	0	-2.92	0	0.01	0
-0.09	0	-2.92	0	0.01	0
-0.12	0	-2.92	0	0.01	0
-0.15	0	-2.92	0	0.01	0
-0.18	0	-2.92	0	0.01	0
-0.2	0	-2.91	0	0.01	0
-0.23	0	-2.91	0	0.01	0
-0.26	0	-2.91	0	0.01	0
-0.29	0	-2.91	0	0.01	0
-0.32	0	-2.9	0	0.01	0
-0.35	0	-2.9	0	0.01	0
-0.38	0	-2.9	0	0.01	0

Table 4 joint three motion

(d) Only joint 4 move $\dot{q}_4 = 0.01 rad / s$

When only joint 4 moves, end effector can only have linear velocity in x, z direction and angular velocity in y direction where $w_y = c = 0.01 rad / s$.

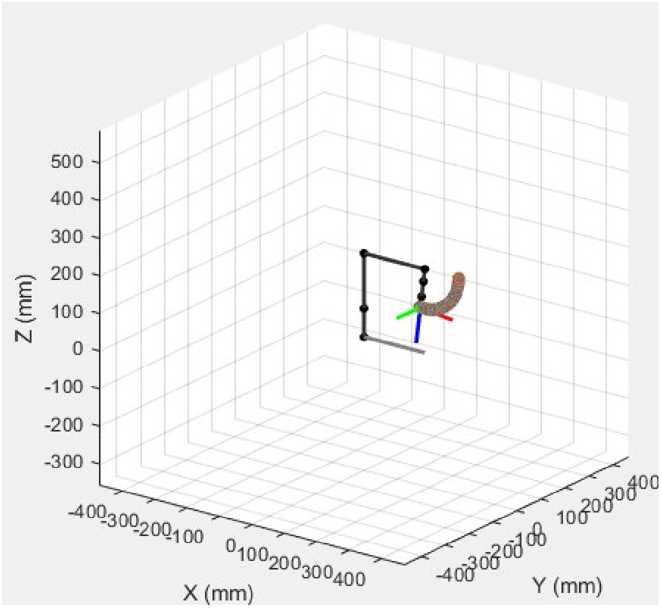


Figure 5 only joint 4 moving motion

There are some data about end effector velocity to prove it.

$e\_evl = \left[v_x \quad v_y \quad v_z \quad w_x \quad w_y \quad w_z \right]$

$\boxed{v_x}$	$\boxed{v_y}$	$\boxed{v_z}$	$\boxed{w_x}$	$\boxed{w_y}$	$\boxed{w_z}$
0	0	-1.05	0	0.01	0
-0.01	0	-1.05	0	0.01	0
-0.02	0	-1.05	0	0.01	0
-0.03	0	-1.05	0	0.01	0
-0.04	0	-1.05	0	0.01	0
-0.05	0	-1.05	0	0.01	0
-0.06	0	-1.05	0	0.01	0
-0.07	0	-1.05	0	0.01	0
-0.08	0	-1.04	0	0.01	0
-0.09	0	-1.04	0	0.01	0
-0.1	0	-1.04	0	0.01	0
-0.12	0	-1.04	0	0.01	0
-0.13	0	-1.04	0	0.01	0
-0.14	0	-1.04	0	0.01	0

Table 5 joint four motion

(e) Only joint 5 move $\dot{q}_5 = 0.01 rad / s$

When only joint 5 moves, end effector can only have angular velocity in x direction where $w_x = c = 0.01 rad / s$.

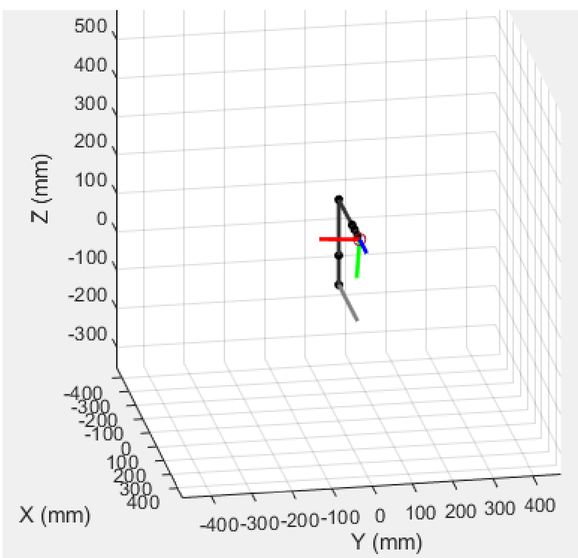


Figure 6 only joint 5 moving motion

There are some data about end effector velocity to prove it.

$e\_evl = \left[v_x \quad v_y \quad v_z \quad w_x \quad w_y \quad w_z \right]$

$\boxed{v_x}$	$\boxed{v_y}$	$\boxed{v_z}$	$\boxed{w_x}$	$\boxed{w_y}$	$\boxed{w_z}$
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0
0	0	0	0.01	0	0

Table 6 joint five motion

(2) all joints moving at a constant velocity

We set all joint angle move at constant speed which is 0.01 radian per second. $\dot{q}_1 = \dot{q}_2 = \dot{q}_3 = \dot{q}_4 = \dot{q}_5 = \pm 0.01 rad / s$

Table 7 joint upper and lower limit

variable	Rotation along which axis	Upper limit	lower limit
θ_1^*	z_0	-1.40	1.40
θ_2^*	z_1	-1.20	1.40
θ_3^*	z_2	-1.80	1.70
θ_4^*	z_3	-1.90	1.70
θ_5^*	z_4	-2	1.5

(a) All joints move from upper individual joint limit to lower joint limit with constant speed.

$$q_{start} = \begin{bmatrix} q_{1\_max} & q_{2\_max} & q_{3\_max} & q_{4\_max} & q_{5\_max} & 0 \end{bmatrix} = [1.4 \quad 1.4 \quad 1.7 \quad 1.7 \quad 1.5 \quad 0]$$

$$\dot{q} = [-0.01 \quad -0.01 \quad -0.01 \quad -0.01 \quad -0.01 \quad 0](rad/s)$$

(b) All joints move from lower individual joint limit to upper joint limit with constant speed.

$$q_{start} = \begin{bmatrix} q_{1\_min} & q_{2\_min} & q_{3\_min} & q_{4\_min} & q_{5\_min} & 0 \end{bmatrix} = [-1.4 \quad -1.2 \quad -1.8 \quad -1.9 \quad -2 \quad 0]$$

$$\dot{q} = [0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0](rad/s)$$

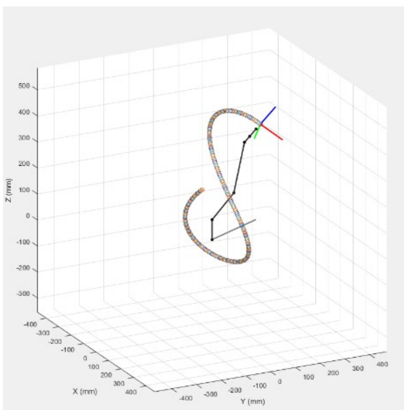


Figure 7 from upper limit to lower limit

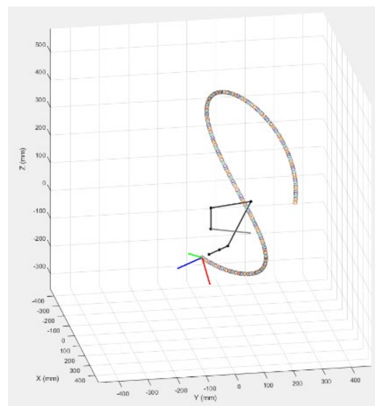


Figure 8 from lower limit to upper limit

2. joint angle trajectory corresponding to required end effector trajectory

(1) pseudocode example

This is **logistics** for drawing a circle trajectory on x-y plane.

q start from zero position

θ for drawing circle start from 0

loop for 250 times

{

tangential velocity changes as thetas changes

[note: model for tangential velocity in circle

Since $\begin{cases} x = r \times \cos\theta \\ y = r \times \sin\theta \end{cases}$ $\begin{cases} v_x = -r \times \sin\theta \times \dot{\theta} \\ v_y = r \times \cos\theta \times \dot{\theta} \end{cases}$]

use inverse kinematics to get angular velocity $\dot{q}$

update q : add angular velocity $\dot{q}$ to q

θ changes from 0 to 2π

use forward kinematics to get position of end effector

Plot end effector

}

(2) circle trajectory of end effector

We all start from zero position $q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

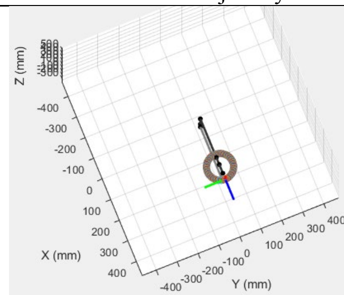
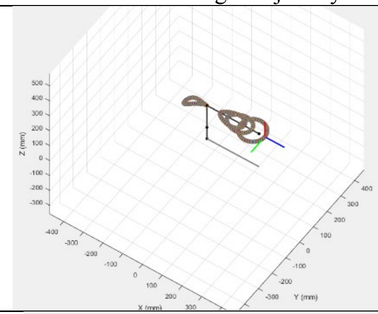
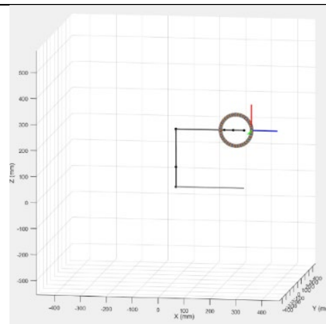
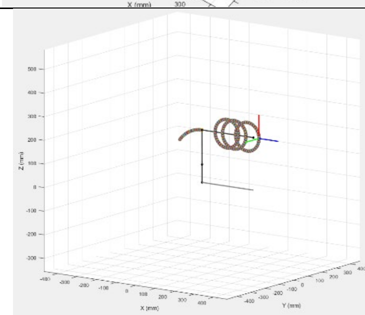
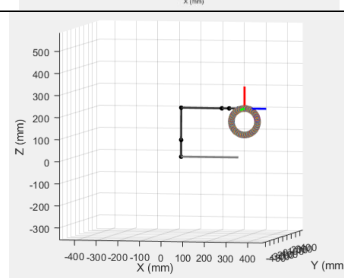
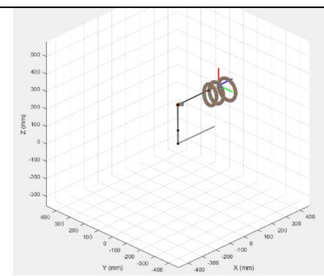
Plane	Circle equation (mm)	End effector trajectory	Joint angle trajectory
x-y plane	$x^2 + y^2 = r^2 = 60^2 = 3600$		
x-z plane	$x^2 + z^2 = r^2 = 60^2 = 3600$		
y-z plane	$y^2 + z^2 = r^2 = 60^2 = 3600$		

Table 8 circles in different plane

(3) line trajectory of end effector

(a) equation of straight line

To plot a straight line, we let the speed to be $v_x = 0.1rad/s$ $v_x = 0.2rad/s$ $v_z = 0.2rad/s$. Also, we start from zero position where $q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

In this way, the equation for straight line can be expressed as equation 1. Combines with our data, we can know express it as shown in equation two.

$$\begin{cases} x = v_x t + x_0 = 0.1t + x_0 \\ y = v_y t + y_0 = 0.2t + y_0 \text{....}equation\_1 \\ z = v_z t + z_0 = 0.2t + z_0 \end{cases} \qquad \begin{cases} x = 0.1t + 292.2021 \\ y = 0.2t + 0.2045 \text{ .....}equation\_2 \\ z = 0.2t + 222.456 \end{cases}$$

(b) Simulation (next page)

(c) Analysis

Obviously, when the trajectory of end effector is a straight line in 3D space, joint angle trajectory is not necessarily a straight line. It is decided by the current configuration and joint.

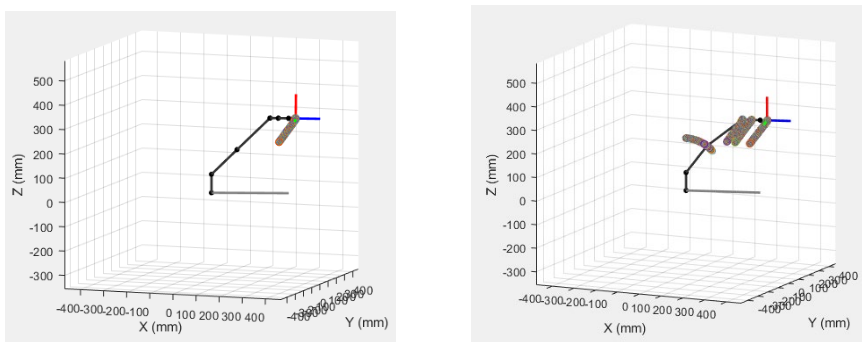


Figure 9 straight line

3. simultaneously controlling translation and orientation of end-effector

(1) pseudocode example

During the example, we control the velocity of end effector to be e_evl and start from zero configuration.

$$e\_evl = \begin{bmatrix} v_x & v_y & v_z & w_x & w_y & w_z \end{bmatrix} = [-1 \quad 1 \quad 0 \quad 0.01 \quad 0.01 \quad 0]$$

Our logistics is shown as below.

q start from zero position
set constant velocity and angular velocity e_evl
loop for 500 times
{
use inverse jacobian to get angular velocity with given tangential velocity
add angular velocity to q
use forward kinematics to get position of end effector
}

(2) simulation

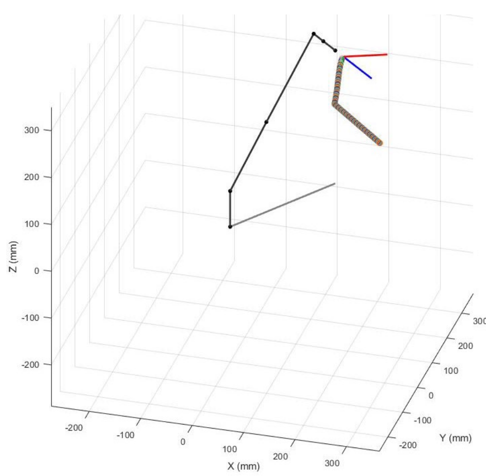


Figure 10 control translation and orientation

If i joint reached its joint limit, q_i remains the same. And the system keeps processing with the q_i and given velocity e_evl. The reason caused the derail of the wanted trajectory.

(3) Conclusion

We find that we can control end effector velocity components v_x, v_y, v_z, w_x and w_y as we want. In other word, we can control end-effector orientation in x_e and y_e direction. However, when lynx is in zero configuration, to control end-effector orientation in z_e direction, we must control end-effector velocity component w_z and v_y at the same time. We cannot control the end-effector velocity component w_z and v_y separately in zero configuration due to physical constraint of lynx itself.

4. Singularity

Singularity means that the robot losses some degree of freedom. There are three kinds of situations:

(1) Unbounded joint velocities

When $q = [q_1 \quad q_2 \quad -\pi/2 \quad 0 \quad 0 \quad 0](rad)$, where q_1 and q_2 are both in joint limit and nonzero, inverse kinematic problem for velocity is solvable. However, the computed angular velocities of joint 2 and joint 3 are unbounded. At these configurations, bounded end-effector velocities corresponds to unbounded joint velocities.

For instance when $q_1 = 0, q_2 = \pi/3$, $q = [0 \quad \pi/3 \quad -\pi/2 \quad 0 \quad 0 \quad 0]$, we can compute the Jacobian matrix (6*5) which is

$$J = \begin{bmatrix} 0 & 220.1873 & 146.5434 & 52.3784 & 0 \\ 378.8038 & 0 & 0 & 0 & 0 \\ 0 & -378.8038 & -252.6801 & -90.7430 & 0 \\ 0 & 0 & 0 & 0 & 0.8661 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0.4999 \end{bmatrix}_{6\times5}$$

Obviously, $rank(J) = 5$, Jacobian matrix is solvable but it is in singularity.

(2) Singularities on the boundaries of workspace

When we are on some points of the boundaries of workspace, inverse kinematic problem using Jacobian matrix is unsolvable. For instance, when $q = [0 \quad 0 \quad -\pi/2 \quad 0 \quad 0 \quad 0]$, $rank(J) = 3$ so that inverse kinematic problem for velocity is unsolvable. At that time, robot is fully stretched and the velocity of end-effector is only comprised of $v_x \quad w_y \quad w_z$. It loses its mobility of v_y, v_z and w_x.

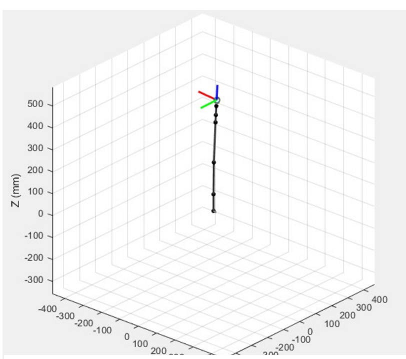


Figure 11 fully stretched robot

$$J = \begin{bmatrix} 0 & 438.15 & 292.1 & 104.775 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8661 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{6\times5}$$

(3) Singularities in z_0 axis

When end effector in z_0 axis, there will be infinite solution for inverse kinematics problem. At that time, the configuration can also be seen as a singularity.

○ Analysis

Forward velocity kinematics works well in every experiment. The velocity and angular velocity of end effector can be represented precisely with input of angular velocity of each joints. The reason of its accuracy and stableness is its derivation from forward kinematics.

Inverse velocity kinematics works not so well if the check of rank of Jacobian and its augmented Jacobian was implemented. Because the program takes six inputs of joint variables, which implies to control six degree of freedom; however, the Jacobian mostly has only five degree of freedom and causes the condition of non-existent inverse of Jacobian. This restricted phenomenon can be seen in the motion in y.

So, without implementing the rank check, the **pseudo inverse of Jacobian can mostly be solved**, and the end effector can reach the desired points; however, we cannot control the every six degree of freedom of end effector in each points due to decrease of the rank of Jacobian.

Certain tasks such as assembly pipeline require robotic arm to complete the mission in an amount of the time to achieve collaboration with other robots. In this context, velocity control would be better than position control, which is harder to program a continuous path and control the time of operation.