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SUMMARY 1

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Solution 1 (first part)**.** I learnt filtering, smoothing and decoding in the first few lecture.

All of the theories are based on the Markov assumption:

Given current state, the future event is conditionally independent to the previous states

$$P(X_n|X_{1:n-1}) = P(X_n|X_{n-1})$$

Filtering:

given the observation until current timestamp, what is the most possible state?

$$P(X_k|Y_{1:k})$$

Smoothing:

given the observations of all timestamps, what is the most possible state at timestamp t?

$$P(X_k|Y_{1:t}) \text{ where } t > k$$

Decoding:

given the observations of all timestamps, what is the most possible states sequence?

$$argmax_{X_{1:t}}(P(X_{1:t}|Y_{1:t})$$

In the homework, I also learnt:

update markov model

$$argmax_{M,T,X}(P(Y_{1:t}))$$

The calculation can be done with forward and backward algorithm:

$$\begin{aligned}\alpha_{k+1}(x_1) &= \alpha_k(x_1)T_{x_1,x_1}M_{x_1,y_{k+1}} \\ &\quad + \alpha_k(x_2)T_{x_2,x_1}M_{x_1,y_{k+1}} \\ \alpha_{k+1}(x_2) &= \alpha_k(x_1)T_{x_1,x_2}M_{x_2,y_{k+1}} \\ &\quad + \alpha_k(x_2)T_{x_2,x_2}M_{x_2,y_{k+1}}\end{aligned}$$

Forward algorithm says: given current state, the probability of next state

For the backward algorithm:

$$\begin{aligned}\beta_k(x_1) &= \beta_{k+1}(x_1)T_{x_1,x_1}M_{x_1,y_{k+1}} \\ &\quad + \beta_{k+1}(x_2)T_{x_1,x_2}M_{x_2,y_{k+1}} \\ \beta_k(x_2) &= \beta_{k+1}(x_1)T_{x_2,x_1}M_{x_1,y_{k+1}} \\ &\quad + \beta_{k+1}(x_2)T_{x_2,x_2}M_{x_2,y_{k+1}}\end{aligned}$$

The backward algorithm says: given current state, the probability of the previous state

Solution 2 (second part)**.** Kalman gain

The big concept is true state can never be got. The only way is to find the optimal estimate of the true state.

With several estimators: internal sensors, external measurements, how to merge all the estimations together to get the optimal estimate?

The system then can be separated into two categories: linear and non-linear.

$$\text{linear :}\hat{X} = aX_1 + bX_1$$

$$\text{non-linear :}\hat{X} = T(a)X_1 + T(b)X_1 \text{ where } T() = \frac{\partial}{\partial}...$$

For linear system, we could use kalman filter since linear transformation of gaussian remains gaussian

For non-linear system: we have two choices:

1. linearized the system, which brings us the extended Kalman filter but the uncertainty populated along with the time

2.Unscented Kalman filter:

Even the dynamic is not linear. The close distribution shape could be got by dense sampling of the distribution

(1) number of sigma points is twice the dimension of the state
(2) propagate the state with previous belief
(3) model the prediction of states with the dynamic model
(4) modified observation using the UT(I don't really understand this part and would love to spend more time on this)
(5) calculate kalman gain