

# Web Science: Social Networks (Part 1 - Graphs)

CS 432/532

Old Dominion University

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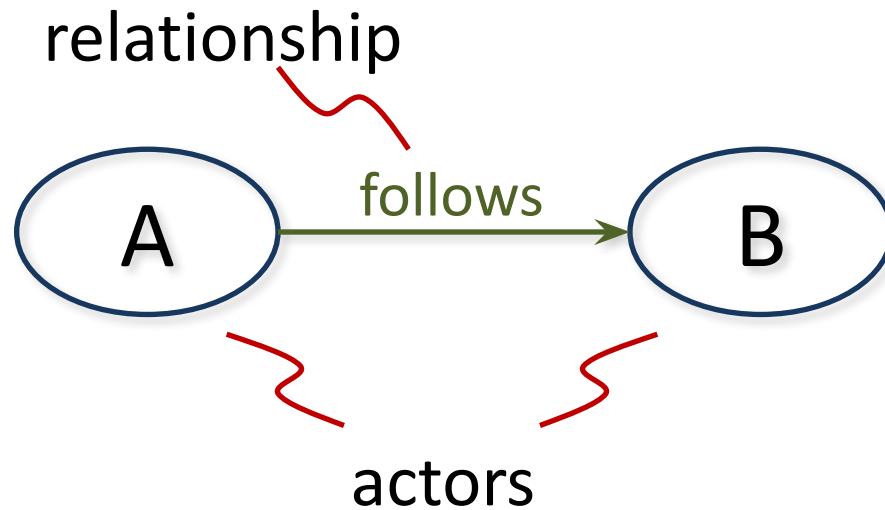
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Most slides based on Ch 2 of  
*Networks, Crowds and Markets* by  
Easley & Kleinberg (2010)  
(abbreviated as EK10)

*Kleinberg is the HITS guy (hubs and authorities)*

# Social Network

- Definition: Graph composed of *actors* (people, organizations, groups) that are tied by *social links*



# Online Social Networks (OSN)

- Digital incarnation of a social network
- Created by social network sites, social networking services
- Web-based services that allows users to:<sup>1</sup>
  - construct (semi-)public profiles within the system
  - "connect" to other users
  - view and traverse lists of connections created by others

<sup>1</sup>Boyd & Ellison (2007) - Social Network Sites: Definition, History, and Scholarship

**facebook**

**orkut**

**digg**

**flickr**



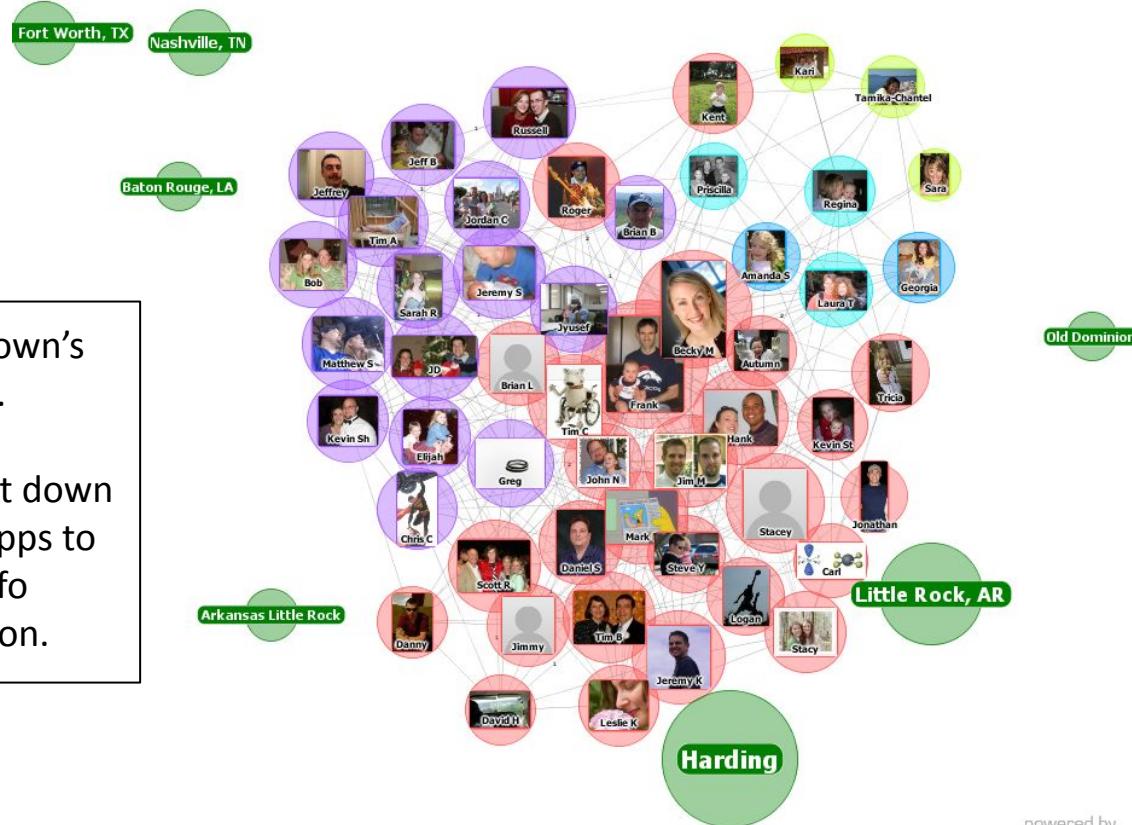
**twitter**

**LinkedIn**

**You Tube**

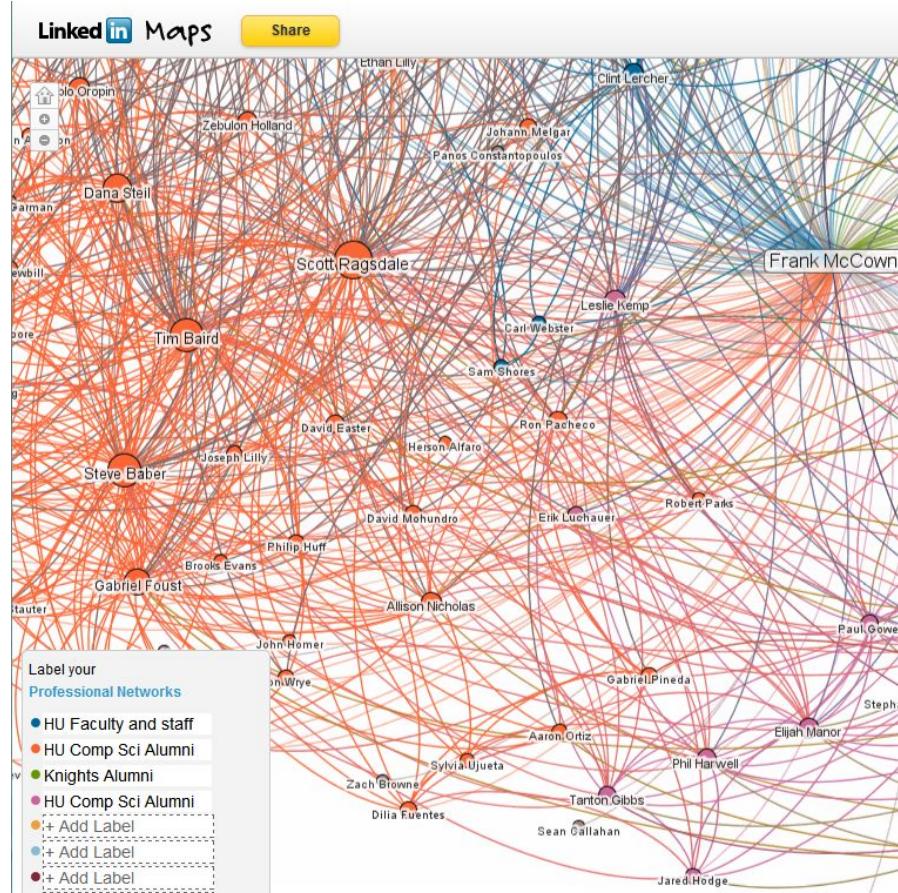
This is part of Dr. McCown's friend group pre-2014.

In 2015, Facebook shut down the API that allowed apps to access your friends' info without their permission.



# Facebook Friends

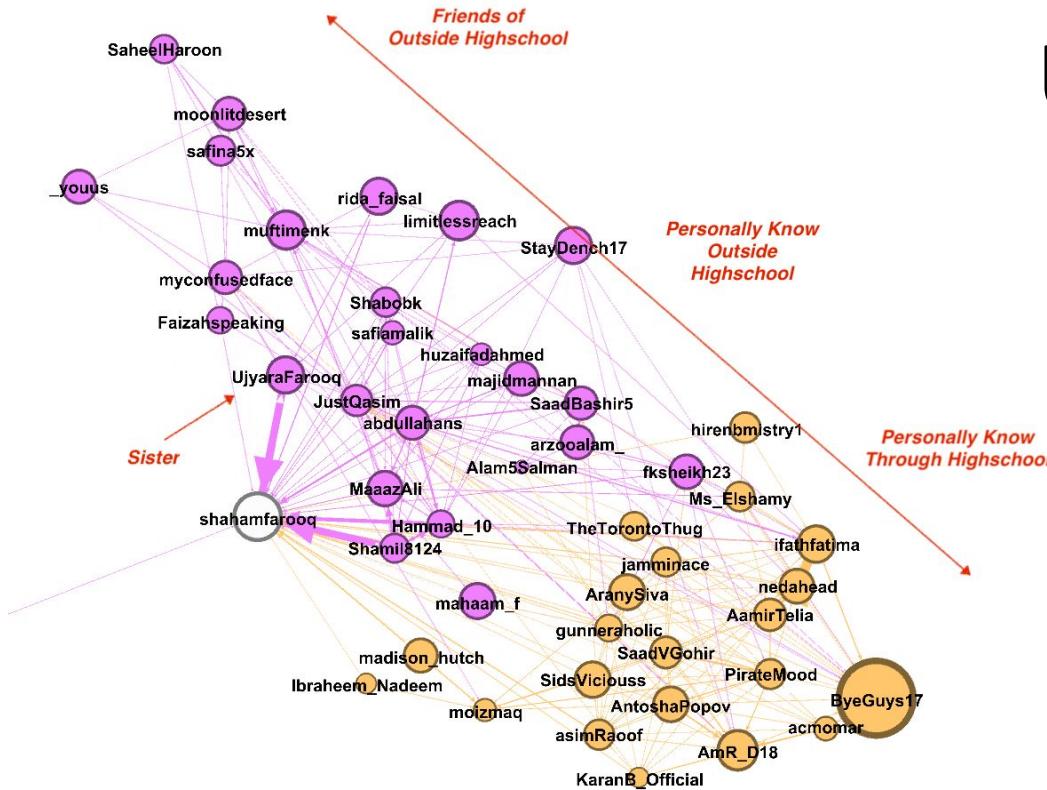
# LinkedIn Maps



[Visualize your LinkedIn network with InMaps, 2011](#)

[LinkedIn Is Quietly Retiring Network Visualization Tool InMaps, 2014](#)

# User-generated Twitter Network



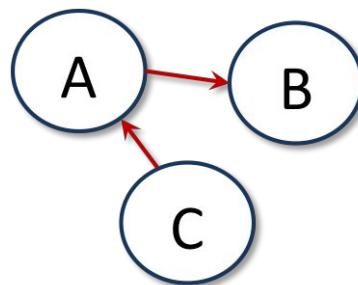
Generating A Twitter Ego-Network & Detecting Communities | by Shaham

# Why Study Social Networks?

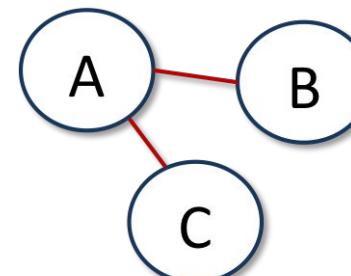
- Social scientists use social networks to study how people interact and develop theories of social behavior. OSNs offer opportunities to study social networks at a larger scale.
- Understanding the structure of OSNs can lead to systems in the future that improve security, leverage trust, improve social interaction, etc.

# Graph Types

Directed



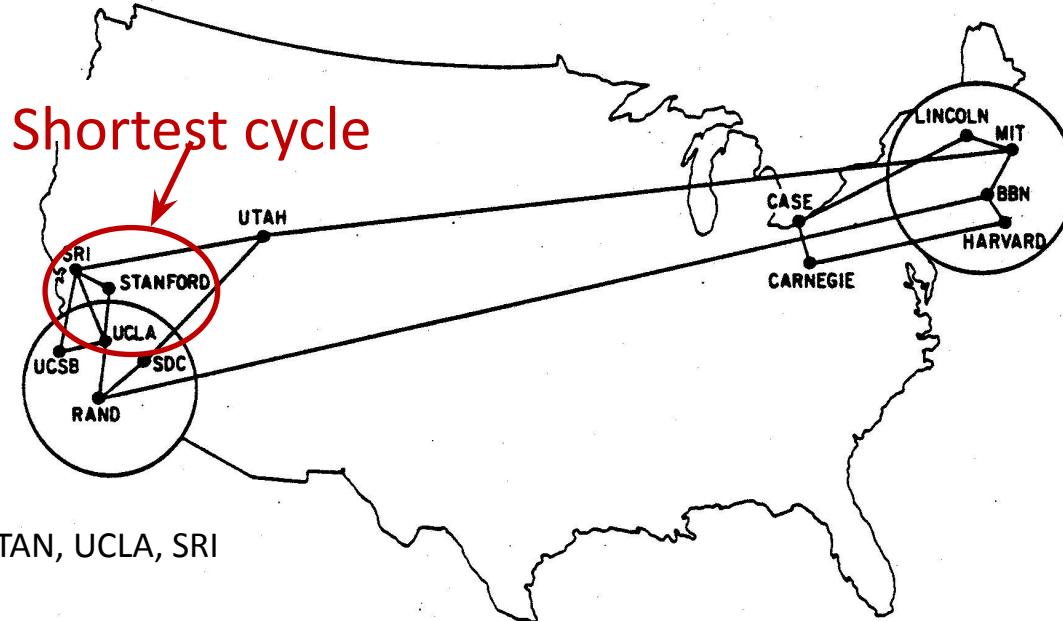
Undirected



e.g., Twitter  
followers

e.g., Facebook  
friends

# Cycles

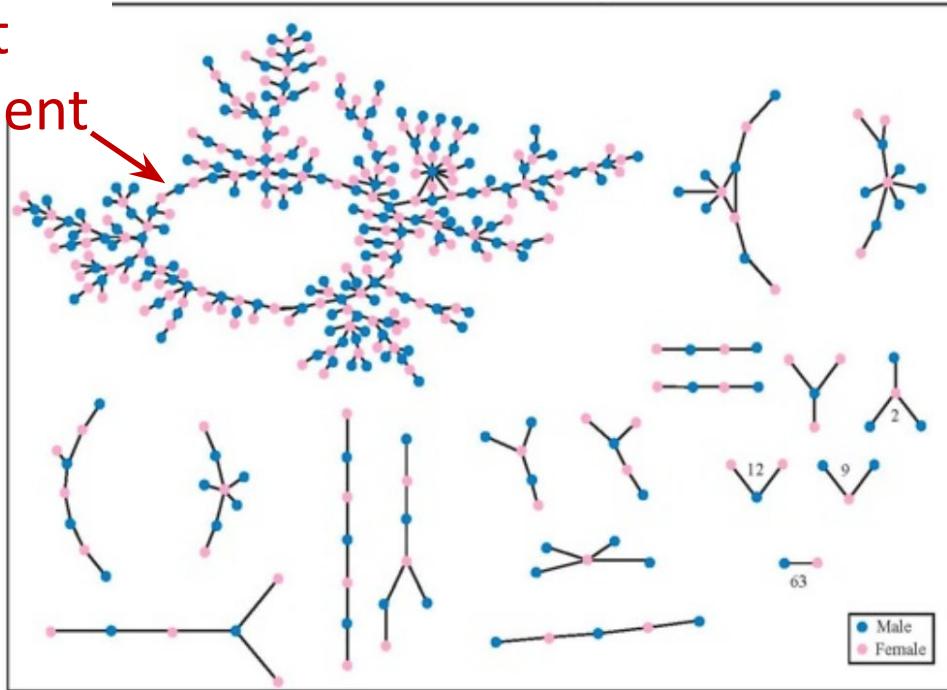


Shortest cycle is SRI, STAN, UCLA, SRI

[EK10] [Figure 2.2](#): A network depicting the sites on the Internet, then known as the Arpanet, in December 1970. (Image from F. Heart, A. McKenzie, J. McQuillan, and D. Walden)

# Components

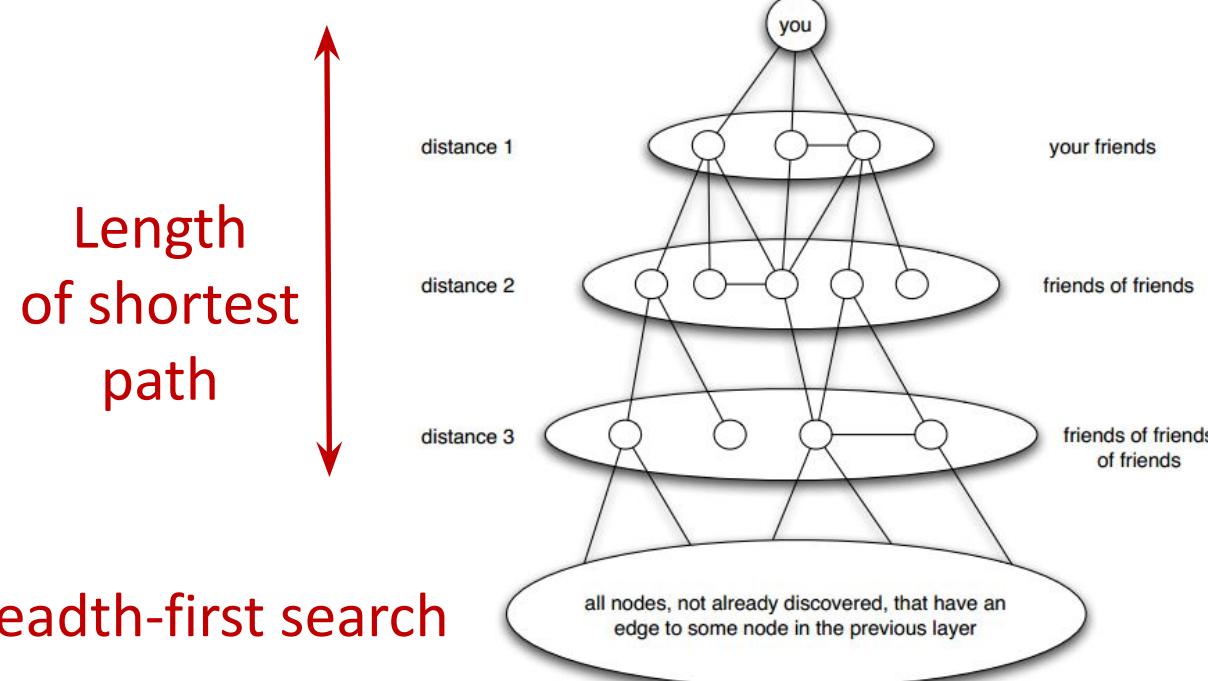
Giant component



[EK10]

Figure 2.7: A network in which the nodes are students in a large American high school, and an edge joins two who had a romantic relationship at some point during the 18-month period in which the study was conducted [49].

# Distance



[EK10] Figure 2.8: Breadth-first search discovers distances to nodes one “layer” at a time; each layer is built of nodes that have an edge to at least one node in the previous layer.

# Small World Network

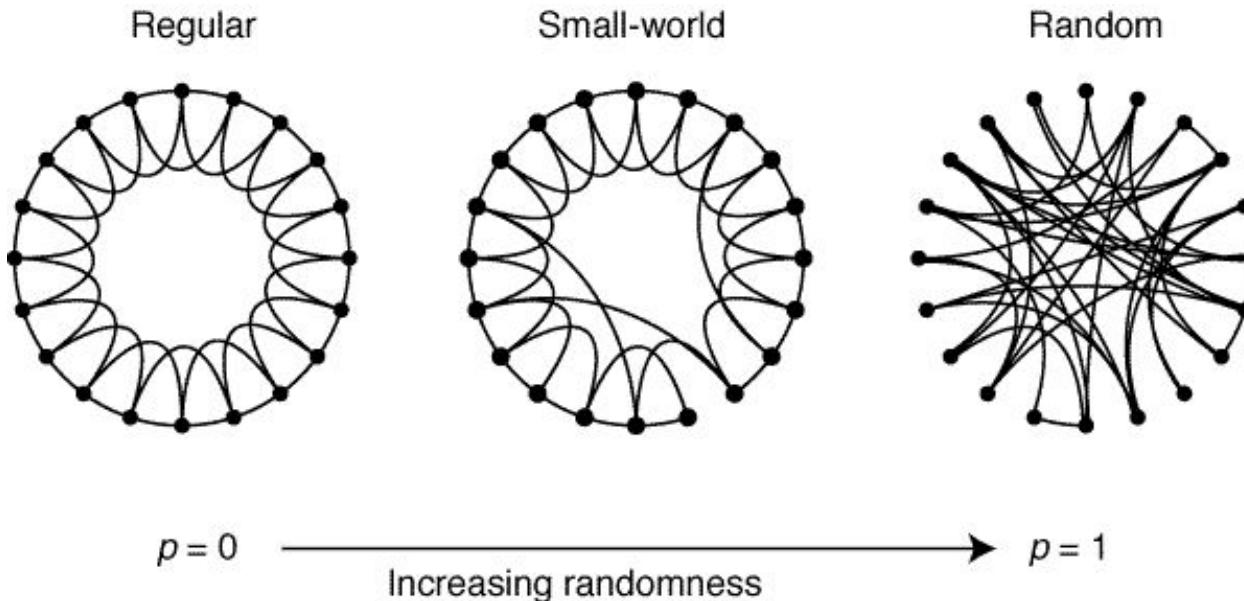
- Six degrees of separation
  - first tested by Stanley Milgram in 1960s who found median length of 6 between two individuals
- OSNs make studying this phenomena a little easier
- Six degrees of Kevin Bacon
- Erdos number



[Six Degrees of Kevin Bacon](#) (Wikipedia) / [The Oracle of Bacon](#)  
[Erdős number](#) (Wikipedia)

[My Erdos number is 3](#)

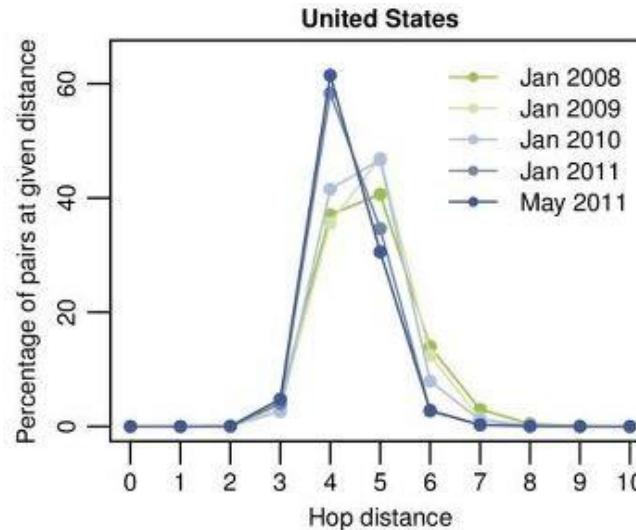
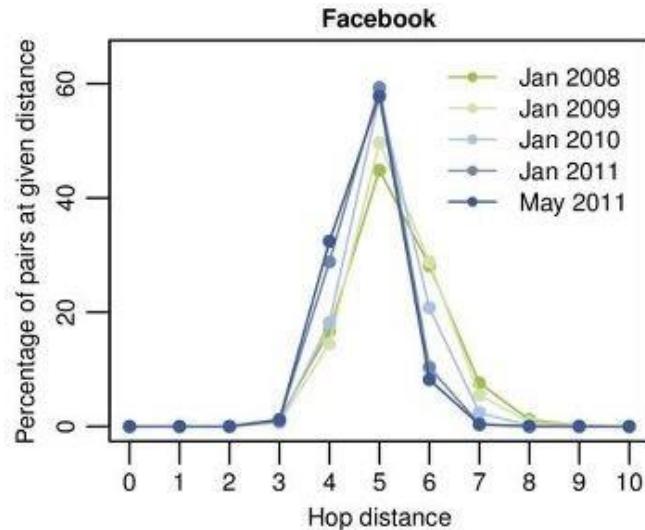
# Regular → Small World → Random



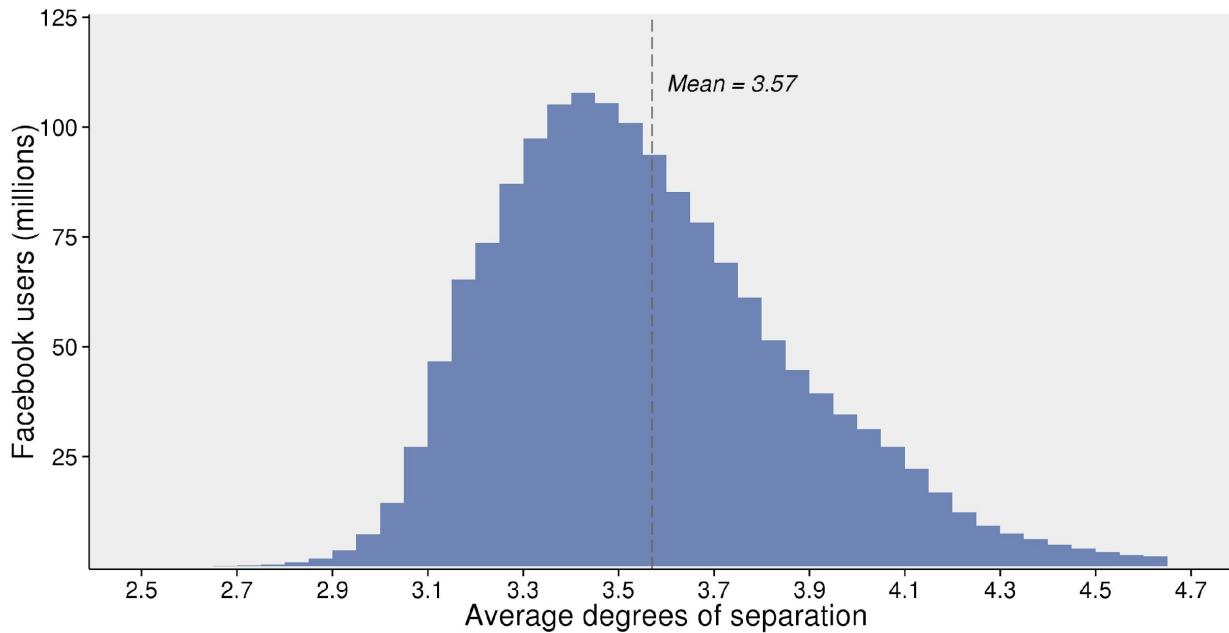
Watts & Strogatz, "[Collective dynamics of 'small-world' networks](#)", *Nature*, 1998 ([pdf](#))

# Four Degrees of Separation

Study of all Facebook users (721 million) in 2011 revealed avg distance of 4.74

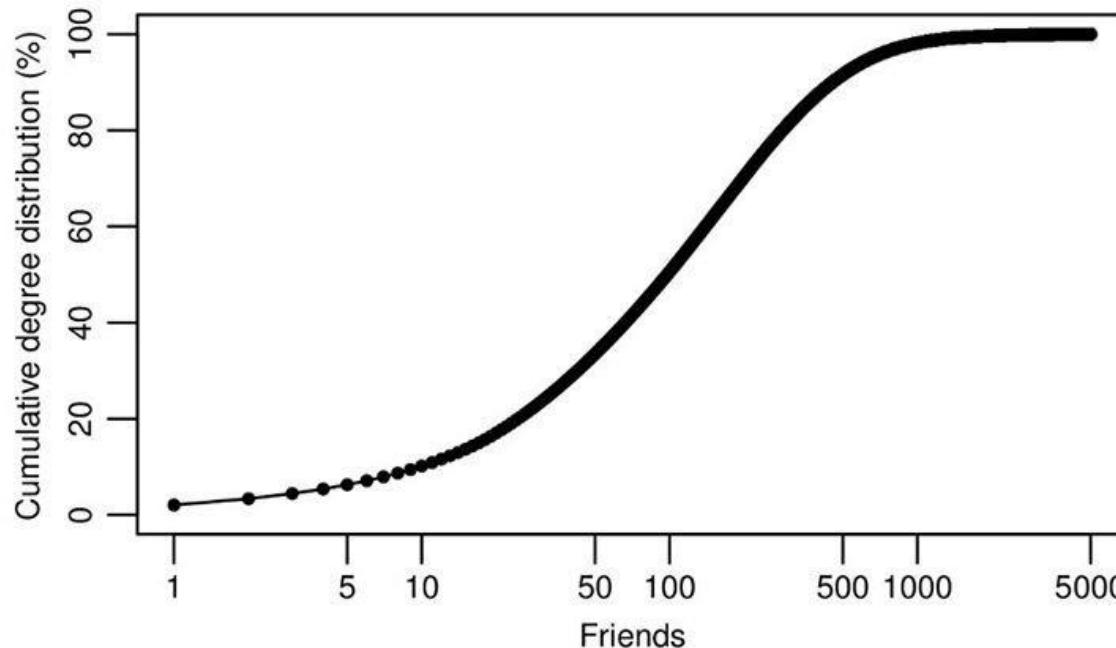


# 2016 Update: 3.57 Degrees



[Three and a half degrees of separation](#)

# How Many Friends?



[Anatomy of Facebook](#)

example of a CDF: [Cumulative distribution function](#) (Wikipedia)

# "Friendship Paradox"

*Fun fact: your friends have more friends than you do.*

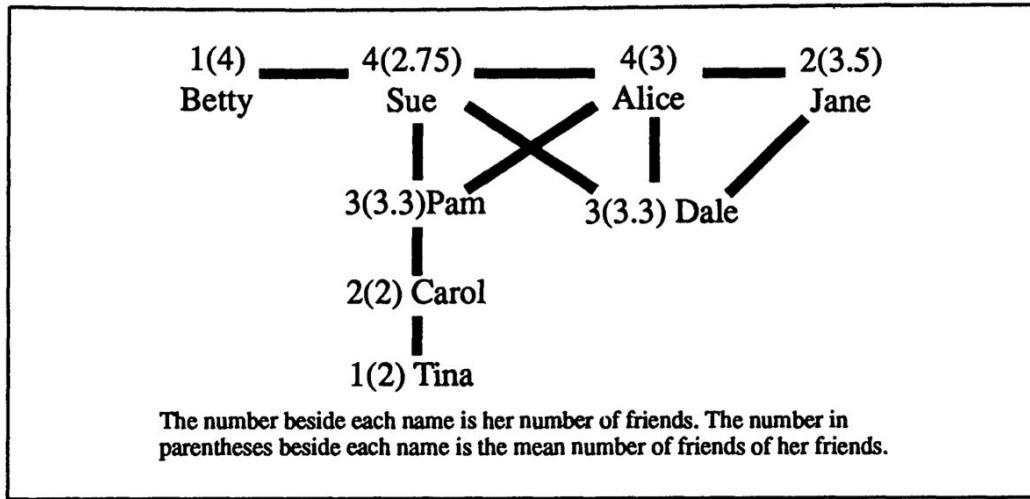


FIG. 1.—Friendships among eight girls at Marketville High School

Feld, ["Why Your Friends Have More Friends Than You Do"](#) ([pdf](#))

[Friendship paradox](#) (Wikipedia)

Your friends are also happier than you: [The happiness paradox: your friends are happier than you](#)

# Sue & Alice Rule Marketville

(Carol is doing ok)

TABLE 1

A SUMMARY OF THE NUMBERS OF FRIENDS AND THE MEAN NUMBERS OF FRIENDS  
OF FRIENDS FOR EACH OF THE GIRLS IN FIGURE 1

	Number of Friends ( $x_i$ )	Total Number of Friends of Her Friends ( $\Sigma x_j$ )	Mean Number of Friends of Her Friends ( $\Sigma x_j/x_i$ )
Betty.....	1	4	4
Sue .....	4	11	2.75
Alice .....	4	12	3
Jane .....	2	7	3.5
Pam .....	3	10	3.3
Dale.....	3	10	3.3
Carol .....	2	4	2
Tina .....	1	2	2
Total.....	20	60	23.92
Mean	2.5*	3†	2.99*

\* For eight girls.

† For 20 friends.

# Complete Network of Marketville

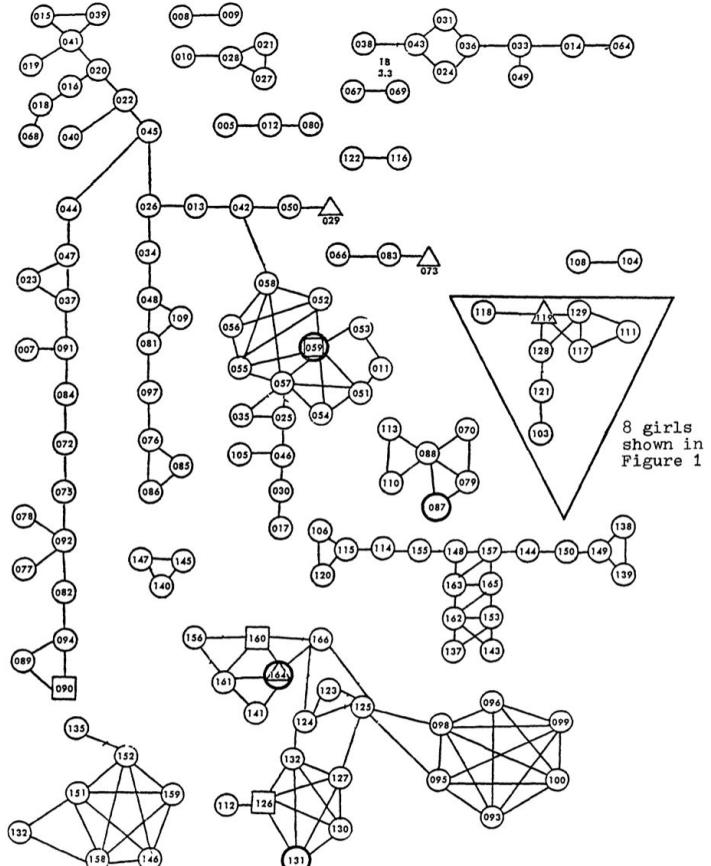
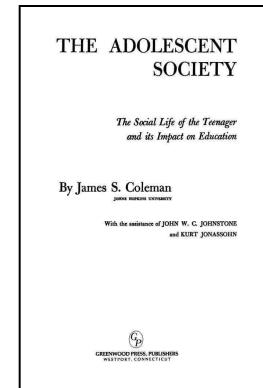


FIG. 2.—Network of reciprocated friendships among Marketville girls; the triangle at right indicates friendships illustrated in fig. 1. (From *The Adolescent Society* by James S. Coleman. © 1961 by the Free Press, a division of Macmillan, Inc. Used with permission.)

In 1961 study, friendship is via bi-directional naming in a questionnaire (i.e., Facebook style).



# Web Science: Social Networks

## (Part 2 - Strong and Weak Ties)

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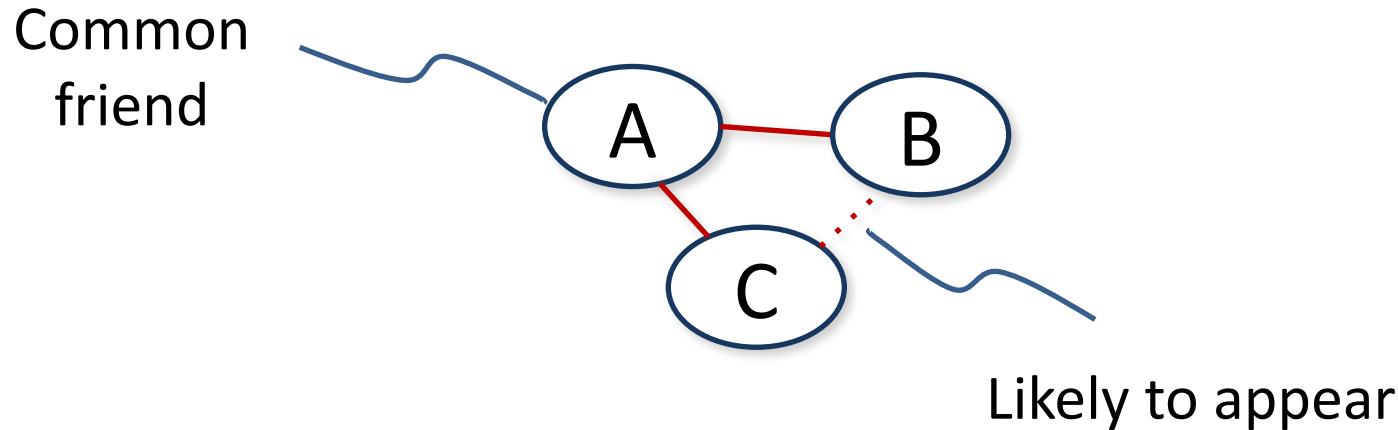
# Network Evolution

- Social networks tend to change over time
- You make new friends, lose contact with others
- You change schools, jobs, etc.
- We're often interested in examining multiple snapshots of the social graph through time

"If two people in a social network have a friend in common, then there is an increased likelihood that they will become friends themselves at some point in the future."

- Anatole Rapoport (1953)

# Triadic Closure



# Reasons for Triadic Closure

- *Opportunity* – A spending time with B & C increases chance they will meet
- *Trust* – Research shows that B & C are more likely to trust each other if they are aware of mutual friend A
- *Incentive* – Research shows that latent stress occurs when mutual friends are not friends with each other

# Measuring Triadic Closure

- *Clustering coefficient* – probability that any two randomly selected friends of a node are also friends

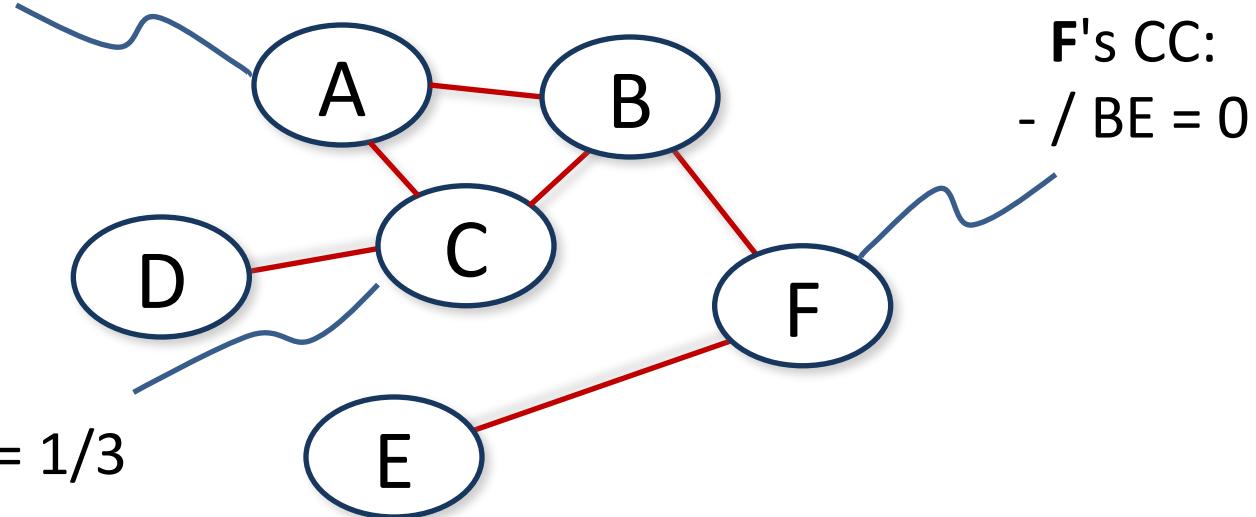
$$CC \text{ for a node} = \frac{\text{\# of friend pairs}}{\text{\# of possible friend pairs}}$$

- $CC = 0$  when **none** of node's friends are friends with each other
- $CC = 1$  when **all** of node's friends are friends with each other

# Clustering Coefficient

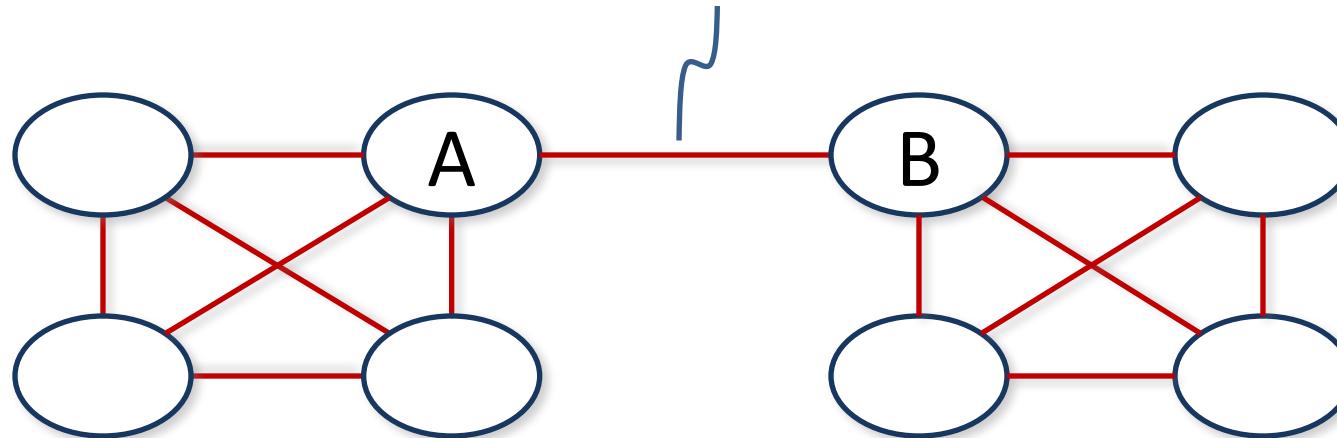
A's CC:

$$BC / BC = 1$$



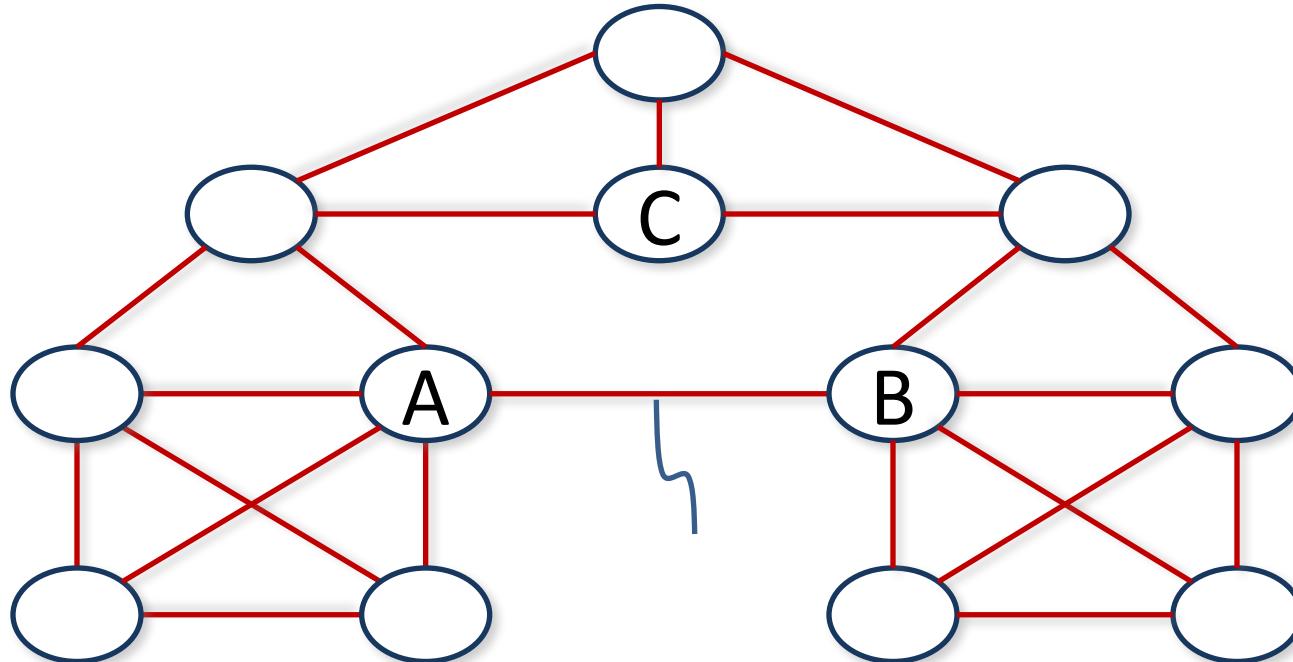
CC for a node = # of friend pairs / # of possible friend pairs

# Bridges



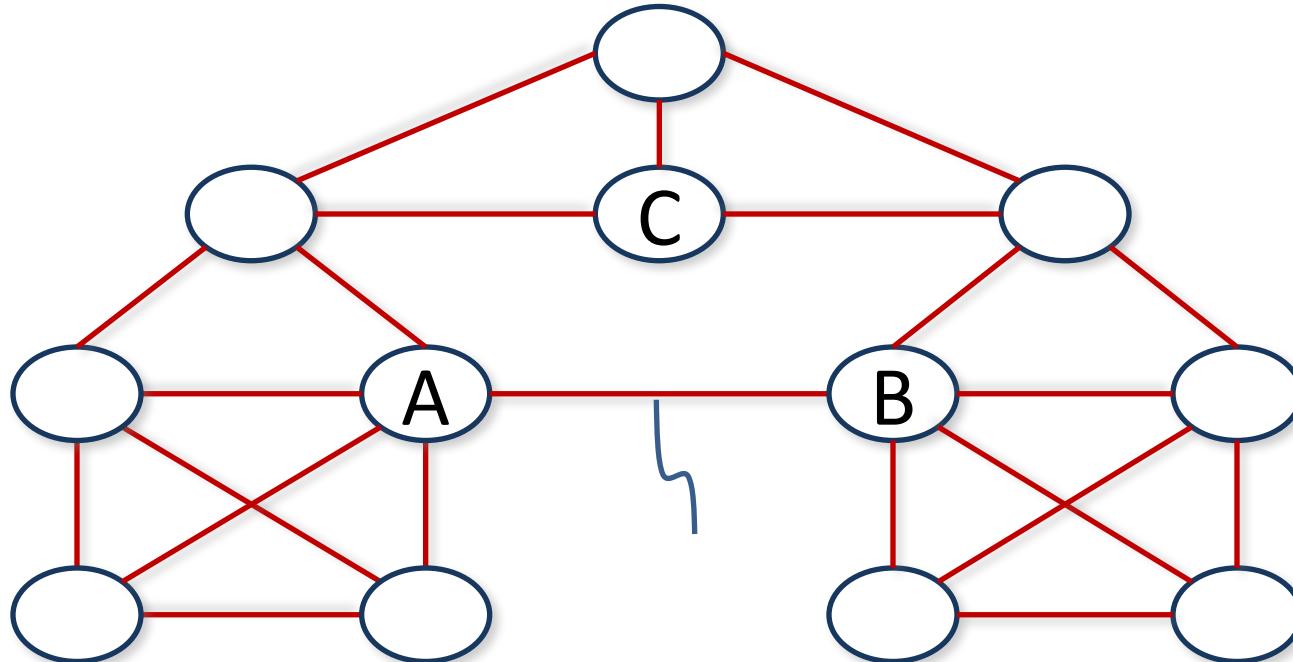
Edge is a **bridge** if removing the edge creates two components

# Local Bridges



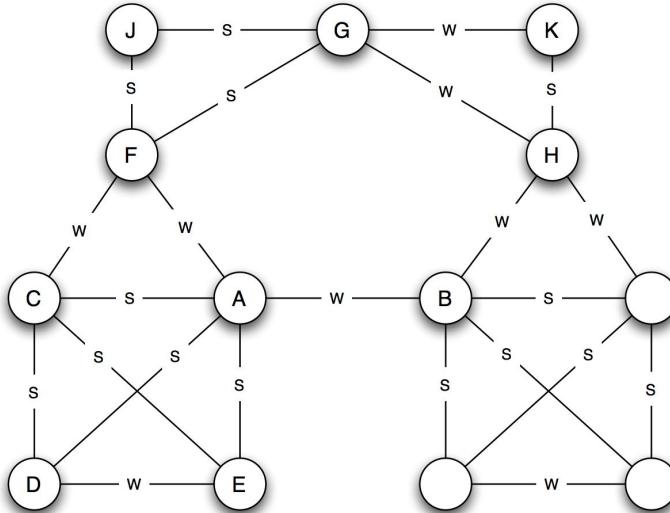
Edge is a **local bridge** if endpoints have no friends in common

# Local Bridges



Removing a local bridge does **not** create two components!

# Strong and Weak Ties

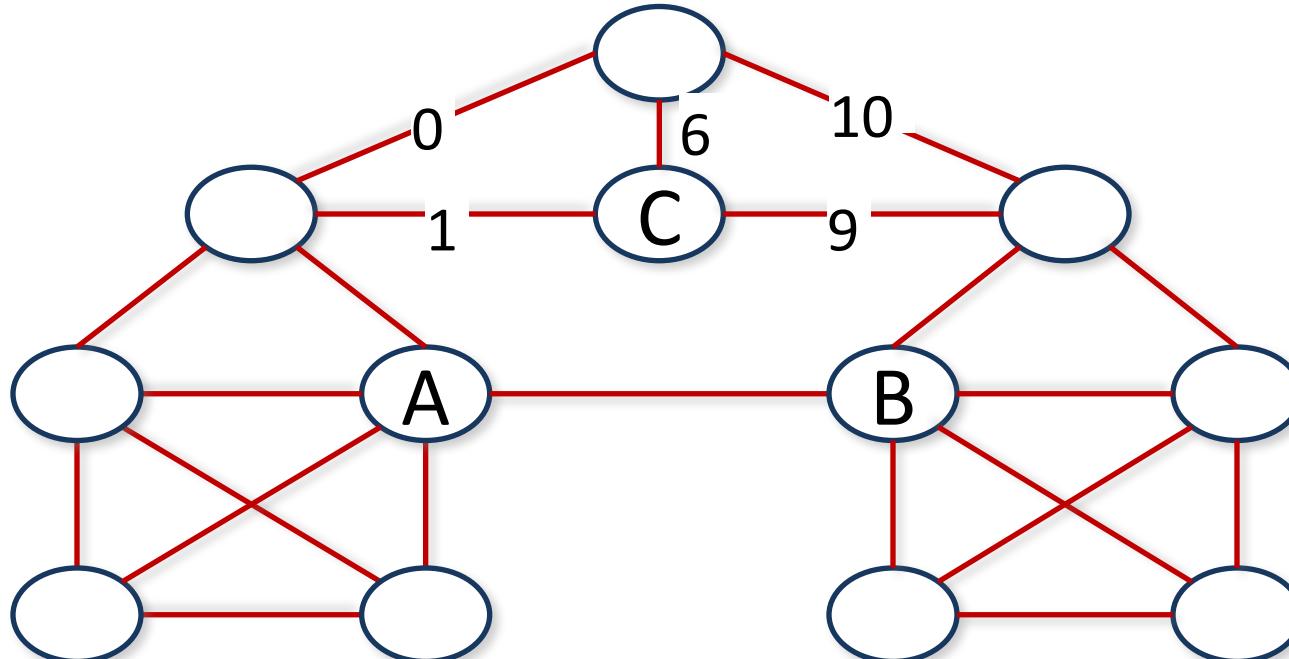


[EK10]

Figure 3.5: Each edge of the social network from Figure 3.4 is labeled here as either a *strong tie* (S) or a *weak tie* (W), to indicate the strength of the relationship. The labeling in the figure satisfies the Strong Triadic Closure Property at each node: if the node has strong ties to two neighbors, then these neighbors must have at least a weak tie between them.

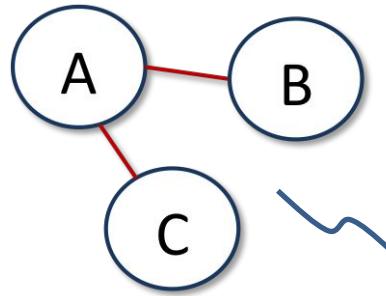
**Not everyone is your BFF!**

# Variable Tie Strength

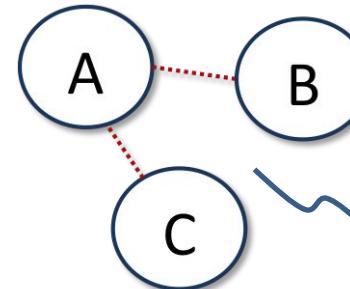


Can use more fine-grained methods to determine tie strength, like number of Facebook messages exchanged

# Tie Strength and Triadic Closure



More likely to  
form



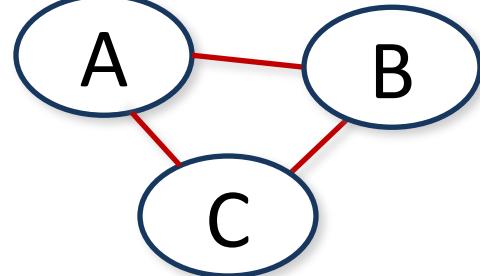
Less likely to  
form

Triadic closure is more likely to form when initial edges are **strong**

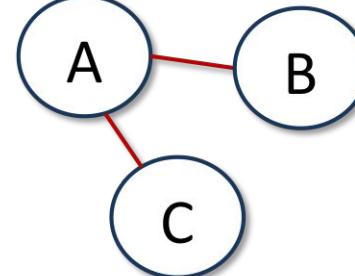
# Strong Triadic Closure Property

Node A satisfies the **Strong Triadic Closure Property** if it has strong ties to two other nodes B and C, and there is some edge between B and C

satisfies

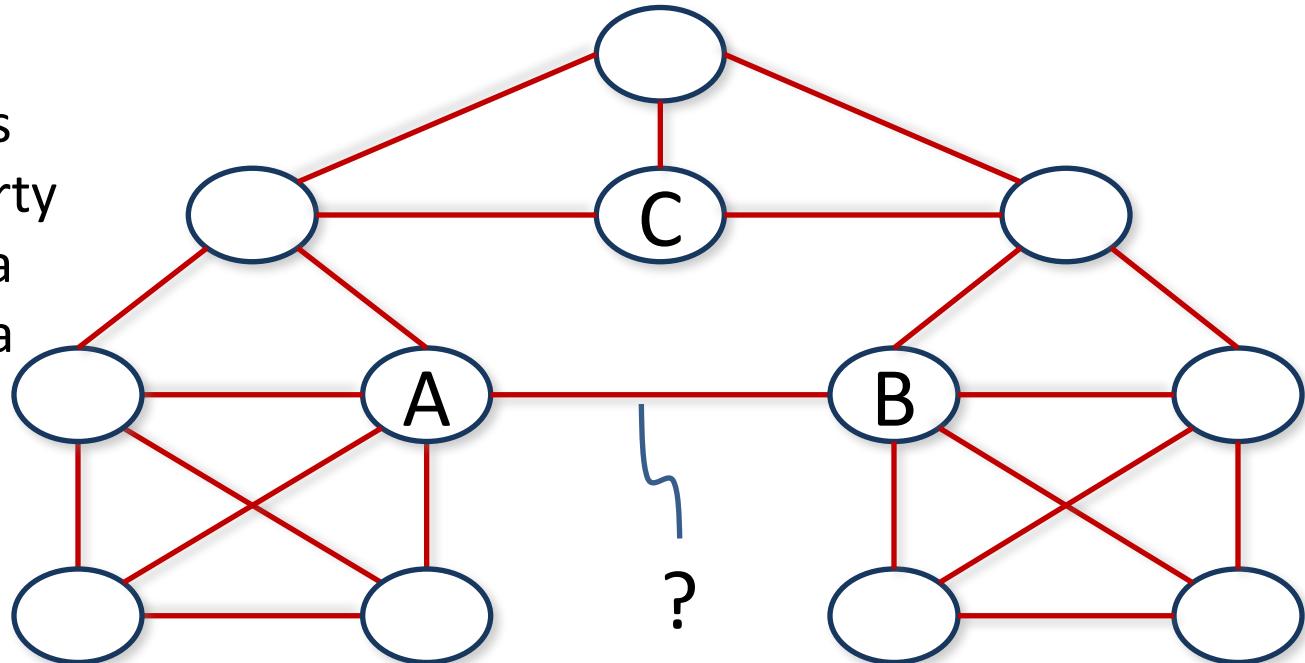


violates



# Strong Triadic Closure and Bridges

Can a node with at least 2 strong edges satisfy the STC property and be involved in a local bridge that is a **strong tie**?



Answer: No! Why not?

# Strong Triadic Closure and Bridges

Can a node with at least 2 strong edges satisfy the STC property and be involved in a local bridge that is a **strong tie**?

[EK10]

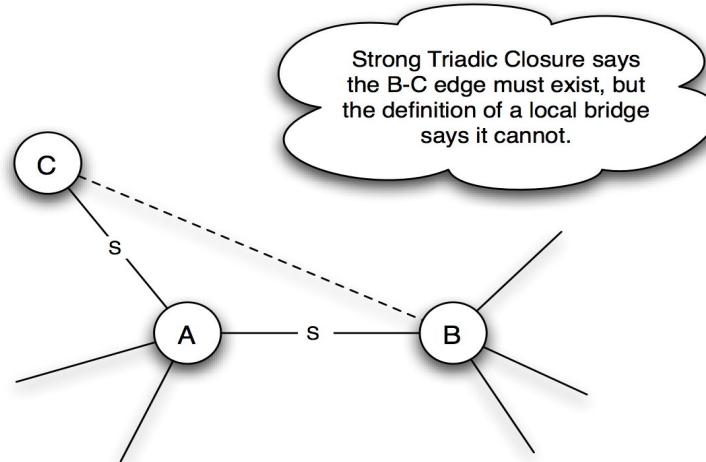


Figure 3.6: If a node satisfies Strong Triadic Closure and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie. The figure illustrates the reason why: if the  $A-B$  edge is a strong tie, then there must also be an edge between  $B$  and  $C$ , meaning that the  $A-B$  edge cannot be a local bridge.

Answer: No! Why not?

# Strong Triadic Closure and Bridges

- STC property says edge must form when two strong edges emanate from the same node
- Therefore a network that satisfies the STC property and has a sufficient number of strong ties will likely have *only weak* local bridges
- Does not always hold in real social networks, but useful theoretical framework in which to understand connections and test theories

# Web Science: Social Networks

## (Part 3 - Tie Strength and Embeddedness)

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# Large-Scale Data Studies

- Put new concepts to work by examining several large-scale studies
- Examples:
  - Cell phone usage
  - Facebook
  - Twitter

# Cell Phone Social Network

- 18 weeks of cell phone data covering 20% of country's population
- Node = cell phone user
- Edge = if two nodes called each other (both directions)
- Tie strength = duration of calls
  - rather than just “strong” or “weak”

Onnela et al., Structure and tie strengths in mobile communication networks (2007)

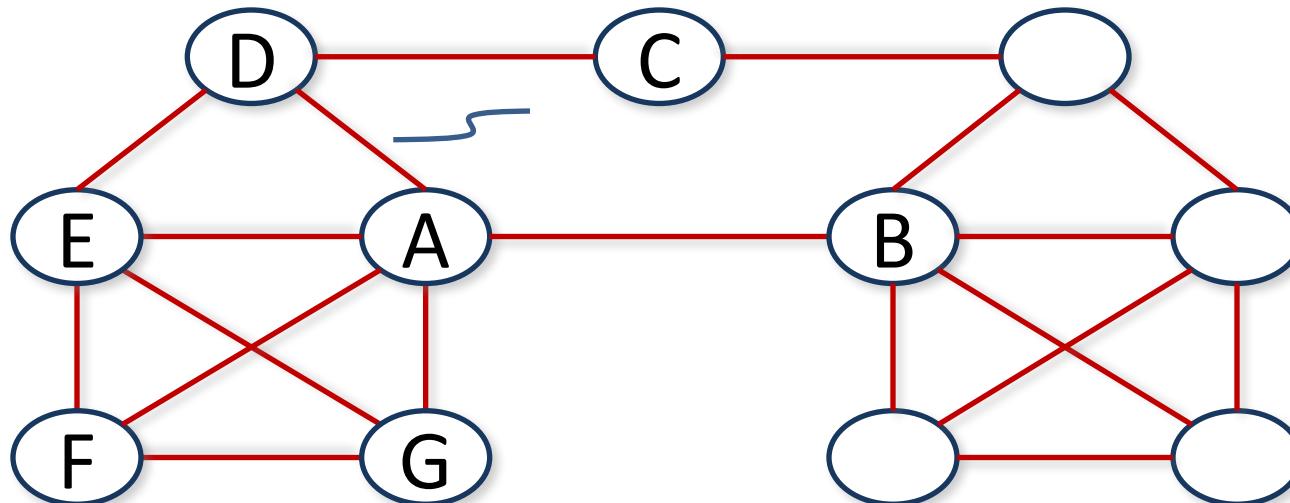
# Cell Phone Social Network

- Instead of determining if edge is a local bridge or not, use neighborhood overlap

$$\text{Neighborhood overlap} = \frac{\text{neighbors of both A and B}}{\text{neighbors of A or B}}$$

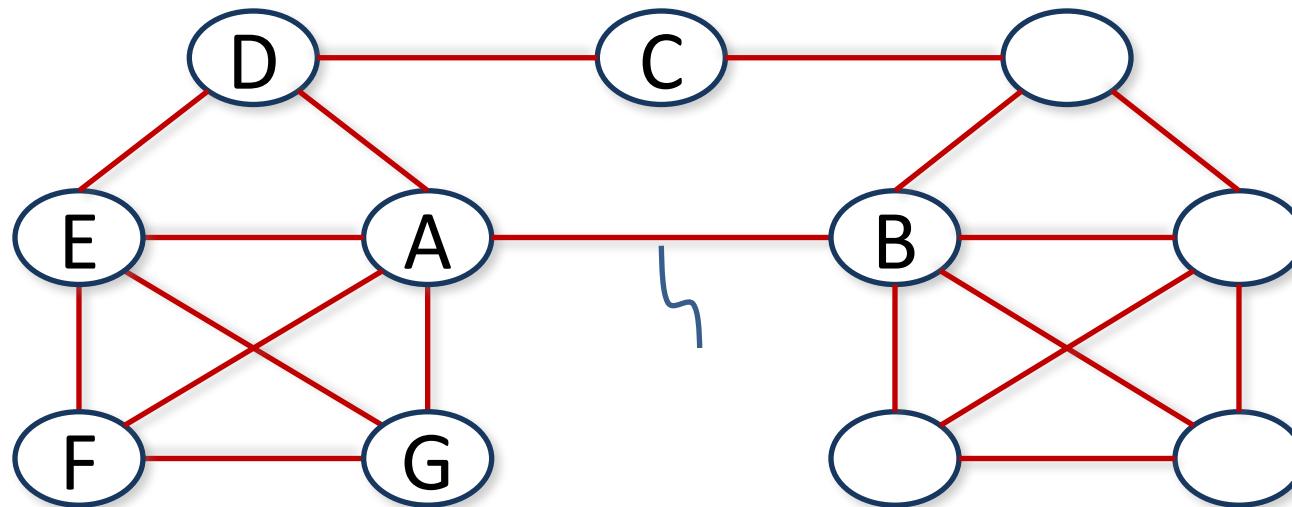
- = 1 if all neighbors of A and B are connected
- = 0 if no neighbors are connected (local bridge)
- Close to 0 is "almost" a local bridge

# Neighborhood Overlap



$$\text{Neighborhood overlap of AD} = \frac{E}{C, E, F, G, B} = 1/5$$

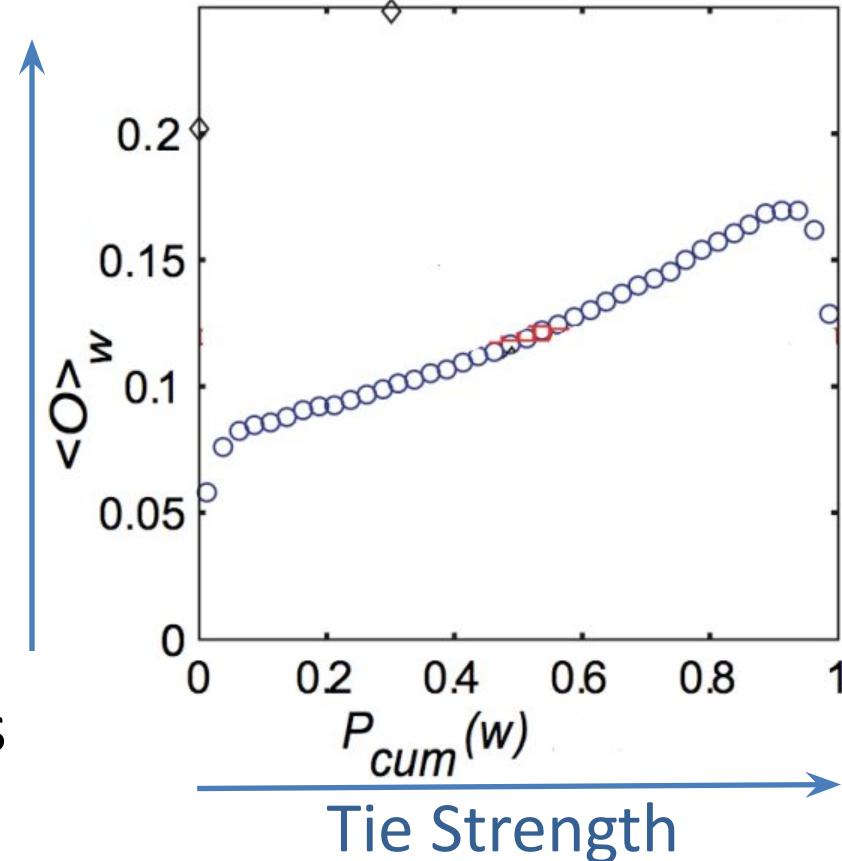
# Neighborhood Overlap



$$\text{Neighborhood overlap of AB} = \frac{-}{\text{D, E, F, G, etc.}} = 0$$

# Neighborhood Overlap

# Onnela et al.'s findings



Q: What's happening here?  
A: We're not sure. Unusual phone usage?

[EK10] Fig 3.7. A plot of the neighborhood overlap of edges as a function of their percentile in the sorted order of all edges by tie strength.

# Removing Ties

- 84% of cell phone graph nodes lay in a single giant component
- Do weak ties link strongly connected components?
  - Removed edges one at a time starting with strongest edges  
→ giant component slowly erodes
  - Removed edges starting with weakest edges → giant component shrank quicker, remnants broke apart quickly once critical weak edges removed

# Tie Strength on Facebook

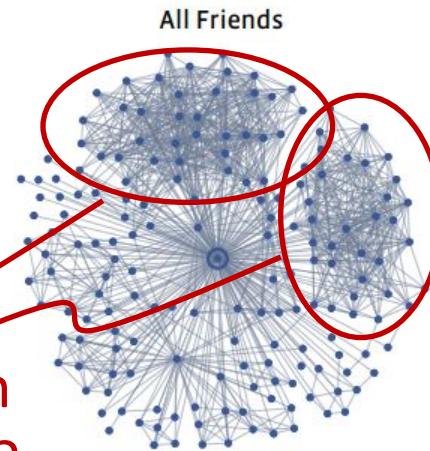
- Marlow et al., Maintained relationships on Facebook, 2009
- Analyzed tie strength of Facebook users using one month of interactions

# Three Categories of Links

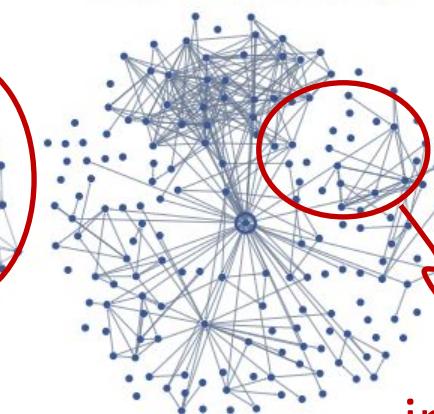
- *Reciprocal/Mutual* – messages sent both ways
- *One-way* – at least one message sent (friend may or may not have reciprocated)
- *Maintained* – user clicked on friend's content (from feed) or profile more than once

# Typical User's Network

College & high school friends?

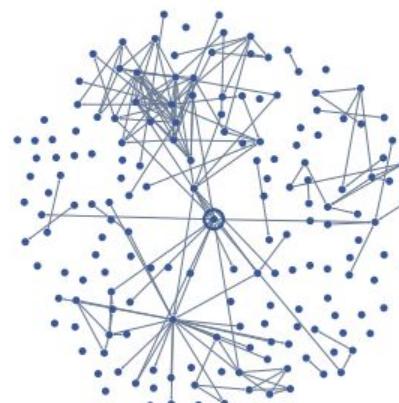


Maintained Relationships

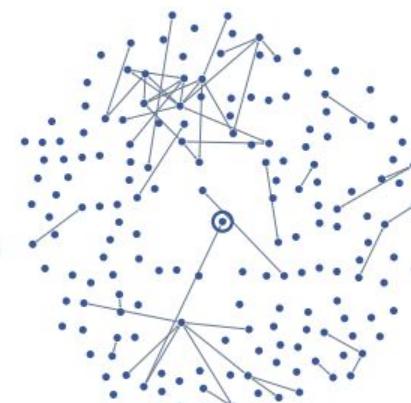


Less interest

One-way Communication



Mutual Communication



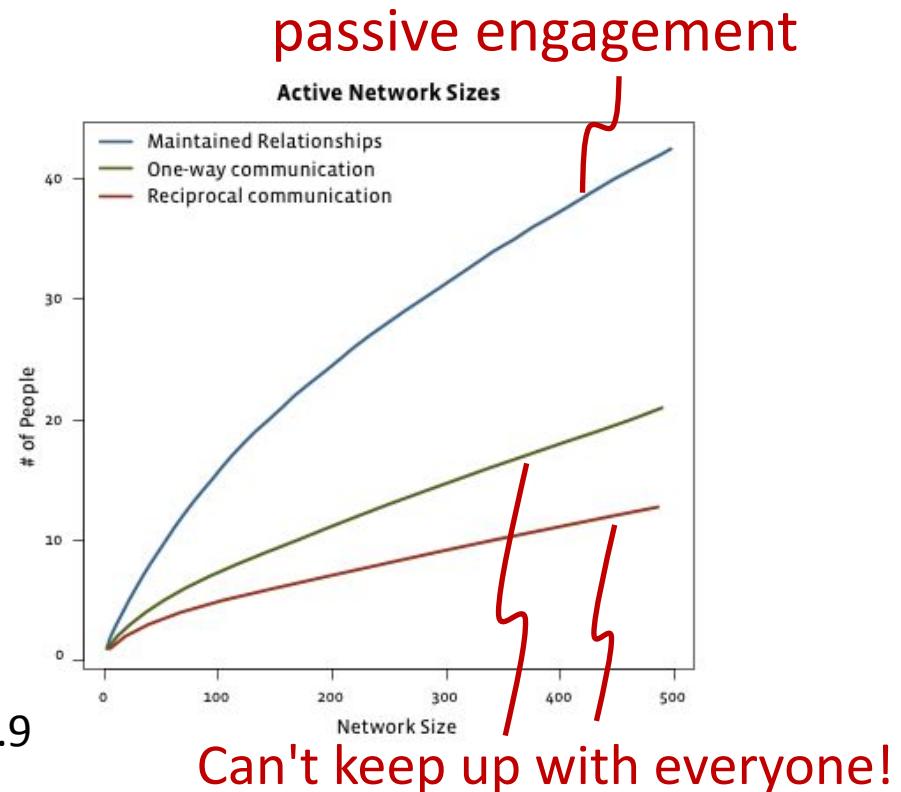
[EK10] Fig 3.8

[Maintained Relationships on Facebook](#)

# Limits to Maintaining Relationships

- *Reciprocal* – messages sent both ways
- *One-way* – at least one message sent
- *Maintained* – user clicked on Friend's content (from feed) or profile more than once

[EK10] Fig 3.9



[Maintained Relationships on Facebook](#)

# Tie Strength on Twitter

- Analyzed tie strength of 211,024 active Twitter users during observation period
- **Followees** – people being followed
- **Friend** – person who user has directed at least 2 tweets to (strong ties)

# Friends vs. Followees

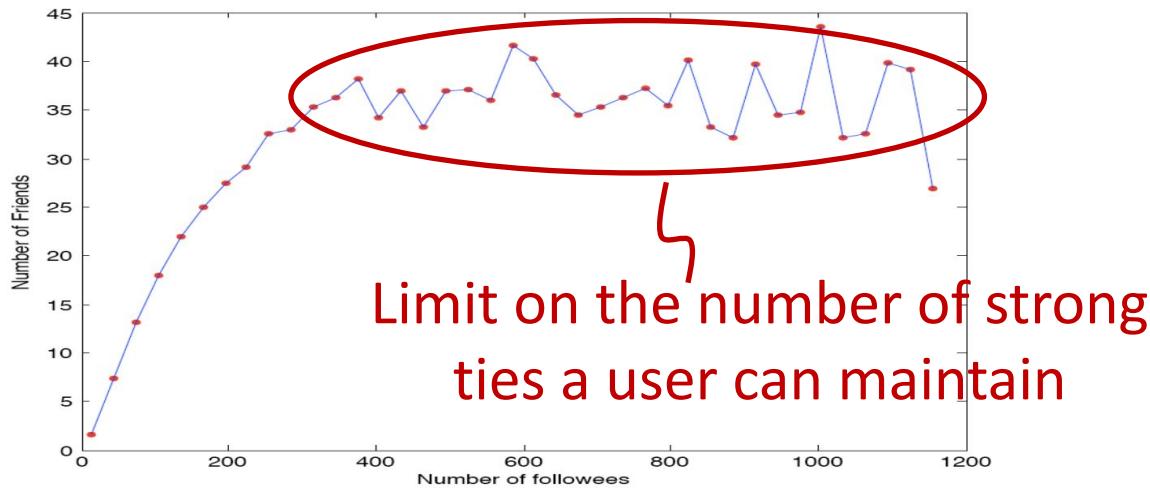
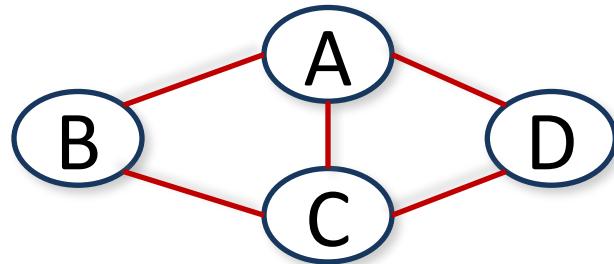


Figure 4: Number of friends as a function of the number of followees. The total number of friends saturates while the number of followees keeps growing due to the minimal effort required to add a followee.

Now let's look at the edges to see  
which *nodes* are “important”...

# Edge Embeddedness

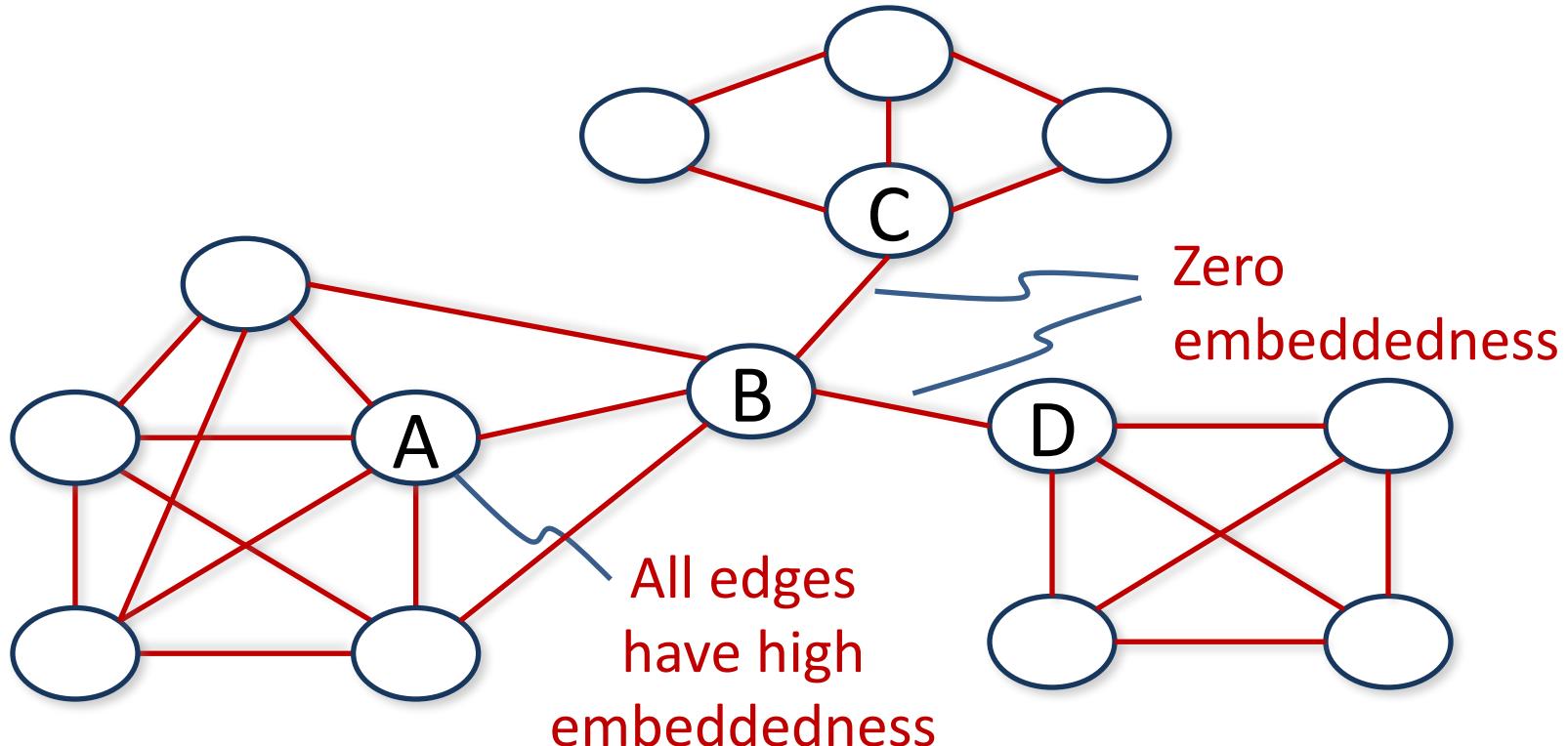
- Defined as the number of common neighbors shared by two endpoints



Embeddedness of A-B = 1  
Embeddedness of A-C = 2

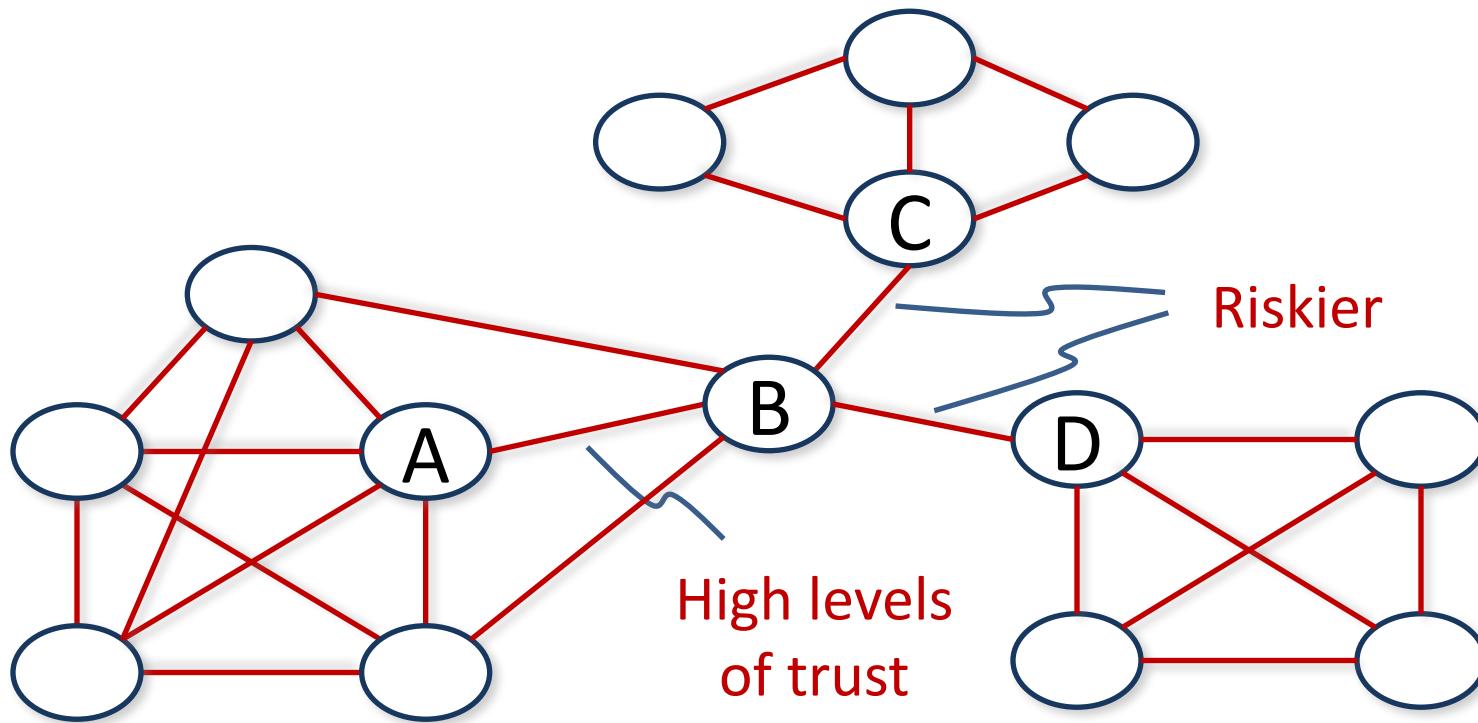
- Equal to numerator in neighborhood overlap
- Local bridges have embeddedness of zero

# Edge Embeddedness Example

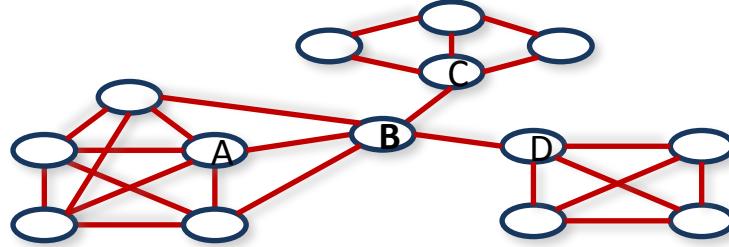


# Embeddedness and Trust

- Research shows that if you and I are connected by an embedded edge (we share a mutual friend), there is more trust and confidence in the integrity of the transactions (social, economic, etc.) between us
- If I cheat you, our mutual friends are likely to find out, and they are less likely to trust me in the future!
- Threat is absent for edges of zero embeddedness; no mutual friends to punish me!

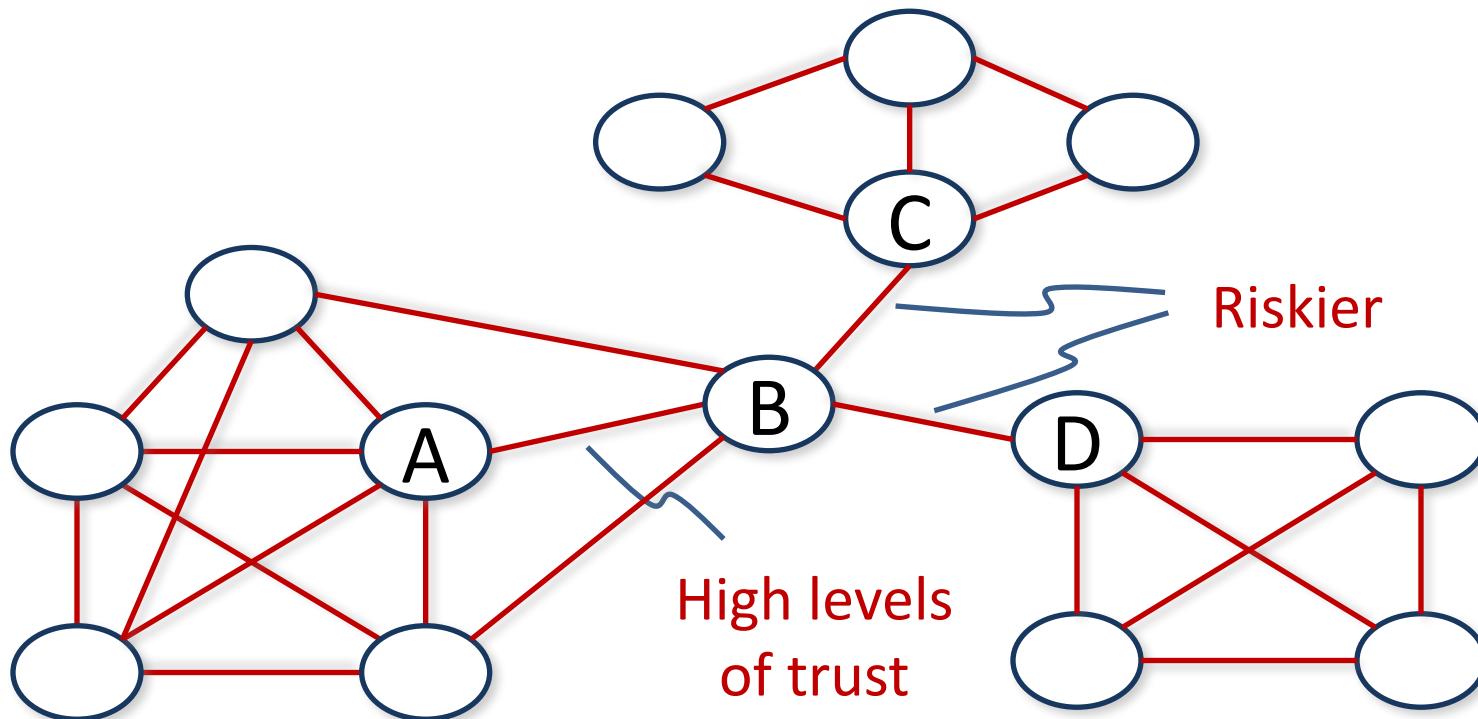


B also must deal with potentially contradictory norms and expectations from various groups



# It's good to be B...

- Node B is at the end of multiple local bridges
- Node B spans a **structural hole** in the network – space between two components that do not interact closely
- If network represents a company or organization, some advantages for B:
  - B has early access to info that originates in various components
  - Having access to this info can amplify B's creativity by combining disparate info into novel ideas
  - B acts as "gatekeeper" and can control flow of information to various components
  - B seems “smart” to everybody else...



Who would you rather be? **A** with trusted relationships or risk-taker **B**?

# Web Science: Social Networks

## (Part 4 - Graph Partitioning)

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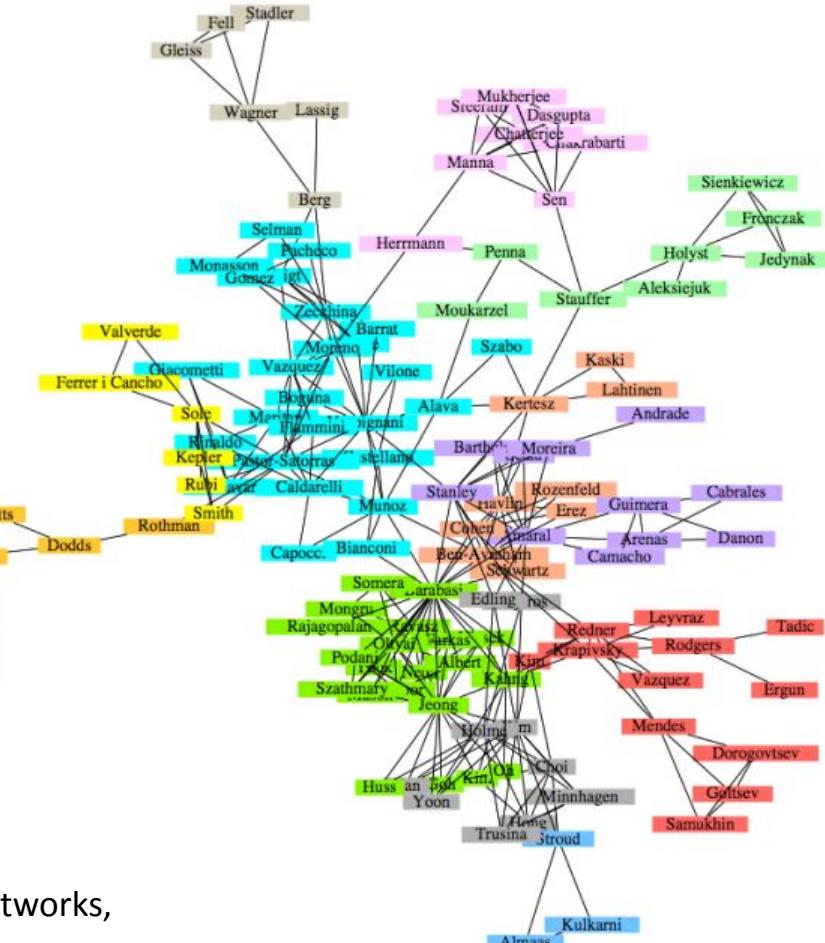
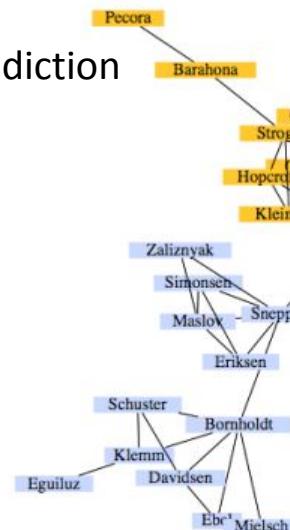
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# Co-authorship network

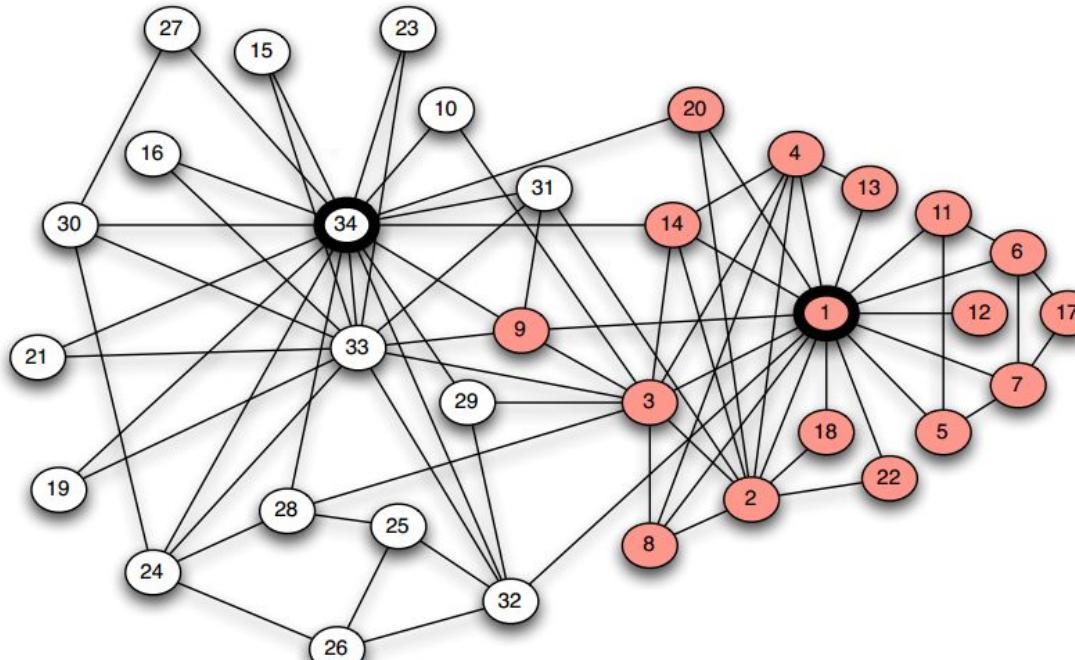
How can the tightly clustered groups be identified mathematically?

1. We can often “eyeball” clusters, important nodes
2. We often are interested in link prediction



Newman & Girvan, Finding and evaluating community structure in networks,  
Physical Review E, 69(2), 2004

# Karate Club splits after a dispute. Can new clubs be identified based on network structure?



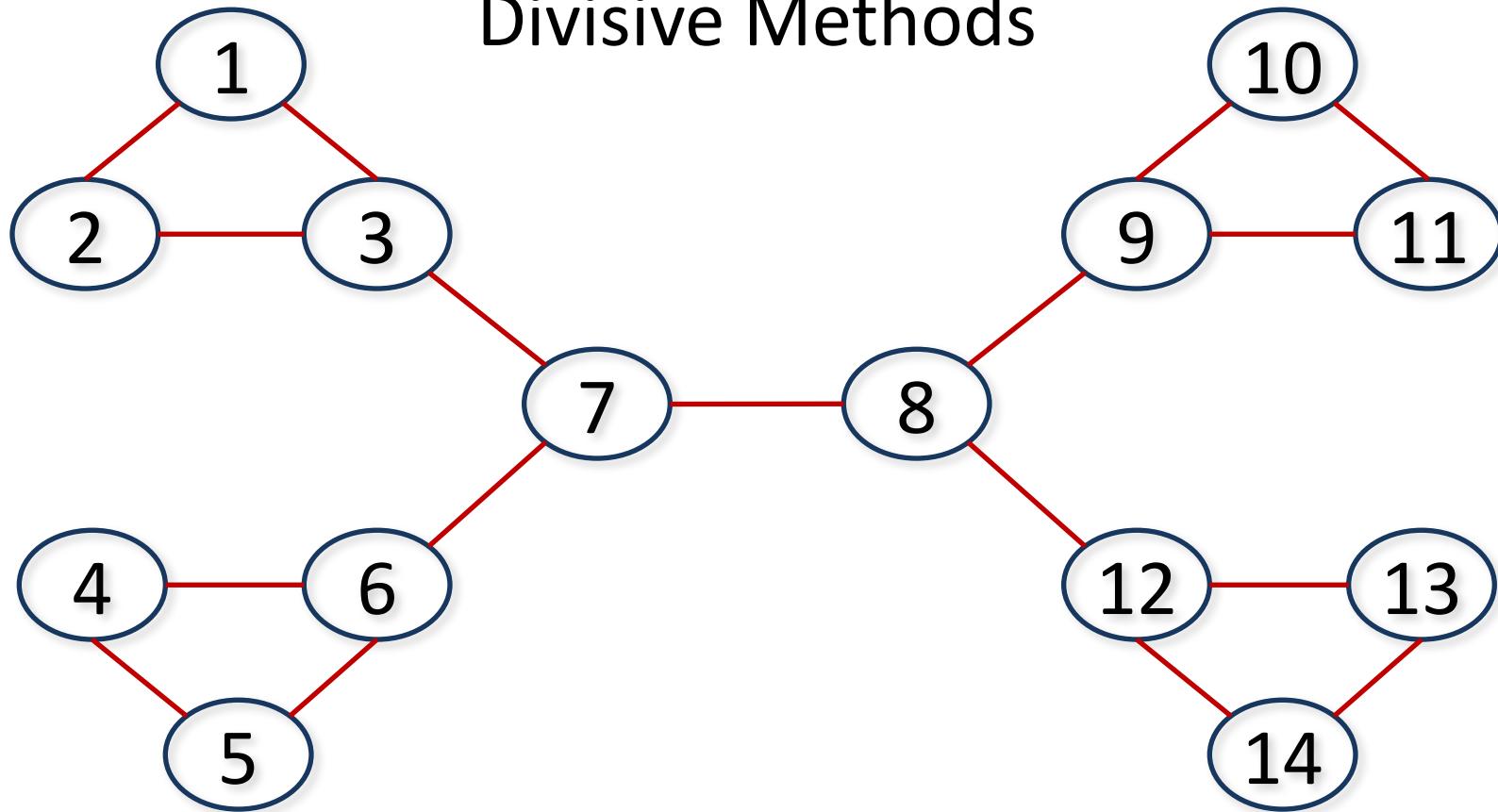
Zachary W. (1977). An information flow model for conflict and fission in small groups. Journal of Anthropological Research, 33, 452-473.

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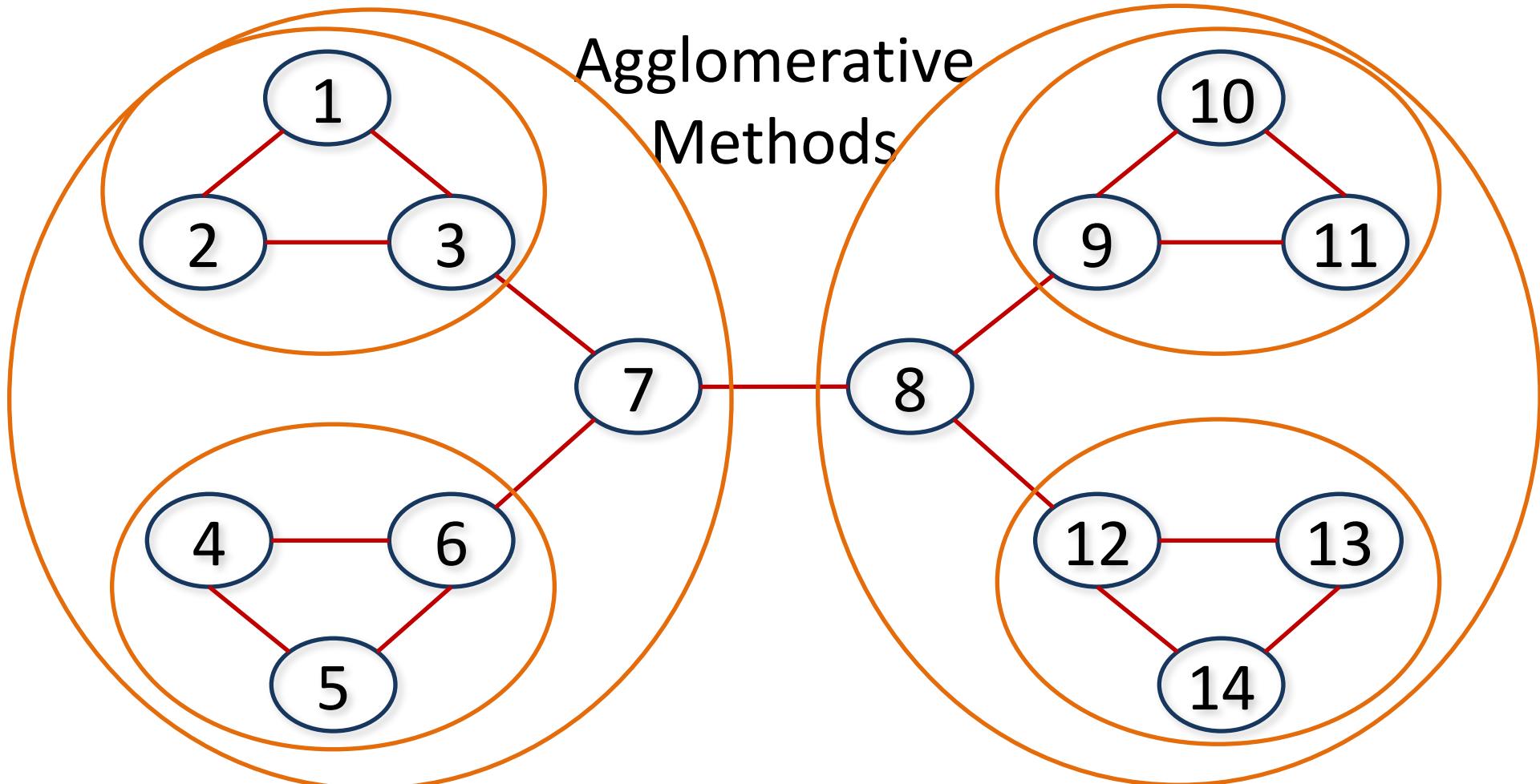
# Graph Partitioning

- Methods to break a network into sets of connected components called *regions*
- Many general approaches
  - **Divisive methods:** Repeatedly identify and remove edges connecting densely connected regions
  - **Agglomerative methods:** Repeatedly identify and merge nodes that likely belong in the same region

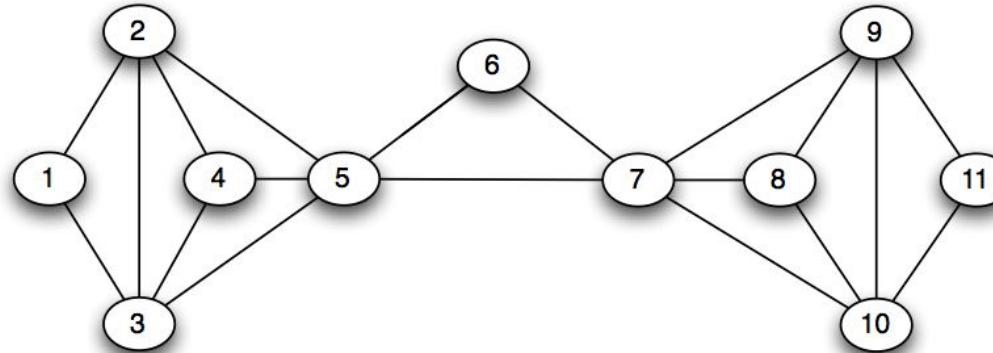
# Divisive Methods



## Agglomerative Methods



# How to split/cluster?



[EK10] Figure 3.15: A network can display tightly-knit regions even when there are no bridges or local bridges along which to separate it.

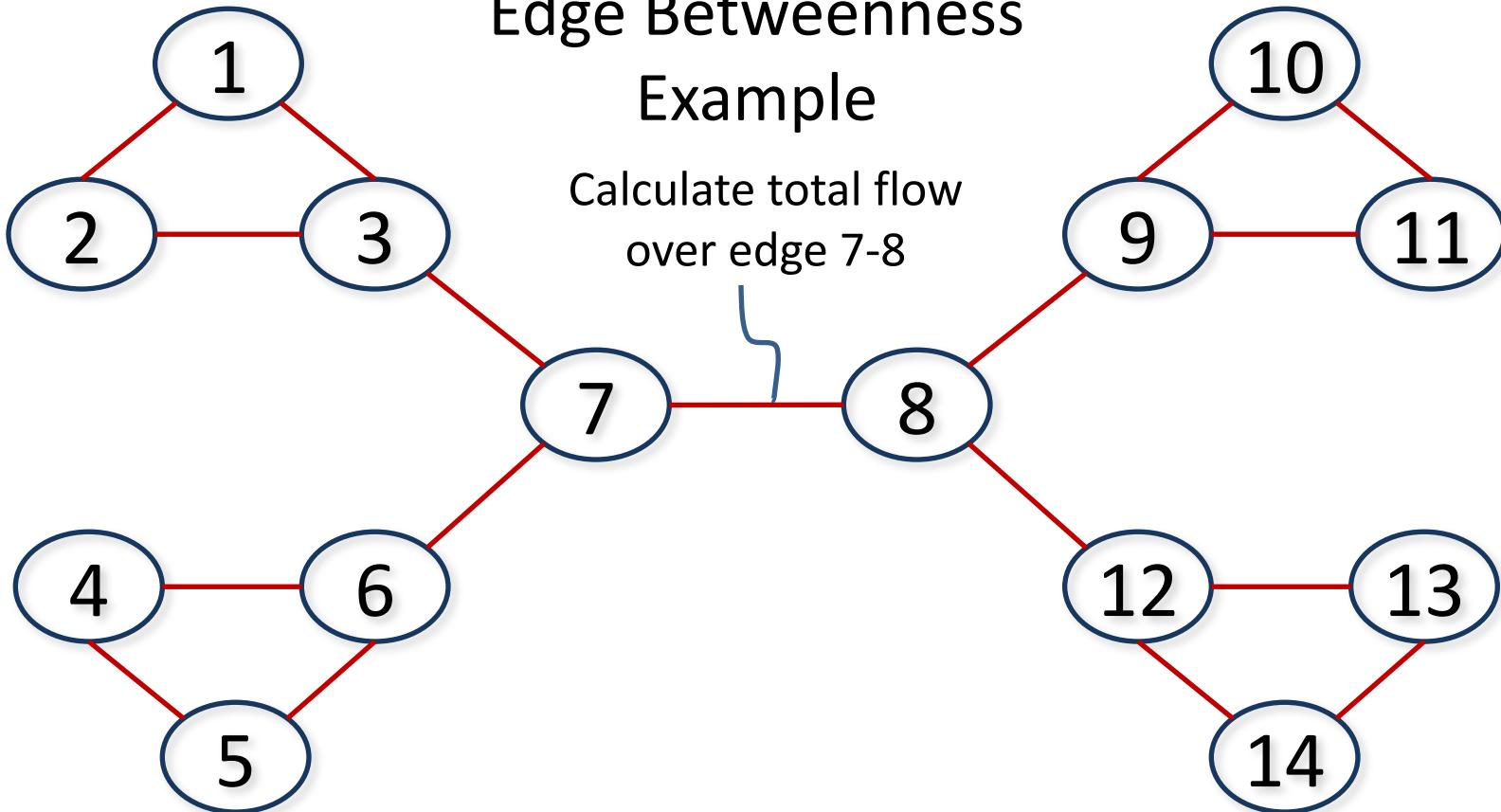
# Girvan-Newman Algorithm

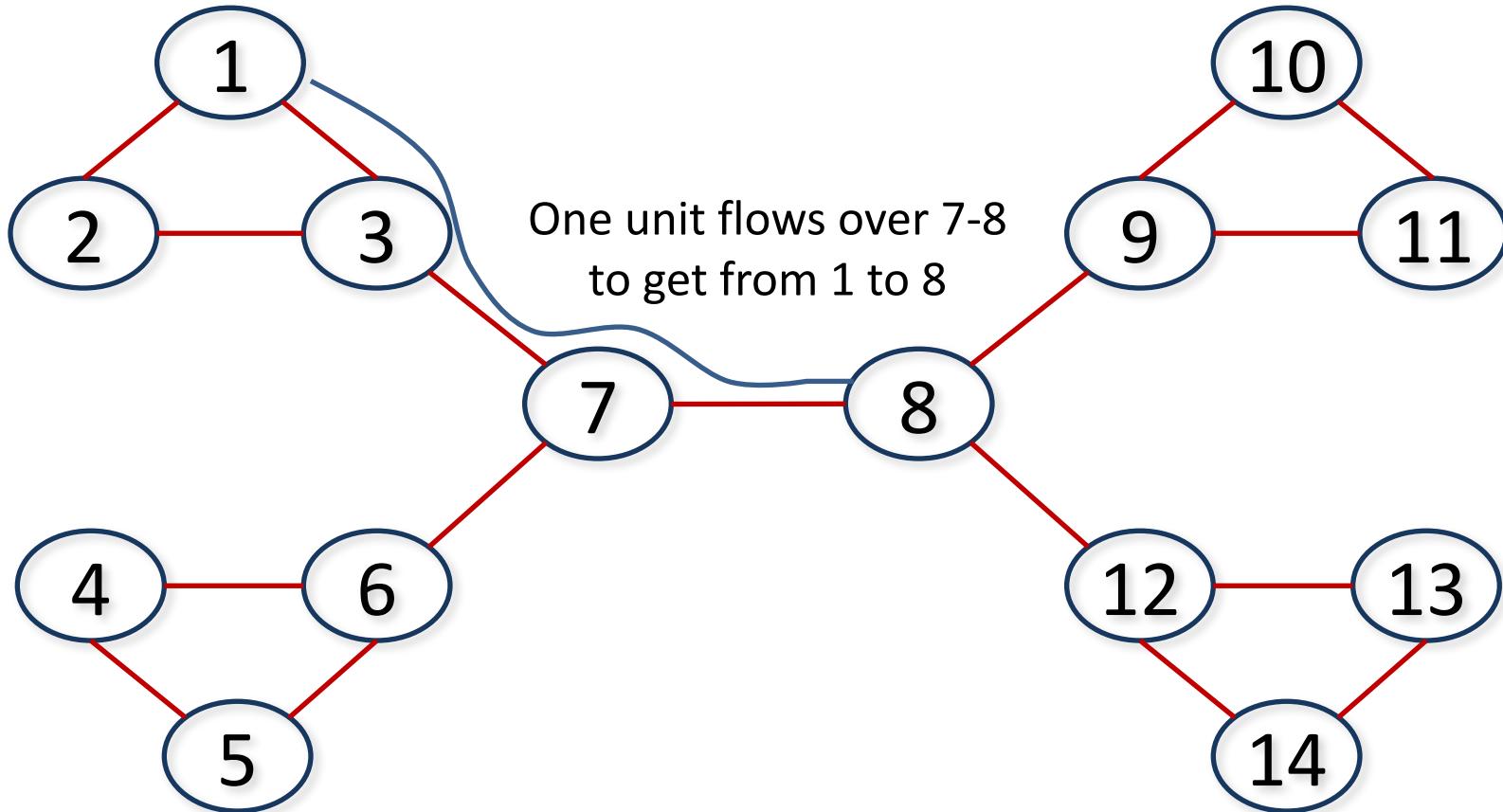
- Divisive method - Uses *edge betweenness* to identify edges to remove
- **Edge betweenness:** Total amount of "flow" an edge carries between all pairs of nodes
  - a single unit of flow between two nodes divides itself evenly among all shortest paths between the nodes ( $1/k$  units flow along each of  $k$  shortest paths)
- Betweenness is a measure of *centrality*, or importance, in a graph

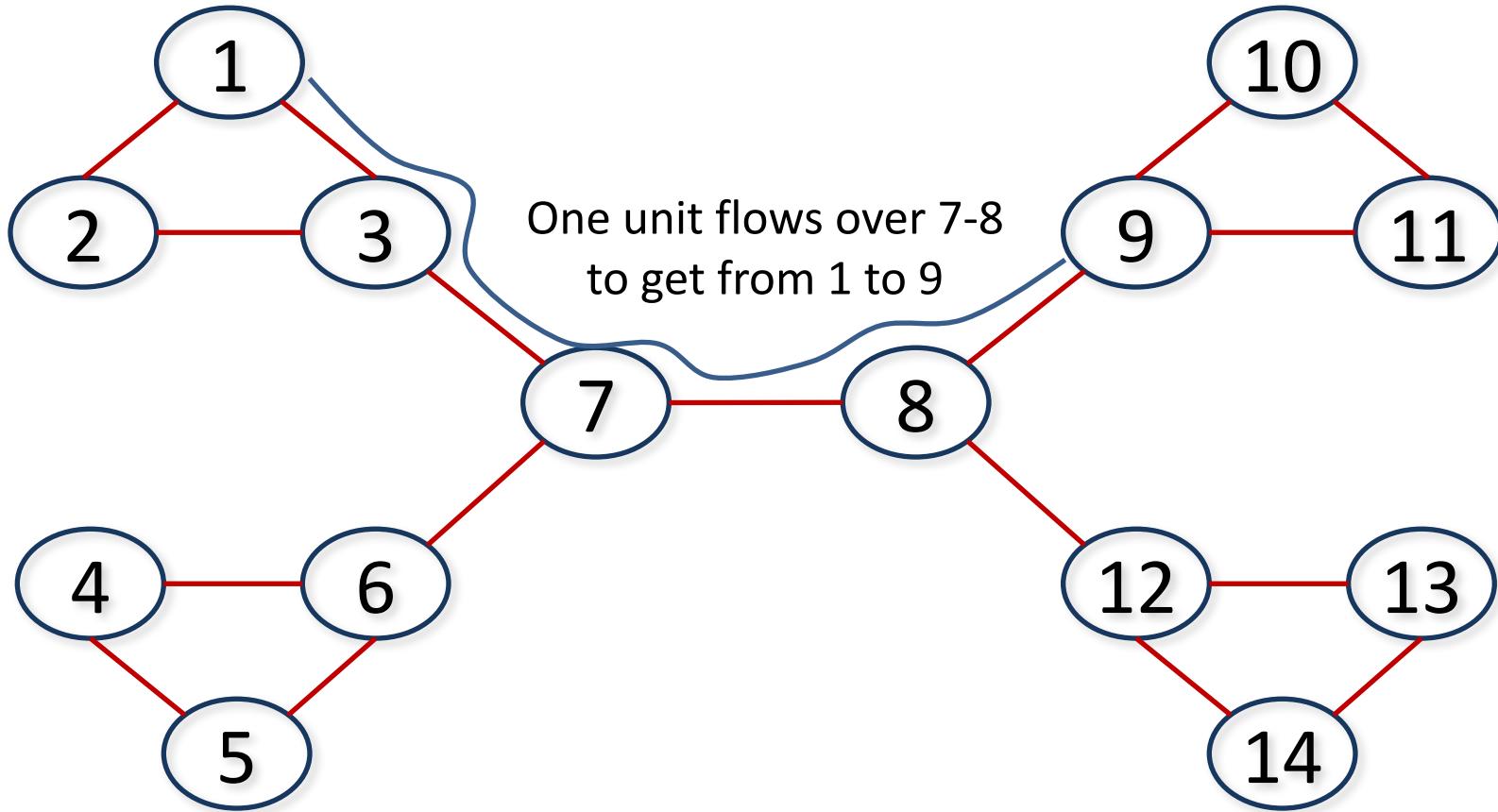
Girvan M. and Newman M. E. J., ["Community structure in social and biological networks"](#), Proc. Natl. Acad. Sci. USA **99**, 7821–7826 (2002)  
[Centrality](#) (Wikipedia)

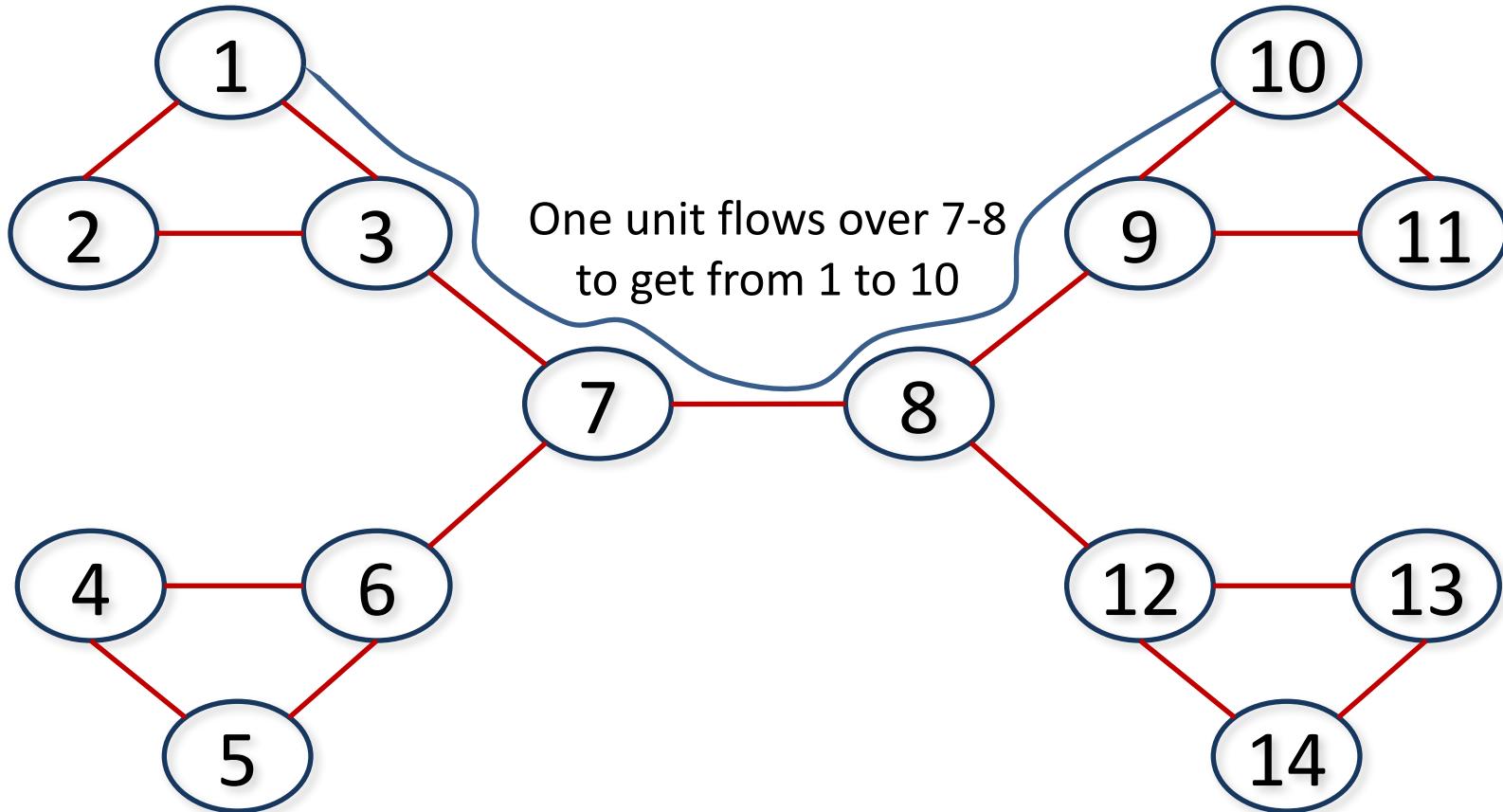
## Edge Betweenness Example

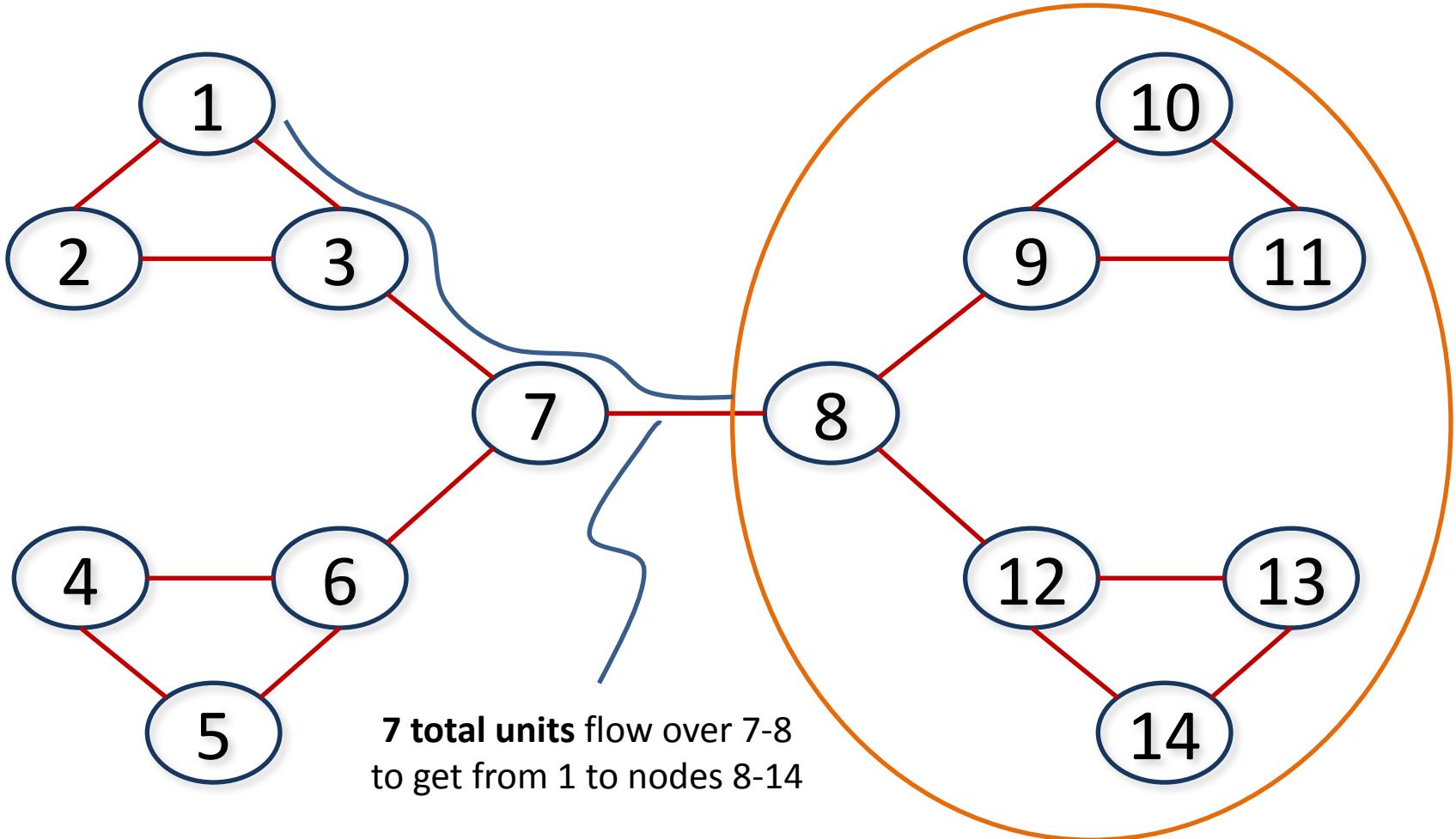
Calculate total flow over edge 7-8

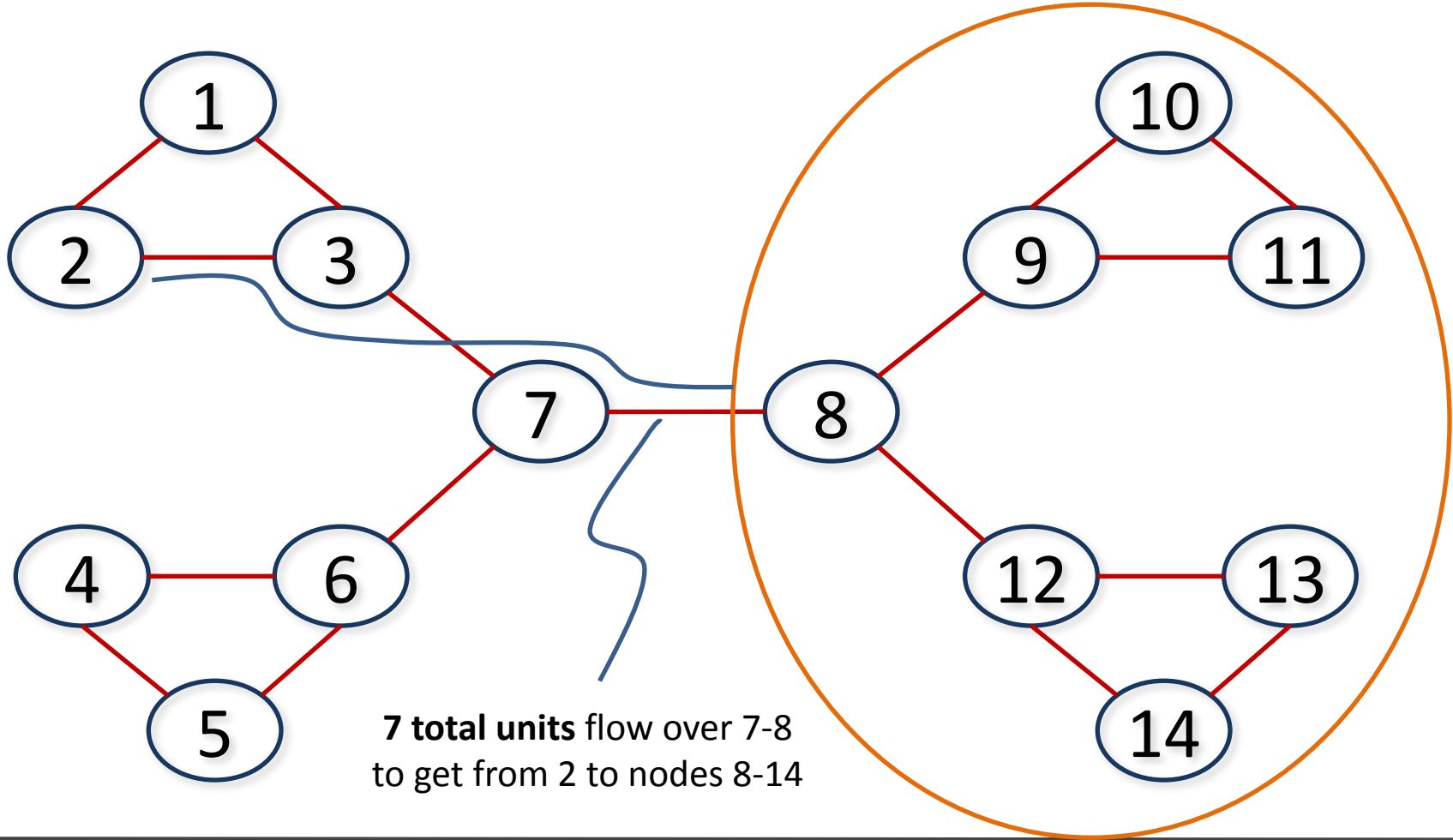


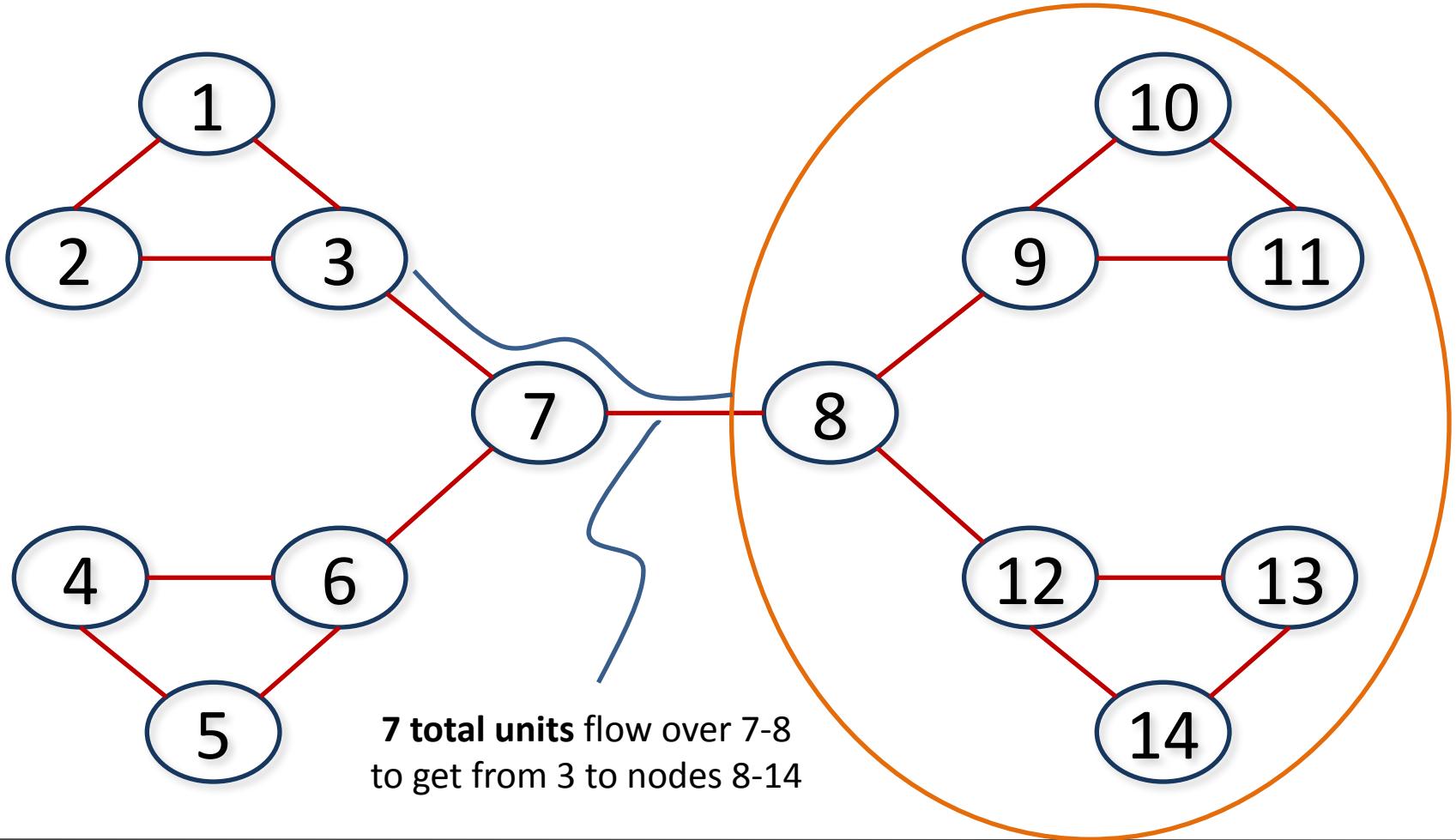




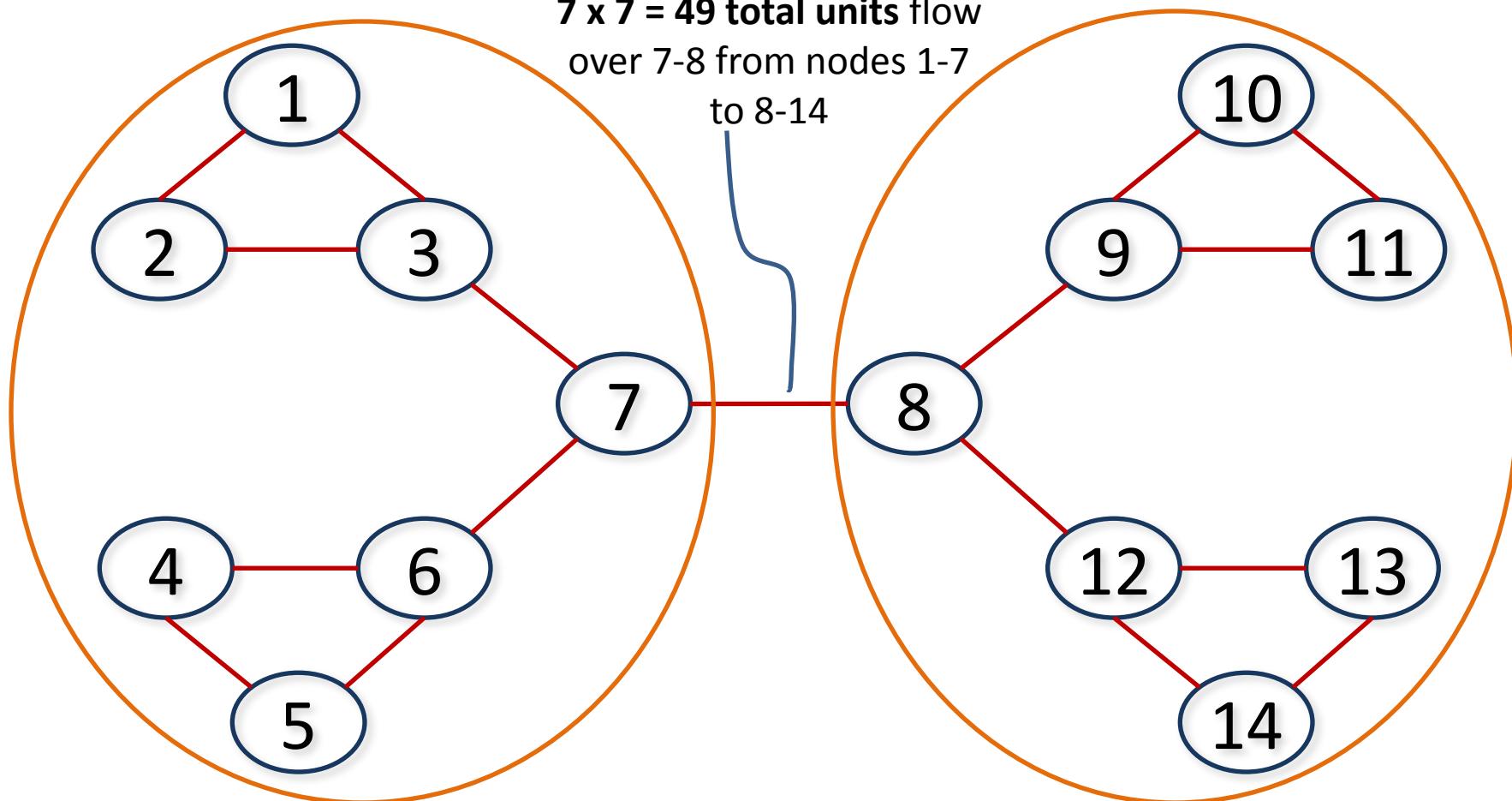


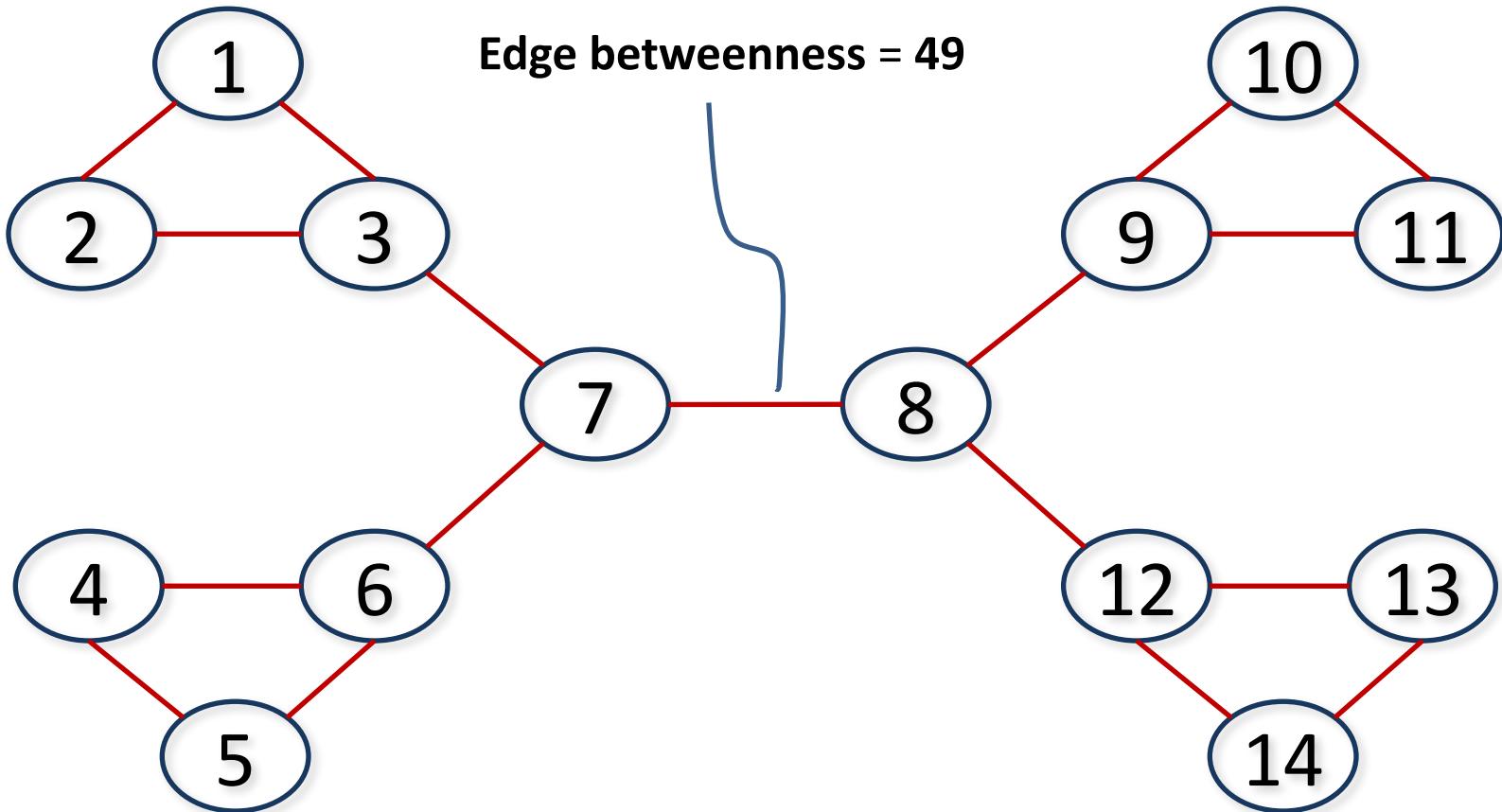


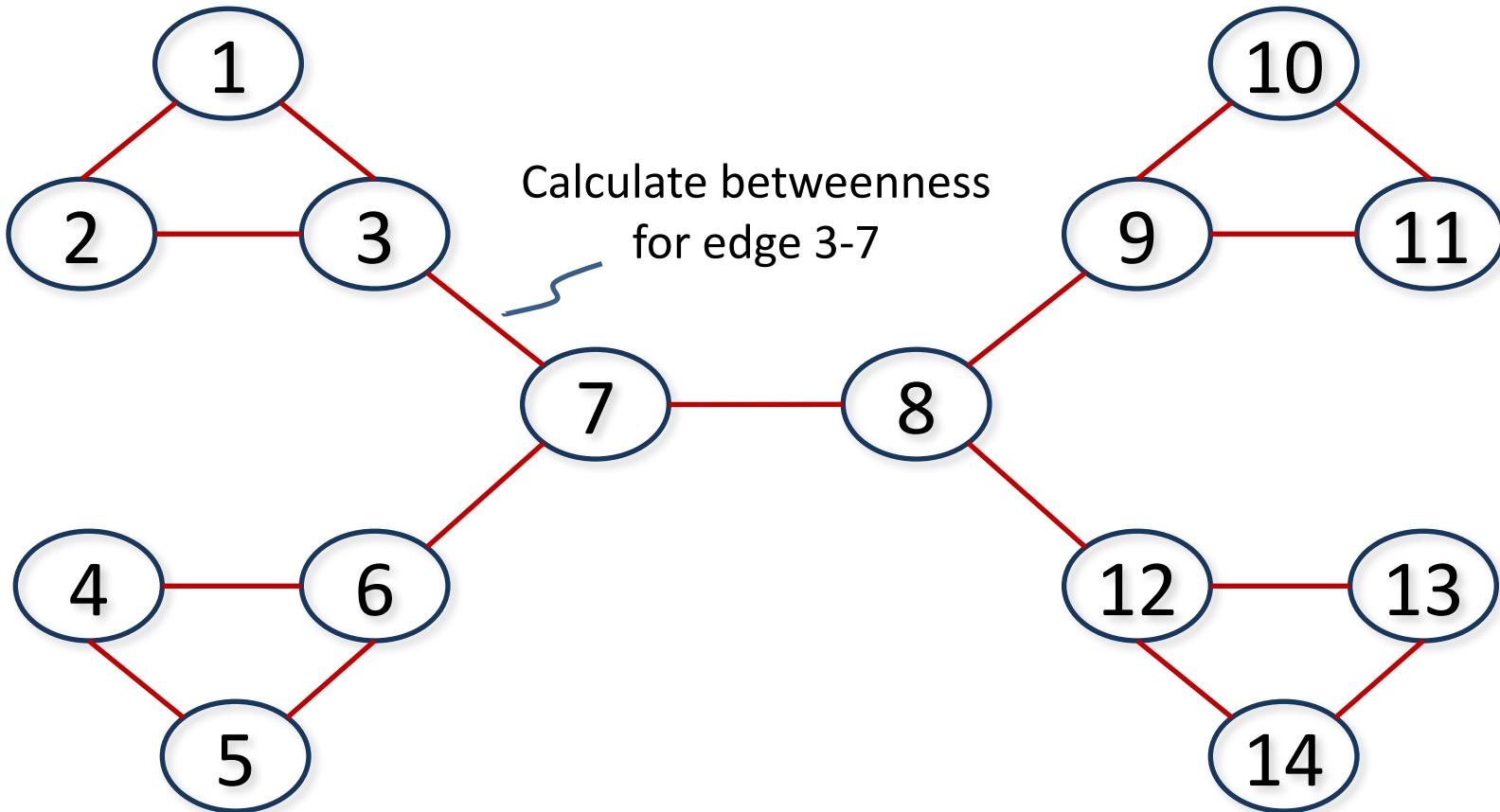


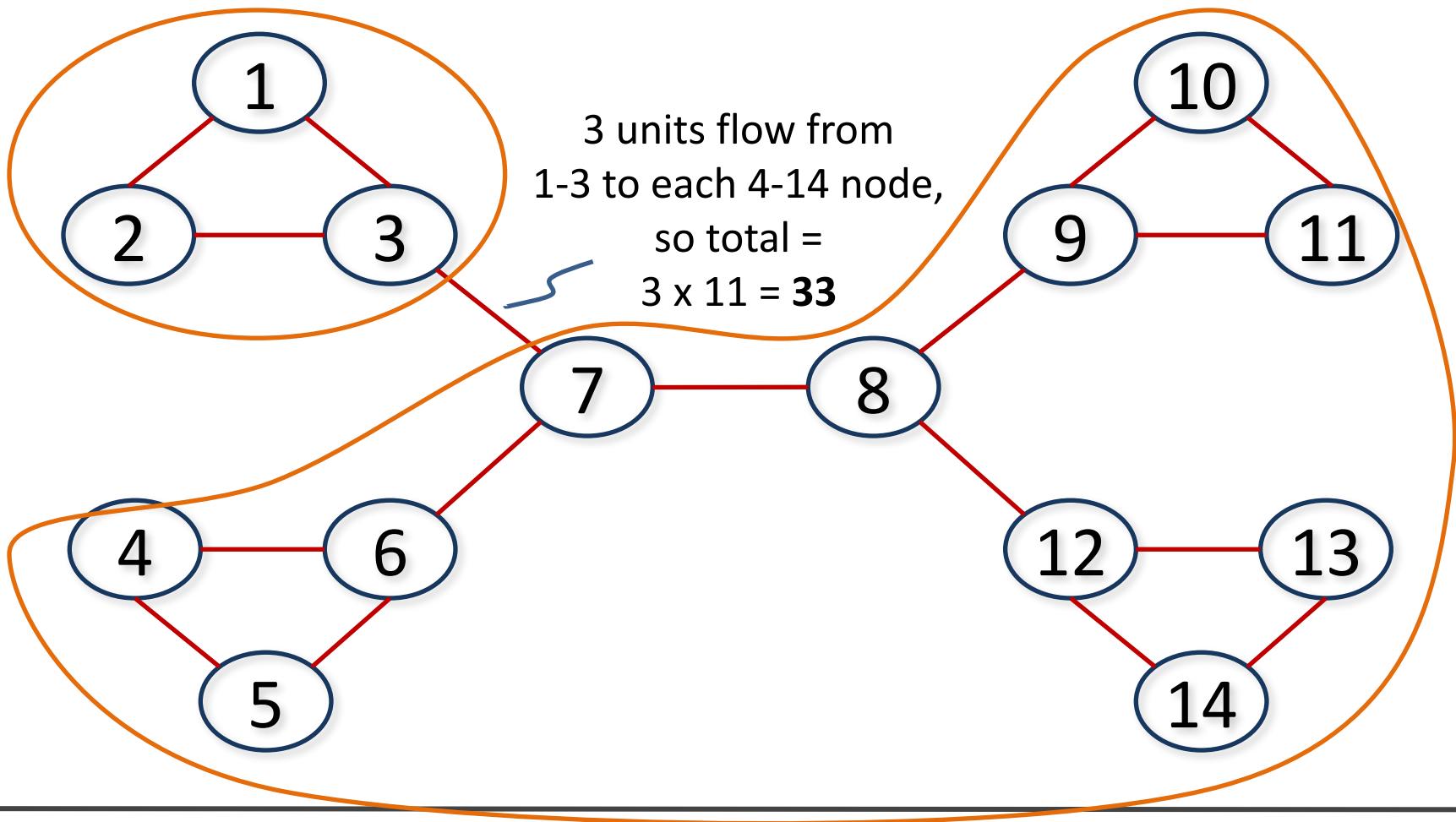


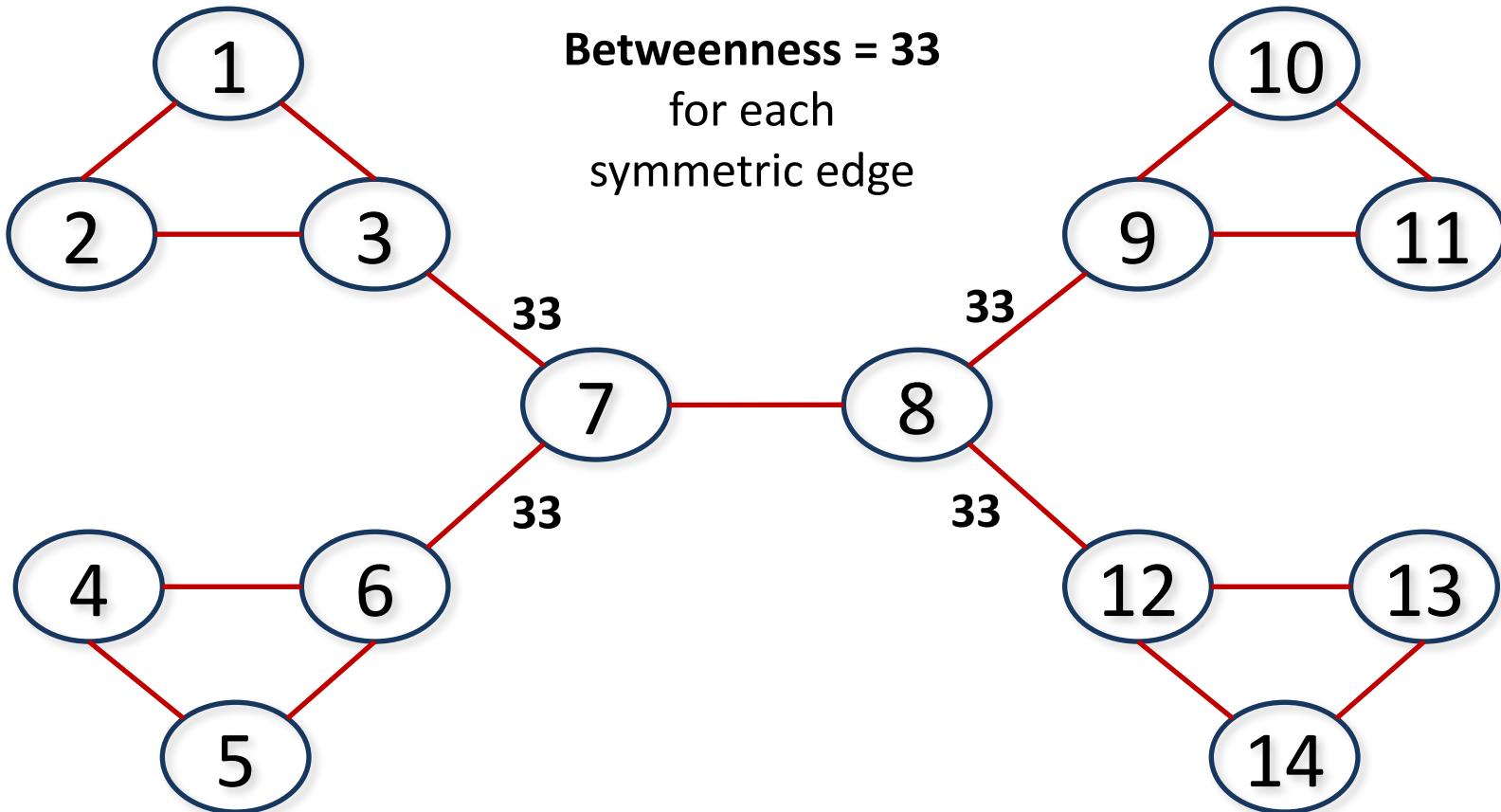
$7 \times 7 = 49$  total units flow  
over 7-8 from nodes 1-7  
to 8-14

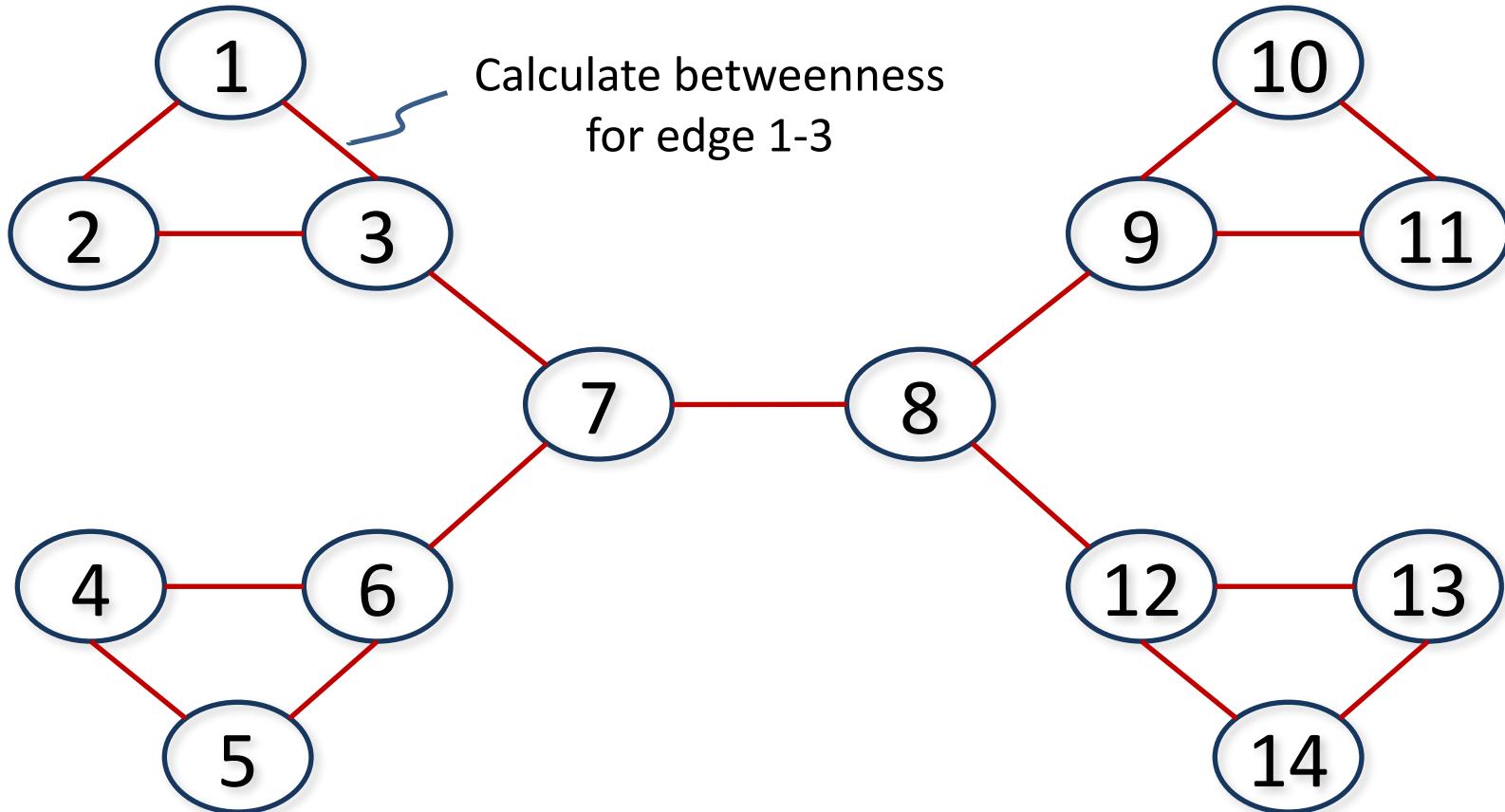


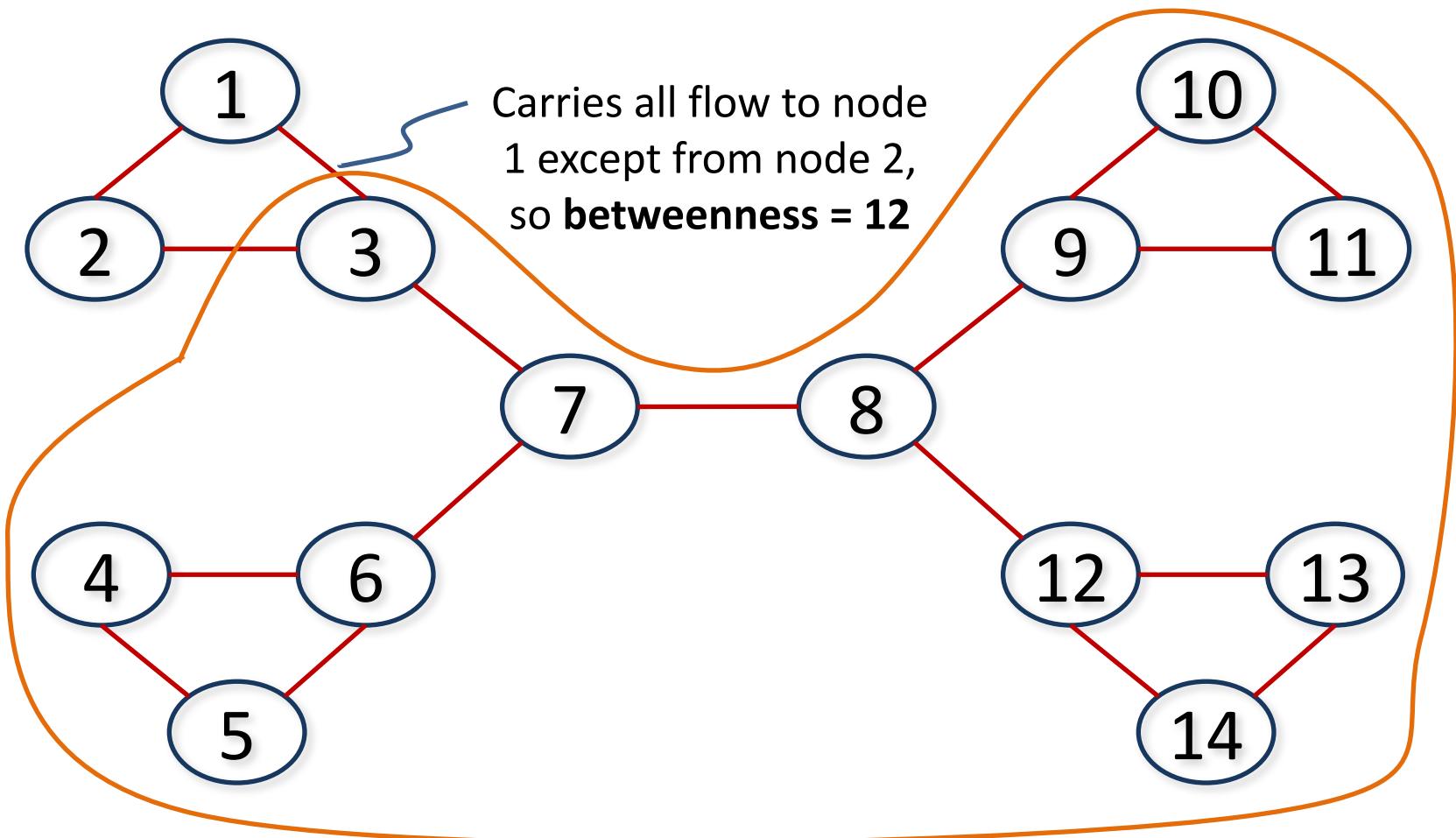


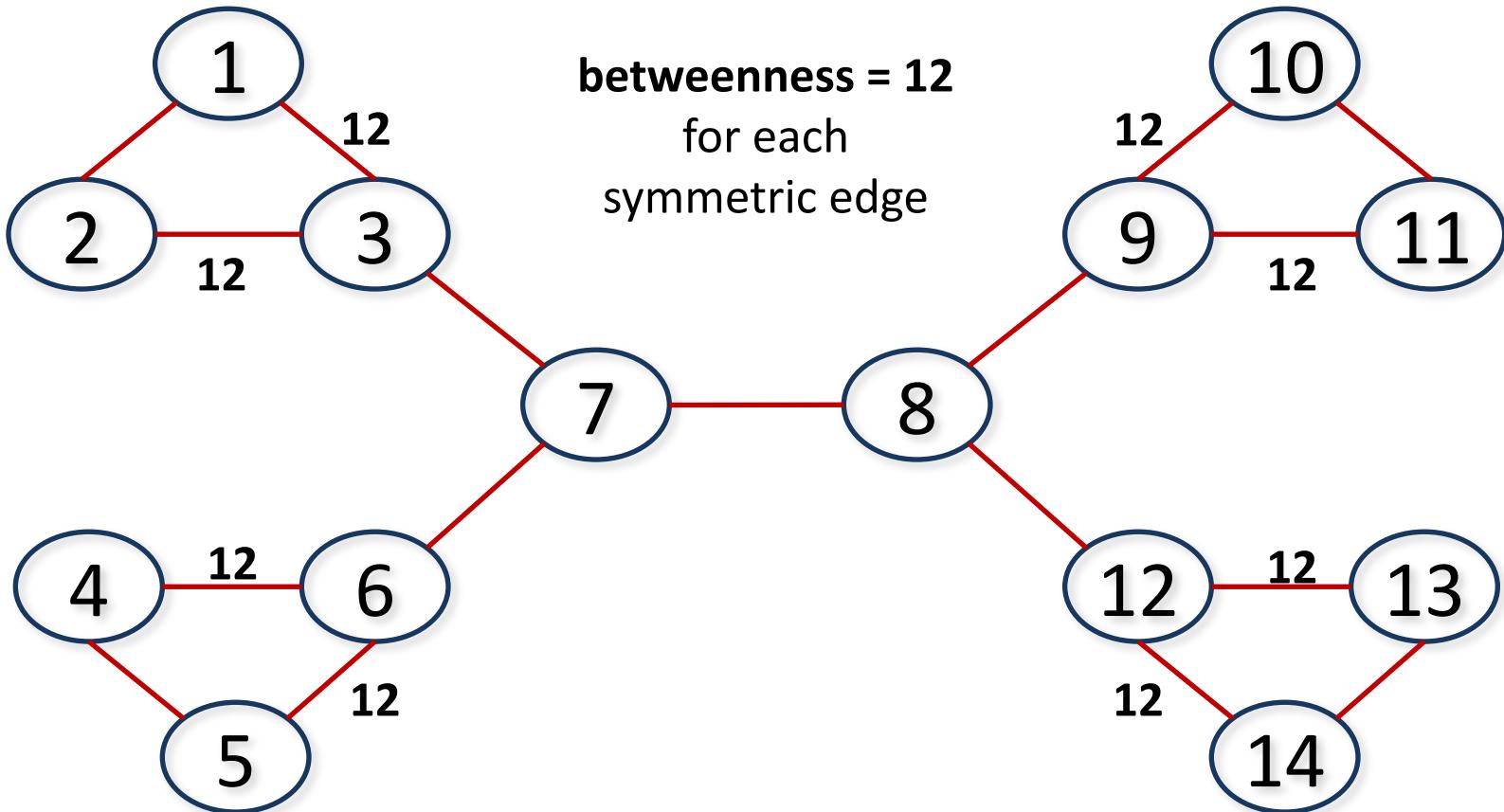


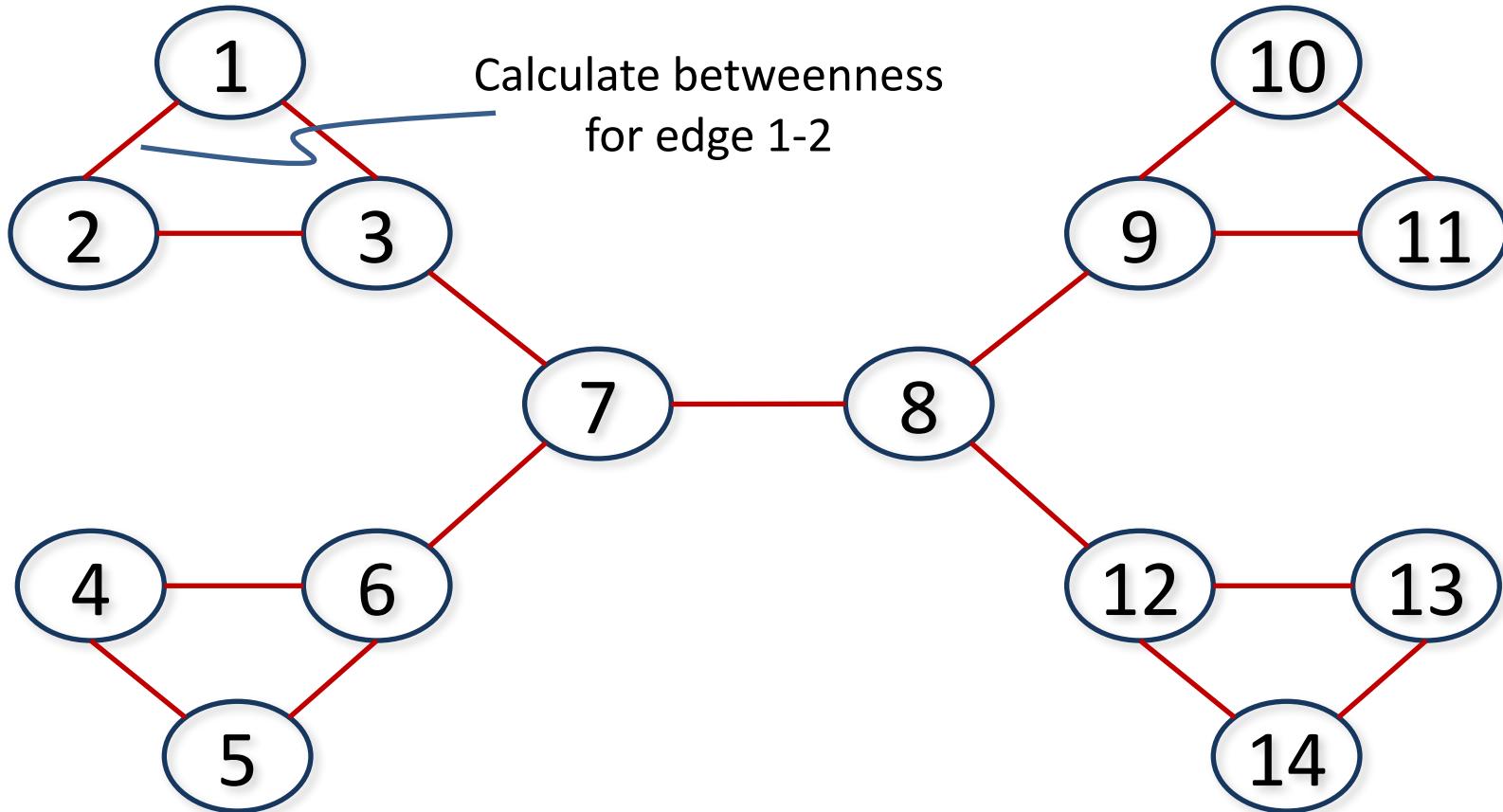


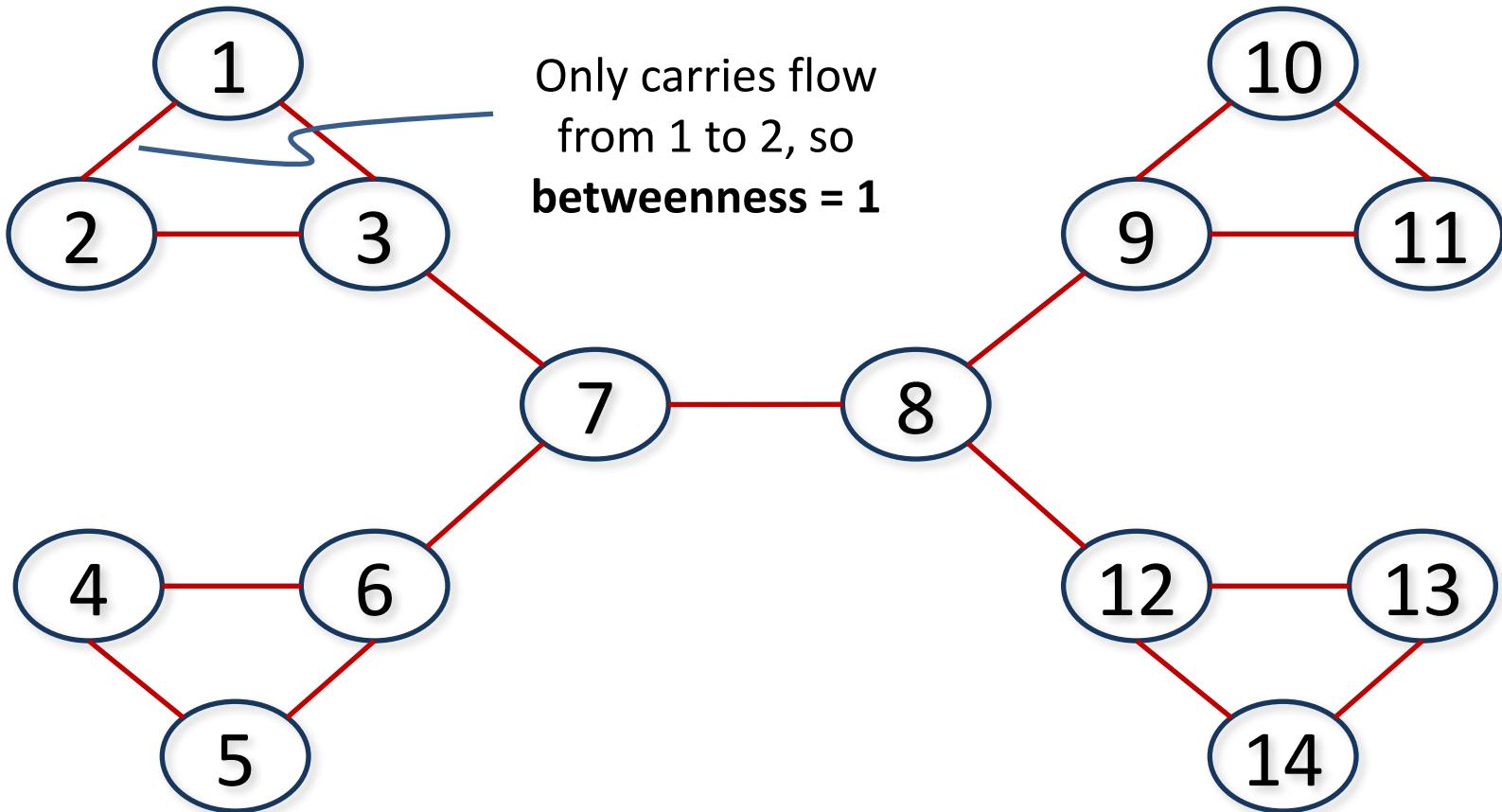


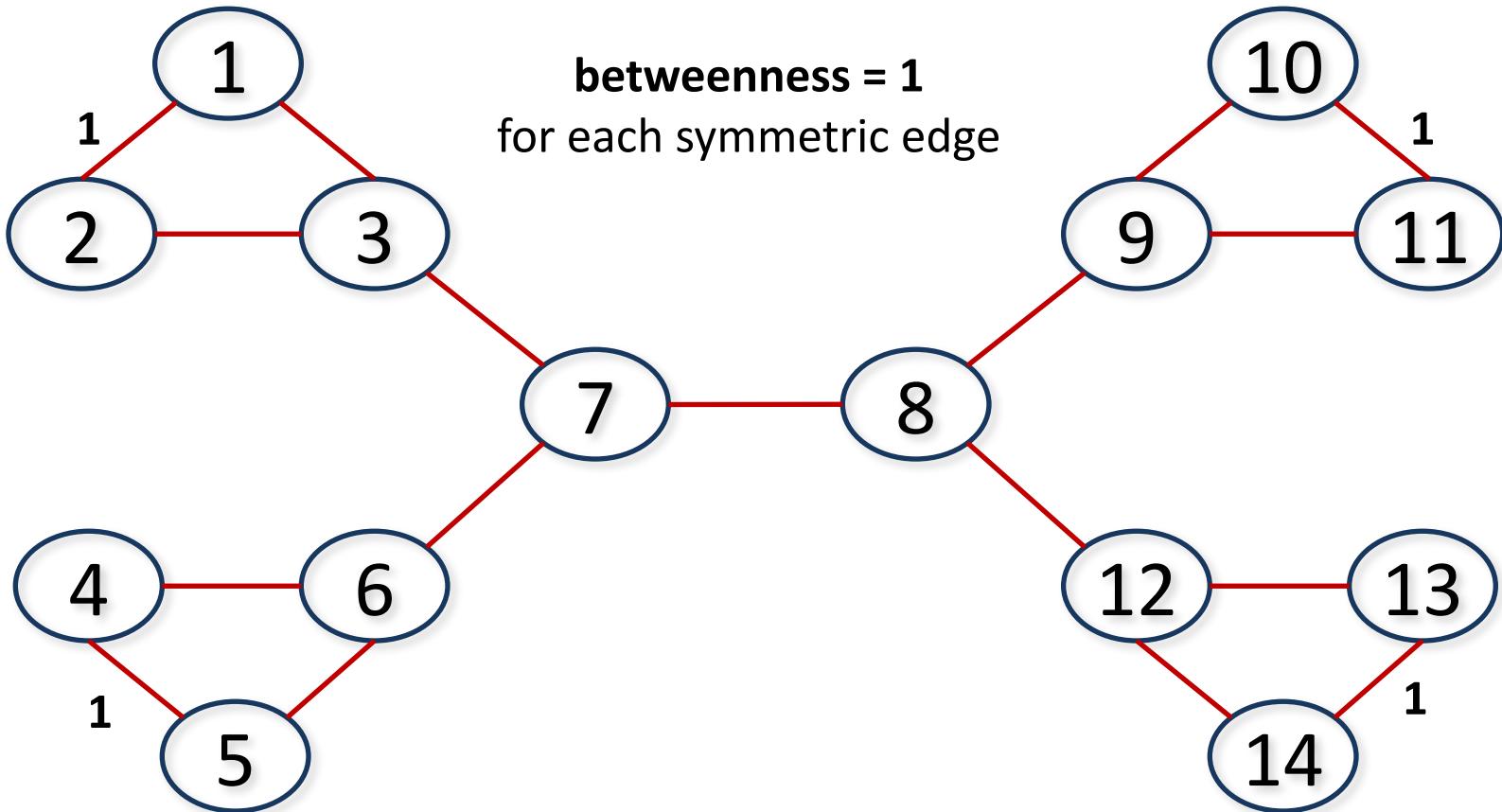


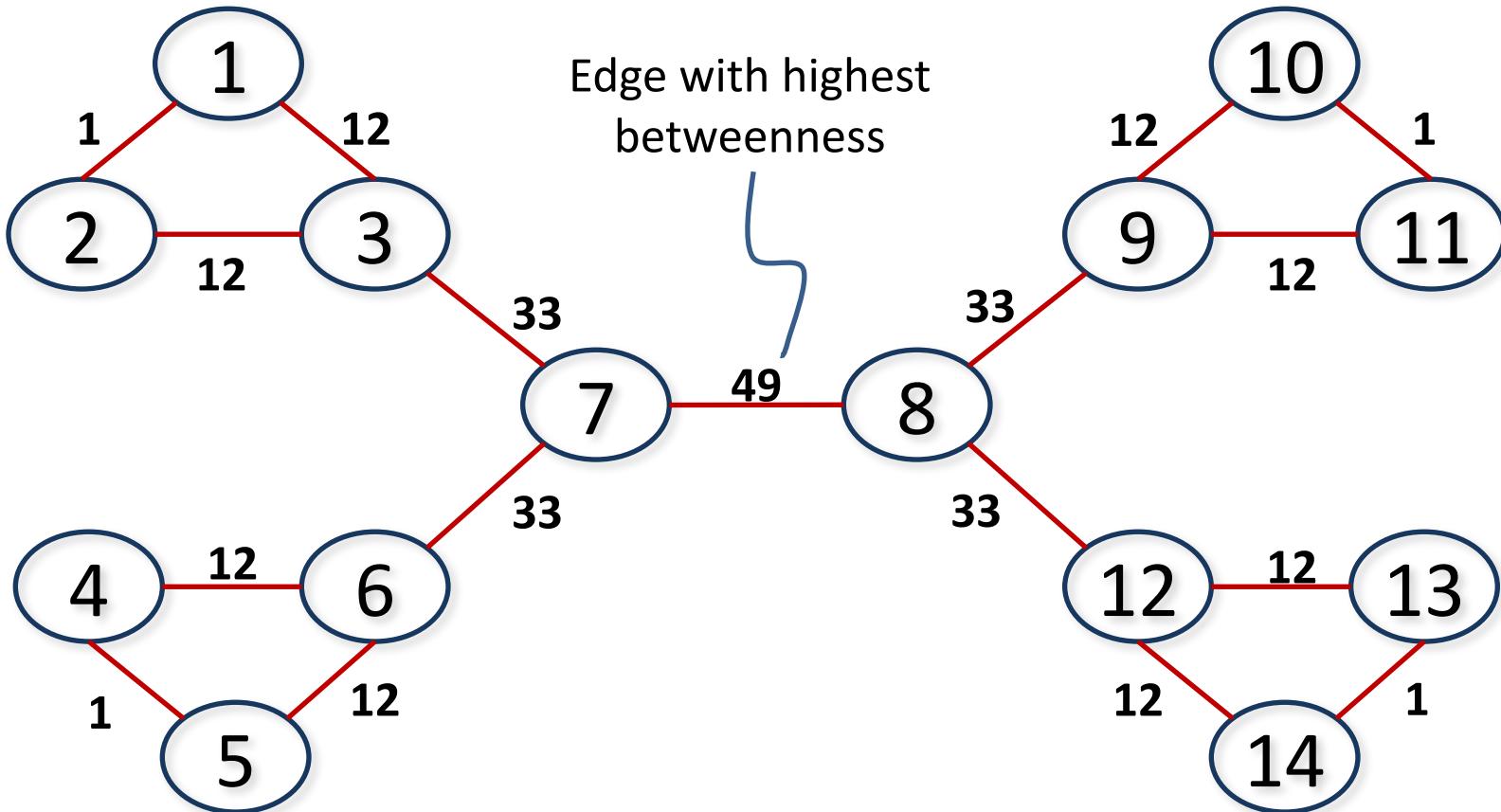












# Girvan-Newman Algorithm

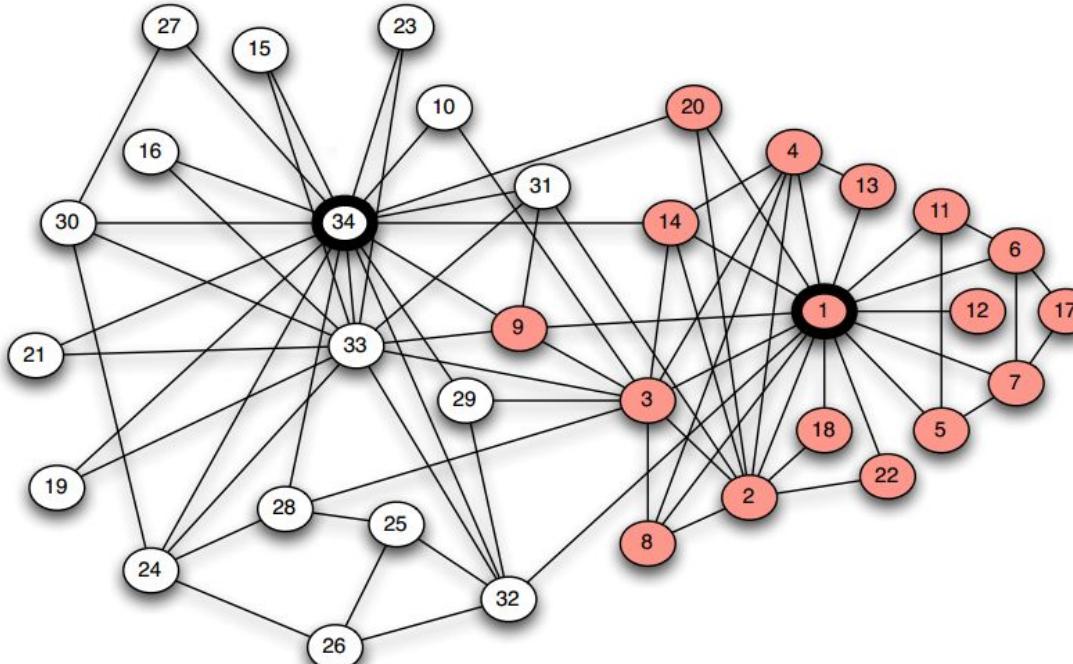
1. Calculate betweenness of all edges
2. Remove the edge(s) with highest betweenness
3. Repeat steps 1 and 2 until graph is partitioned into as many regions as desired

# Girvan-Newman Pseudocode

```
Given graph G,  
while( edgeCount > 0 and clusterCount < maximumClusterThreshold )  
{  
Step 1    allEdgesBetweennessList = G.getBetweennessValues()  
            maximumBetweennessValue = getMaximum(allEdgeBetweennessList)  
  
Step 2    edgeWithMaximumBetweennessValue =  
            getEgdeWithMaximumBetweennessValue(maximumBetweennessValue)  
  
            G.remove(edgeWithMaximumBetweennessValue)  
}
```

Step 3

# Karate Club splits after a dispute. Can new clubs be identified based on network structure?

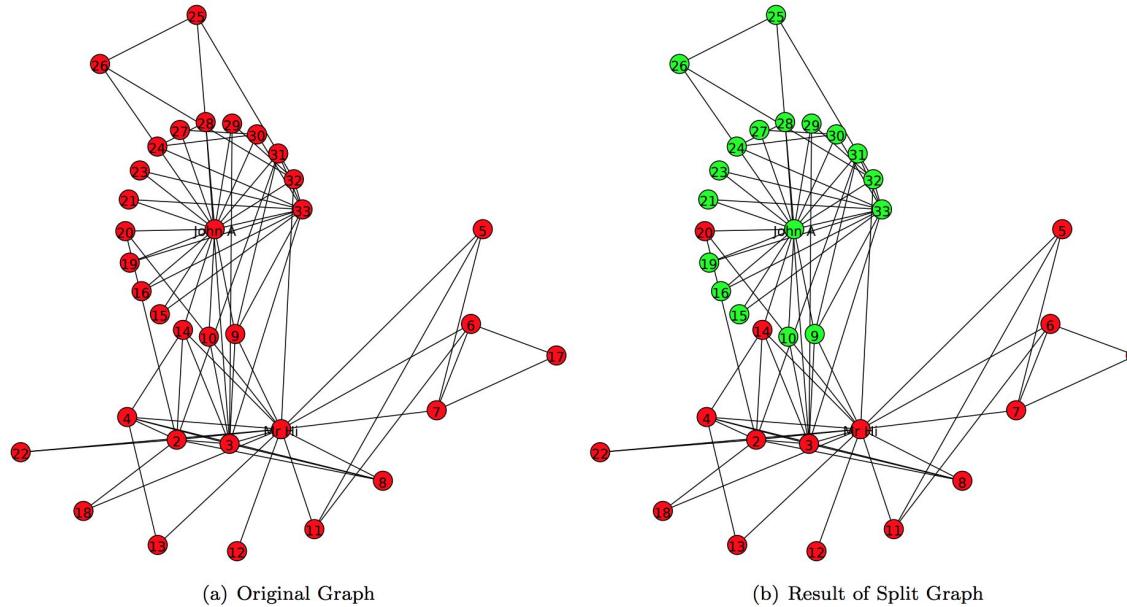


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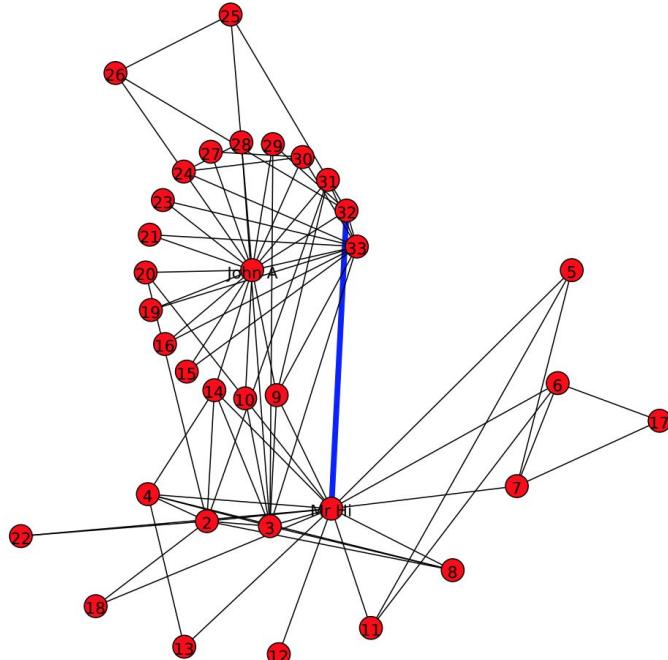
# Girvan-Newman run on Karate-Club graph

Figure 1: Zachary's Karate Club Graphs

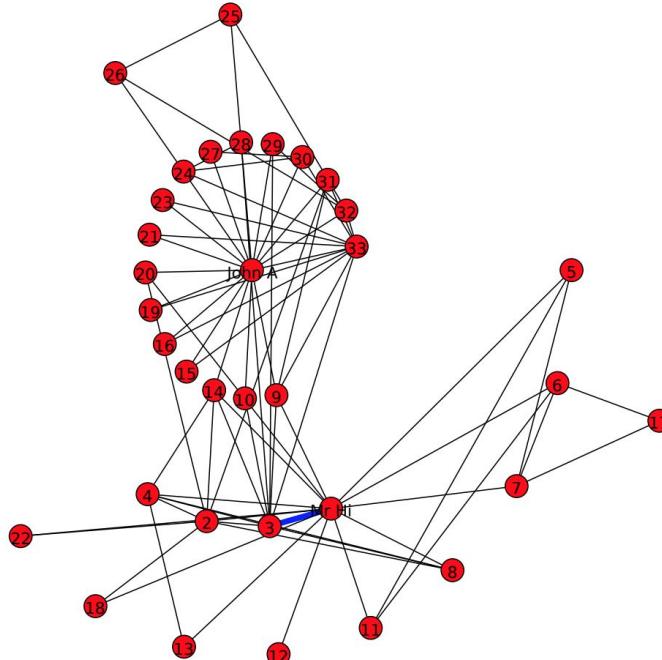


Graphs generated by Alexander Nwala, Fall 2014

# Girvan-Newman run on Karate-Club graph

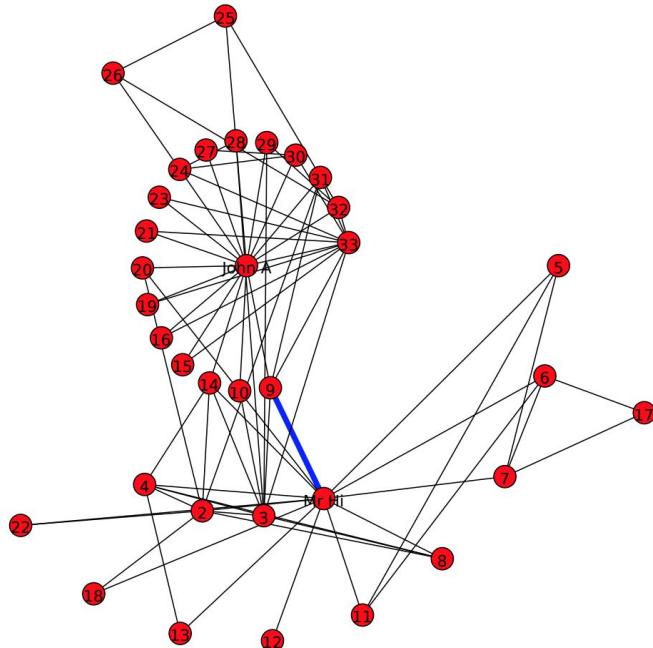


(a) Iteration: 0, Clusters: 1

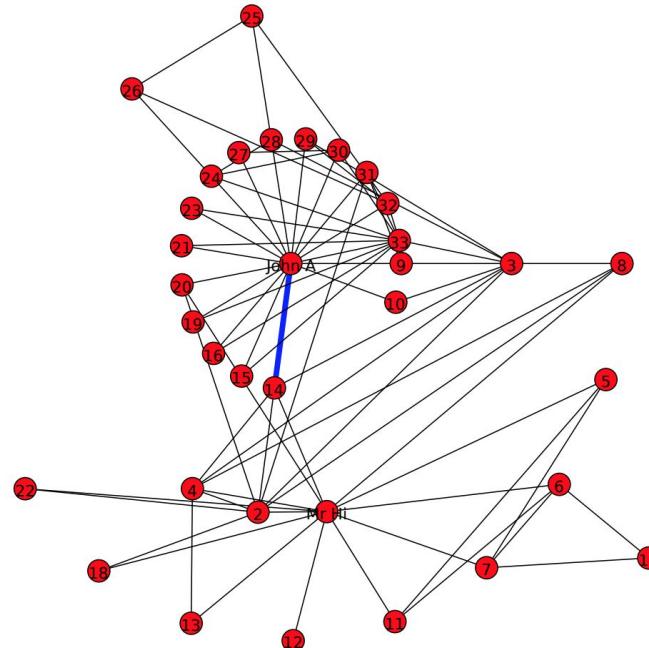


(b) Iteration: 1, Clusters: 1

# Girvan-Newman run on Karate-Club graph

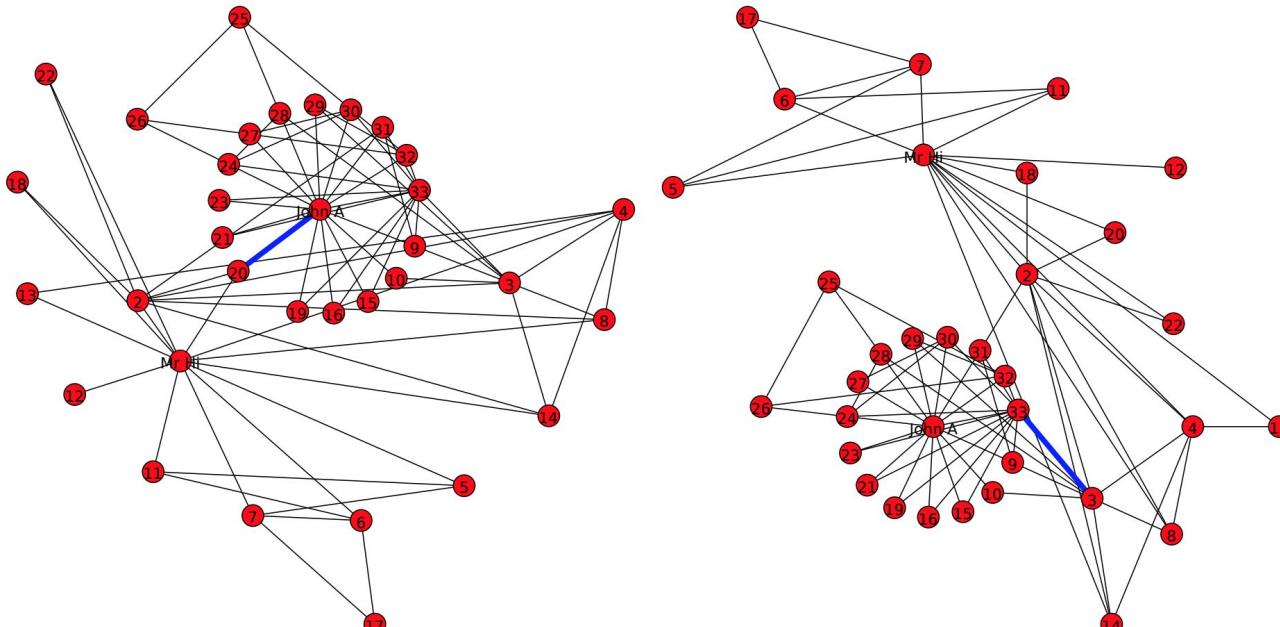


(c) Iteration: 2, Clusters: 1

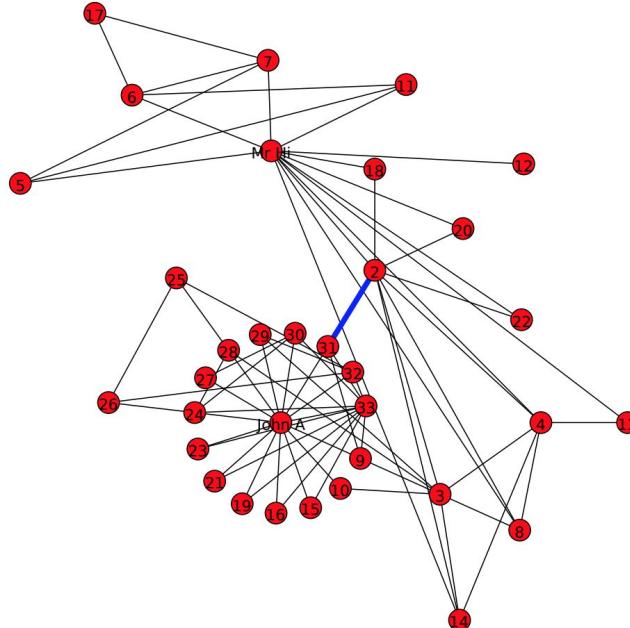


(d) Iteration: 3, Clusters: 1

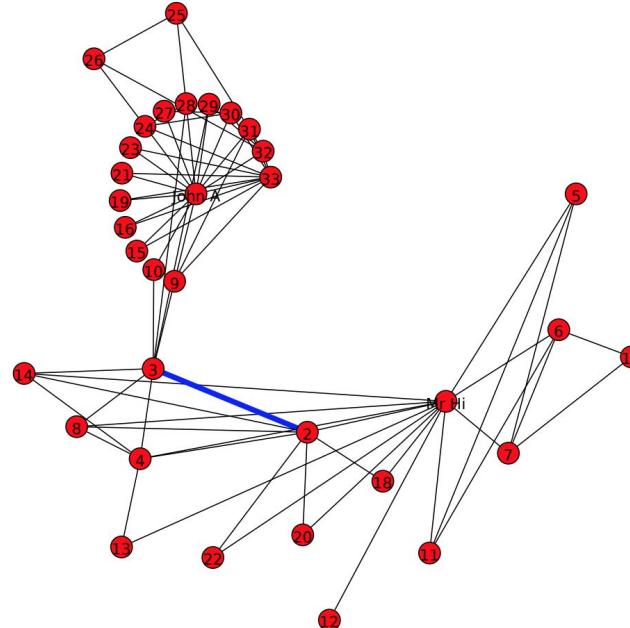
# Girvan-Newman run on Karate-Club graph



# Girvan-Newman run on Karate-Club graph

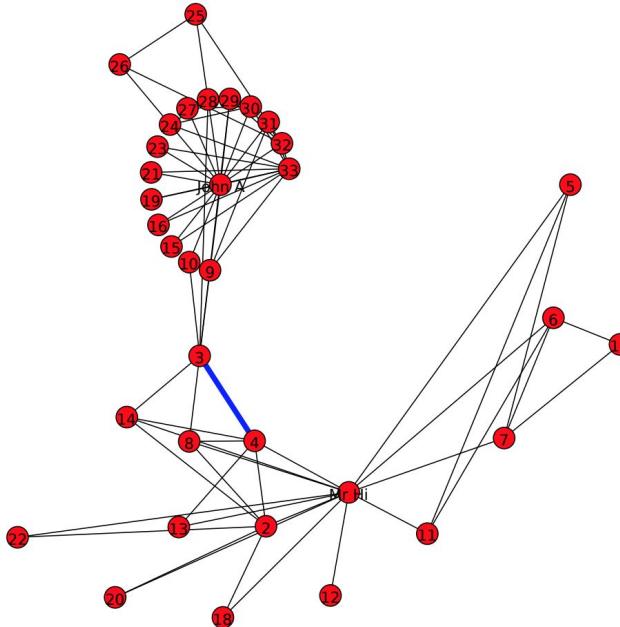


(c) Iteration: 6, Clusters: 1

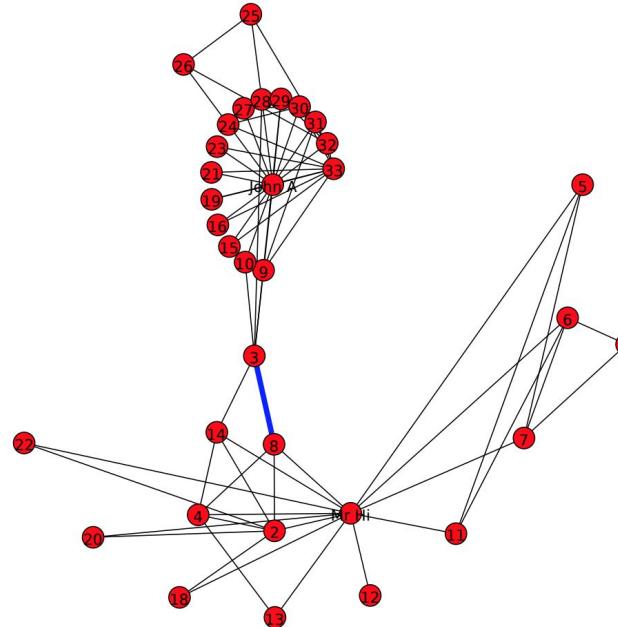


(d) Iteration: 7, Clusters: 1

# Girvan-Newman run on Karate-Club graph

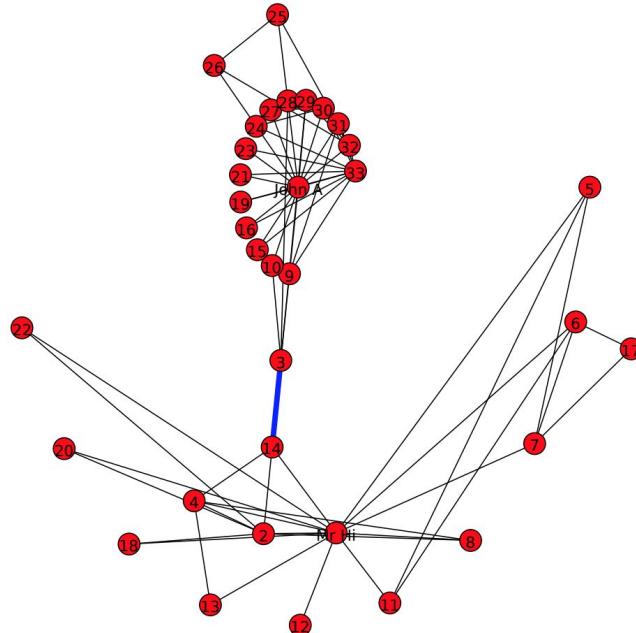


(a) Iteration: 8, Clusters: 1

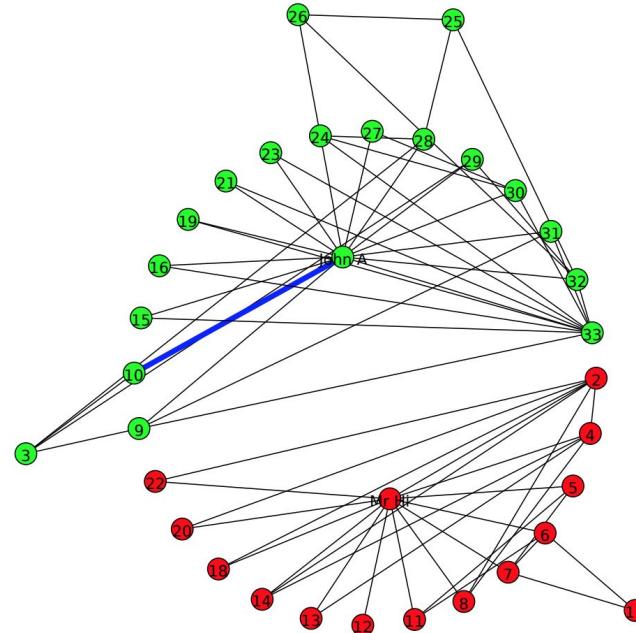


(b) Iteration: 9, Clusters: 1

# Girvan-Newman run on Karate-Club graph



(c) Iteration: 10, Clusters: 1



(d) Iteration: 11, Clusters: 2

# Objectives

- Describe the friendship paradox.
- Explain the triadic closure in social networks.
- Explain how edge embeddedness affects trust.
- Explain the edge betweenness property.
- Explain the steps in the Girvan-Newman graph partitioning algorithm.