Federal University of Rio de Janeiro

UFRJ - Time Feliz ^−^

Chris Ciafrino, Gustavo Miguel e Letícia Freire

adapted from KTH ACM Contest Template Library

2019

Contest (1)

template.cpp
29 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

using lint = long long;
using ldouble = long double;

const double PI = static_cast<double>(acosl(-1.0));

// Retorna -1 se a < b, 0 se a = b e 1 se a > b.
int cmp_double(double a, double b = 0, double eps = 1e-9) {
    return a + eps > b ? b + eps > a ? 0 : 1 : -1;
}

//be careful with cin optimization
string read_string() {
    char *str;
    scanf("%ms", &str);
    string result(str);
    free(str);
    return result;
}

int main() {
    ios_base::sync_with_stdio(0), cin.tie(0), cout.tie(0);
    cin.exceptions(cin.failbit);

    return 0;
}
```

hash.sh
1 lines

```
tr -d '[:space:]' | md5sum
```

hash-cpp.sh
1 lines

```
cpp -P -fpreprocessed | tr -d '[:space:]' | md5sum
```

vimrc
29 lines

```
set nocp ai bs=2 hls ic is lbr ls=2 mouse=a nu ru sc scs
    ↪smd so=3 sw=4 ts=4
filetype plugin indent on
syn on
map gA m'ggVG"+y''

com -range=% -nargs=1 P exe "<line1>,<line2>!".<q-args> |y|
    ↪sil u|echom @"
com -range=% Hash <line1>,<line2>P tr -d '[:space:]' |
    ↪md5sum
au FileType cpp com! -buffer -range=% Hash <line1>,<line2>P
    ↪ cpp -dD -P -fpreprocessed | tr -d '[:space:]' |
    ↪md5sum

:autocmd BufNewFile *.cpp 0r /etc/vim/templates/cp.cpp

" shift+arrow selection
nmap <S-Up> v<Up>
nmap <S-Down> v<Down>
nmap <S-Left> v<Left>
nmap <S-Right> v<Right>
vmap <S-Up> <Up>
vmap <S-Down> <Down>
vmap <S-Left> <Left>
vmap <S-Right> <Right>
imap <S-Up> <Esc>v<Up>
imap <S-Down> <Esc>v<Down>
imap <S-Left> <Esc>v<Left>
imap <S-Right> <Esc>v<Right>
vmap <C-c> y<Esc>i
vmap <C-x> d<Esc>i
map <C-v> pi
imap <C-v> <Esc>pi
imap <C-z> <Esc>ui
```

troubleshoot.txt
62 lines

```
Pre-submit:
Write down your thoughts, even if they don't completely
    ↪solve the problem.
Stay organized (don't leave papers all over the place)!
Give your variables (and files) useful names!
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Remove debug output.
Make sure to submit the right file.
You should know what your code is doing ...

Wrong answer:
Read the full problem statement again.
Have you understood the problem correctly?
Are you sure your algorithm works?
Try writing a slow (but correct) solution.
Can your algorithm handle the whole range of input?
Did you consider corner cases (n=1) or other special cases?
Print your solution! Print debug output, as well.
Is your output format correct? (including whitespace)
Are you clearing all data structures between test cases?
Any uninitialized variables?
Any undefined behavior (array out of bounds)?
Any overflows or NaNs (shifting ll by 64 bits or more)?
Confusing N and M, i and j, etc.?
Confusing ++i and i++?
Correctly account for numbers close to (but not) zero.
For line sweeping over polygons, correctly deal with
    ↪vertices.
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some test cases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Rewrite your solution from the start or let a teammate do
    ↪it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
```

Do you have any possible infinite loops?
What's your complexity? Large TL does not mean that
    ↪something simple (like NlogN) isn't intended.
Are you copying a lot of unnecessary data? (References)
Avoid vector, map. (use arrays/unordered_map)
How big is the input and output? (consider FastI)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need
    ↪?
Are you clearing all data structures between test cases?
Delete pointers?

Mathematics (2)

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.2 Master theorem

Given a recurrence of the form $T(n) = aT(\frac{n}{b}) + f(n)$ where $a \geq 1$, $b > 1$.

1) If $f(n) = \mathcal{O}(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$, then

$$T(n) = \Theta(n^{\log_b a})$$

2) If $f(n) = \Theta(n^{\log_b a})$, then

$$T(n) = \Theta(n^{\log_b a} \log n)$$

3) If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$ (and $af(\frac{n}{b}) \leq cf(n)$ for some $c < 1$ for all n sufficiently large), then

$$T(n) = \Theta(f(n))$$

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\dfrac{a + b}{a - b} = \dfrac{\tan \dfrac{\alpha + \beta}{2}}{\tan \dfrac{\alpha - \beta}{2}}$

Pick's: A polygon on an integer grid strictly containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$. (Nothing similar in higher dimensions)
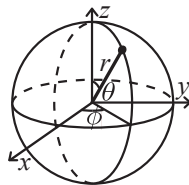
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and
$A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.4.3 Spherical coordinates

$$x = r \sin \theta \cos \phi \qquad r = \sqrt{x^2 + y^2 + z^2}$$
$$y = r \sin \theta \sin \phi \qquad \theta = \text{acos}(z/\sqrt{x^2 + y^2 + z^2})$$
$$z = r \cos \theta \qquad \phi = \text{atan2}(y, x)$$

2.4.4 Centroid of a polygon

The x coordinate of the centroid of a polygon is given by $\frac{1}{3A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$, where A is twice the signed area of the polygon.

2.5 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$

$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) \qquad \int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n + 1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance
$\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$
where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Gambler's Ruin

Em um jogo no qual ganhamos cada aposta com probabilidade p e perdemos com probabilidade $q := 1 - p$, paramos quando ganhamos B ou perdemos A. Então $Prob(\text{ganhar B}) = \frac{1-(p/q)^B}{1-(p/q)^{A+B}}$.

2.8.2 Bertrand's ballot theorem

In an election where candidate A receives p votes and candidate B receives q votes with $p > q$, the probability that A will be strictly ahead of B throughout the count is $\frac{p-q}{p+q}$. If draw is a possible outcome, the probability will be equal to $\frac{p+1-q}{p+1}$, to find how many possible outcomes for both cases just multiply by $\binom{p+q}{q}$

2.8.3 Discrete distributions
Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \le p \le 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \ \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\text{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1}, \ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \ \sigma^2 = \lambda$$

2.8.4 Continuous distributions
Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an absorbing chain if
1. there is at least one absorbing state and
2. it is possible to go from any state to at least one absorbing state in a finite number of steps.
A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki}t_k$.

Data Structures (3)

HashMap.h
Description: Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 (if provided).   7 lines

```cpp
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash {
  const uint64_t C = ll(2e18 * M_PI) + 71; // large odd
      ↪number
  ll operator()(ll x) const { return __builtin_bswap64(x*C)
      ↪; }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h
    ↪({},{},{},{},{1<<16}); // hash-cpp-all = 1443
    ↪bc6c3b4062862ebb83553bc131a0
```

OrderStatisticTree.h
Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element.
Time: $\mathcal{O}(\log N)$ 16 lines

```cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
```

```
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9));
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
} // hash-cpp-all = 782797f91ca134bf996558987dbf1924
```

DSU.h
Description: Disjoint-set data structure
Time: $\mathcal{O}(\alpha(N))$
<div align="right">20 lines</div>

```
struct UF {
    int n;
    vector<int> parent, rank;
    UF(int _n): n(_n), parent(n), rank(n, 0) {
        iota(parent.begin(), parent.end(), 0);
    }
    int find(int v) {
        if (parent[v] == v) return v;
        return parent[v] = find(parent[v]);
    }
    int unite(int a, int b) {
        a = find(a);
        b = find(b);
        if (a == b) return a;
        if (rank[a] > rank[b]) swap(a, b);
        parent[a] = b;
        if (rank[a] == rank[b]) ++rank[b];
        return b;
    }
}; // hash-cpp-all = b237fabe1fcbfbf7f52205b112487f5e
```

DSURoll.h
Description: Disjoint-set data structure with undo.
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$
<div align="right">21 lines</div>

```
struct RollbackUF {
    vector<int> e; vector<pair<int,int>> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return st.size(); }
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool unite(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
}; // hash-cpp-all = 7ddf1d63541b7bda1fc6daed3c938fb6
```

MinQueue.h
Description: Structure that supports all operations of a queue and get the minimum/maximum active value in the queue. Useful for sliding window 1D and 2D. For 2D problems, you will need to pre-compute another matrix, by making a row-wise traversal, and calculating the min-/max value beginning in each cell. Then you just make a column-wise traverse as they were each an independent array.

Time: $\mathcal{O}(1)$
<div align="right">24 lines</div>

```
template<typename T>
struct minQueue {
    int lx, rx, sum;
    deque<pair<T, T>> q;
    minQueue() { lx = 1; rx = 0; sum = 0; }
    void clear() { lx = 1, rx = 0, sum = 0; q.clear(); }
    void push(T delta) {
        // q.back().first + sum <= delta for a maxQueue
        while(!q.empty() && q.back().first + sum >= delta)
            q.pop_back();
        q.emplace_back(delta - sum, ++rx);
    }
    void pop() {
        if (!q.empty() && q.front().second == lx++)
            q.pop_front();
    }
    void add(T delta) {
        sum += delta;
    }
    T getMin() {
        return q.front().first + sum;
    }
    int size() { return rx-lx+1; }
}; // hash-cpp-all = d40e772246502e3ab2ec99a1b0943803
```

LazySegmentTree.h
Description: Better SegTree. Range Sum, can be extended to max/min/product/gcd, pay attention to propagate, f and update functions when extending. Be careful with each initialization aswell.
<div align="right">50 lines</div>

```
template<typename T, typename Q>
struct segtree_t {
    int n;
    vector<T> tree;
    vector<Q> lazy, og;
    segtree_t(int N) : n(N), tree(4*N), lazy(4*N) {}
    segtree_t(const vector<Q> &other) :
            n(other.size()), og(other),
            tree(4*n), lazy(4*n) {
        build(1, 0, n-1);
    }
    T f(const T &a, const T &b) { return (a + b); }
    T build(int v, int l, int r) {
        lazy[v] = 0;
        if (l == r) return tree[v] = og[l];
        int m = l + (r - l)/2;
        return tree[v] = f(build(2*v,l, m), build(2*v+1, m
            ↪+1, r));
    }
    void propagate(int v, int l, int r) {
        if (!lazy[v]) return;
        int m = l + (r - l)/2;
        tree[2*v] += lazy[v] * (m - l + 1);
        tree[2*v+1] += lazy[v] * (r - (m+1) + 1);
        lazy[2*v] += lazy[v];
        lazy[2*v+1] += lazy[v];
        lazy[v] = 0;
    }
    T query(int a, int b) { return query(a, b, 1, 0, n-1);
        ↪}
    T query(int a, int b, int v, int l, int r) {
        if (b < l || r < a) return 0;
        if (a <= l && r <= b) return tree[v];
        propagate(v,l, r);
        int m = l + (r - l)/2;
```

```
        return f(query(a, b, 2*v,l, m), query(a, b, 2*v+1,
            ↪m+1, r));
    }
    T update(int a, int b, Q delta) { return update(a, b,
        ↪delta, 1, 0, n-1); }
    T update(int a, int b, Q delta, int v, int l, int r) {
        if (b < l || r < a) return tree[v];
        if (a <= l && r <= b) {
            tree[v] += delta * (r-l+1);
            lazy[v] += delta;
            return tree[v];
        }
        propagate(v,l, r);
        int m = l + (r - l)/2;
        return tree[v] = f(update(a, b, delta, 2*v, l, m),
            update(a, b, delta, 2*v+1, m+1, r));
    }
};
// hash-cpp-all = 3af20e17da7a1a0e4e2d3ac3d108286a
```

DynamicSegTree.h
Description: Dynamic Segment Tree with lazy propagation. Allows range query, range update (increment and assignment). For assignment change all += to = in push and update functions.
Usage: vector<int> a;
node *segtree = build(0, n, a);
<div align="right">79 lines</div>

```
struct node {
    node *l, *r;
    lint minv, sumv, lazy;
    int lx, rx;
};

void push(node *v) {
    if(v != nullptr && v->lazy) {
        v->minv += v->lazy;
        v->sumv += v->lazy * (v->rx - v->lx + 1);
        if(v->l) v->l->lazy += v->lazy;
        if(v->r) v->r->lazy += v->lazy;
        v->lazy = 0;
    }
}

void update(node *v, int lx, int rx, lint delta) {
    push(v);
    if(rx < v->lx || v->rx < lx) return;
    if(lx <= v->lx && v->rx <= rx) {
        v->lazy += delta;
        push(v);
        return;
    }
    update(v->l, lx, rx, delta);
    update(v->r, lx, rx, delta);
    push(v->l);
    v->minv = min(v->l->minv, v->r->minv);
    v->sumv = v->l->sumv + v->r->sumv;
}

// without propagation, way faster in practice
void upd(node *v, int lx, int rx, lint delta) {
    if(rx < v->lx || v->rx < lx) return;
    if(v->lx == v->rx) {
        v->lazy += delta;
        v->minv += delta;
        v->sumv += delta;
        return;
    }
```

```cpp
        upd(v->l, lx, rx, delta);
        upd(v->r, lx, rx, delta);
        v->minv = min(v->l->minv, v->r->minv) + v->lazy;
        v->sumv = v->l->sumv + v->r->sumv + v->lazy * (v->rx - v
            ↪->lx + 1);
    }

    lint mquery(node *v, int lx, int rx) {
        push(v);
        if(rx < v->lx || v->rx < lx) return 1e16;
        if(lx <= v->lx && v->rx <= rx) return v->minv;
        return min(mquery(v->l, lx, rx), mquery(v->r, lx, rx));
    }

    lint squery(node *v, int lx, int rx) {
        push(v);
        if(rx < v->lx || v->rx < lx) return 0;
        if(lx <= v->lx && v->rx <= rx) return v->sumv;
        return squery(v->l, lx, rx) + squery(v->r, lx, rx);
    }

    node *build(int lx, int rx, vector<int>& a) {
        node *v = new node();
        v->lx = lx; v->rx = rx;
        if(lx == rx) {
            v->lazy = 0;
            v->l = v->r = nullptr;
            v->minv = v->sumv = a[lx];
        }
        else {
            v->l = build(lx, (lx + rx)/2, a);
            v->r = build((lx + rx)/2 + 1, rx, a);
            v->minv = min(v->l->minv, v->r->minv);
            v->sumv = v->l->sumv + v->r->sumv;
            v->lazy = 0;
        }
        return v;
    }

    // hash-cpp-all = 8b99e908bdf223001f42b55d15ce52c6
```

MergeSortTree.h
Description: Build segment tree where each node stores a sorted version of the underlying range.
Time: $\mathcal{O}\left(\log^2 N\right)$

<div align="right">37 lines</div>

```cpp
struct MergeSortTree {
    vector<int> v, id;
    vector<vector<int>> tree;
    MergeSortTree(vector<int> &v) : v(v), tree(4*(v.size()
        ↪+1)) {
        for(int i = 0; i < v.size(); ++i) id.push_back(i);
        sort(id.begin(), id.end(), [&v](int i, int j) {
            ↪return v[i] < v[j]; });
        make_tree(1, 0, v.size()-1);
    }
    void make_tree(int id, int left, int right) {
        if (left == right)
            tree[id].push_back(id[left]);
        else {
            int mid = (left + right)/2;
            make_tree(2*id, left, mid);
            make_tree(2*id+1, mid+1, right);
            tree[id] = vector<int>(right - left + 1);
            merge(tree[2*i].begin(), tree[2*i].end(),
                tree[2*id+1].begin(), tree[2*id+1].end(),
                    tree[id].begin());
```

```cpp
        }
    }
    // how many elements in this node have id in the range
        ↪[a,b]
    int how_many(int id, int a, int b) {
        return (int)(upper_bound(tree[id].begin(), tree[id
            ↪].end(), b)
            - lower_bound(tree[id].begin(), tree[id].end(),
                ↪ a));
    }
    int query(int id, int left, int right, int a, int b,
        ↪int x) {
        if (left == right) return v[tree[id].back()];
        int mid = (left + right)/2;
        int lcount = how_many(2*id, a, b);
        if (lcount >= x) return query(2*id, left, mid, a, b
            ↪, x);
        else return query(2*id+1, mid+1, right, a, b, x -
            ↪lcount);
    }
    int kth(int a, int b, int k) {
        return query(1, 0, v.size()-1, a, b, k);
    }
}; // hash-cpp-all = 01e250d36257c202f6f6713e170d49d3
```

Mo.h
Description: Mo's algorithm example problem: Count how many elements appear at least two times in given range $[l, r]$. For path queries on trees, flatten the tree by DFSing and pushing even-depth nodes at entry and odd-depth nodes at exit. If you need to squeeze Mo's in the TL and Q is greater than N, consider Hilbert Curves. Will work much faster.
Time: $(n + q)sqrt(n)$

<div align="right">62 lines</div>

```cpp
struct query_t { int l, r, id; };
int n, m, total = 0; // elements, queries, result.
const int sqn = sqrt(n), maxv = 1000000;
vector<int> values(n), freq(2*maxv), result(m);
vector<query_t> queries(m);

sort(queries.begin(), queries.end(), [sqn](const query_t &a
    ↪, const query_t &b) {
    if (a.l/sqn != b.l/sqn) return a.l < b.l;
    return a.r < b.r;
});

/*
Mo's using Hilbert Curves.

constexpr int lgn = 20, maxn = 1 << lgn;
struct query_t {
    int l, r, id;
    inline lint order() const {
        int x = l, y = r;
        lint d = 0;
        for (int s = 1 << (lgn - 1); s; s >>= 1) {
            bool rx = x & s, ry = y & s;
            d = d << 2 | rx * 3 ^ static_cast<int>(ry);
            if (!ry) {
                if (rx) {
                    x = maxn - x;
                    y = maxn - y;
                }
                swap(x, y);
            }
        }
        return d;
```

```cpp
    }
};
sort(queries.begin(), queries.end(), [&](const query_t &a,
    ↪const query_t &b) {
    return a.order() < b.order();
});
*/

int l = 0, r = -1;
for(query_t &q : queries) {
    auto add = [&](int i) {
        // Change if needed
        ++freq[values[i]];
        if (freq[values[i]] == 2) total += 2;
        else if (freq[values[i]] > 2) ++total;
    };
    auto del = [&](int i) {
        // Change if needed
        --freq[values[i]];
        if (freq[values[i]] == 1) total -= 2;
        else if (freq[values[i]] > 1) --total;
    };
    while(r < q.r) add(++r);
    while(l > q.l) add(--l);
    while(r > q.r) del(r--);
    while(l < q.l) del(l++);
    result[q.id] = total;
}

// hash-cpp-all = 33f45f767453beb8f0b1c28702606ed7
```

RMQ.h
Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time. Returns a pair that holds the answer, first element is the value and the second is the index, obviously doesn't work with sum or similar queries.
Usage: RMQ<int> rmq(values);
rmq.query(inclusive, inclusive);
Time: $\mathcal{O}\left(|V| \log |V| + Q\right)$

<div align="right">33 lines</div>

```cpp
// change cmp for max query or similar
template<typename T, typename Cmp=less<pair<T, int>>>
struct RMQ {
    Cmp cmp;
    vector<vector<pair<T, int>>> table;
    RMQ(const vector<T> &values) {
        int n = values.size();
        table.resize(__lg(n)+1);
        table[0].resize(n);
        for (int i = 0; i < n; ++i) table[0][i] = {values[i], i
            ↪};
        for (int l = 1; l < (int)table.size(); ++l) {
            table[l].resize(n - (1<<l) + 1);
            for (int i = 0; i + (1<<l) <= n; ++i) {
                table[l][i] = min(table[l-1][i], table[l-1][i
                    ↪+(1<<(l-1))], cmp); // Change if max
                //table[l][i].first = (table[l-1][i].first +
                    ↪table[l-1][i+(1<<(l-1))].first); //
                    ↪example of sum
            }

        }
    }
    pair<T, int> query(int a, int b) { // min query
        int l = __lg(b-a+1);
        return min(table[l][a], table[l][b-(1<<l)+1], cmp);
    }
```

```cpp
int sum_query(int a, int b) {
    int l = b-a+1, ret = 0;
    for (int i = (int)table.size(); i >= 0; --i)
        if ((1 << i) <= l) {
            ret += table[i][a].first; a += (1 << i);
            l = b - a + 1;
        }
    return ret;
}
}; // hash-cpp-all = a4b96ac4510d8a21d788aadcb7621b46
```

FenwickTree.cpp
Description: Classic FT with linear initialization. All queries are [a, b). get(pos) function returns the element at index pos in O(1) amortized. lowerbound(sum) returns the largest i in [0, n] st query(i) <= sum. Returns -1 if no such i exists (sum < 0). Can be used as an ordered set on indices in [0, n) by using the tree as a 0/1 array: update(index, +1) is equivalent to insert(index); be careful not to re-insert. get(index) provides whether index is present or not. query(index) provides the count of elements < index. lowerbound(k) finds the k-th smallest element (0-indexed).
Time: Both operations are $\mathcal{O}(\log N)$.

46 lines
```cpp
template<typename T> struct FT {
    T tree_sum;
    const int n;
    vector<T> tree;
    FT(int _n) : n(_n), tree(n) {}
    FT(vector<T> &og) : n(og.size()+1), tree(n+1), tree_sum
        ↪(0) {
        for (int i = 1; i <= n; ++i) {
            tree_sum += og[i-1];
            tree[i] = og[i-1];
            for (int k = (i&-i) >> 1; k > 0; k >>= 1)
                tree[i] += tree[i-k];
        }
    }
    void update(int idx, const T delta) {
        tree_sum += delta;
        for (;idx <= n; idx += idx&-idx)
            tree[idx] += delta;
    }
    T query(int idx){
        T ret = 0;
        for (; idx; idx -= idx&-idx) ret += tree[i];
        return ret;
    }
    T query_suffix(int idx) { return tree_sum - query(idx);
        ↪ }
    T query(int a, int b) { return query(b) - query(a); }
    T get(int pos) {
        int above = pos + 1;
        T sum = tree[above];
        above -= above & -above;
        while (pos != above) {
            sum -= tree[pos];
            above -= above&-above;
        }
        return sum;
    }
    int lower_bound(T sum) {
        if (sum < 0) return -1;
        int prefix = 0;
        for (int k = 31 - __builtin_clz(n); k >= 0; k--)
            if (prefix + (1 << k) <= n && tree[prefix + (1
                ↪<< k)] <= sum) {
                prefix += 1 << k;
```
```cpp
                sum -= tree[prefix];
            }
        return prefix;
    }
}; // hash-cpp-all = 9e38aa30b5e889fa8da0535bdb515c1d
```

FenwickTree.h
Description: N-D Fenwick Tree that supports range sum query with point update.
Usage: FT<int, 10, 10> tree; -> 2D BIT
Time: Both operations are $\mathcal{O}\left((\log N)^D\right)$.

18 lines
```cpp
template <class T, int ...Ns> struct FT {
    T val = 0;
    void update(T v) { val += v; }
    T query() { return val; }
};
template <class T, int N, int... Ns> struct FT<T, N, Ns...>
    ↪ {
    FT<T,Ns...> tree[N+1];
    template<typename... Args> void update(int pos, Args...
        ↪args) {
        for (; pos <= N; pos += (pos&-pos)) tree[pos].update(
            ↪args...);
    }
    template<typename... Args> T sum(int r, Args... args) {
        T res = 0; for (; r; r -= (r&-r))
            res += tree[r].query(args...);
        return res;
    }
    template<typename... Args> T query(int l, int r, Args...
        args) { return sum(r,args...)-sum(l-1,args...); }
}; // hash-cpp-all = 16528286222355229a8e00f569d57b84
```

LazyFenwickTree.h
Description: 1D Fenwick Tree with range increment
"FenwickTree.h"

14 lines
```cpp
template<class T, int SZ> struct FT_t {
    FT<T,SZ> bit[2]; // piecewise linear functions
    // let cum[x] = sum_{i=1}^{x}a[i]
    void update(int right, T delta) { // add delta to a[1..
        ↪right]
        // if x <= right, cum[x] += delta*x
        bit[1].update(1, delta), bit[1].update(right+1, -
            ↪delta);
        // if x > right, cum[x] += delta*right
        bit[0].update(right+1, right*delta);
    }
    void update(int left, int right, T delta) {
        update(left-1,-delta), update(right, delta); }
    T sum(int x) { return bit[1].sum(x)*x+bit[0].sum(x); }
    T query(int x, int y) { return sum(y)-sum(x-1); }
}; // hash-cpp-all = e5de08e1588b1a67a24548e7ebe684ea
```

MisofTree.h
Description: A simple treedata structure for inserting, erasing, and querying the nth largest element.
Time: $\mathcal{O}(\alpha(N))$

15 lines
```cpp
const int BITS = 15;
struct misof_tree{
    int cnt[BITS][1<<BITS];
    misof_tree() {memset(cnt, 0, sizeof cnt);}
    void add(int x, int dv) {
        for (int i = 0; i < BITS; cnt[i++][x] += dv, x >>=
            ↪1); }
```
```cpp
    void del(int x, int dv) {
        for (int i = 0; i < BITS; cnt[i++][x] -= dv, x >>=
            ↪1); }
    int nth(int n) {
        int r = 0, i = BITS;
        while(i--) if (cnt[i][r <<= 1] <= n)
            n -= cnt[i][r], r |= 1;
        return r;
    }
}; // hash-cpp-all = 8c50f4c6f10e1ba44cd8a7679881cc1b
```

LineContainer.h
Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming.
Time: $\mathcal{O}(\log N)$

32 lines
```cpp
bool Q;
struct Line {
    mutable lint k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};

struct LineContainer : multiset<Line> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const lint inf = LLONG_MAX;
    lint div(lint a, lint b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(lint k, lint m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y
            ↪));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    lint query(lint x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
}; // hash-cpp-all = bcdecf9b510ec34e057f36b6e1d02a21
```

Matrix.h
Description: Basic operations on square matrices.
Usage: Matrix<int, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;

28 lines
```cpp
template<class T, int N> struct Matrix {
    typedef Matrix M;
    array<array<T, N>, N> d{};
    M operator*(const M &m) const {
        M a;
        for(int i = 0; i < N; ++i)
            for(int j = 0; j < N; ++j)
                for(int k = 0; k < N; ++k) a.d[i][j] += d[i][k]*m
                    ↪.d[k][j];
        return a;
    }
```

```cpp
    }
  vector<T> operator*(const vector<T> &vec) const {
    vector<T> ret(N);
    for(int i = 0; i < N; ++i)
        for(int j = 0; j < N; ++j) ret[i] += d[i][j] * vec[
            ↪ j];
    return ret;
  }
  M operator^(T p) const {
    assert(p >= 0);
    M a, b(*this);
    for(int i = 0; i < N; ++i) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
}; // hash-cpp-all = ac78976eee0ad16cad5450c4dfecd3a0
```

Treap3.h
Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

69 lines

```cpp
const int N = ; typedef int num;
num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];

void calc(int u) {
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
}

void unlaze(int u) {
    if (!u) return;
    // code here, no recursion
}

void split_val(int u, num x, int &lx, int &rx) {
    unlaze(u); if (!u) return (void)(lx = rx = 0);
    if (X[u] <= x) {
        split_val(R[u], x, lx, rx);
        R[u] = lx;
        lx = u;
    }
    else {
        split_val(L[u], x, lx, rx);
        L[u] = rx;
        rx = u;
    }
    calc(u);
}
void split_sz(int u, int s, int &lx, int &rx) {
    unlaze(u); if (!u) return (void)(lx = rx = 0);
    if (sz[L[u]] < s) {
        split_sz(R[u], s-sz[L[u]]-1, lx, rx);
        R[u] = lx;
        lx = u;
    }
    else {
        split_sz(L[u], s, lx, rx);
        L[u] = rx;
        rx = u;
    }
    calc(u);
```

```cpp
}

int merge(int lx, int rx) {
    unlaze(lx); unlaze(rx); if (!lx || !rx) return lx+rx;
    int u;
    if (Y[lx] > Y[rx]) {
        R[lx] = merge(R[lx], rx);
        u = lx;
    }
    else {
        L[rx] = merge(lx, L[rx]);
        u = rx;
    }
    calc(u);
    return u;
}

void build(int n = N-1) {
    for (int i = en = 1; i <= n; ++i) {
        Y[i] = i;
        sz[i] = 1;
        L[i] = R[i] = 0;
    }
    random_shuffle(Y + 1, Y + n + 1);
}

// hash-cpp-all = 3584d09d8794275b37f50c27be4d14e6
```

LCT.cpp
Description: Link-Cut Tree. Supports BBST = like augmentation, can fully replace Heavylight Decomposition.

106 lines

```cpp
struct T {
  bool rr;
  T *son[2], *pf, *fa;
} f1[N], *ff = f1, *f[N], *null;

void downdate(T *x) {
  if (x -> rr) {
    x -> son[0] -> rr = !x -> son[0] -> rr;
    x -> son[1] -> rr = !x -> son[1] -> rr;
    swap(x -> son[0], x -> son[1]);
    x -> rr = false;
  }
  // add stuff
}

void update(T *x) {
  // add stuff
}

void rotate(T *x, bool t) { // hash-cpp-1
  T *y = x -> fa, *z = y -> fa;
  if (z != null)  z -> son[z -> son[1] == y] = x;
  x -> fa = z;
  y -> son[t] = x -> son[!t];
  x -> son[!t] -> fa = y;
  x -> son[!t] = y;
  y -> fa = x;
  update(y);
} // hash-cpp-1 = 28958e1067126a5892dcaa67307d2f1d

void xiao(T *x) {
  if (x -> fa != null)  xiao(x -> fa), x -> pf = x -> fa ->
      ↪ pf;
  downdate(x);
}
```

```cpp
void splay(T *x) { // hash-cpp-2
  xiao(x);
  T *y, *z;
  while (x -> fa != null) {
    y = x -> fa; z = y -> fa;
    bool t1 = (y -> son[1] == x), t2 = (z -> son[1] == y);
    if (z != null) {
      if (t1 == t2) rotate(y, t2), rotate(x, t1);
      else   rotate(x, t1), rotate(x, t2);
    }else rotate(x, t1);
  }
  update(x);
} // hash-cpp-2 = 0bc1a3b77275f92cebc947211444fdb7

void access(T *x) { // hash-cpp-3
  splay(x);
  x -> son[1] -> pf = x;
  x -> son[1] -> fa = null;
  x -> son[1] = null;
  update(x);
  while (x -> pf != null) {
    splay(x -> pf);
    x -> pf -> son[1] -> pf = x -> pf;
    x -> pf -> son[1] -> fa = null;
    x -> pf -> son[1] = x;
    x -> fa = x -> pf;
    splay(x);
  }
  x -> rr = true;
} // hash-cpp-3 = db89159f01a2099d67e93163c3bfa384

bool Cut(T *x, T *y) { // hash-cpp-4
  access(x);
  access(y);
  downdate(y);
  downdate(x);
  if (y -> son[1] != x || x -> son[0] != null)
    return false;
  y -> son[1] = null;
  x -> fa = x -> pf = null;
  update(x);
  update(y);
  return true;
} // hash-cpp-4 = 42850d63565f84698378e8c2c23df1fe

bool Connected(T *x, T *y) {
  access(x);
  access(y);
  return x == y || x -> fa != null;
}

bool Link(T *x, T *y) {
  if (Connected(x, y))
    return false;
  access(x);
  access(y);
  x -> pf = y;
  return true;
}

int main() {
  read(n); read(m); read(q);
  null = new T; null -> son[0] = null -> son[1] = null ->
      ↪ fa = null -> pf = null;
  for (int i = 1; i <= n; i++) {
    f[i] = ++ff;
```

```
    f[i] -> son[0] = f[i] -> son[1] = f[i] -> fa = f[i] ->
        ↪pf = null;
    f[i] -> rr = false;
  }
  // init null and f[i]
}
```

SplayTree.h
99 lines

```
//const int N = ;
//typedef int num;
int en = 1;
int p[N], sz[N];
int C[N][2]; // {left, right} children
num X[N];

// atualize os valores associados aos nos que podem ser
    ↪calculados a partir dos filhos
void calc(int u) {
  sz[u] = sz[C[u][0]] + 1 + sz[C[u][1]];
}

// Puxa o filho dir de u para ficar em sua posicao e o
    ↪retorna
int rotate(int u, int dir) {
  int v = C[u][dir];
  C[u][dir] = C[v][!dir];
  if(C[u][dir]) p[C[u][dir]] = u;
  C[v][!dir] = u;
  p[v] = p[u];
  if(p[v]) C[p[v]][C[p[v]][1] == u] = v;
  p[u] = v;
  calc(u);
  calc(v);
  return v;
}
// Traz o no u a raiz
void splay(int u) {
  while(p[u]) {
    int v = p[u], w = p[p[u]];
    int du = C[v][1] == u;
    if(!w)
      rotate(v, du);
    else {
      int dv = (C[w][1] == v);
      if(du == dv) {
        rotate(w, dv);
        rotate(v, du);
      } else {
        rotate(v, du);
        rotate(w, dv);
      }
    }
  }
}
// retorna um no com valor x, ou outro no se n foi
    ↪encontrado (n eh floor nem ceiling)
int find_val(int u, num x) {
  int v = u;
  while(u && X[u] != x) {
    v = u;
    if(x < X[u]) u = C[u][0];
    else u = C[u][1];
  }
  if(!u) u = v;
  splay(u);
  return u;
}
```

```
}
// retorna o s-esimo no (0-indexed)
int find_sz(int u, int s) {
  while(sz[C[u][0]] != s) {
    if(sz[C[u][0]] < s) {
      s -= sz[C[u][0]] + 1;
      u = C[u][1];
    } else u = C[u][0];
  }
  splay(u);
  return u;
}
// junte duas splays, assume que elementos l <= elementos r
int merge(int l, int r) {
  if(!l || !r) return l + r;
  while(C[l][1]) l = C[l][1];
  splay(l);
  assert(!C[l][1]);
  C[l][1] = r;
  p[r] = l;
  calc(l);
  return l;
}
// Adiciona no x a splay u e retorna x
int add(int u, int x) {
  int v = 0;
  while(u) v = u, u = C[u][X[x] >= X[u]];
  if(v) { C[v][X[x] >= X[v]] = x; p[x] = v; }
  splay(x);
  return x;
}
// chame isso 1 vez no inicio
void init() {
  en = 1;
}
// Cria um novo no
int new_node(num val) {
  int i = en++;
  assert(i < N);
  C[i][0] = C[i][1] = p[i] = 0;
  sz[i] = 1;
  X[i] = val;
  return i;
} // hash-cpp-all = 30e14f2069467aa6b27d51912e95775b
```

Wavelet.h
Description: Segment tree on values instead of indices. kth return
the largest number in 0-indexed interval. count return the number of
elements of a[i, j) that belong in [x, y].
Time: $\mathcal{O}(\log(n))$
27 lines

```
template<int SZ> struct Wavelet {
  vector<int> L[SZ], R[SZ];
  void build(vector<int> a, int v=1, int lx=0, int rx=SZ-1)
      ↪ {
    if (lx == rx) return;
    L[v] = R[v] = {0};
    vector<int> A[2]; int mid = lx + (rx-lx)/2;
    for(auto &t:a) {
      A[t>mid].pb(t);
      L[v].pb(A[0].size()), R[v].pb(A[1].size());
    }
    build(A[0],2*v,lx,mid), build(A[1],2*v+1,mid+1,rx);
  }
  int kth(int i,int j,int k,int v=1,int lx=0,int rx=SZ-1) {
      ↪ // [i, j)!!
    if (lx == rx) return lx;
```

```
    int mid = lx + (rx - lx)/2, t = L[v][j]-L[v][i];
    if (t >= k) return kth(L[v][i],
        L[v][j],k,2*v,lx,mid);
    return kth(R[v][i],
      R[v][j],k-t,2*v+1,mid+1,rx);
  }
  int count(int i,int j,int x, int y, int v=1,int lx=0,int
      ↪rx=SZ-1) {
    if (y < lx || rx < x) return 0; //count(i, j, x, y)
        ↪retorna o numero de elementos
    if (x <= lx && rx <= y) return j - i; // de a[i, j) que
        ↪ pertencem a [x, y]
    int mid = lx + (rx - lx)/2;
    return count(L[v][i], L[v][j], x, y, 2*v, lx, mid) +
        ↪count(i-L[v][i], j-L[v][j], x, y, 2*v+1, mid+1, rx
        ↪);
  }
}; // hash-cpp-all = be31505bd8f415e099ddcbcc85e617b9
```

Numerical (4)

GoldenSectionSearch.h
Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one
local minimum. The maximum error in the result is eps. Works equally
well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b - a)/\epsilon))$
14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
} // hash-cpp-all = 31d45b514727a298955001a74bb9b9fa
```

Polynomial.h
17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = a.size(); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    for(int i = 1; i < a.size(); ++i) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i = a.size()-1; i--;) c = a[i], a[i] = a[i+1]*
        ↪x0+b, b=c;
    a.pop_back();
  }
}; // hash-cpp-all = 84593c332febd5a0502a84570aa64c30
```

PolyRoots.h
Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h"   23 lines
```cpp
vector<double> poly_roots(Poly p, double xmin, double xmax)
    ↪ {
  if ((p.a).size() == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(dr.begin(), dr.end());
  for(int i = 0; i < dr.size()-1; ++i) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      for(int it = 0; it < 60; ++it) { // while (h - l > 1e
          ↪-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
} // hash-cpp-all = 49396af6a482b97394e6b2e412a6069c
```

PolyInterpolate.h
Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0]*x^0+...+a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$
                                                                  13 lines
```cpp
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  for(int k = 0; k < n-1; ++k) for(int i = k+1; i < n; ++i)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  for(int k = 0; k < n; ++k) for(int i = 0; i < n; ++i) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
} // hash-cpp-all = 97a266204931196ab2c1a2081e6f2f60
```

Lagrange.h
Description: Lagrange Polynomials.
Time: $\mathcal{O}\left(N\right)$
"ModPow.h", "ModInv.h", "Factorial.h"   30 lines
```cpp
template<typename T>
struct Lagrange {
  const int n;
  vector<T> f, den;
  Lagrange(vector<T> other) : f(other), n(other.size()) {
    den.resize(n);
    for(int i = 0; i < n; ++i) {
      f[i] = (f[i] % mod + mod) % mod;
      den[i] = ifact[n-i-1] * ifact[i] % mod;
      if((n-i-1) % 2 == 1)
        den[i] = (mod - den[i]) % mod;
```

```cpp
  }
}
T interpolate(T x) {
  x %= mod;
  vector<T> l, r;
  l.resize(n); r.resize(n);
  l[0] = r[n-1] = 1;
  for (int i = 1; i < n; ++i)
    l[i] = l[i-1] * (x - (i-1) + mod) % mod;
  for (int i = n-2; i >= 0; --i)
    r[i] = r[i+1] * (x - (i+1) + mod) % mod;
  T ans = 0;
  for (int i = 0; i < n; ++i) {
    T coef = l[i] * r[i] % mod;
    ans = (ans + coef * f[i] % mod * den[i]) % mod;
  }
  return ans;
}
}; // hash-cpp-all = 31ad4afb396146045d8ab630178912af
```

BerlekampMassey.h
Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$
"../number-theory/ModPow.h"   18 lines
```cpp
vector<lint> BerlekampMassey(vector<lint> s) {
  int n = s.size(), L = 0, m = 0;
  vector<lint> C(n), B(n), T;
  C[0] = B[0] = 1;
  lint b = 1;
  for(int i = 0; i < n; ++i) { ++m;
    lint d = s[i] % mod;
    for(int j = 1; j <= L; ++j) d = (d + C[j] * s[i - j]) %
        ↪ mod;
    if (!d) continue;
    T = C; lint coef = d * modpow(b, mod-2) % mod;
    for(int j = m; j < n; ++j) C[j] = (C[j] - coef * B[j -
        ↪m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
  C.resize(L + 1); C.erase(C.begin());
  for(lint &x : C) x = (mod - x) % mod;
  return C;
} // hash-cpp-all = 4b649f4e587445f59ded78eedd937af4
```

LinearRecurrence.h
Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$
                                                                  22 lines
```cpp
typedef vector<lint> Poly;
lint linearRec(Poly S, Poly tr, lint k) { // hash-cpp-1
  int n = tr.size();
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    for(int i = 0; i < n+1; ++i) for(int j = 0; j < n+1; ++
        ↪j)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
```

```cpp
    for (int i = 2 * n; i > n; --i) for(int j = 0; j < n;
        ↪++j)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
          ↪mod;
    res.resize(n + 1);
    return res;
  };
  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  lint res = 0;
  for(int i = 0; i < n; ++i) res = (res + pol[i + 1] * S[i
      ↪]) % mod;
  return res;
} // hash-cpp-1 = e5c828081215d4d0def337e161d7c0ec
```

HillClimbing.h
Description: Poor man's optimization for unimodal functions.
                                                                  16 lines
```cpp
typedef array<double, 2> P;

double func(P p);

pair<double, P> hillClimb(P start) {
  pair<double, P> cur(func(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    for(int j = 0; j < 100; ++j) for(int dx = -1; dx < 2;
        ↪++dx) for(int dy = -1; dy < 2; ++dy) {
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(func(p), p));
    }
  }
  return cur;
} // hash-cpp-all = ac5d8e54c13316850419d034af305ebb
```

Integrate.h
Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.
                                                                  7 lines
```cpp
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  for(int i = 1; i < n*2; ++i)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
} // hash-cpp-all = a89f8870779936ce986c40a84367af33
```

IntegrateAdaptive.h
Description: Fast integration using an adaptive Simpson's rule.
Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`
                                                                  15 lines
```cpp
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
```

```
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2,
    ↪S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
} // hash-cpp-all = 5ce1b1578b88e19cb895f828aad45477
```

Determinant.h
Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$
<div align="right">15 lines</div>

```
double det(vector<vector<double>> &a) {
  int n = a.size(); double res = 1;
  for(int i = 0; i < n; ++i) {
    int b = i;
    for(int j = i+1; j < n; ++j) if (fabs(a[j][i]) > fabs(a
      ↪[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    for(int j = i+1; j < n; ++j) {
      double v = a[j][i] / a[i][i];
      if (v != 0) for(int k = i+1; k < n; ++k) a[j][k] -= v
        ↪ * a[i][k];
    }
  }
  return res;
} // hash-cpp-all = 5906bc97b263956b316da1cff94cee0b
```

IntDeterminant.h
Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$
<div align="right">18 lines</div>

```
const lint mod = 12345;
lint det(vector<vector<lint>>& a) {
  int n = a.size(); lint ans = 1;
  for(int i = 0; i < n; ++i) {
    for(int j = i+1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        lint t = a[i][i] / a[j][i];
        if (t) for(int k = i; k < n; ++k)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
} // hash-cpp-all = 6ddd70c56d5503da62fc2a3b03ab8df3
```

Elimination.h
Description: Gaussian elimination
<div align="right">23 lines</div>

```
using T = double;
constexpr T EPS = 1e-8;

T elimination(vector<vector<double>> &m, int rows) { //
  ↪return the determinant
  int r = 0; T det = 1;                    // MODIFIES the input
  for (int c = 0; c < rows && r < rows; ++c) {
```

```
    int p = r;
    for (int i = r+1; i < rows; ++i)
      if (fabs(m[i][c]) > fabs(m[p][c])) p=i;
    if (fabs(m[p][c]) < EPS){ det = 0; continue; }
    swap(m[p], m[r]);
    det = -det;
    T s = 1.0 / m[r][c], t; det *= m[r][c];
    for(int j = 0; j < C; ++j) m[r][j] *= s; // make
      ↪leading term in row 1
    for(int i = 0; i < rows; ++i)
      if (i != r){
        t = m[i][c];
        for(int j = 0; j < C; ++j) m[i][j] -= t * m[r][j];
      }
    ++r;
  }
  return det;
} // hash-cpp-all = 6bf7c77ee9924912326017117030246c
```

Math-Simplex.cpp
Description: Simplex algorithm. WARNING- segfaults on empty (size 0) max cx st Ax<=b, x>=0 do 2 phases; 1st check feasibility; 2nd check boundedness and ans
<div align="right">40 lines</div>

```
vector<double> simplex(vector<vector<double> > A, vector<
  ↪double> b, vector<double> c) {
  int n = (int) A.size(), m = (int) A[0].size()+1, r = n, s
    ↪ = m-1;
  vector<vector<double> > D = vector<vector<double> > (n+2,
    ↪ vector<double>(m+1));
  vector<int> ix = vector<int> (n+m);
  for (int i=0; i<n+m; i++) ix[i] = i;
  for (int i=0; i<n; i++) {
    for (int j=0; j<m-1; j++)D[i][j]=-A[i][j];
    D[i][m-1] = 1;
    D[i][m] = b[i];
    if (D[r][m] > D[i][m]) r = i;
  }
  for (int j=0; j<m-1; j++) D[n][j]=c[j];
  D[n+1][m-1] = -1; int z = 0;
  for (double d;;) {
    if (r < n) {
      swap(ix[s], ix[r+m]);
      D[r][s] = 1.0/D[r][s];
      for (int j=0; j<=m; j++) if (j!=s) D[r][j] *= -D[r][s
        ↪];
      for(int i=0; i<=n+1; i++)if(i!=r) {
        for (int j=0; j<=m; j++) if(j!=s) D[i][j] += D[r][j
          ↪] * D[i][s];
        D[i][s] *= D[r][s];
      }
    }
    r = -1; s = -1;
    for (int j=0; j <m; j++) if (s<0 || ix[s]>ix[j]) {
      if (D[n+1][j]>eps || D[n+1][j]>-eps && D[n][j]>eps) s
        ↪ = j;
    }
    if (s < 0) break;
    for (int i=0; i<n; i++) if(D[i][s]<-eps) {
      if (r < 0 || (d = D[r][m]/D[r][s]-D[i][m]/D[i][s]) <
        ↪-eps
        || d < eps && ix[r+m] > ix[i+m]) r=i;
    }
    if (r < 0) return vector<double>(); // unbounded
  }
  if (D[n+1][m] < -eps) return vector<double>(); //
    ↪infeasible
```

```
  vector<double> x(m-1);
  for (int i = m; i < n+m; i ++) if (ix[i] < m-1) x[ix[i]]
    ↪= D[i-m][m];
  printf("%.2lf\n", D[n][m]);
  return x; // ans: D[n][m]
} // hash-cpp-all = 70201709abdff05eff90d9393c756b95
```

SolveLinear.h
Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$
<div align="right">36 lines</div>

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd> &A, vd &b, vd &x) {
  int n = A.size(), m = x.size(), rank = 0, br, bc;
  if (n) assert(A[0].size() == m);
  vector<int> col(m); iota(col.begin(), col.end(), 0);
  for(int i = 0; i < n; ++i) {
    double v, bv = 0;
    for(int r = i; r < n; ++r) for(int c = i; c < m; ++c)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      for(int j = i; j < n; ++j) if (fabs(b[j]) > eps)
        ↪return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for(int j = 0; j < n; ++j) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    for(int j = i+1; j < n; ++j) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      for(int k = i+1; k < m; ++k) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }
  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    for(int j = 0; j < i; ++j) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = 2654db9ae0ca64c0f3e32879d85e35d5
```

SolveLinear2.h
Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:
"SolveLinear.h"
<div align="right">8 lines</div>

```
for(int j = 0; j < n; ++j) if (j != i) // instead of for(
  ↪int j = i+1; j < n; ++j)
// ... then at the end:
x.assign(m, undefined);
for(int i = 0; i < rank; ++i) {
  for(int j = rank; j < m; ++j) if (fabs(A[i][j]) > eps)
    ↪goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
// hash-cpp-all = c8e85a5f8fc2c9ae6fc5672997b15cda
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}\left(n^2 m\right)$

34 lines

```cpp
typedef bitset<1000> bs;

int solveLinear(vector<bs> &A, vector<int> &b, bs& x, int m
    ↪) {
  int n = A.size(), rank = 0, br;
  assert(m <= x.size());
  vector<int> col(m); iota(col.begin(), col.end(), 0);
  for(int i = 0; i < n; ++i) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for(int j = 0; j < n; ++j) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    for(int j = i+1; j < n; ++j) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    for(int j = 0; j < i; ++j) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = 71d8713aa9eab9f9d77a9e46d9caed1f
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank $< n$). Can easily be extended to prime moduli; for prime powers, foreatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}\left(n^3\right)$

35 lines

```cpp
int matInv(vector<vector<double>>& A) {
  int n = A.size(); vector<int> col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  for(int i = 0; i < n; ++i) tmp[i][i] = 1, col[i] = i;

  for(int i = 0; i < n; ++i) { // hash-cpp-1
    int r = i, c = i;
    for(int j = i; j < n; ++j) for(int k = i; k < n; ++k)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    for(int j = 0; j < n; ++j)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    for(int j = i+1; j < n; ++j) {
      double f = A[j][i] / v;
```

```cpp
      A[j][i] = 0;
      for(int k = i+1; k < n; ++k) A[j][k] -= f*A[i][k];
      for(int k = 0; k < n; ++k) tmp[j][k] -= f*tmp[i][k];
    }
    for(int j = i+1; j < n; ++j) A[i][j] /= v;
    for(int j = 0; j < n; ++j) tmp[i][j] /= v;
    A[i][i] = 1;
  } // hash-cpp-1 = b5c37a0147222a30250f8eb364b7dd25

  for (int i = n-1; i > 0; --i) for(int j = 0; j < i; ++j)
      ↪{ // hash-cpp-2
    double v = A[j][i];
    for(int k = 0; k < n; ++k) tmp[j][k] -= v*tmp[i][k];
  }

  for(int i = 0; i < n; ++i) for(int j = 0; j < n; ++j) A[
      ↪col[i]][col[j]] = tmp[i][j];
  return n;
} // hash-cpp-2 = cb1e282dd60fc93e07018380693a681b
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \; 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\},$$
$$\{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$

26 lines

```cpp
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T> &
    ↪super,
    const vector<T> &sub, vector<T> b) {
  int n = b.size(); vector<int> tr(n);
  for(int i = 0; i < n-1; ++i) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
        ↪== 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
```

```cpp
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
} // hash-cpp-all = d0855fb63594fa47d372bf1a8c3078f9
```

NewtonMethod.h

Description: Root find method

12 lines

```cpp
double f(double x) { return (x*x) - 4; }
double df(double x) { return 2*x; }
double root(double x0) {
    const double eps = 1E-15;
    double x = x0;
    while (1) {
        double nx = x - (f(x)/df(x));
        if (abs(x - nx) < eps) break;
        x = nx;
    }
    return x;
} // hash-cpp-all = c064c4edb7b9963d922729cce3ce22e2
```

NewtonSQRT.h

Description: Square root find method

21 lines

```cpp
double sqrt_newton(double n) {
    const double eps = 1E-15;
    double x = 1;
    while (1) {
        double nx = (x + n / x) / 2;
        if (abs(x - nx) < eps) break;
        x = nx;
    }
    return x;
}
int isqrt_newton(int n) {
    int x = 1;
    bool decreased = false;
    while (1) {
        int nx = (x + n / x) >> 1;
        if (x == nx || nx > x && decreased) break;
        decreased = nx < x;
        x = nx;
    }
    return x;
} // hash-cpp-all = 25ff801e8601f802f249e0b08ecdf4bf
```

4.1 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. Useful for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $\left(\sum a_i^2 + \sum b_i^2\right)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use long doubles/NTT/FFTMod.

Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

36 lines

```cpp
typedef complex<long double> doublex;
struct FFT {
    vector<doublex> fft(vector<doublex> y, bool invert =
        ↪false) {
        const int N = y.size(); assert(N == (N&-N));
        vector<lint> rev(N);
        for (int i = 1; i < N; ++i) {
```

```
            rev[i] = (rev[i>>1]>>1) | (i&1 ? N>>1 : 0);
            if (rev[i] < i) swap(y[i], y[rev[i]]);
        }
        vector<doublex> rootni(N/2);
        for (lint n = 2; n <= N; n *= 2) {
            const doublex rootn = polar(1.0, (invert ? +1.0
                ↪ : -1.0) * 2.0*acos(-1.0)/n);
            rootni[0] = 1.0;
            for (lint i = 1; i < n/2; ++i) rootni[i] =
                ↪rootni[i-1] * rootn;
            for (lint left = 0; left != N; left += n) {
                const lint mid = left + n/2;
                for (lint i = 0; i < n/2; ++i) {
                    const doublex temp = rootni[i] * y[mid
                        ↪+ i];
                    y[mid + i] = y[left + i] - temp; y[left
                        ↪ + i] += temp;
                }
            }
        }
        } if (invert) for (auto &v : y) v /= (doublex)N;
        return move(y);
    }
    uint nextpow2(uint v) { return v ? 1 << __lg(2*v-1) :
        ↪1; }
    vector<doublex> convolution(vector<doublex> a, vector<
        ↪doublex> b) {
        const lint n = max((int)a.size()+(int)b.size()-1,
            ↪0), n2 = nextpow2(n);
        a.resize(n2); b.resize(n2);
        vector<doublex> fa = fft(move(a)), fb = fft(move(b
            ↪)), &fc = fa;
        for (lint i = 0; i < n2; ++i) fc[i] = fc[i] * fb[i
            ↪];
        vector<doublex> c = fft(move(fc), true);
        c.resize(n);
        return move(c);
    }
} fft;
// hash-cpp-all = 26c9ae5b309bb520a31e6e6531b4cb6b
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

85 lines

```
typedef unsigned int uint;
typedef long double ldouble;

template<typename T, typename U, typename B> struct
    ↪ModularFFT {
    inline T ifmod(U v, T mod) { return v >= (U)mod ? v -
        ↪mod : v; }
    T pow(T x, U y, T p) {
        T ret = 1, x2p = x;
        while (y) {
            if (y % 2) ret = (B)ret * x2p % p;
            y /= 2; x2p = (B)x2p * x2p % p;
        }
        return ret;
    }
    vector<T> fft(vector<T> y, T mod, T gen, bool invert =
        ↪false) {
        int N = y.size(); assert(N == (N&-N));
        if (N == 0) return move(y);
        vector<int> rev(N);
```

```
        for (int i = 1; i < N; ++i) {
            rev[i] = (rev[i>>1]>>1) | (i&1 ? N>>1 : 0);
            if (rev[i] < i) swap(y[i], y[rev[i]]);
        }
        assert((mod-1)%N == 0);
        T rootN = pow(gen, (mod-1)/N, mod);
        if (invert) rootN = pow(rootN, mod-2, mod);
        vector<T> rootni(N/2);
        for (int n = 2; n <= N; n *= 2) {
            T rootn = pow(rootN, N/n, mod);
            rootni[0] = 1;
            for (int i = 1; i < n/2; ++i) rootni[i] = (B)
                ↪rootni[i-1] * rootn % mod;
            for (int left = 0; left != N; left += n) {
                int mid = left + n/2;
                for (int i = 0; i < n/2; ++i) {
                    T temp = (B)rootni[i] * y[mid+i] % mod;
                    y[mid+i] = ifmod((U)y[left+i] + mod -
                        ↪temp, mod);
                    y[left+i] = ifmod((U)y[left+i] + temp,
                        ↪mod);
                }
            }
        }
        if (invert) {
            T invN = pow(N, mod-2, mod);
            for (T &v : y) v = (B)v * invN % mod;
        }
        return move(y);
    }
    vector<T> convolution(vector<T> a, vector<T> b, T mod,
        ↪T gen) {
        int N = a.size()+b.size()-1, N2 = nextpow2(N);
        a.resize(N2); b.resize(N2);
        vector<T> fa = fft(move(a), mod, gen), fb = fft(
            ↪move(b), mod, gen), &fc = fa;
        for (int i = 0; i < N2; ++i) fc[i] = (B)fc[i] * fb[
            ↪i] % mod;
        vector<T> c = fft(move(fc), mod, gen, true);
        c.resize(N); return move(c);
    }
    vector<T> self_convolution(vector<T> a, T mod, T gen) {
        int N = 2*a.size()-1, N2 = nextpow2(N);
        a.resize(N2);
        vector<T> fc = fft(move(a), mod, gen);
        for (int i = 0; i < N2; ++i) fc[i] = (B)fc[i] * fc[
            ↪i] % mod;
        vector<T> c = fft(move(fc), mod, gen, true);
        c.resize(N); return move(c);
    }
    uint nextpow2(uint v) { return v ? 1 << __lg(2*v-1) :
        ↪1; }
};

const int mod = 998244353, mod_gen = 3;

vector<int> convolute(const vector<int> &a, const vector<
    ↪int> &b) {
    if (a.empty() || b.empty()) return {};
    ModularFFT<int, uint, lint> modular_fft;
    return modular_fft.convolution(a, b, mod, mod_gen);
}

vector<int> convolute_all(const vector<vector<int>> &polys,
    ↪ int begin,
                          int end) {
    if (end - begin == 0) return {1};
    else if (end - begin == 1) return polys[begin];
```

```
    else {
        int mid = begin + (end - begin) / 2;
        return convolute(convolute_all(polys, begin, mid),
                         convolute_all(polys, mid, end));
    }
}

vector<int> convolute_all(const vector<vector<int>> &polys)
    ↪ {
    return convolute_all(polys, 0, (int)polys.size());
} // hash-cpp-all = dbcea8e7b4d75d1274c7284d384a5a64
```

NumberTheoreticTransform.h

Description: Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$

"../number-theory/modpow.h"   32 lines

```
const lint mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
    ↪ 21
// and 483 << 21 (same root). The last two are > 10^9.

typedef vector<lint> vl;
void ntt(vl& a, vl& rt, vl& rev, int n) {
    for(int i = 0; i < n; ++i) if (i < rev[i]) swap(a[i], a[
        ↪rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) for(int j = 0; j < k
            ↪; ++j) {
            lint z = rt[j + k] * a[i + j + k] % mod, &ai = a[i
                ↪+ j];
            a[i + j + k] = (z > ai ? ai - z + mod : ai - z);
            ai += (ai + z >= mod ? z - mod : z);
        }
}

vl conv(const vl& a, const vl& b) {
    if (a.empty() || b.empty())
        return {};
    int s = a.size()+b.size()-1, B = 32 - __builtin_clz(s), n
        ↪ = 1 << B;
    vl L(a), R(b), out(n), rt(n, 1), rev(n);
    L.resize(n), R.resize(n);
    for(int i = 0; i < n; ++i) rev[i] = (rev[i / 2] | (i & 1)
        ↪ << B) / 2;
    lint curL = mod / 2, inv = modpow(n, mod - 2);
    for (int k = 2; k < n; k *= 2) {
        lint z[] = {1, modpow(root, curL /= 2)};
        for(int i = k; i < 2*k; ++i) rt[i] = rt[i / 2] * z[i &
            ↪1] % mod;
    }
    ntt(L, rt, rev, n); ntt(R, rt, rev, n);
    for(int i = 0; i <n; ++i) out[-i & (n-1)] = L[i] * R[i] %
        ↪ mod * inv % mod;
    ntt(out, rt, rev, n);
    return {out.begin(), out.begin() + s};
} // hash-cpp-all = 1f6be88c85faaf9505586299f0b01d29
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

16 lines

```
void FST(vector<int> &a, bool inv) { // hash-cpp-1
```

```cpp
  for (int n = a.size(), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) for(int j = i; j
      ↪< i+step; ++j) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for(auto &x : a) x /= a.size(); // XOR only
} // hash-cpp-1 = a4980de468052607447174d1308c276b
vector<int> conv(vector<int> a, vector<int> b) { // hash-
  ↪cpp-2
  FST(a, 0); FST(b, 0);
  for(int i = 0; i < a.size(); ++i) a[i] *= b[i];
  FST(a, 1); return a;
} // hash-cpp-2 = 733c60843e71a1333215a8d28f020966
```

4.1.1   Table of non-trigonometric integrals

Some useful integrals are:

$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\arctan\frac{x}{a}$
$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\ln\frac{x-a}{x+a}$
$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}$
$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\arcsin\frac{x}{a}$
$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\ln\left(u+\sqrt{x^2-a^2}\right)$
$\int \frac{dx}{x\sqrt{x^2-a^2}}$	$\frac{1}{a}\operatorname{arcsec}\left\|\frac{u}{a}\right\|$
$\int \frac{dx}{x\sqrt{x^2+a^2}}$	$-\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
$\int \frac{dx}{x\sqrt{a^2+x^2}}$	$-\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

4.1.2   Table of trigonometric integrals

A list of common and not-so-common trigonometric integrals:

$\int \tan x\,dx$	$-\ln\|\cos x\|$
$\int \cot x\,dx$	$\ln\|\sin x\|$
$\int \sec x\,dx$	$\ln\|\sec x+\tan x\|$
$\int \csc x\,dx$	$\ln\|\csc x-\cot x\|$
$\int \sec^2 x\,dx$	$\tan x$
$\int \csc^2 x\,dx$	$\cot x$
$\int \sin^n x\,dx$	$\frac{-\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}\int\sin^{n-2}x\,dx$
$\int \cos^n x\,dx$	$\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}\int\cos^{n-2}x\,dx$
$\int \arcsin x\,dx$	$x\arcsin x+\sqrt{1-x^2}$
$\int \arccos x\,dx$	$x\arccos x-\sqrt{1-x^2}$
$\int \arctan x\,dx$	$x\arctan x-\frac{1}{2}\ln\|1-x^2\|$

4.1.3   Determinants

$\det(A)=\sum_{\sigma\in S_n}\operatorname{sgn}(\sigma)\prod_{i=1}^n a_{i,\sigma(i)}$

4.1.4   Common integral substitutions
And finally, a list of common substitutions:

$\int F(\sqrt{ax+b})dx$	$u=\sqrt{ax+b}$	$\frac{2}{a}\int uF(u)du$
$\int F(\sqrt{a^2-x^2})dx$	$x=a\sin u$	$a\int F(a\cos u)\cos u\,du$
$\int F(\sqrt{x^2+a^2})dx$	$x=a\tan u$	$a\int F(a\sec u)\sec^2 u\,du$
$\int F(\sqrt{x^2-a^2})dx$	$x=a\sec u$	$a\int F(a\tan u)\sec u\tan u\,du$
$\int F(e^{ax})dx$	$u=e^{ax}$	$\frac{1}{a}\int\frac{F(u)}{u}du$
$\int F(\ln x)dx$	$u=\ln x$	$\int F(u)e^u du$

4.1.5   Generating functions
A list of generating functions for useful sequences:

$(1,1,1,1,1,1,\ldots)$	$\frac{1}{1-z}$
$(1,-1,1,-1,1,-1,\ldots)$	$\frac{1}{1+z}$
$(1,0,1,0,1,0,\ldots)$	$\frac{1}{1-z^2}$
$(1,0,\ldots,0,1,0,1,0,\ldots,0,1,0,\ldots)$	$\frac{1}{1-z^2}$
$(1,2,3,4,5,6,\ldots)$	$\frac{1}{(1-z)^2}$
$(1,\binom{m+1}{m},\binom{m+2}{m},\binom{m+3}{m},\ldots)$	$\frac{1}{(1-z)^{m+1}}$
$(1,c,\binom{c+1}{2},\binom{c+2}{3},\ldots)$	$\frac{1}{(1-z)^c}$
$(1,c,c^2,c^3,\ldots)$	$\frac{1}{1-cz}$
$(0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots)$	$\ln\frac{1}{1-z}$

A neat manipulation trick is:

$$\frac{1}{1-z}G(z)=\sum_n\sum_{k\le n}g_k z^n$$

4.1.6   Polyominoes
How many free (rotation, reflection), one-sided (rotation) and fixed n-ominoes are there?

n	3	4	5	6	7	8	9	10
free	2	5	12	35	108	369	1.285	4.655
one-sided	2	7	18	60	196	704	2.500	9.189
fixed	6	19	63	216	760	2.725	9.910	36.446

Number theory (5)

5.1   Modular arithmetic

ModularArithmetic.h
Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.
59 lines

```cpp
template <int MOD_>  struct modnum {
private:
  using lint = long long;
  lint v;
  static int modinv(int a, int m) {
    a %= m;
    assert(a);
    return a == 1 ? 1 : int(m - lint(modinv(m, a)) * lint(m
      ↪) / a);
  }
public:
  static constexpr int MOD = MOD_;
  modnum() : v(0) {}
  modnum(lint v_) : v(int(v_ % MOD)) { if (v < 0) v += MOD;
    ↪ }
  explicit operator int() const { return v; }
  friend std::ostream &operator<<(std::ostream& out, const
    ↪modnum& n) { return out << int(n); }
  friend std::istream &operator>>(std::istream& in, modnum&
    ↪ n) { lint v_; in >> v_; n = modnum(v_); return in;
    ↪}
  friend bool operator==(const modnum& a, const modnum& b)
    ↪{ return a.v == b.v; }
  friend bool operator!=(const modnum& a, const modnum& b)
    ↪{ return a.v != b.v; }
  modnum inv() const {
    modnum res;
    res.v = modinv(v, MOD);
    return res;
  }
  modnum neg() const {
    modnum res;
    res.v = v ? MOD-v : 0;
    return res;
  }
  modnum operator-() const { return neg(); }
  modnum operator+() const { return modnum(*this); }
  modnum& operator+=(const modnum& o) {
    v += o.v;
    if (v >= MOD) v -= MOD;
    return *this;
  }
  modnum& operator-=(const modnum& o) {
    v -= o.v;
    if (v < 0) v += MOD;
    return *this;
  }
  modnum& operator*=(const modnum& o) {
    v = int(lint(v) * lint(o.v) % MOD);
    return *this;
  }
  modnum& operator/=(const modnum& o) { return *this *= o.
    ↪inv(); }
  friend modnum operator+(const modnum& a, const modnum& b)
    ↪ { return modnum(a) += b; }
  friend modnum operator-(const modnum& a, const modnum& b)
    ↪ { return modnum(a) -= b; }
  friend modnum operator*(const modnum& a, const modnum& b)
    ↪ { return modnum(a) *= b; }
  friend modnum operator/(const modnum& a, const modnum& b)
    ↪ { return modnum(a) /= b; }
};

template <typename T> T pow(T a, lint b) {
  assert(b >= 0);
  T r = 1; while (b) { if (b & 1) r *= a; b >>= 1; a *= a;
    ↪} return r;
}

using num = modnum<int(1e9)+7>;

// hash-cpp-all = 56537454341667fa4f164b1506b9ed93
```

PairNumTemplate.h
Description: Support pairs operations using modnum template. Pretty good for string hashing.
43 lines

```cpp
template <typename T, typename U> struct pairnum {
  T t; U u;
  pairnum() : t(0), u(0) {}
  pairnum(long long v) : t(v), u(v) {}
  pairnum(const T& t_, const U& u_) : t(t_), u(u_) {}
  friend std::ostream& operator << (std::ostream& out,
      ↪const pairnum& n) { return out << '(' << n.t << ','
      ↪<< ' ' << n.u << ')'; }
  friend std::istream& operator >> (std::istream& in,
      ↪pairnum& n) { long long v; in >> v; n = pairnum(v);
      ↪return in; }
  friend bool operator == (const pairnum& a, const pairnum&
      ↪ b) { return a.t == b.t && a.u == b.u; }
  friend bool operator != (const pairnum& a, const pairnum&
      ↪ b) { return a.t != b.t || a.u != b.u; }
  pairnum inv() const {
    return pairnum(t.inv(), u.inv());
  }
  pairnum neg() const {
    return pairnum(t.neg(), u.neg());
  }
  pairnum operator- () const {
    return pairnum(-t, -u);
  }
  pairnum operator+ () const {
    return pairnum(+t, +u);
  }
  pairnum& operator += (const pairnum& o) {
    t += o.t; u += o.u;
    return *this;
  }
  pairnum& operator -= (const pairnum& o) {
    t -= o.t; u -= o.u;
    return *this;
  }
  pairnum& operator *= (const pairnum& o) {
    t *= o.t; u *= o.u;
    return *this;
  }
  pairnum& operator /= (const pairnum& o) {
    t /= o.t; u /= o.u;
    return *this;
  }
  friend pairnum operator + (const pairnum& a, const
      ↪pairnum& b) { return pairnum(a) += b; }
  friend pairnum operator - (const pairnum& a, const
      ↪pairnum& b) { return pairnum(a) -= b; }
  friend pairnum operator * (const pairnum& a, const
      ↪pairnum& b) { return pairnum(a) *= b; }
  friend pairnum operator / (const pairnum& a, const
      ↪pairnum& b) { return pairnum(a) /= b; }
};
// hash-cpp-all = 229a89dc1bd3c18584636921c098ebdc
```

ModInv.h
Description: Find x such that $ax \equiv 1 \pmod m$. The inverse only exists if a and m are coprimes.
18 lines

```cpp
template<typename T>
T modinv(T a, T m) {
  assert(m > 0);
  if (m == 1) return 0;
  a %= m;
  if (a < 0) a += m;
```

```cpp
  assert(a != 0);
  if (a == 1) return 1;
  return m - modinv(m, a) * m/a;
}

// Iff mod is prime
lint modinv(lint a) { return modpow(a % mod, mod-2); }

// const lint mod = 1000000007, LIM = 200000;
lint* inv = new lint[LIM] - 1; inv[1] = 1;
for(int i = 2; i < LIM; ++i) inv[i] = mod - (mod/i) * inv[
    ↪mod%i] % mod;
// hash-cpp-all = 00205250d7354b57c72ec90fcf5947ff
```

Modpow.h
13 lines

```cpp
lint modpow(lint a, lint e){
    if(e == 0) return 1;
    if(e & 1) return (a*modpow(a,e-1)) % mod;
    lint c = modpow(a, e>>1);
    return (c*c) % mod;
}


lint modpow(lint b, lint e) {
  lint ret = 1;
  for (int i = 1; i <= e; i *= 2, b = b * b % mod)
    if (i & e) ret = ret * b % mod;
  return ret;
} // hash-cpp-all = ba3b4f88d538ef63e2d071661d1e3106
```

ModSum.h
Description: Sums of mod'ed arithmetic progressions.
$\mathrm{modsum(to,\ c,\ k,\ m)} = \sum_{i=0}^{to-1} (ki+c)\%m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.
19 lines

```cpp
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
  }
  return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} // hash-cpp-all = 8d6e082e0ea6be867eaea12670d08dcc
```

ModMul.cpp
Description: Modular multiplication operation
10 lines

```cpp
lint modMul(lint a, lint b){
    lint ret = 0;
    a %= mod;
    while (b){
        if (b & 1) ret = (ret + a) % mod;
        a = (2 * a) % mod;
        b >>= 1;
    }
```

```cpp
  return ret;
} // hash-cpp-all = f741d07bbdfa19949a4d645f2c519ecd
```

ModMulLL.h
Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b < c < 2^{63}$.
Time: $\mathcal{O}(1)$ for mod_mul, $\mathcal{O}(\log b)$ for mod_pow
13 lines

```cpp
typedef unsigned long long ull;
typedef long double ld;

ull mod_mul(ull a, ull b, ull M) {
    lint ret = a * b - M * ull(ld(a) * ld(b) / ld(M));
    return ret + M * (ret < 0) - M * (ret >= (lint)M);
}
ull mod_pow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = mod_mul(b, b, mod), e >>= 1)
        if (e & 1) ans = mod_mul(ans, b, mod);
    return ans;
} // hash-cpp-all = 6ecbeac391f4533c348906f0d41e9ede
```

ModSqrt.h
Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p
"ModPow.h"
23 lines

```cpp
lint sqrt(lint a, lint p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  lint s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  lint x = modpow(a, (s + 1) / 2, p);
  lint b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    lint t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    lint gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
} // hash-cpp-all = 09107ec523e930fa8ba3787d3729bde9
```

MulOrder.h
Description: Find the smallest integer k such that $a^k \pmod m = 1$. $0 < k < m$.
8 lines

```cpp
int mulOrder(int x, int y){
    if (__gcd(x, y) != 1) return 0;
    lint p = phi(y);
    pair<int,int> k = factorize(x);
    for (auto &t : k)
        while(p % t.first == 0 && modpow(x, p/t.first, p)
            ↪== 1) p /= t.first;
    return P;
} // hash-cpp-all = b3fb0f17b93555f29edba04fd05433b9
```

Quadratic.h
Description: Solve $x^2 \equiv n \mod p (0 \leq a < p)$ where p is prime in $O(\log p)$.

<div align="right">18 lines</div>

```cpp
struct quadric {
  void multiply(lint &c, lint &d, lint a, lint b, lint w,
      ↪lint p) { // hash-cpp-1
    int cc = (a * c + b * d % p * w) % p;
    int dd = (a * d + b * c) % p; c = cc, d = dd; }
  bool solve(int n, int p, int &x) {
    if (n == 0) return x = 0, true; if (p == 2) return x =
      ↪1, true;
    if (mod_pow(n, p / 2, p) == p - 1) return false;
    lint c = 1, d = 0, b = 1, a, w;
    do { a = rand() % p; w = (a * a - n + p) % p;
      if (w == 0) return x = a, true;
    } while (mod_pow(w, p / 2, p) != p - 1);
    for (int times = (p + 1) / 2; times; times >>= 1) {
      if (times & 1) multiply (c, d, a, b, w, p);
      multiply (a, b, a, b, w, p);
    }
    return x = c, true;
    } // hash-cpp-1 = 7b06e39b96dbf9618c8735bc05ee61f4
};
```

5.2   Primality

Sieve.h
Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored.

<div align="right">12 lines</div>

```cpp
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vector<int> run_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vector<int> primes;
  for(int i = 2; i < lim; ++i) if (isprime[i])
    primes.push_back(i);
  return primes;
} // hash-cpp-all = 589ae49892b13171ea507398a9543dff
```

LinearSieve.h
Description: Prime sieve for generating all primes up to a certain limit.
Time: $\mathcal{O}(n)$

<div align="right">19 lines</div>

```cpp
vector<int> least = {0, 1};
vector<int> primes;
int precalculated = 1;
void LinearSieve(int n) {
  n = max(n, 1);
  least.assign(n + 1, 0);
  primes.clear();
  for (int i = 2; i <= n; i++) {
    if (least[i] == 0) {
      least[i] = i;
      primes.push_back(i);
    }
    for (int p : primes) {
      if (p > least[i] || i * p > n) break;
      least[i * p] = p;
    }
  }
  precalculated = n;
}
```

```
} // hash-cpp-all = 126ac7f141d28a888e2d52e4be549215
```

Mobius.h
Description: If g and f are arithmetic functions . Return 0 if divisible by any perfect square, 1 if has an even quantity of prime numbers and -1 if has an odd quantity of primes.
Time: $\mathcal{O}(sqrt(n))$

<div align="right">11 lines</div>

```cpp
template<typename T>
T mobius(T n) {
    T p = 0, aux = n;
    for (int i = 2; i*i <= n; ++i) {
        if (n % i == 0) {
            n /= i;
            p += 1;
            if (n % i == 0) return 0;
        }
    }
    return (p&1 ? 1 : -1);
} // hash-cpp-all = c2cf445d5148aab42f5f697c3d61f4bb
```

MobiusSieve.h
Description: Pre calculate all mobius values.
Time: $\mathcal{O}(N))$

<div align="right">19 lines</div>

```cpp
vector<int> mobius, lp;
void run_sieve(int n) {
    mobius.assign(n+1, -1);
    lp.assign(n+1, 0);
    mobius[1] = 1;
    vector<int> prime;
    for (int i = 2; i <= n; ++i) {
        if (!lp[i]) {
            lp[i] = i;
            prime.push_back(i);
        }
        for (int p : prime) {
            if (p > lp[i] || p*i > n) break;
            if (i % p == 0) mobius[i*p] = 0;
            lp[p*i] = p;
            mobius[p*i] *= mobius[i];
        }
    }
} // hash-cpp-all = 2a1f951748ce65147cdd07f255ac814b
```

MillerRabin.h
Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to 2^{64}; for larger numbers, extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h"
<div align="right">12 lines</div>

```cpp
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
  vector<ull> A = {2, 325, 9375, 28178, 450775, 9780504,
      ↪1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for(ull a : A) {   // ^ count trailing zeroes
    ull p = mod_pow(a, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = mod_mul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
} // hash-cpp-all = 208a221ccf7017821fec2f13aed4687c
```

Factorize.h
Description: Get all factors of n.

<div align="right">17 lines</div>

```cpp
vector<pair<int, int>> factorize(int value) {
    vector<pair<int, int>> result;
    for (int p = 2; p*p <= value; ++p)
        if (value % p == 0) {
            int exp = 0;
            while (value % p == 0) {
                value /= p;
                ++exp;
            }
            result.emplace_back(p, exp);
        }
    if (value != 1) {
        result.emplace_back(value, 1);
        value = 1;
    }
    return result;
} // hash-cpp-all = 46ea351907e7fba012d0082844c7c198
```

PollardRho.h
Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$ gcd calls, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h"
<div align="right">19 lines</div>

```cpp
ull pollard(ull n) {
  auto f = [n](ull x) { return (mod_mul(x, x, n) + 1) % n;
      ↪};
  if (!(n & 1)) return 2;
  for (ull i = 2;; i++) {
    ull x = i, y = f(x), p;
    while ((p = __gcd(n + y - x, n)) == 1)
      x = f(x), y = f(f(y));
    if (p != n) return p;
  }
}

vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n/x);
  l.insert(l.end(), all(r));
  return l;
} // hash-cpp-all = f5adaa4517c8c7f5812dd65047dab785
```

5.3   Divisibility

ExtendedEuclidean.h
Description: Finds two integers x and y, such that $ax+by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are co-prime, then x is the inverse of $a \pmod b$.

<div align="right">10 lines</div>

```cpp
template<typename T>
T egcd(T a, T b, T &x, T &y) {
    if (a == 0) {
        x = 0, y = 1;
        return b;
    }
    T p = b/a, g = egcd(b - p * a, a, y, x);
    x -= y * p;
    return g;
} // hash-cpp-all = 7bbefb658dff0adc3293dd80efb22a86
```

DiophantineEquation.h

Description: Check if a the Diophantine Equation $ax + by = c$ has solution.
<div align="right">39 lines</div>

```cpp
template<typename T>
bool diophantine(T a, T b, T c, T &x, T &y, T &g) { // hash
    ↪-cpp-1
    if (a == 0 && b == 0) {
        if (c == 0) {
            x = y = g = 0;
            return true;
        }
        return false;
    }
    if (a == 0) {
        if (c % b == 0) {
            x = 0;
            y = c / b;
            g = abs(b);
            return true;
        }
        return false;
    }
    if (b == 0) {
        if (c % a == 0) {
            x = c / a;
            y = 0;
            g = abs(a);
            return true;
        }
        return false;
    } // hash-cpp-1 = b6de1e1af6bb4f670fb53e9f8abf08b5
// hash-cpp-2
    g = egcd<lint>(a, b, x, y);
    if (c % g != 0) return false;
    T dx = c / a;
    c -= dx * a;
    T dy = c / b;
    c -= dy * b;
    x = dx + (T) ((__int128) x * (c / g) % b);
    y = dy + (T) ((__int128) y * (c / g) % a);
    g = abs(g);
    return true; // |x|, |y| <= max(|a|, |b|, |c|)
} // hash-cpp-2 = a8604c857ce66f7c6cb5d318ece21e1c
```

Divisors.h

Description: Get all divisors of n.
<div align="right">15 lines</div>

```cpp
vector<int> divisors(int n) {
    vector<int> ret, ret1;
    for (int i = 1; i*i <= n; ++i) {
        if (n % i == 0) {
            ret.push_back(i);
            int d = n / i;
            if (d != i) ret1.push_back(d);
        }
    }
    if (!ret1.empty()) {
        reverse(ret1.begin(), ret1.end());
        ret.insert(ret.end(), ret1.begin(), ret1.end());
    }
    return ret;
} // hash-cpp-all = 325815a4263d6fd7fac1bf3aee29d4d6
```

Pell.h

Description: Find the smallest integer root of $x^2 - ny^2 = 1$ when n is not a square number, with the solution set $x_{k+1} = x_0 x_k + n y_0 y_k$, $y_{k+1} = x_0 y_k + y_0 x_k$.
<div align="right">16 lines</div>

```cpp
pair<int,int> Pell(int n) {
    int p0 = 0, p1 = 1, q0 = 1, q1 = 0;
    int a0 = (int)sqrt(n), a1 = a0, a2 = a0;
    if(a0 * a0 == n) return {-1, -1};
    int g1 = 0, h1 = 1;
    while (1) {
        int g2 = -g1 + a1 * h1;
        int h2 = (n - g2 * g2)/h1;
        a2 = (g2 + a0)/h2;
        int p2 = a1 * p1 + p0;
        int q2 = a1 * q1 + q0;
        if (p2*p2 - n*q2*q2 == 1) return {p2, q2};
        a1 = a2; g1 = g2; h1 = h2; p0 = p1;
        p1 = p2; q0 = q1; q1 = q2;
    }
} // hash-cpp-all = 21e64f801a6641a3f2537e6f2f604860
```

PrimeFactors.h

Description: Find all prime factors of n.

`"Sieve.h"`
<div align="right">13 lines</div>

```cpp
vector<int> primeFac(int n){
    vector<int> factors;
    int idx = 0, prime_factors = primes[idx];
    while (prime_factors * prime_factors <= n){
        while (n % prime_factors == 0) {
            n /= prime_factors;
            factors.push_back(prime_factors);
        }
        prime_factors = primes[++idx];
    }
    if (n != 1) factors.push_back(n);
    return factors;
} // hash-cpp-all = 36e8e616befeeb19a350d9ad4e50cb0e
```

NumDiv.h

Description: Count the number of divisors of n.

`"Sieve.h"`
<div align="right">15 lines</div>

```cpp
int NumDiv(lint n){
    int idx = 0, prime_factors = primes[idx];
    lint ans = 1;
    while (prime_factors * prime_factors <= n) {
        int power = 0;
        while (n % prime_factors == 0) {
            n /= prime_factors;
            ++power;
        }
        ans *= 1ll * (power + 1);
        prime_factors = primes[++idx];
    }
    if (n != 1) ans *= 2;
    return ans;
} // hash-cpp-all = b91d21d80ce02a37a2911bcc84a23458
```

SumDiv.h

Description: Sum of all divisors of n.

`"Sieve.h"`
<div align="right">15 lines</div>

```cpp
lint sum(int n){
    int idx = 0, prime_factors = primes[idx];
    lint ans = 1;
    while (prime_factors * prime_factors <= n){
        int power = 0;
```

```cpp
        while (n % prime_factors == 0) {
            n /= prime_factors;
            power++;
        }
        ans *= ((lint)pow((double)prime_factors, power+1.0)
            ↪-1)/(prime_factors-1);
        prime_factors = primes[++idx];
    }
    if (n != 1) ans *= ((lint)pow((double)n, 2.0)-1)/(n-1);
    return ans;
} // hash-cpp-all = 5a0480423099b07ead604a8709ede140
```

Bezout.h

Description: Let $d := mdc(a, b)$. Then, there exist a pair x and y such that $ax + by = d$.
<div align="right">5 lines</div>

```cpp
pair<int, int> find_bezout(int x, int y) {
    if (y == 0) return bezout(1, 0);
    pair<int, int> g = find_bezout(y, x % y);
    return {g.second, g.first - (x/y) * g.second};
} // hash-cpp-all = d5ea908f84c746952727ecfe20a4f6f4
```

EulerPhi.h
<div align="right">16 lines</div>

```cpp
template<typename T>
T phi(T n){
    T aux, result;
    aux = result = n;
    for (T i = 2; i*i <= n; ++i)
        if (aux % i == 0) {
            while (aux % i == 0) aux /= i;
            result /= i;
            result *= (i-1)
        }
    if (aux > 1) {
        result /= aux;
        result *= (aux-1);
    }
    return result;
} // hash-cpp-all = fc45b1c8035350209d822304f9012ebe
```

phiFunction.h

Description: *Euler's totient* or *Euler's phi* function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. The *cototient* is $n - \phi(n)$. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p} \ \forall a$.
<div align="right">7 lines</div>

```cpp
const int LIM = (int)(1e5) * 5;
vector<int> phi(LIM);
void calculatePhi() {
    for(int i = 0; i < LIM; ++i) phi[i] = i&1 ? i : i/2;
    for(int i = 3; i < LIM; i += 2) if(phi[i] == i)
        for(int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} // hash-cpp-all = 44cd69787f2beeb7e7ba2d15baabdabe
```

DiscreteLogarithm.h

Description: Returns the smallest $x \geq 0$ s.t. $a^x = b \pmod{m}$. a and m must be coprime.
Time: $\mathcal{O}\left(\sqrt{m}\right)$
<div align="right">11 lines</div>

```cpp
lint modLog(lint a, lint b, lint m) {
```

```cpp
    assert(__gcd(a, m) == 1);
    lint n = (lint) sqrt(m) + 1, e = 1, x = 1, res =
        ↪LLONG_MAX;
    unordered_map<lint, lint> f;
    for(int i = 0; i < n; ++i) e = e * a % m;
    for(int i = 0; i < n; ++i) x = x * e % m, f.emplace(x,
        ↪i + 1);
    for(int i = 0; i < n; ++i)
        if (f.count(b = b * a % m))
            res = min(res, f[b] * n - i - 1);
    return res;
} // hash-cpp-all = 4e6790ea248af84e0e24fd996ab7b22f
```

Legendre.h
Description: Given an integer n and a prime number p, find the largest x such that p^x divides $n!$.

8 lines

```cpp
int legendre(int n, int p){
    int ret = 0, prod = p;
    while (prod <= n) {
        ret += n/prod;
        prod *= p;
    }
    return ret;
} // hash-cpp-all = 81613f762a8ec7c41ca9f6db5e02878a
```

GroupOrder.h
Description: Calculate the order of a in Z_n. A group Z_n is cyclic if, and only if n = $1, 2, 4, p^k$ or $2p^k$, being p an odd prime number.
Time: $\mathcal{O}\left(sqrt(n)log(n)\right)$

19 lines

```cpp
vector<int> divisors(int n) {
    vector<int> result, aux;
    for (int i = 1; i*i <= n; ++i) {
        if (n % i == 0) {
            result.push_back(i);
            if (i*i != n) aux.push_back(n/i);
        }
    }
    for (int i = aux.size()-1; i+1; --i) result.push_back(
        ↪aux[i]);
    return result;
}

template<typename T>
T order(T a, T n) {
    vector<T> d = divisors(phi(n));
    for (int i : v)
        if (modpow(a, i, n) == 1) return i;
    return -1;
} // hash-cpp-all = 58e88b0fcf540ae4af20160a1770b1e7
```

Bet.h
Description: Given fractions $a \leq b$ with non-negative numerators and denominators, finds fraction f with lowest denominator such that $a \leq f \leq b$. Should work with all numbers less than 2^{62}.

9 lines

```cpp
lint cdiv(lint x, lint y) { return (x+y-1)/y; }
lint bet(lint P, lint A, lint L, lint R) {
    if (A == 0) return L == 0 ? 0 : -1;
    lint c = cdiv(L,A); if (A*c <= R) return c;
    lint B = P % A; // P = k*A+B, L <= A(x-Ky)-By <= R
    // => -R <= By % A <= -L
    auto y = bet(A,B, A-R % A, A-L%A);
    return y == -1 ? y : cdiv(L+B*y, A) + P/A*y;
} // hash-cpp-all = af25bf14a5ba55995c68ff1852cf4e06
```

PrimitiveRoots.h
Description: That is, g is a primitive root mod n if for every number x coprime to n there is an integer z s.t. $x \equiv g^z \pmod{n}$

17 lines

```cpp
template<typename T>
T primitive_roots(T p) {
    T n = p - 1;
    vector<T> factors;
    for (int i = 2; i*i <= n; ++i) if (n % i == 0) {
        factors.push_back(i);
        while (n % i == 0) n /= i;
    }
    if (n > 1) factors.push_back(n);
    for (int i = 2; i <= p; ++i) {
        bool works = true;
        for (int j = 0; j < factors.size() && works; ++j)
            works &= modpow(i, (p-1)/factors[j], p) != 1;
        if (works) return i;
    }
    return -1;
} // hash-cpp-all = 0133e9368db22d6251359e2b964e566b
```

5.4 Chinese remainder theorem

ChineseRemainder.h
Description: Chinese Remainder Theorem.
crt(a, m, b, n) computes x such that $x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m,n)$. Assumes $mn < 2^{62}$.
Time: $\mathcal{O}\left(\log(n)\right) - \mathcal{O}\left(n\log(LCM(m))\right)$

22 lines

```cpp
template<typename T>
T crt(T a, T m, T b, T n, T &x, T &y) { // hash-cpp-1
    if (n > m) swap(a, b), swap(m, n);
    T g = egcd(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
} // hash-cpp-1 = 7913facb67d55ef46cdf5f2ba5862ed5

template<typename T> // Solve system up to n congruences
T crt_system(vector<T> &a, vector<T> &m, int n) {
    for (int i = 0; i < n; ++i)
        a[i] = (a[i] % m[i] + m[i]) % m[i];
    T ret = a.front(), lcm = m.front();
    for (int i = 1; i < n; ++i) {
        T x, y;
        ret = crt(ret, lcm, a[i], m[i], x, y);
        T d = egcd(lcm, m[i], x = 0, y = 0);
        lcm = lcm * m[i] / d;
    }
    return ret;
}
```

5.5 Fractions

Fractions.h
Description: Template that helps deal with fractions.

31 lines

```cpp
struct frac { // hash-cpp-1
    lint n, d;
    frac() { n = 0, d = 1; }
    frac(lint _n, lint _d) {
        n = _n, d = _d;
        lint g = __gcd(n,d); n /= g, d /= g;
        if (d < 0) n *= -1, d *= -1;
    }
    frac(lint _n) : frac(_n,1) {}
    friend frac abs(frac F) { return frac(abs(F.n),F.d); }
```

```cpp
    friend bool operator<(const frac& l, const frac& r) {
        ↪return l.n*r.d < r.n*l.d; }
    friend bool operator==(const frac& l, const frac& r) {
        ↪return l.n == r.n && l.d == r.d; }
    friend bool operator!=(const frac& l, const frac& r) {
        ↪return !(l == r); }
    friend frac operator+(const frac& l, const frac& r) {
        ↪return frac(l.n*r.d+r.n*l.d,l.d*r.d); }
    friend frac operator-(const frac& l, const frac& r) {
        ↪return frac(l.n*r.d-r.n*l.d,l.d*r.d); }
    friend frac operator*(const frac& l, const frac& r) {
        ↪return frac(l.n*r.n,l.d*r.d); }
    friend frac operator*(const frac& l, int r) { return l*
        ↪frac(r,1); }
    friend frac operator*(int r, const frac& l) { return l*
        ↪r; }
    friend frac operator/(const frac& l, const frac& r) {
        ↪return l*frac(r.d,r.n); }
    friend frac operator/(const frac& l, const int& r) {
        ↪return l/frac(r,1); }
    friend frac operator/(const int& l, const frac& r) {
        ↪return frac(l,1)/r; }
    friend frac& operator+=(frac& l, const frac& r) {
        ↪return l = l+r; }
    friend frac& operator-=(frac& l, const frac& r) {
        ↪return l = l-r; }
    template<class T> friend frac& operator*=(frac& l,
        ↪const T& r) { return l = l*r; }
    template<class T> friend frac& operator/=(frac& l,
        ↪const T& r) { return l = l/r; }
    friend ostream& operator<<(ostream& strm, const frac& a
        ↪) {
        strm << a.n;
        if (a.d != 1) strm << "/" << a.d;
        return strm;
    }
};
```

ContinuedFractions.h
Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$. For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}\left(\log N\right)$

21 lines

```cpp
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<lint, lint> approximate(d x, lint N) { // hash-cpp-1
    lint LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y =
        ↪ x;
    for (;;) {
        lint lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q :
            ↪inf),
            a = (lint)floor(y), b = min(a, lim),
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives
                ↪us a
            // better approximation; if b = a/2, we *may* have
                ↪one.
            // Return {P, Q} here for a more canonical
                ↪approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q
                ↪)) ?
            {NP, NQ} : {P, Q};
        }
        if (abs(y = 1/(y - (d)a)) > 3*N) {
```

```
        return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
} // hash-cpp-1 = e3f27076ec30785f7826aabd1eb5ac59
```

FracBinarySearch.h
Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); //` `{1,3}`
Time: $\mathcal{O}(\log(N))$
<div align="right">24 lines</div>

```cpp
struct Frac { lint p, q; };

template<class F>
Frac fracBS(F f, lint N) { // hash-cpp-1
  bool dir = 1, A = 1, B = 1;
  Frac left{0, 1}, right{1, 1}; // Set right to 1/0 to
      ↪search (0, N]
  assert(!f(left)); assert(f(right));
  while (A || B) {
    lint adv = 0, step = 1; // move right if dir, else left
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{left.p * adv + right.p, left.q * adv + right
          ↪.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    right.p += left.p * adv;
    right.q += left.q * adv;
    dir = !dir;
    swap(left, right);
    A = B; B = !!adv;
  }
  return dir ? right : left;
} // hash-cpp-1 = 66f3c71eb28df4393cd2a2abbea9345e
```

5.5.1 Pythagorean Triples
The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ \ b = k \cdot (2mn), \ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.5.2 Primitive Roots
It only exists when n is $2, 4, p^k, 2p^k$, where p odd prime. If g is a primitive root, all primitive roots are of the form g^k where $k, \phi(p)$ are coprime (hence there are $\phi(\phi(p))$ primitive roots).

5.5.3 Chicken McNugget theorem
Sejam x e y dois inteiros coprimos, o maior inteiro que não pode ser escrito como $ax + by$ é $\frac{(x-1)(y-1)}{2}$

5.5.4 Bézout's identity
For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a, b)}, y - \frac{ka}{\gcd(a, b)} \right), \quad k \in \mathbb{Z}$$

5.5.5 Möbius Inversion Formula
Se $F(n) = \sum_{d|n} f(d)$, então $f(n) = \sum_{d|n} \mu(d)F(n/d)$.

5.6 Primes
$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than $1\,000\,000$.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.6.1 Sum of primes
For any multiplicative f:

$$S(n, p) = S(n, p-1) - f(p) \cdot (S(n/p, p-1) - S(p-1, p-1))$$

5.6.2 Wilson's theorem
Seja $n > 1$. Então $n | (n-1)! + 1$ sse n é primo.

5.6.3 Wolstenholme's theorem
Seja $p > 3$ um número primo. Então o numerador do número $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{p-1}$ é divisível por p^2.

5.6.4 Estimates
$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.6.5 Prime counting function ($\pi(x)$)
The prime counting function is asymptotic to $\frac{x}{\log x}$, by the prime number theorem.

x	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$\pi(x)$	4	25	168	1.229	9.592	78.498	664.579	5.761.455

Combinatorial (6)

6.1 Permutations
6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

Factorial.h
Description: Precalculate factorials
<div align="right">21 lines</div>

```cpp
void init(int lim) {
    fact.resize(lim + 1);
    fact[0] = 1;
    for (int i = 1; i <= lim; ++i)
        fact[i] = (lint)i * fact[i-1] % mod;
    ifact.resize(lim + 1);
    ifact[lim] = modinv(fact[lim], mod);
    for (int i = lim-1; i >= 0; --i)
        ifact[i] = (lint)(i+1) * ifact[i+1] % mod;
}

void init(int lim) { // modtemplate!
  fact = {1};
  for(int i = 1; i <= lim; i++)
    fact.push_back(i * fact[i-1]);
  ifact.resize(fact.size());
  ifact.back() = 1/fact.back();
  for(int i = (int)ifact.size()-1; i > 0; i--)
    ifact[i-1] = i * ifact[i];
}
// hash-cpp-all = d32b7a3615f384703ff8ba505e4fb14e
```

numPerm.h
Description: Number of permutations
<div align="right">6 lines</div>

```cpp
lint num_perm(int n, int r) {
    if (r < 0 || n < r) return 0;
    lint ret = 1;
    for (int i = n; i > n-r; --i) ret *= i;
    return ret;
} // hash-cpp-all = 9063aaab522de1bd1cbb483b1e4d6a39
```

6.1.2 Binomials
nCr.h
Description: ^nC_r
<div align="right">13 lines</div>

```cpp
lint ncr(lint n, lint r){
    if(r < 0 || n < 0) return 0;
    if(n < r) return 0;
    lint a = fact[n];
    a = (a * invfact[r]) % mod;
    a = (a * invfact[n-r]) % mod;
    return a;
}

num ncr(int n, int k){
  if(k < 0 || k > n) return 0;
```

```
    return fact[n] * ifact[k] * ifact[n-k];
} // hash-cpp-all = 321ddb6eb353b8c75a4c0be672ceb75d
```

Catalan.h
Description: Pre calculate Catalan numbers.
<ModTemplate.h>   9 lines
```
num catalan[MAX];
void pre() {
    catalan[0] = catalan[1] = 1;
    for (int i = 2; i <= n; ++i) {
        catalan[i] = 0;
        for (int j = 0; j < i; ++j)
            catalan[i] += catalan[j] * catalan[i-j-1];
    }
} // hash-cpp-all = e99e44501c3c9cd841cf3a61de1a8e6b
```

Multinomial.h
Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.
                 7 lines
```
lint multinomial(vector<int>& v) {
    lint c = 1, m = v.empty() ? 1 : v[0];
    for (int i = 1 < v.size(); ++i)
        for (int j = 0; j < v[i]; ++j)
            c = c * ++m / (j+1);
    return c;
} // hash-cpp-all = 864cdb12b60507bb64330bca4f60b112
```

PascalTriangle.h
Description: Pre-compute all binomial coefficient
Time: $\mathcal{O}\left(n^2\right)$
                 9 lines
```
void init() {
    c[0][0] = 1;
    for (int i = 0; i < n; ++i) {
        c[i][0] = c[i][i] = 1;
        for (int j = 1; j < i; ++j)
            c[i][j] = c[i-1][j-1] + c[i-1][j];
    }
```

6.1.3 Derangements
`// hash-cpp-all = 0ec6a097f990e14d2a4eaf3f588f1c05`
Permutações de um conjunto tais que nenhum dos elementos aparecem em sua posição original.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Cycles
Suponha que $g_S(n)$ é o número de n-permutações quais o tamanho do ciclo pertence ao conjunto S. Então

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

6.1.5 Inclusion-Exclusion Principle
Sejam $A_1, A_2, ..., A_n$ conjuntos. Então o número de elementos da união $A_1 \cup A_2 \cup ... \cup A_n$ é

$$\left|\bigcup_{i=1}^n A_i\right| = \sum_{\substack{I \subseteq \{1,2,...,n\} \\ I \neq \emptyset}} (-1)^{|I|+1} \left|\bigcap_{i \in I} A_i\right|$$

6.1.6 The twelvefold way (from Stanley)
How many functions $f : N \to X$ are there?

N	X	Any f	Injective	Surjective
dist.	dist.	x^n	$\frac{x!}{(x-n)!}$	$x!\{^n_x\}$
indist.	dist.	$\binom{x+n-1}{n}$	$\binom{x}{n}$	$\binom{n-1}{n-x}$
dist.	indist.	$\{^n_1\} + \ldots + \{^n_x\}$	$[n \le x]$	$\{^n_k\}$
indist.	indist.	$p_1(n) + \ldots p_x(n)$	$[n \le x]$	$p_x(n)$

Where $\binom{a}{b} = \frac{1}{b!}(a)_b$, $p_x(n)$ is the number of ways to partition the integer n using x summand and $\{^n_x\}$ is the number of ways to partition a set of n elements into x subsets (aka Stirling number of the second kind).

6.1.7 Involutions
Uma involução é uma permutação com ciclo de tamanho máximo 2, e é a sua própria inversa.

$$a(n) = a(n-1) + (n-1)a(n-2)$$

$$a(0) = a(1) = 1$$

1, 1, 2, 4, 10, 26, 76, 232, 764, 2620, 9496, 35696, 140152

6.1.8 Burnside
Seja $A : GX \to X$ uma ação. Defina:

- $w \coloneqq$ número de órbitas em X.
- $S_x \coloneqq \{g \in G \mid g \cdot x = x\}$
- $F_g \coloneqq \{x \in X \mid g \cdot x = x\}$

Então $w = \frac{1}{|G|} \sum_{x \in X} |S_x| = \frac{1}{|G|} \sum_{g \in G} |F_g|$.

6.2 Partitions and subsets
6.2.1 Partition function
Número de formas de escrever n como a soma de inteiros positivos, independente da ordem deles.

$$p(0) = 1, \ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Lucas's theorem
Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$.

6.3 General purpose numbers
6.3.1 Bernoulli numbers
EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0,$
frac142, $\ldots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Fórmula de Euler-Maclaurin para somas infinitas:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind
Número de permutações em n itens com k ciclos.

$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k), c(0,0) = 1$

$\sum_{k=0}^n c(n,k)x^k = x(x+1)\ldots(x+n-1)$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers
Número de permutações $\pi \in S_n$ na qual exatamente k elementos são maiores que os anteriores. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n, n-1) = 1$$

$$E(n,k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j}(k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partições de n elementos distintos em exatamente k grupos.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.3.5 Bell numbers

Número total de partições de n elementos distintos.
$B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$

$$\mathcal{B}_{n+1} = \sum_{k=0}^{n}\binom{n}{k}\mathcal{B}_k$$

Também é possível calcular usando Stirling numbers of the second kind,

$$B_n = \sum_{k=0}^{n}S(n,k)$$

Já para p primo,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

\# em n vertices: n^{n-2}
\# em k árvores existentes de tamanho n_i:
$n_1 n_2 \cdots n_k n^{k-2}$
\# de grau d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$
\# florestas com exatamente k árvores enraizadas:

$$\binom{n}{k}k \cdot n^{n-k-1}$$

.

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2}C_n, \ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in a $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children) or $2n+1$ elements.
- ordered trees with $n+1$ vertices.
- \# ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subsequence.

6.3.8 Super Catalan numbers

The number of monotonic lattice paths of a nxn grid that do not touch the diagonal.

$$S(n) = \frac{3(2n-3)S(n-1) - (n-3)S(n-2)}{n}$$

$$S(1) = S(2) = 1$$

1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049, 518859

6.3.9 Motzkin numbers

Number of ways of drawing any number of nonintersecting chords among n points on a circle. Number of lattice paths from (0, 0) to $(n, 0)$ never going below the x-axis, using only steps NE, E, SE.

$$M(n) = \frac{3(n-1)M(n-2) + (2n+1)M(n-1)}{n+2}$$

$$M(0) = M(1) = 1$$

1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835, 113634

6.3.10 Narayana numbers

Number of lattice paths from (0,0) to $(2n,0)$ never going below the x-axis, using only steps NE and SE, and with k peaks.

$$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k-1}$$

$$N(n,1) = N(n,n) = 1$$

$$\sum_{k=1}^{n}N(n,k) = C_n$$

1, 1, 1, 1, 3, 1, 1, 6, 6, 1, 1, 10, 20, 10, 1, 1, 15, 50

6.3.11 Schroder numbers

Number of lattice paths from (0, 0) to (n, n) using only steps N,NE,E, never going above the diagonal. Number of lattice paths from (0, 0) to $(2n, 0)$ using only steps NE, SE and double east EE, never going below the x-axis. Twice the Super Catalan number, except for the first term.
1, 2, 6, 22, 90, 394, 1806, 8558, 41586, 206098

6.3.12 Triangles

Given rods of length 1, ..., n,

$$T(n) = \frac{1}{24}\begin{cases} n(n-2)(2n-5) & n \text{ even} \\ (n-1)(n-3)(2n-1) & n \text{ odd} \end{cases}$$

is the number of distinct triangles (positive are) that can be constructed, i.e., the of 3-subsets of $[n]$ s.t. $x \le y \le z$ and $z \ne x + y$.

6.4 Game Theory

A game can be reduced to Nim if it is a finite impartial game. Nim and its variants include:

6.4.1 Nim

Let $X = \bigoplus_{i=1}^{n} x_i$, then $(x_i)_{i=1}^{n}$ is a winning position iff $X \ne 0$. Find a move by picking k such that $x_k > x_k \oplus X$.

6.4.2 Misère Nim

Regular Nim, except that the last player to move *loses*. Play regular Nim until there is only one pile of size larger than 1, reduce it to 0 or 1 such that there is an odd number of piles. The second player wins $(a_1, \ldots, a_n)$ if 1) there is a pile $a_i > 1$ and $\oplus_{i=1}^{n}a_i = 0$ or 2) all $a_i \le 1$ and $\oplus_{i=1}^{n}a_i = 1$.

6.4.3 Staircase Nim

Stones are moved down a staircase and only removed from the last pile. $(x_i)_{i=1}^{n}$ is an L-position if $(x_{2i-1})_{i=1}^{n/2}$ is (i.e. only look at odd-numbered piles).

6.4.4 Moore's Nim$_k$

The player may remove from at most k piles (Nim = Nim$_1$). Expand the piles in base 2, do a carry-less addition in base $k+1$ (i.e. the number of ones in each column should be divisible by $k+1$).

6.4.5   Dim$^+$

The number of removed stones must be a divisor of the pile size. The Sprague-Grundy function is $k + 1$ where 2^k is the largest power of 2 dividing the pile size.

6.4.6   Aliquot Game

Same as above, except the divisor should be proper (hence 1 is also a terminal state, but watch out for size 0 piles). Now the Sprague-Grundy function is just k.

6.4.7   Nim (at most half)

Write $n + 1 = 2^m y$ with m maximal, then the Sprague-Grundy function of n is $(y - 1)/2$.

6.4.8   Lasker's Nim

Players may alternatively split a pile into two new non-empty piles. $g(4k + 1) = 4k + 1$, $g(4k + 2) = 4k + 2$, $g(4k + 3) = 4k + 4$, $g(4k + 4) = 4k + 3$ $(k \geq 0)$.

6.4.9   Hackenbush on Trees

A tree with stalks $(x_i)_{i=1}^n$ may be replaced with a single stalk with length $\bigoplus_{i=1}^n x_i$.

Nim-Product.cpp
Description: Nim Product.
<div align="right">17 lines</div>

```cpp
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
  if (_nimProd2[i][j]) return _nimProd2[i][j];
  if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
  int a = (i&j) & -(i&j);
  return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i
    ↪ ^ a) | (a-1), (j ^ a) | (i & (a-1)));
}
ull nimProd(ull x, ull y) {
  ull res = 0;
  for (int i = 0; x >> i; i++)
    if ((x >> i) & 1)
      for (int j = 0; y >> j; j++)
        if ((y >> j) & 1)
          res ^= nimProd2(i, j);
  return res;
} // hash-cpp-all = e0411498c7a77d77ae793efab5500851
```

RandomWalk.h
Description: Probability of reaching N(winning) Variation - Loser gives a coin to the winner
<ModPow.h>
<div align="right">6 lines</div>

```cpp
// pmf = probability of moving forward
double random_walk(double p, int i, int n) {
    double q = 1 - p;
    if (fabs(p - q) < EPS) return 1.0 * i/n;
    return (1 - modpow(q/p, i))/(1 - modpow(q/p, n));
} // hash-cpp-all = 71c0095f96b65c6e75a9016180a4c3b5
```

Partitions.cpp
Description: Fills array with partition function p(n) for 0 <= i <= n
<div align="right">17 lines</div>

```cpp
array<int, 122> part; // 121 is max partition that will fit
    ↪ into int
void partition(int n) {
    part[0] = 1;
    for (int i = 1; i <= n; ++i) {
        part[i] = 0;
        for (int k = 1, x; k <= i; ++k) {
            x = i - k * (3*k-1)/2;
            if (x < 0) break;
            if (k&1) part[i] += part[x];
            else part[i] -= part[x];
            x = i - k * (3*k+1)/2;
            if (x < 0) break;
            if (k&1) part[i] += part[x];
            else part[i] -= part[x];
        }
    }
} // hash-cpp-all = 3b7c36794ae71cf38eae6fa9a135ace2
```

Lucas.h
Description: Lucas theorem
Time: $\mathcal{O}\left(log_p(n)mod_inverse()\right)$
<ModTemplate.h>, <nCr.h>
<div align="right">11 lines</div>

```cpp
num lucas(lint n, lint m) {
  num c = 1;
  while (n || m) {
    lint x = n % MOD, y = m % MOD;
    if (x < y) return 0;
    c = c * ncr(x, y);
    n /= MOD;
    m /= MOD;
  }
  return c;
} // hash-cpp-all = fb6ac919315ce14ff27894d42b0fd271
```

Graph (7)

7.1   Fundamentals

BellmanFord.h
Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$
<div align="right">21 lines</div>

```cpp
const lint inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { lint dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int
    ↪ s) {
  nodes[s].dist = 0;
  sort(eds.begin(), eds.end(), [](Ed a, Ed b) { return a.s
    ↪ () < b.s(); });
  int lim = nodes.size() / 2 + 2; // /3+100 with shuffled
    ↪ vertices
  for(int i = 0; i < lim; ++i) for(auto &ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    lint d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
```

```cpp
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  for(int i = 0; i < lim; ++i) for(auto &e : eds)
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
} // hash-cpp-all = 62f3d4db997360483e6628d5373994af
```

FloydWarshall.h
Description: Calculates all-pairs shortest path in a directed graph that might have negative edge distances. Input is an distance matrix m, where $m[i][j] = $ inf if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$
<div align="right">16 lines</div>

```cpp
const lint inf = 1LL << 62;
void floydWarshall(vector<vector<lint>>& m) {
  int n = m.size();
  for (int i = 0; i < n; ++i) m[i][i] = min(m[i][i], {});
  for (int k = 0; k < n; ++k)
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        if (m[i][k] != inf && m[k][j] != inf) {
          auto newDist = max(m[i][k] + m[k][j], -inf);
          m[i][j] = min(m[i][j], newDist);
        }
  for (int k = 0; k < n; ++k) if (m[k][k] < 0)
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        if (m[i][k] != inf && m[k][j] != inf) m[i][j] =
          ↪ -inf;
} // hash-cpp-all = 578e31a61dfb8557ef1e1f4c611b2815
```

TopoSort.h
Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V| + |E|)$
<div align="right">14 lines</div>

```cpp
vector<int> topo_sort(const vector<vector<int>> &g) {
  vector<int> indeg(g.size()), ret;
  for(auto &li : g) for(auto &x : li) indeg[x]++;
  queue<int> q; // use priority queue for lexic. smallest
    ↪ ans.
  for(int i = 0; i < g.size(); ++i) if (indeg[i] == 0) q.
    ↪ push(-i);
  while (!q.empty()) {
    int i = -q.front(); // top() for priority queue
    ret.push_back(i);
    q.pop();
    for(auto &x : g[i])
      if (--indeg[x] == 0) q.push(-x);
  }
  return ret;
} // hash-cpp-all = d2ba1ef7b98de4bab3212a9a20c7220d
```

CutVertices.h
<div align="right">31 lines</div>

```cpp
vector<int> cut, mark, low, par;
vector<vector<int>> edges;
int Time = 0;

void dfs(int v, int p) {
    int cnt = 0;
    par[v] = p;
```

```cpp
    low[v] = mark[v] = Time++;
    for (int u : edges[v]) {
        if (mark[u] == -1) {
            par[u] = v;
            dfs(u, v);
            low[v] = min(low[v], low[u]);
            if (low[u] >= mark[v]) cnt++;
            //if (low[u] > mark[v]) u-v bridge
        }
        else if (u != par[v]) low[v] = min(low[v], mark[u])
            ↪;
    }
    if (cnt > 1 || (mark[v] != 0 && cnt > 0)) cut[v] = 1;
}
void solve(int n) {
    cut.resize(n, 0);
    mark.resize(n, -1);
    low.resize(n, 0);
    par.resize(n, 0);
    for (int i = 0; i < n; ++i)
        if (mark[i] == -1) {
            Time = 0;
            dfs(i, i);
        }
} // hash-cpp-all = 23e6fcdbd3ffa84a303354844e44c8bb
```

Bridges.h
Description: Find bridges in an undirected graph G. Do not forget to set the first level as 1. (level[0] = 1)

19 lines

```cpp
vector<vector<int>> edges;
vector<int> level, dp;
int bridge = 0;

void dfs(int v, int p) {
    dp[v] = 0;
    for (int u : edges[v]) {
        if (level[u] == 0) {
            level[u] = level[v] + 1;
            dfs(u, v);
            dp[v] += dp[u];
        }
        else if (level[u] < level[v]) dp[v]++;
        else if (level[u] > level[v]) dp[v]--;
    }
    dp[v]--;
    if (level[v] > 1 && dp[v] == 0) // Edge_vp is a bridge
        bridge++;
} // hash-cpp-all = 990615e56d90abaddbb7130047b6dd79
```

Dijkstra.cpp
Description: Calculates the shortest path between start node and every other node in the graph

19 lines

```cpp
void dijkstra(vector<vector<pii>> &graph, vector<int> &dist
    ↪, int start){
    vector<bool> vis(n, 0);
    for(int i = 0; i < n; i++) dist[i] = INF;
    priority_queue <pii, vector<pii>, greater<pii>> q;
    q.push({dist[start] = 0,start});
    while(!q.empty()) {
        int u=q.top().nd;
        q.pop();
        vis[u]=1;
        for(pii p: graph[u]){
            int e=p.st, v=p.nd;
            if (vis[v]) continue;
```

```cpp
            int new_dist=dist[u]+e;
            if(new_dist<dist[v]){
                q.push({dist[v] = new_dist,v});
            }
        }
    }
} // hash-cpp-all = dca271572a4b037e16e5d9002cc482c3
```

Prim.h
Description: Find the minimum spanning tree. Better for dense graphs.
Time: $\mathcal{O}(E \log V)$

25 lines

```cpp
struct prim_t {
    int n;
    vector<vector<pair<int,int>>> edges;
    vector<bool> seen;
    priority_queue<pair<int, int>> pq;
    prim_t(int _n) : n(_n), edges(n), seen(n, false) {}
    void process(int u) { //inicializa com process(0)
        seen[u] = true;
        for (int j = 0; j < (int) edges[u].size(); j++) {
            pair<int, int> v = edges[u][j];
            if (!seen[v.first]) pq.push(make_pair(-v.second
                ↪, -v.first));
        }
    }
    int solve() {
        int mst_cost = 0;
        while (!pq.empty()) {
            pair<int,int> front = pq.top();
            pq.pop();
            int u = -front.second, w = -front.first;
            if (!seen[u]) mst_cost += w;
            process(u);
        }
        return mst_cost;
    }
}; // hash-cpp-all = 49bf0dc36deec9090635c411e8c89507
```

Kruskal.h
Description: Find the minimum spanning tree. Better for sparse graphs.
Time: $\mathcal{O}(E \log E)$

12 lines

```cpp
template<typename T>
T kruskal(vector<pair<T, pair<int,int>>> &edges) {
    sort(edges.begin(), edges.end());
    T cost = 0;
    UF dsu(edges.size());
    for (auto &e : edges)
        if (dsu.find(e.second.first) != dsu.find(e.second.
            ↪second)) {
            dsu.unite(e.second.first, e.second.second);
            cost += e.first;
        }
    return cost;
} // hash-cpp-all = f407f7a7396721b7868a52e8cf876e95
```

SPFA.h
Description: Shortest Path Faster Algorithm.
Time: $\mathcal{O}(E)$

26 lines

```cpp
int s; // source node
int dis[100100], f[100100], head[100100];
vector<pair<int,int>> edges[100100];
vector<int> q;
```

```cpp
void spfa() {
    q.push_back(s);
    memset(dis, 127,sizeof(dis));
    memset(f,0,sizeof(f));
    f[s]=1,dis[s]=0;
    int inf=dis[s+1];
    for (int i=0;i<q.size();++i){
        int now=q[i];
        f[now]=0;
        for(auto u:edges[now]){
            int next=u.first;
            int cost=u.second;
            if (dis[next]>dis[now]+cost) {
                dis[next]=dis[now]+cost;
                if (!f[next]) {
                    f[next] = 1;
                    q.push_back(next);
                }
            }
        }
    }
} // hash-cpp-all = 13d3c7613f2899f018199619dc20bfd3
```

7.2   Euler walk

EulerWalk.h
Description: Eulerian undirected/directed path/cycle algorithm. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, also put it->second in s (and then ret).
Time: $\mathcal{O}(E)$ where E is the number of edges.

17 lines

```cpp
using pii = pair<int,int>;

vector<int> eulerWalk(vector<vector<pii>>& gr, int nedges,
    ↪int src=0) {
    int n = gr.size();
    vector<int> D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = gr[x].
            ↪size();
        if (it == end){ ret.push_back(x); s.pop_back();
            ↪continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        }}
    for(auto &x : D) if (x < 0 || ret.size() != nedges+1)
        ↪return {};
    return {ret.rbegin(), ret.rend()};
} // hash-cpp-all = 400c6e63c2e9553cfc3b4909f8898483
```

7.3   Network flow

PushRelabel.h
Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}(V^2\sqrt{E})$ Better for dense graphs - Slower than Dinic (in practice)

47 lines

```cpp
typedef lint Flow;

struct PushRelabel {
    struct edge_t {
        int dest, back;
```

```
  Flow f, c;
};
vector<vector<edge_t>> g;
vector<Flow> ec;
vector<edge_t*> cur;
vector<vector<int>> hs; vector<int> H;
PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n)
  ↪{}
void add_edge(int s, int t, Flow cap, Flow rcap=0) {
  if (s == t) return;
  g[s].push_back({t, g[t].size(), 0, cap});
  g[t].push_back({s, g[s].size(), 0, rcap});
}
void add_flow(edge_t& e, Flow f) {
  edge_t &back = g[e.dest][e.back];
  if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
  e.f += f; e.c -= f; ec[e.dest] += f;
  back.f -= f; back.c += f; ec[back.dest] -= f;
}
Flow maxflow(int s, int t) {
  int v = g.size(); H[s] = v; ec[t] = 1;
  vector<int> co(2*v); co[0] = v-1;
  for(int i = 0; i < v; ++i) cur[i] = g[i].data();
  for(auto &e : g[s]) add_flow(e, e.c);
  for (int hi = 0;;) {
    while (hs[hi].empty()) if (!hi--) return -ec[s];
    int u = hs[hi].back(); hs[hi].pop_back();
    while (ec[u] > 0)  // discharge u
      if (cur[u] == g[u].data() + g[u].size()) {
        H[u] = 1e9;
        for(auto &e : g[u]) if (e.c && H[u] > H[e.dest
          ↪]+1)
          H[u] = H[e.dest]+1, cur[u] = &e;
        if (++co[H[u]], !--co[hi] && hi < v)
          for(int i = 0; i < v; ++i) if (hi < H[i] && H[i
            ↪] < v)
            --co[H[i]], H[i] = v + 1;
        hi = H[u];
      } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
        add_flow(*cur[u], min(ec[u], cur[u]->c));
      else ++cur[u];
  }
}
bool leftOfMintCut(int a) { return H[a] >= g.size(); }
}; // hash-cpp-all = c4c114b51fa640b1ca9b9ad83a73ad56
```

Dinic.h

Description: Flow algorithm with complexity $O(VE \log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching. To obtain the actual flow, look at positive values only.

57 lines

```
template<typename T = lint>
struct Dinic {
  struct Edge {
    int to, rev; T c, f;
  };
  vector<int> lvl, ptr, q;
  vector<int> partition; //call findMinCut before use it
  vector<pair<pair<int,int>,int>> cut; //u,v,c
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n),partition(n),
    ↪cut(0) {}
  void addEdge(int a, int b, T c, int rcap = 0) {
    adj[a].push_back({b, adj[b].size(), c, 0});
    adj[b].push_back({a, adj[a].size() - 1, rcap, 0});
  }
  T dfs(int v, int t, T f) {
```

```
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < adj[v].size(); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (T p = dfs(e.to, t, min(f, e.c - e.f))) {
          e.f += p, adj[e.to][e.rev].f -= p;
          return p;
        }
    }
    return 0;
  }
  T maxflow(int s, int t) {
    T flow = 0; q[0] = s;
    for(int L = 0; L < 31; ++L) do { // 'int L=30' maybe
      ↪faster for random data
      lvl = ptr = vector<int>(q.size());
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for(Edge &e : adj[v])
          if (!lvl[e.to] && (e.c - e.f) >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (T p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  //only if you want the edges of the cut
  void dfsMC(int u){
    partition[u] = 1;
    for (Edge &e : adj[u])
      if (!partition[e.to])
        if (e.c - e.f == 0)
          cut.push_back({{u,e.to},e.c});
        else if (e.c - e.f > 0) dfsMC(e.to);
  }
  //only if you want the edges of the cut
  vector<pair<pair<int,int>,int>> findMinCut(int u,int t){
    maxflow(u,t); //DONT call again if you already called
      ↪it
    dfsMC(u);
    return cut;
  }
}; // hash-cpp-all = 00d2fea8e1f8098f86dde65c62d7131f
```

HLPP.h

Description: Highest label preflow push algorithm. Use it only if you really need the fastest maxflow algo. One limitation of the HLPP implementation is that you can't recover the weights for the full flow - use Dinic's for this.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$. Faster than Dinic with scaling(in practice).

79 lines

```
template <int MAXN, class T = int> struct HLPP {
  const T INF = numeric_limits<T>::max();
  struct edge_t { int to, rev; T flow; };
  int s = MAXN - 1, t = MAXN - 2;
  vector<edge_t> adj[MAXN];
  vector<int> lst[MAXN], gap[MAXN];
  T excess[MAXN];
  int highest, height[MAXN], cnt[MAXN], work;
  void addEdge(int from, int to, int flow, bool
    ↪isDirected = true) {
    adj[from].push_back({to, adj[to].size(), flow});
    adj[to].push_back({from, adj[from].size() - 1,
      ↪isDirected ? 0 : flow});
  }
  void updHeight(int v, int nh) {
```

```
    work++;
    if (height[v] != MAXN) cnt[height[v]]--;
    height[v] = nh;
    if (nh == MAXN) return;
    cnt[nh]++, highest = nh;
    gap[nh].push_back(v);
    if (excess[v] > 0) lst[nh].push_back(v);
  }
  void globalRelabel() {
    work = 0;
    fill(height.begin(), height.end(), MAXN);
    fill(cnt.begin(), cnt.end(), 0);
    for (int i = 0; i < highest; i++)
      lst[i].clear(), gap[i].clear();
    height[t] = 0;
    queue<int> q({t});
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (auto &e : adj[v])
        if (height[e.to] == MAXN && adj[e.to][e.rev
          ↪].flow > 0)
          q.push(e.to), updHeight(e.to, height[v]
            ↪ + 1);
      highest = height[v];
    }
  }
  void push(int v, edge_t &e) {
    if (excess[e.to] == 0)
      lst[height[e.to]].push_back(e.to);
    T df = min(excess[v], e.flow);
    e.flow -= df, adj[e.to][e.rev].flow += df;
    excess[v] -= df, excess[e.to] += df;
  }
  void discharge(int v) {
    int nh = MAXN;
    for (auto &e : adj[v]) {
      if (e.flow > 0) {
        if (height[v] == height[e.to] + 1) {
          push(v, e);
          if (excess[v] <= 0) return;
        }
        else nh = min(nh, height[e.to] + 1);
      }
    }
    if (cnt[height[v]] > 1) updHeight(v, nh);
    else {
      for (int i = height[v]; i <= highest; i++) {
        for (auto j : gap[i]) updHeight(j, MAXN);
        gap[i].clear();
      }
    }
  }
  T maxflow(int heur_n = MAXN) {
    fill(excess.begin(), excess.end(), 0);
    excess[s] = INF, excess[t] = -INF;
    globalRelabel();
    for (auto &e : adj[s]) push(s, e);
    for (; highest >= 0; highest--) {
      while (!lst[highest].empty()) {
        int v = lst[highest].back();
        lst[highest].pop_back();
        discharge(v);
        if (work > 4 * heur_n) globalRelabel();
      }
    }
    return excess[t] + INF;
  }
}; // hash-cpp-all = fa1d36a82b0ee3ea819a0f3cd2e1c3cb
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.

Usage: `unordered_map<int, T> graph;`
`graph[a][b] += c; //adds edge from a to b with capacity c,`
`use "+=" NOT "="`
<div align="right">33 lines</div>

```cpp
template<class T> T edmondsKarp(vector<unordered_map<int, T
    ↪>> &graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vector<int> par(graph.size()), q = par;
  for (;;) {
    fill(par.begin(),par.end(), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;
    for (int i = 0; i < ptr; ++i) {
      int x = q[i];
      for (pair<int,int> e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);
    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
};
// hash-cpp-all = 61d8900b275a8485d1f54c130eee76fa
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.
<div align="right">1 lines</div>

```cpp
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not.

Time: Approximately $\mathcal{O}\left(E^2\right)$ faster than Kactl's on practice

`<bits/extc++.h>` // don't forget!
<div align="right">73 lines</div>

```cpp
template <typename flow_t = int, typename cost_t = long
    ↪long>
struct MCMF_SSPA { // hash-cpp-1
  int N;
  vector<vector<int>> adj;
  struct edge_t {
    int dest; flow_t cap; cost_t cost;
  };
  vector<edge_t> edges;
  vector<char> seen;
  vector<cost_t> pi;
  vector<int> prv;
```

```cpp
  explicit MCMF_SSPA(int N_) : N(N_), adj(N), pi(N, 0), prv
      ↪(N) {}
  void addEdge(int from, int to, flow_t cap, cost_t cost) {
    assert(cap >= 0);
    int e = int(edges.size());
    edges.emplace_back(edge_t{to, cap, cost});
    edges.emplace_back(edge_t{from, 0, -cost});
    adj[from].push_back(e);
    adj[to].push_back(e+1);
  }
  const cost_t INF_COST = numeric_limits<cost_t>::max() /
      ↪4;
  const flow_t INF_FLOW = numeric_limits<flow_t>::max() /
      ↪4;
  vector<cost_t> dist;
  __gnu_pbds::priority_queue<pair<cost_t, int>> q;
  vector<typename decltype(q)::point_iterator> its;
// hash-cpp-1 = 8aca97b902d3c8e2ff81879aff6726b7
  void path(int s) { // hash-cpp-2
    dist.assign(N, INF_COST);
    dist[s] = 0;
    its.assign(N, q.end());
    its[s] = q.push({0, s});
    while (!q.empty()) {
      int i = q.top().second; q.pop();
      cost_t d = dist[i];
      for (int e : adj[i]) {
        if (edges[e].cap) {
          int j = edges[e].dest;
          cost_t nd = d + edges[e].cost;
          if (nd < dist[j]) {
            dist[j] = nd;
            prv[j] = e;
            if (its[j] == q.end()) its[j] = q.push({-(dist[
                ↪j] - pi[j]), j});
            else q.modify(its[j], {-(dist[j] - pi[j]), j});
          }
        }
      }
    }
    swap(pi, dist);
  } // hash-cpp-2 = e0e5e63209e5bf3bf43cf2446879454e
  pair<flow_t, cost_t> maxflow(int s, int t) { // hash-cpp
      ↪-3
    assert(s != t);
    flow_t totFlow = 0; cost_t totCost = 0;
    while (path(s), pi[t] < INF_COST) {
      flow_t curFlow = numeric_limits<flow_t>::max();
      for (int cur = t; cur != s; ) {
        int e = prv[cur];
        int nxt = edges[e^1].dest;
        curFlow = min(curFlow, edges[e].cap);
        cur = nxt;
      }
      totFlow += curFlow;
      totCost += pi[t] * curFlow;
      for (int cur = t; cur != s; ) {
        int e = prv[cur];
        int nxt = edges[e^1].dest;
        edges[e].cap -= curFlow;
        edges[e^1].cap += curFlow;
        cur = nxt;
      }
    }
    return {totFlow, totCost};
  } // hash-cpp-3 = f023f1f510c6212c3225362b96a23efc
};
```

StoerWagner.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $\mathcal{O}\left(V^3\right)$
<div align="right">30 lines</div>

```cpp
pair<int, vector<int>> GetMinCut(vector<vector<int>> &
    ↪weights) {
  int N = weights.size();
  vector<int> used(N), cut, best_cut;
  int best_weight = -1;
  for (int phase = N-1; phase >= 0; phase--) { // hash-cpp
      ↪-1
    vector<int> w = weights[0], added = used;
    int prev, k = 0;
    for (int i = 0; i < phase; ++i){
      prev = k;
      k = -1;
      for (int j = 1; j < N; ++j)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase-1) {
        for (int j = 0; j < N; ++j) weights[prev][j] +=
            ↪weights[k][j];
        for (int j = 0; j < N; ++j) weights[j][prev] =
            ↪weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        for (int j = 0; j < N; ++j)
          w[j] += weights[k][j];
        added[k] = true;
      }
    }
  } // hash-cpp-1 = 134b05ab04bdf6f5735abb5acd44401c
  return {best_weight, best_cut};
}
```

7.4 Matching

HopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: `vector<int> btoa(m, -1); hopcroftKarp(g, btoa);`

Time: $\mathcal{O}\left(\sqrt{V}E\right)$
<div align="right">42 lines</div>

```cpp
bool dfs(int a, int layer, const vector<vector<int>> &g,
    ↪vector<int> &btoa, vector<int> &A, vector<int> &B) {
    ↪// hash-cpp-1
  if (A[a] != layer) return 0;
  A[a] = -1;
  for(auto &b : g[a]) if (B[b] == layer + 1) {
    B[b] = -1;
    if (btoa[b] == -1 || dfs(btoa[b], layer+2, g, btoa, A,
        ↪B))
      return btoa[b] = a, 1;
  }
  return 0;
} // hash-cpp-1 = 1707b0c00c4eecb14a7d272f189c7330

int hopcroftKarp(const vector<vector<int>> &g, vector<int>
    ↪&btoa) { // hash-cpp-2
  int res = 0;
```

```cpp
    vector<int> A(g.size()), B(btoa.size()), cur, next;
    for (;;) {
        fill(A.begin(), A.end(), 0);
        fill(B.begin(), B.end(), -1);
        cur.clear();
        for(auto &a : btoa) if(a != -1) A[a] = -1;
        for (int a = 0; a < g.size(); ++a) if(A[a] == 0) cur.
            ↪push_back(a);
        for (int lay = 1;; lay += 2) {
            bool islast = 0;
            next.clear();
            for(auto &a : cur) for(auto &b : g[a]) {
                if (btoa[b] == -1) {
                    B[b] = lay;
                    islast = 1;
                }
                else if (btoa[b] != a && B[b] == -1) {
                    B[b] = lay;
                    next.push_back(btoa[b]);
                }
            }
            if (islast) break;
            if (next.empty()) return res;
            for(auto &a : next) A[a] = lay+1;
            cur.swap(next);
        }
        for(int a = 0; a < g.size(); ++a)
            res += dfs(a, 0, g, btoa, A, B);
    }
} // hash-cpp-2 = dfd1b7085dec9a0ed4688fd1a144fb31
```

MCBM.h

Description: Fast Kuhn! Simple maximum cardinality bipartite matching algorithm. Fast and reliable maximum cardinality matching solver, better than DFSMatching and sometimes even faster than hopcroftKarp (Crazy heuristic huh). This implementation has got an $O(n^2)$ worst case on a sparse graph. Shuffling the edges and vertices ordering might fix it. Good Luck. $R[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Time: $\mathcal{O}(VE)$ worst case with shuffling I guess
                                                                    41 lines

```cpp
struct BipartiteMatcher {
    vector<vector<int>> edges;
    vector<int> L, R, seen;
    BipartiteMatcher(int n, int m) :
    edges(n), L(n, -1), R(m, -1), seen(n) {}
    void addEdge(int a, int b) { edges[a].push_back(b); }
    void improve() {
        mt19937 rng(chrono::steady_clock::now().
            ↪time_since_epoch().count());
        for (int i = 0; i < edges.size(); ++i)
            shuffle(edges[i].begin(), edges[i].end(), rng);
    }
    bool find(int v) {
        if (seen[v]) return false;
        seen[v] = true;
        for (int u : edges[v])
            if (R[u] == -1) {
                L[v] = u;
                R[u] = v;
                return true;
            }
        for (int u : edges[v])
            if (find(R[u])) {
                L[v] = u;
                R[u] = v;
                return true;
```

```cpp
            }
        return false;
    }
    int match() {
        int ok = true;
        while (ok--) {
            fill(seen.begin(), seen.end(), 0);
            for (int i = 0; i < (int)L.size(); ++i)
                if (L[i] == -1) ok |= find(i);
        }
        int ret = 0;
        for (int i = 0; i < L.size(); ++i)
            ret += (L[i] != -1);
        return ret;
    }
}; // hash-cpp-all = ae31ab3d5b8fb2f2301c8fe9827c3e8a
```

WeightedMatching.h

Description: Min cost bipartite matching. Negate costs for max cost.
Time: $\mathcal{O}(N^3)$
                                                                    66 lines

```cpp
typedef vector<double> vd;
bool zero(double x) { return fabs(x) < 1e-10; }
double MinCostMatching(const vector<vd>& cost, vector<int>&
    ↪ L, vector<int>& R) {
    int n = cost.size(), mated = 0;
    vd dist(n), u(n), v(n);
    vector<int> dad(n), seen(n);
    for(int i = 0; i < n; ++i) {
        u[i] = cost[i][0];
        for(int j = 1; j < n; ++j) u[i] = min(u[i], cost[i][j])
            ↪;
    }
    for(int j = 0; j < n; ++j) {
        v[j] = cost[0][j] - u[0];
        for(int i = 1; i < n; ++i) v[j] = min(v[j], cost[i][j]
            ↪- u[i]);
    }
    L = R = vector<int>(n, -1);
    for(int i = 0; i < n; ++i) for(int j = 0; j < n; ++j) {
        if (R[j] != -1) continue;
        if (zero(cost[i][j] - u[i] - v[j])) {
            L[i] = j; R[j] = i;
            mated++;
            break;
        }
    }
    for (; mated < n; mated++) { // until solution is
        ↪feasible
        int s = 0;
        while (L[s] != -1) s++;
        fill(dad.begin(), dad.end(), -1);
        fill(seen.begin(), seen.end(), 0);
        for(int k = 0; k < n; ++k)
            dist[k] = cost[s][k] - u[s] - v[k];
        int j = 0;
        for (;;) {
            j = -1;
            for(int k = 0; k < n; ++k) {
                if (seen[k]) continue;
                if (j == -1 || dist[k] < dist[j]) j = k;
            }
            seen[j] = 1;
            int i = R[j];
            if (i == -1) break;
            for(int k = 0; k < n; ++k) {
                if (seen[k]) continue;
```

```cpp
                auto new_dist = dist[j] + cost[i][k] - u[i] - v[k];
                if (dist[k] > new_dist) {
                    dist[k] = new_dist;
                    dad[k] = j;
                }
            }
        }
        for(int k = 0; k < n; ++k) {
            if (k == j || !seen[k]) continue;
            auto w = dist[k] - dist[j];
            v[k] += w, u[R[k]] -= w;
        }
        u[s] += dist[j];
        while (dad[j] >= 0) {
            int d = dad[j];
            R[j] = R[d], L[R[j]] = j;
            j = d;
        }
        R[j] = s, L[s] = j;
    }
    auto value = vd(1)[0];
    for(int i = 0; i < n; ++i) value += cost[i][L[i]];
    return value;
} // hash-cpp-all = a1f5b2e9bf9a0f175eb2c5f099825802
```

GeneralMatching.h

Description: Maximum Matching for general graphs (undirected and non bipartite) using Edmond's Blossom Algorithm.
Time: $\mathcal{O}(EV^2)$
                                                                    70 lines

```cpp
struct blossom_t {
    int t, n; // 1-based indexing!!
    vector<vector<int>> edges;
    vector<int> seen, parent, og, match, aux, Q;
    blossom_t(int _n) : n(_n), edges(n+1), seen(n+1),
        parent(n+1), og(n+1), match(n+1), aux(n+10), t(0)
        ↪{}
    void addEdge(int u, int v) {
        edges[u].push_back(v);
        edges[v].push_back(u);
    }
    void augment(int u, int v) {
        int pv = v, nv; // flip states of edges on u-v path
        do {
            pv = parent[v]; nv = match[pv];
            match[v] = pv; match[pv] = v;
            v = nv;
        } while(u != pv);
    }
    int lca(int v, int w) { // find LCA in O(dist)
        ++t;
        while (1) {
            if (v) {
                if (aux[v] == t) return v; aux[v] = t;
                v = og[parent[match[v]]];
            }
            swap(v, w);
        }
    }
    void blossom(int v, int w, int a) {
        while (og[v] != a) {
            parent[v] = w; w = match[v]; // go other way
                ↪around cycle
            if(seen[w] == 1) Q.push_back(w), seen[w] = 0;
            og[v] = og[w] = a;       // merge into supernode
            v = parent[w];
        }
    }
```

```cpp
    }
    bool bfs(int u) {
        for (int i = 1; i <= n; ++i) seen[i] = -1, og[i] =
            ↪i;
        Q = vector<int>(); Q.push_back(u); seen[u] = 0;
        for(int i = 0; i < Q.size(); ++i) {
            int v = Q[i];
            for(auto &x : edges[v]) {
                if (seen[x] == -1) {
                    parent[x] = v; seen[x] = 1;
                    if (!match[x]) return augment(u, x),
                        ↪true;
                    Q.push_back(match[x]); seen[match[x]] =
                        ↪ 0;
                } else if (seen[x] == 0 && og[v] != og[x])
                    ↪{
                    int a = lca(og[v], og[x]);
                    blossom(x, v, a); blossom(v, x, a);
                }
            }
        }
        return false;
    }
    int find_match() {
        int ans = 0;
        // find random matching (not necessary, constant
            ↪improvement)
        vector<int> V(n-1); iota(V.begin(), V.end(), 1);
        shuffle(V.begin(), V.end(), mt19937(0x94949));
        for(auto &x : V) if(!match[x])
            for(auto &y : edges[x]) if (!match[y]) {
                match[x] = y, match[y] = x;
                ++ans; break;
            }
        for(int i = 1; i <= n; ++i)
            if (!match[i] && bfs(i))
                ++ans;
        return ans;
    }
}; // hash-cpp-all = 7603b5274164025932e18a2a9a22ccc8
```

MaximumIndependentSet.h
Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.
<div align="right">1 lines</div>

```cpp
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

MinimumVertexCover.h
Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.
"DFSMatching.h", "MCBM.h"
<div align="right">19 lines</div>

```cpp
vector<int> cover(vector<vector<int>>& g, int n, int m,
    ↪BipartiteMatcher &B) {
    int res = B.match();
    vector<bool> lfound(n, true), seen(m);
    for(int &it : B.R) if (it != -1) lfound[it] = false;
    vector<int> q, cover;
    for(int i = 0; i < n; ++i) if (lfound[i]) q.push_back(i
        ↪);
    while (!q.empty()) {
        int i = q.back(); q.pop_back();
        lfound[i] = 1;
        for(int e : g[i]) if (!seen[e] && B.R[e] != -1) {
            seen[e] = true;
            q.push_back(B.R[e]);
```

```cpp
        }
    }
    for(int i = 0; i < n; ++i) if (!lfound[i]) cover.
        ↪push_back(i);
    for(int i = 0; i < m; ++i) if (seen[i]) cover.push_back
        ↪(n+i);
    assert(cover.size() == res);
    return cover;
} // hash-cpp-all = 809bd9a1ad39bf4f56e712f4528fc539
```

Hungarian.h
Description: finds min cost to complete n jobs w/ m workers each worker is assigned to at most one job (n <= m)
<div align="right">29 lines</div>

```cpp
int HungarianMatch(const vector<vector<int>> &a) { // cost
    ↪array, negative values are ok
    int n = a.size()-1, m = a[0].size()-1; // jobs 1..n,
        ↪workers 1..m
    vector<int> u(n+1), v(m+1), p(m+1); // p[j] -> job
        ↪picked by worker j
    for(int i = 1; i <= n; ++i) { // find alternating path
        ↪with job i
        p[0] = i; int j0 = 0;
        vector<int> dist(m+1, INT_MAX), pre(m+1,-1); //
            ↪dist, previous vertex on shortest path
        vector<bool> done(m+1, false);
        do {
            done[j0] = true;
            int i0 = p[j0], j1; int delta = INT_MAX;
            for(int j = 1; j <= m; ++j) if (!done[j]) {
                auto cur = a[i0][j]-u[i0]-v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] =
                    ↪j0;
                if (dist[j] < delta) delta = dist[j], j1 =
                    ↪j;
            }
            for(int j = 0; j <= m; ++j) { // just dijkstra
                ↪with potentials
                if (done[j]) u[p[j]] += delta, v[j] -=
                    ↪delta;
                else dist[j] -= delta;
            }
            j0 = j1;
        } while (p[j0]);
        do { // update values on alternating path
            int j1 = pre[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }
    return -v[0]; // min cost
} // hash-cpp-all = 6fbac6d7eb27b2806f36b5cc91b951d3
```

7.5 DFS algorithms

CentroidDecomposition.cpp
Description: Divide and Conquer on Trees.
<div align="right">34 lines</div>

```cpp
struct centroid_t {
    vector<bool> seen;
    vector<int> sz, level, par_tree, closest;
    vector<vector<int>> edges, dist, parent;
    centroid_t(vector<vector<int>> &e, int n) : edges(e),
        ↪seen(n),
        sz(n), level(n), par_tree(n), closest(n, INT_MAX/2)
        ↪,
        dist(n, vector<int>(20)), parent(n, vector<int>(20)
        ↪) {
```

```cpp
        build(0, -1); }
    void dfs(int v, int p, int parc, int lvl) {
        sz[v] = 1;
        parent[v][lvl] = parc;
        dist[v][lvl] = 1 + dist[p][lvl];
        for (int u : edges[v]) {
            if (u == p || seen[u]) continue;
            dfs(u, v, parc, lvl);
            sz[v] += sz[u];
        }
    }
    int get_centroid(int v, int p, int tsz) {
        for (int u : edges[v])
            if (!seen[u] && u != p && sz[u] > tsz/2)
                return get_centroid(u, v, tsz);
        return v;
    }
    void build(int v, int p, int lvl = 0) {
        dfs(v, -1, p, lvl);
        int x = get_centroid(v, v, sz[v]);
        seen[x] = 1;
        par_tree[x] = p;
        level[x] = 1 + lvl;
        for (int u : edges[x])
            if (!seen[u]) build(u, x, 1 + lvl);
    }
}; // hash-cpp-all = 114b96449cf7cb6efe28752355d4ea5b
```

Tarjan.h
Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: cnt_of[i] holds the component index of a node (a component only has edges to components with lower index). ncnt will contain the number of components.
Time: $\mathcal{O}(E + V)$
<div align="right">29 lines</div>

```cpp
struct tarjan_t {
    int n, ncnt = 0, time = 0;
    vector<vector<int>> edges;
    vector<int> preorder_of, cnt_of, order;
    stack<int> stack_t;
    tarjan_t(int n): n(n), edges(n), preorder_of(n, 0),
        ↪cnt_of(n, -1) {}
    int dfs(int u) {
        int reach = preorder_of[u] = ++time, v;
        stack_t.push(u);
        for (int v : edges[u])
            if (cnt_of[v] == -1)
                reach = min(reach, preorder_of[v]?:dfs(v));
        if (reach == preorder_of[u]) {
            do {
                v = stack_t.top();
                stack_t.pop();
                order.push_back(v);
                cnt_of[v] = ncnt;
            } while (v != u);
            ++ncnt;
        }
        return preorder_of[u] = reach;
    }
    void solve() {
        time = ncnt = 0;
        for (int i = 0; i < (int)edges.size(); ++i)
            if (cnt_of[i] == -1) dfs(i);
    }
```

```
}; // hash-cpp-all = 0bc997e2bf8643f6aa44e5aa31c7384a
```

BiconnectedComponents.h
Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: int eid = 0; ed.resize(N);
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++);
}
Time: $\mathcal{O}(E+V)$
46 lines

```cpp
typedef vector<int> vi;
typedef vector<vector<pair<int,int>>> vii;

vector<int> num, st;
vii ed;
int Time;

int dfs(int at, int par,vector<vector<int>> &comps) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto &pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me){
        st.push_back(e);
      }
    } else {
      int si = st.size();
      int up = dfs(y, e, comps);
      top = min(top, up);
      if (up == me) {

        st.push_back(e);
        comps.push_back(vector<int>());
        for(int i=st.size()-1;i>=si;i--){
          comps[comps.size()-1].push_back(st[i]);
        }
        st.resize(si);
        cont_comp++;
      }
      else if (up < me){ st.push_back(e);}
      else { cont_comp++;comps.push_back({e});/* e is a
          ↪bridge */ }
    }
  }
  return top;
}

vector<vector<int>> bicomps() {
  // returns components and its edges ids
  vector<vector<int>> comps;
  num.assign(ed.size(), 0);
  for (int i = 0; i < ed.size(); ++i)
    if (!num[i]) dfs(i, -1, comps);
  return comps;
} // hash-cpp-all = 3e7f07e94a887065fdfa6d0cdc978102
```

2sat.h
Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|\|c)\&\&(d\|\|\|!b)\&\&...$ becomes true, or reports that it is un-satisfiable. Negated variables are represented by bit-inversions ($\sim$x).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.
51 lines

```cpp
struct TwoSat {
  int N;
  vector<vector<int>> gr;
  vector<int> values; // 0 = false, 1 = true
  TwoSat(int n = 0) : N(n), gr(2*n) {}
  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }
  void either(int f, int j) { // hash-cpp-1
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f^1].push_back(j);
    gr[j^1].push_back(f);
  } // hash-cpp-1 = 1140d4116e06cfd5efce120090e3f131
  void set_value(int x) { either(x, x); }
  void at_most_one(const vector<int>& li) { // (optional)
      ↪// hash-cpp-2
    if (li.size() <= 1) return;
    int cur = ~li[0];
    for (int i = 2; i < li.size(); ++i) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  } // hash-cpp-2 = d1cd651b7bb790d3aba3c4895427d962
  vector<int> val, comp, z; int time = 0;
  int dfs(int i) { // hash-cpp-3
    int low = val[i] = ++time, x; z.push_back(i);
    for (auto e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    ++time;
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = time;
      if (values[x>>1] == -1)
        values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
  } // hash-cpp-3 = 9daa11ba272442daba9b26ba87433109
  bool solve() { // hash-cpp-4
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    for (int i = 0; i < 2*N; ++i) if (!comp[i]) dfs(i);
    for (int i = 0; i < N; ++i) if (comp[2*i] == comp[2*i
        ↪+1]) return 0;
    return 1;
  } // hash-cpp-4 = 49f5aec465cba73979ba291353751689
};
```

Cycles.h
Description: Cycle Detection (Detects a cycle in a directed or undirected graph.)
Time: $\mathcal{O}(V)$
25 lines

```cpp
bool detectCycle(vector<vector<int>> &edges, bool
    ↪undirected) {
  vector<int> seen(n, 0), parent(n), stack_t;
  for (int i = 0; i < edges.size(); ++i) {
    if (seen[i] == 2) continue;
    stack_t.push_back(i);
    while(!stack_t.empty()) {
      int u = stack_t.back();
      stack_t.pop_back();
      if (seen[u] == 1) seen[u] = 2;
      else {
        stack_t.push_back(u);
        seen[u] = 1;
        for (int w : edges[u]) {
          if (seen[w] == 0) {
            parent[w] = u;
            stack_t.push_back(w);
          }
          else if (seen[w] == 1 && (!undirected
              ↪|| w != parent[u]))
            return true;
        }
      }
    }
  }
}
// hash-cpp-all = 7ff93a874ccce87f8fcc944ce4adc144
```

7.6 Heuristics

MaximalCliques.h
Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Possible optimization: on the top-most recursion level, ignore 'cands', and go through nodes in order of increasing degree, where degrees go down as nodes are removed.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs
12 lines

```cpp
typedef bitset<128> B;
template<class F>
void cliques(vector<B> &eds, F f, B P = ~B(), B X={}, B R
    ↪={}) { // hash-cpp-1
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  for(int i = 0; i < eds.size(); ++i) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
} // hash-cpp-1 = 1dc1acd20ad3a69c17c07ce840d575ca
```

MaximumClique.h
Description: Finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.
49 lines

```cpp
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit = 0.025, pk = 0;
  struct Vertex { int i, d = 0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
```

```cpp
  vector<vector<int>> C;
  vector<int> qmax, q, S, old;
  void init(vv& r) {
    for(auto& v : r) v.d = 0;
    for(auto& v : r) for(auto& j : r) v.d += e[v.i][j.i];
    sort(r.begin(), r.end(), [](auto a, auto b) { return a.
      ↪d > b.d; });
    int mxD = r[0].d;
    for(int i = 0; i < r.size(); ++i) r[i].d = min(i, mxD)
      ↪+ 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (R.size()) {
      if (q.size() + R.back().d <= qmax.size()) return;
      q.push_back(R.back().i);
      vv T;
      for(auto& v : R) if (e[R.back().i][v.i]) T.push_back
        ↪({v.i});
      if (T.size()) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(qmax.size() - q.size
          ↪() + 1, 1);
        C[1].clear(), C[2].clear();
        for(auto& v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(C[k].begin(), C[k].end(), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        for(int k = mnk; k <= mxk; ++k) for(auto& i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (q.size() > qmax.size()) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vector<int> maxClique() { init(V), expand(V); return qmax
    ↪; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(C.size()),
    ↪old(S) {
    for(int i = 0; i < e.size(); ++i) V.push_back({i});
  }
}; // hash-cpp-all = 0fb921df39bfda2151477954b30fd256
```

Cycle-Counting.cpp
Description: Counts 3 and 4 cycles

<bits/stdc++.h>   62 lines
```cpp
#define P 1000000007
#define N 110000

int n, m;
vector <int> go[N], lk[N];

int w[N];
int circle3(){ // hash-cpp-1
  int ans=0;
  for (int i = 1; i <= n; i++)
    w[i]=0;

  for (int x = 1; x <= n; x++) {
    for(int y:lk[x])w[y]=1;
```

```cpp
    for(int y:lk[x])for(int z:lk[y])if(w[z]){
      ans=(ans+go[x].size()+go[y].size()+go[z].size()-6)%P;
    }

    for(int y:lk[x])w[y]=0;
  }
  return ans;
} // hash-cpp-1 = 719dcec935e20551fd984c12c3bfa3ba

int deg[N], pos[N], id[N];

int circle4(){ // hash-cpp-2
  for (int i = 1; i <= n; i++)
    w[i]=0;
  int ans=0;
  for (int x = 1; x <= n; x++) {
    for(int y:go[x])for(int z:lk[y])if(pos[z]>pos[x]){
      ans=(ans+w[z])%P;
      w[z]++;
    }
    for(int y:go[x])for(int z:lk[y])w[z]=0;
  }
  return ans;
} // hash-cpp-2 = 39b3aaf47e9fdc4dfff3fdfdf22d3a8e

inline bool cmp(const int &x,const int &y){
  return deg[x]<deg[y];
}

void init() {
  scanf("%d%d", &n, &m);
  for (int i = 1; i <= n; i++)
    deg[i] = 0, go[i].clear(), lk[i].clear();;
  while (m--) {
    int a,b;
    scanf("%d%d",&a,&b);
    deg[a]++;deg[b]++;
    go[a].push_back(b);go[b].push_back(a);
  }
  for (int i = 1; i <= n; i++)
    id[i] = i;
  sort(id+1,id+1+n,cmp);
  for (int i = 1; i <= n; i++) pos[id[i]]=i;
  for (int x = 1; x <= n; x++)
    for(int y:go[x])
      if(pos[y]>pos[x])lk[x].push_back(y);
}
```

7.7   Trees

Tree.h
Description: Structure that handles tree's, can find its diameter points, diameter length, center vertices, etc;
                                                                   42 lines
```cpp
struct tree_t {
    int n;
    vector<vector<int>> edges;
    vector<int> parent, dist;
    pair<int, int> center, diameter;
    tree_t(vector<vector<int>> g) : n(g.size()), parent(n),
      ↪ dist(n) {
        edges = g;
        diameter = {1, 1};
    }
    void dfs(int v, int p) {
        for (int u : edges[v]) {
            if (u == p) continue;
            parent[u] = v;
```

```cpp
            dist[u] = dist[v] + 1;
            dfs(u, v);
        }
    }
    pair<int,int> find_diameter() { // diameter start->
      ↪finish point
        parent[0] = -1;
        dist[0] = 0;
        dfs(0, 0);
        for (int i = 0; i < n; ++i)
            if (dist[i] > dist[diameter.first]) diameter.
              ↪first = i;
        parent[diameter.first] = -1;
        dist[diameter.first] = 0;
        dfs(diameter.first, diameter.first);
        for (int i = 0; i < n; ++i)
            if (dist[i] > dist[diameter.second]) diameter.
              ↪second = i;
        return diameter;
    }
    int get_diameter() { // length of diameter
        diameter = find_diameter();
        return dist[diameter.second];
    }
    pair<int,int> find_center() {
        diameter = find_diameter();
        int k = diameter.second, length = dist[diameter.
          ↪second];
        for (int i = 0; i < length/2; ++i) k = parent[k];
        if (length%2) return center = {k, parent[k]}; //
          ↪two centers
        else return center = {k, -1}; // k is the only
          ↪center of the tree
    }
}; // hash-cpp-all = efc11e16a1306de29644c4ce6907baba
```

LCA.cpp
Description: Solve lowest common ancestor queries using binary jumps. Can also find the distance between two nodes.
Time: $\mathcal{O}(N + Q)$
                                                                   64 lines
```cpp
struct lca_t {
    int logn, preorderpos;
    vector<int> invpreorder, height;
    vector<vector<int>> edges, jump_binary;
    lca_t(int n, vector<vector<int>>& adj) : edges(adj),
        height(n), invpreorder(n) {
        while((1 << (logn+1)) <= n) ++logn;
        jump_binary.assign(n, vector<int>(n+1, 0));
        dfs(0, -1, 0);
    }
    void dfs(int v, int p, int h) {
        invpreorder[v] = preorderpos++;
        height[v] = h;
        jump_binary[v][0] = (p == -1) ? v : p;
        for (int l = 1; l <= logn; ++l)
            jump_binary[v][l] = jump_binary[jump_binary[v][
              ↪l-1]][l-1];
        for (int u : edges[v]) {
            if (u == p) continue;
            dfs(u, v, h+1);
        }
    }
    int climb(int v, int dist) {
        for (int l = 0; l <= logn; ++l)
            if (dist & (1 << l)) v = jump_binary[v][l];
        return v;
    }
```

```cpp
    }
    int get_kth_ancestor(int v, int k) {
        for (int i = 0; 1 << i <= k; ++i)
            if (k >> i & 1) {
                v = jump_binary[v][i];
                if (v < 0) break;
            }
        return v;
    }
    int query(int a, int b) {
        if (height[a] < height[b]) swap(a, b);
        a = climb(a, height[a] - height[b]);
        if (a == b) return a;
        for (int l = logn; l >= 0; --l)
            if (jump_binary[a][l] != jump_binary[b][l]) {
                a = jump_binary[a][l];
                b = jump_binary[b][l];
            }
        return jump_binary[a][0];
    }
    int dist(int a, int b) {
        return height[a] + height[b] - 2 * height[query(a,b
            ↪)];
    }
    bool is_parent(int p, int v) {
        if (height[p] > height[v]) return false;
        return p == climb(v, height[v] - height[p]);
    }
    bool on_path(int x, int a, int b) {
        int v = query(a, b);
        return is_parent(v, x) && (is_parent(x, a) ||
            ↪is_parent(x, b));
    }
    int get_kth_on_path(int a, int b, int k) {
        int v = query(a, b);
        int x = height[a] - height[v];
        int y = height[b] - height[v];
        if (k < x) return get_kth_ancestor(a, k);
        else return get_kth_ancestor(b, x + y - k);
    }
}; // hash-cpp-all = 0abf3037d10eee2c5f81489f43769204
```

LCA.h
Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected. Can also find the distance between two nodes.
Usage: lca_t lca(undirGraph);
lca.query(firstNode, secondNode);
lca.dist(firstNode, secondNode);
Time: $\mathcal{O}(N \log N + Q)$

46 lines

```cpp
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) {
    int N = V.size(), on = 1, depth = 1;
    while (on < N) on *= 2, depth++;
    jmp.assign(depth, V);
    for(int i = 0; i < depth-1; ++i) for(int j = 0; j < N;
        ↪++j)
      jmp[i+1][j] = min(jmp[i][j],
        jmp[i][min(N - 1, j + (1 << i))]);
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
```

```cpp
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};

struct lca_t {
  int n;
  vector<int> depth, order;
  vector<vector<int>> edges;
  vector<pair<int,int>> temp;
  RMQ<pair<int,int>> rmq;
  lca_t(vector<vector<int>>& g) : n(g.size()),
    edges(g), depth(n), order(n), rmq(dfs(0,-1)) {}
  vector<pair<int,int>> dfs(int v, int p) {
    order[v] = temp.size();
    depth[v] = 1 + depth[p];
    temp.push_back({depth[v], v});
    for (int u : edges[v]) {
      if (u == p) continue;
      dfs(u, v);
      temp.push_back({depth[v], v});
    }
    return temp;
  }
  int query(int a, int b) {
    a = order[a]; b = order[b];
    if (a > b) swap(a, b);
    return rmq.query(a, b).second;
  }
  int dist(int a, int b) {
    return depth[a] + depth[b] - 2*depth[query(a, b)];
  }
}; // hash-cpp-all = 4897fe0ab4353cd05392511138d3759f
```

CompressTree.h
Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h"   20 lines

```cpp
vector<pair<int,int>> compressTree(lca_t &lca, const vector
    ↪<int>& subset) {
  static vector<int> rev; rev.resize(lca.height.size());
  vector<int> li = subset, &T = lca.invpreorder;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(li.begin(), li.end(), cmp);
  int m = li.size()-1;
  for (int i = 0; i < m; ++i) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.query(a, b));
  }
  sort(li.begin(), li.end(), cmp);
  li.erase(unique(li.begin(), li.end()), li.end());
  for (int i = 0; i < li.size(); ++i) rev[li[i]] = i;
  vector<pair<int,int>> ret = {0, li[0]};
  for (int i = 0; i < li.size()-1; ++i) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.query(a, b)], b);
  }
  return ret;
} // hash-cpp-all = 4f28d7f851dd0cb96e0b9e9538bcc079
```

Tree-Isomorphism.h
Time: $\mathcal{O}(N \log(N))$

52 lines

```cpp
map<vector<int>, int> delta;
```

```cpp
struct tree_t {
  int n;
  pair<int,int> centroid;
  vector<vector<int>> edges;
  vector<int> sz;
  tree_t(vector<vector<int>>& graph) :
    edges(graph), sz(edges.size()) {}
  int dfs_sz(int v, int p) {
    sz[v] = 1;
    for (int u : edges[v]) {
      if (u == p) continue;
      sz[v] += dfs_sz(u, v);
    }
    return sz[v];
  }
  int dfs(int tsz, int v, int p) {
    for (int u : edges[v]) {
      if (u == p) continue;
      if (2*sz[u] <= tsz) continue;
      return dfs(tsz, u, v);
    }
    return centroid.first = v;
  }
  pair<int,int> find_centroid(int v) {
    int tsz = dfs_sz(v, -1);
    centroid.second = dfs(tsz, v, -1);
    for (int u : edges[centroid.first]) {
      if (2*sz[u] == tsz)
        centroid.second = u;
    }
    return centroid;
  }
  int hash_it(int v, int p) {
    vector<int> offset;
    for (int u : edges[v]) {
      if (u == p) continue;
      offset.push_back(hash_it(u, v));
    }
    sort(offset.begin(), offset.end());
    if (!delta.count(offset))
      delta[offset] = int(delta.size());
    return delta[offset];
  }
  lint get_hash(int v = 0) {
    pair<int,int> cent = find_centroid(v);
    lint x = hash_it(cent.first, -1), y = hash_it(cent.
        ↪second, -1);
    if (x > y) swap(x, y);
    return (x << 30) + y;
  }
}; // hash-cpp-all = 92e59fd174d98fae157272b14c6b43ee
```

LineTree.h
Description: Performs a preprocessing to enable querying the maximum/minimum edge weight on any path in a tree in constant time.
Time: $\mathcal{O}(n \log(n))$

<RMQ.h>   75 lines

```cpp
struct UF {
    vector<int> parent, size, left, right;
    UF(int n) : parent(n), size(n, 1), left(n), right(n) {
        for (int i = 0; i < n; i++)
            parent[i] = left[i] = right[i] = i;
    }
    int find(int x) {
        return x == parent[x] ? x : parent[x] = find(parent
            ↪[x]);
    }
```

```cpp
    pair<int, int> unite(int x, int y) {
        x = find(x);
        y = find(y);
        assert(x != y);
        if (size[x] < size[y]) swap(x, y);
        parent[y] = x;
        size[x] += size[y];
        pair<int,int> result = {right[x], left[y]};
        right[x] = right[y];
        return result;
    }
};

template<typename T>
struct linetree_t {
  struct edge_t {
    int u, v; T w;
    edge_t() {}
    edge_t(int a, int b, T c) : u(a), v(b), w(c) {}
    bool operator<(const edge_t &other) const {
      return w < other.w;
    }
  };
  int n;
  const T limit = numeric_limits<T>::min();
  vector<int> index, line;
  vector<edge_t> edges; vector<T> line_w;
  unique_ptr<RMQ<T>> rmq;
  linetree_t(int _n) : n(_n), index(n) {}
  void addEdge(int from, int to, T weight) {
    edges.emplace_back(from, to, weight);
  }
  void make_tree() {
    sort(edges.begin(), edges.end());
    UF dsu(n);
    vector<int> next_v(n, -1), has_prev(n);
    vector<T> next_w(n, limit);
    for (edge_t& e : edges) {
      pair<int, int> united = dsu.unite(e.u, e.v);
      next_v[united.first] = united.second;
      has_prev[united.second] = 1;
      next_w[united.first] = e.w;
    }
    int start = -1;
    for (int i = 0; i < n; ++i)
      if (!has_prev[i]) {
        start = i;
        break;
      }
    while(start >= 0) {
      line.push_back(start);
      if (next_v[start] >= 0)
        line_w.push_back(next_w[start]);
      start = next_v[start];
    }
    for (int i = 0; i < n; ++i)
      index[line[i]] = i;
    rmq.reset(new RMQ<T>(line_w));
  }
  T query(int a, int b) {
    if (a == b) return limit;
    a = index[a], b = index[b];
    if (a > b) swap(a, b);
    return rmq->query(a-1, b-1).first;
  }
}; // hash-cpp-all = 96ccfd04e4ec32ca1a67d9f1044fbe61
```

7.8 Functional Graphs

Lumberjack.h
Description: Called lumberjack technique, solve functional graphs problems for digraphs, it's also pretty good for dp on trees. Consists in go cutting the leaves until there is no leaves, only cycles. For that we keep a processing queue of the leaves, note that during this processing time we go through all the childrens of v before reaching a vertex v, therefore we can compute some infos about the children, like subtree of a given vertex
<div align="right">53 lines</div>

```cpp
vector<int> deg, subtree, order, par, parincycles, idxcycle
    ↪, size, st, depth, cycles[MAXN];
vector<bool> seen, incycle,
int numcycle;

void bfs() {
    queue<int> q;
    for (int i = 0; i < n; ++i)
      if (!indeg[i]) {
        q.push(i);
        seen[i] = true;
      }
    while(!q.empty()) {
      int v = q.front(); q.pop();
      order.push_back(v);
      ++subtree[v];
      int curpar = par[v];
      indeg[curpar]--;
      subtree[curpar] += subtree[v];
      if (!indeg[curpar]) {
        q.push(curpar);
        seen[curpar] = true;
      }
    }
    numcycles = 0;
    for (int i = 0; i < n; ++i)
      if (!seen[i]) find_cycle(i);
    for (int i = order.size()-1; i >= 0; --i) {
      int v = order[i], curpar = par[v];
      parincycle[v] = parincycle[curpar];
      cycle[v] = cycle[curpar];
      incycle[v] = false;
      idxcycle[v] = -1;
      depth[v] = 1 + depth[curpar];
    }
}
void find_cycle(int u) {
    int idx = ++numcycle, cur = 0, par = u;
    st[idx] = u;
    size[idx] = 0;
    cycles[idx].clear();
    while (!seen[u]) {
      seen[u] = incycle[u] = true;
      parincycle[u] = u;
      cycle[u] = idx;
      idxcycle[u] = cur;
      cycles[idx].push_back(u);
      ++size[idx];
      depth[u] = 0;
      ++subtree[u];
      u = par[u];
      ++cur;
    }
} // hash-cpp-all = b06e5ba4ca64514fa44ec522f5d2c650
```

Lumberjack2.h
Description: Called lumberjack technique, solve functional graphs problems for graphs, it's also pretty good for dp on trees. Consists in go cutting the leaves until there is no leaves, only cycles. For that we keep a processing queue of the leaves, note that during this processing time we go through all the childrens of v before reaching a vertex v, therefore we can compute some infos about the children, like subtree of a given vertex
<div align="right">60 lines</div>

```cpp
vector<int> deg, subtree, order, par, parincycles, idxcycle
    ↪, sz, st, depth, cycles[MAXN];
vector<bool> seen, incycle,

void bfs() {
    queue<int> q;
    for (int i = 0; i < n; ++i)
      if (deg[i] == 1) {
        q.push(i);
        seen[i] = true;
      }
    while(!q.empty()) {
      int v = q.front(); q.pop();
      order.push_back(v);
      ++subtree[v];
      int curpar = find_par(v);
      par[v] = curpar;
      deg[curpar]--;
      subtree[curpar] += subtree[v];
      if (deg[curpar] == 1) {
        q.push(curpar);
        seen[curpar] = true;
      }
    }
    numcycles = 0;
    for (int i = 0; i < n; ++i)
      if (!seen[i]) find_cycle(i);
    for (int i = order.size()-1; i >= 0; --i) {
      int v = order[i], curpar = par[v];
      parincycle[v] = parincycle[curpar];
      cycle[v] = cycle[curpar];
      incycle[v] = false;
      idxcycle[v] = -1;
      depth[v] = 1 + depth[curpar];
    }
}
void find_cycle(int u) {
    int idx = ++numcycle, cur = 0, par = u;
    st[idx] = u;
    sz[idx] = 0;
    cycles[idx].clear();
    while (!seen[u]) {
      seen[u] = incycle[u] = true;
      par[u] = find_par(u);
      if (par[u] == -1) par[u] = par;
      parincycle[u] = u;
      cycle[u] = idx;
      idxcycle[u] = cur;
      cycles[idx].push_back(u);
      ++sz[idx];
      depth[u] = 0;
      ++subtree[u];
      u = par[u];
      ++cur;
    }
}
int find_par(int u) {
    for (int v : graph[u]) if (!seen[v])
      return v;
```

```
    return -1;
} // hash-cpp-all = 4d3018be44435c8594f1cc1131202c81
```

7.9   Other

kthShortestPath.h
Description: Find Kth shortest path from s to t.
Time: $\mathcal{O}\left((V+E)lg(V)*k\right)$
                                                                    21 lines
```cpp
int getCost(vector<vector<pair<int,int>>> &G, int s, int t,
    ↪ int k) {
  int n = G.size();
  vector<int> dist(n, INF), count(n, 0);
  priority_queue<pair<int,int>, vector<pair<int,int>>,
      ↪greater<pair<int,int>>> Q;
 Q.push({0, s});
 while (!Q.empty() && (count[t] < k)) {
   pair<int,int> v = Q.top();
   int u = v.second, w = v.first;
   Q.pop();
   if ((dist[u] == INF) || (w > dist[u])) { // remove
       ↪equal paths
     count[u] += 1;
     dist[u] = w;
   }
   if (count[u] <= k)
     for (int x : G[u]) {
     int v = x.first, w = x.second;
     Q.push({dist[u] + w, v});
     }
 }
  return dist[t];
} // hash-cpp-all = b611794901cec100dd9015bce082d108
```

MatrixTreeMST.h
Description: Returns the number of msts in undirected weighted graph
using the Matrix Tree theorem.
Time: $\mathcal{O}\left(N^3\right)$
                                                                    89 lines
```cpp
lint det(vector<vector<lint>> a, int n, int p) {
  lint ans = 1;
  for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) a
      ↪[i][j] %= p;
  for(int i = 1; i < n; ++i) {
    for(int j = i+1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        lint t = a[i][i] / a[j][i];
        if (t) for(int k = i; k < n; ++k) {
          a[i][k] = (a[i][k] - a[j][k] * t) % p;
          a[i][k] %= p;
        }
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % p;
    if (!ans) return 0;
  }
  return (ans + p) % p;
}
```

```cpp
struct edge_t {
  int u, v, w;
  bool operator<(const edge_t& o) const {
    return w < o.w;
  }
};
const int N = 101;
```

```cpp
int edgenum = 0;
vector<edge_t> edge;
vector<bool> seen;
vector<int> g[N];
vector<vector<lint>> p, deg;

void addEdge(int u, int v, int d){
  edge_t E = { u, v, d };
  edge[++edgenum] = E;
}

lint MST_count(int n, lint MOD) {
  sort(edge.begin()+1, edge.begin()+edgenum+1);
  int pre = edge[1].w;
  lint ans = 1;
  UF a(n+1), b(n+1);
  seen = vector<bool>(n+1, false);
  deg = vector<vector<lint>>(n+1, vector<lint>(n+1));
  for (int i = 0; i <= n; i++) g[i].clear();
  for (int t = 1; t <= edgenum+1; ++t) {
    if (edge[t].w != pre || t == edgenum + 1) {
      for (int i = 1, k; i <= n; i++) if (seen[i]) {
        k = b.find(i);
        g[k].push_back(i);
        seen[i] = false;
      }
      for (int i = 1; i <= n; ++i)
        if (g[i].size()) {
          p = vector<vector<lint>>(n+1, vector<lint>(n+1));
          for (int j = 0; j < g[i].size(); j++)
          for (int k = j+1, x, y; k < g[i].size(); ++k) {
            x = g[i][j];
            y = g[i][k];
            p[j][k] = p[k][j] = -deg[x][y];
            p[j][j] += deg[x][y];
            p[k][k] += deg[x][y];
          }
          ans = ans*det(p, g[i].size(), MOD) % MOD;
          for (int j = 0; j < g[i].size(); ++j) a.par[g[i][
              ↪j]] = i;
        }
      deg = vector<vector<lint>>(n+1, vector<lint>(n+1));
      for (int i = 1; i <= n; ++i) {
        b.par[i] = a.find(i);
        g[i].clear();
      }
      if (t == edgenum+1) break;
      pre = edge[t].w;
    }
    int x = a.find(edge[t].u);
    int y = a.find(edge[t].v);
    if (x == y) continue;
    seen[x] = seen[y] = true;
    b.unite(x, y);
    deg[x][y]++; deg[y][x]++;
  }
  if (!edgenum) return 0;
  for (int i = 2; i <= n; i++)
    if (b.find(i) != b.find(1))
      return 0;
  return ans;
} // hash-cpp-all = d2ed36c54cf26fb9fdf6b7bf5009816c
```

Boruvka.h
                                                                    31 lines
```cpp
struct edge_t {
  int u, v, w, id;
```

```cpp
  edge_t() {};
  edge_t(int u, int v, int w = 0, int id = 0) : u(u), v(v),
      ↪ w(w), id(id) {};
  bool operator<(edge_t &o) const { return w < other.w; };
};

vector<edge_t> Boruvka(vector<edge_t> &edges, int n) {
  vector<edge_t> mst, best(n);
  UF dsu(n);
  int f = 1;
  while (f) {
    f = 0;
    for (int i = 0; i < n; ++i) best[i] = edge_t(i, i, INF)
        ↪;
    for (edge_t e : edges) {
      int pu = dsu.find(e.u), pv = dsu.find(e.v);
      if (pu == pv) continue;
      if (e < best[pu]) best[pu] = e;
      if (e < best[pv]) best[pv] = e;
    }
    for (int i = 0; i < n; ++i) {
      edge_t e = best[dsu.find(i)];
      if (e.w != INF) {
        dsu.unite(e.u, e.v);
        mst.push_back(e);
        f = 1;
      }
    }
  }
  return mst;
} // hash-cpp-all = 4740f286007bddb541dae6f88340d41d
```

ManhattanMST.h
Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights $w(p,q) = |p.x - q.x| + |p.y - q.y|$. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}\left(NlogN\right)$
<UnionFind.h>
                                                                    28 lines
```cpp
typedef Point<int> P;
pair<vector<array<int, 3>>, int> manhattanMST(vector<P> ps)
    ↪ {
  vector<int> id(ps.size());
  iota(id.begin(), id.end(), 0);
  vector<array<int, 3>> edges;
  for(int k = 0; k < 4; ++k) {
    sort(id.begin(), id.end(), [&](int i, int j) {
        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for(auto& i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
              it != sweep.end(); sweep.erase(it
              ↪++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    if (k & 1) for(auto& p : ps) p.x = -p.x;
    else for(auto& p : ps) swap(p.x, p.y);
  }
  sort(edges.begin(), edges.end());
  UF uf(ps.size());
  int cost = 0;
```

```cpp
    for (auto e: edges) if (uf.unite(e[1], e[2])) cost += e
        ↪[0];
    return {edges, cost};
} // hash-cpp-all = de81704447870021010c8019913b976a
```

SteinerTree.h
Description: Find the cost of the smallest tree containing all elements of terminal ts for a non-negative undirected graph
Time: $\mathcal{O}\left(3^t n + 2^t n^2 + n^3\right)$

                                           25 lines

```cpp
//TODO: Check what is a terminal...

int Steiner(vector<vector<int>> &g, vector<int> &ts) {
    int n = g.size(), m = ts.size();
    if (m < 2) return 0;
    vector<vector<int>> dp(1<<m, vector<int>(n));
    for(int k = 0; k < n; ++k)
        for(int i = 0; i < n; ++i)
            for(int j = 0; j < n; ++j)
                g[i][j] = min(g[i][j], g[i][k] + g[k][j]);
    for(int i = 0; i < m; ++i)
        for(int j = 0; j < n; ++j)
            dp[1<<i][j] = g[ts[i]][j];
    for (int i = 1; i < (1<<m); ++i) if (((i-1)&i) != 0) {
        for (int j = 0; j < n; ++j) {
            dp[i][j] = INF;
            for (int k = (i-1)&i; k > 0; k = (k-1)&i)
                dp[i][j] = min(dp[i][j], dp[k][j] + dp[i^k
                    ↪][j]);
        }
        for (int j = 0; j < n; ++j)
            for (int k = 0; k < n; ++k)
                dp[i][j] = min(dp[i][j], dp[i][k] + g[k][j
                    ↪]);
    }
    return dp[(1<<m)-1][ts[0]];
} // hash-cpp-all = 3bb8ba31a1df9c80e44832d553fbf877
```

Pruefer.cpp
Description: Given a tree, construct its pruefer sequence

                                           37 lines

```cpp
struct pruefer_t {
    vector<vector<int>> adj;
    vector<int> parent;
    pruefer_t(int _n) : adj(n), parent(n) {}
    void dfs (int u) {
        for (int i = 0; i < adj[u].size(); ++i) {
            if (i != parent[u]) {
                parent[i] = v;
                dfs(i);
            }
        }
    }
    vector<int> pruefer() {
        int n = adj.size();
        parent.resize(n);
        parent[n-1] = -1;
        dfs(n-1);
        int one_leaf = -1;
        vector<int> degree(n), ret(n-2);
        for (int i = 0; i < n; ++i) {
            degree[i] = adj[i].size();
            if (degree[i] == 1 && one_leaf == -1) one_leaf
                ↪= i;
        }
        int leaf = one_leaf;
        for (int i = 0; i < n-2; ++i) {
```

```cpp
            int next = parent[leaf];
            ret[i] = next;
            if (--degree[next] == 1 && next < one_leaf)
                ↪leaf = next;
            else {
                ++one_leaf;
                while (degree[one_leaf] != 1) ++one_leaf;
                leaf = one_leaf;
            }
        }
        return ret;
    }
}; // hash-cpp-all = 9617131fb6492a5a9ac2ba9ace41373d
```

ErdosGallai.h
Description: Check if an array of degrees can represent a graph
Time: if sorted $\mathcal{O}(n)$, otherwise $\mathcal{O}(n\log(n))$

                                     15 lines

```cpp
bool EG(vector<int> deg) {
    sort(deg.begin(), deg.end(), greater<int>());
    vector<long long> dp(deg.size());
    int n = deg.size(), p = n-1;
    for(int i = 0; i < n; i++)
        dp[i] = deg[i] + (i > 0 ? dp[i-1] : 0);
    for(int k = 1; k <= n; k++) {
        while(p >= 0 && deg[p] < k) p--;
        long long sum;
        if (p >= k-1) sum = (p-k+1)*1ll*k + dp[n-1] - dp[p
            ↪];
        else sum = dp[n-1] - dp[k-1];
        if (dp[k-1] > k*(k-1LL) + sum) return 0;
    }
    return dp[n-1] % 2 == 0;
} // hash-cpp-all = 56391b5d1b51835fc41d647ab44f510d
```

MisraGries.h
Description: Finds a $\max_i \deg(i) + 1$-edge coloring where there all incident edges have distinct colors. Finding a D-edge coloring is NP-hard.

                                     48 lines

```cpp
struct edge { int to, color, rev; };

struct MisraGries {
    int N, K = 0;
    vector<vector<int>> F;
    vector<vector<edge>> graph;
    MisraGries(int n) : N(n), graph(n) {}
    // add an undirected edge, NO DUPLICATES ALLOWED
    void addEdge(int u, int v) {
        graph[u].push_back({v, -1, (int) graph[v].size()});
        graph[v].push_back({u, -1, (int) graph[u].size()-1});
    }
    void color(int v, int i) {
        vector<int> fan = { i };
        vector<bool> used(graph[v].size());
        used[i] = true;
        for (int j = 0; j < (int) graph[v].size(); j++)
            if (!used[j] && graph[v][j].col >= 0 && F[graph[v][
                ↪fan.back()].to][graph[v][j].col] < 0)
                used[j] = true, fan.push_back(j), j = -1;
        int c = 0; while (F[v][c] >= 0) c++;
        int d = 0; while (F[graph[v][fan.back()].to][d] >= 0) d
            ↪++;
        int w = v, a = d, k = 0, ccol;
        while (true) {
            swap(F[w][c], F[w][d]);
            if (F[w][c] >= 0) graph[w][F[w][c]].col = c;
            if (F[w][d] >= 0) graph[w][F[w][d]].col = d;
```

```cpp
            if (F[w][a^=c^d] < 0) break;
            w = graph[w][F[w][a]].to;
        }
        do {
            Edge &e = graph[v][fan[k]];
            ccol = F[e.to][d] < 0 ? d : graph[v][fan[k+1]].col;
            if (e.col >= 0) F[e.to][e.col] = -1;
            F[e.to][ccol] = e.rev;
            F[v][ccol] = fan[k];
            e.col = graph[e.to][e.rev].col = ccol;
            k++;
        } while (ccol != d);
    }
    // finds a K-edge-coloringgraph
    void color() {
        for(int v = 0; v < N; ++v)
            K = max(K, (int)graph[v].size() + 1);
        F = vector<vector<int>>(N, vector<int>(K, -1));
        for(int v = 0; v < N; ++v) for (int i = graph[v].size()
            ↪; i--; )
            if (graph[v][i].col < 0) color(v, i);
    }
}; // hash-cpp-all = b27b0c0eeabb94e7f648f63f003a6867
```

Directed-MST.cpp
Description: Edmonds' algorithm for finding the weight of the minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFind.h"   48 lines

```cpp
struct edge_t { int a, b; lint w; };
struct Node {
    edge_t key;
    Node *l, *r;
    lint delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    edge_t top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

lint dmst(int n, int r, vector<edge_t>& g) {
    UF uf(n);
    vector<Node*> heap(n);
    for(auto &e : g) heap[e.b] = merge(heap[e.b], new Node{
        ↪e});
    lint res = 0;
    vector<int> seen(n, -1), path(n);
    seen[r] = r;
    for(int s = 0; s < n; ++s) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            path[qi++] = u, seen[u] = s;
            if (!heap[u]) return -1;
            edge_t e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
```

```cpp
        res += e.w, u = uf.find(e.a);
        if (seen[u] == s) {
            Node* cyc = 0;
            do cyc = merge(cyc, heap[w = path[--qi]]);
            while (uf.unite(u, w));
            u = uf.find(u);
            heap[u] = cyc, seen[u] = -1;
        }
    }
}
return res;
} // hash-cpp-all = e458358149fc17fa01ff22451b250c4b
```

Graph-Dominator-Tree.cpp
Description: Dominator Tree.

107 lines

```cpp
#define N 110000 //max number of vertices

vector<int> succ[N], prod[N], bucket[N], dom_t[N];
int semi[N], anc[N], idom[N], best[N], fa[N], tmp_idom[N];
int dfn[N], redfn[N];
int child[N], size[N];
int timestamp;

void dfs(int now) { // hash-cpp-1
  dfn[now] = ++timestamp;
  redfn[timestamp] = now;
  anc[timestamp] = idom[timestamp] = child[timestamp] =
    ↪size[timestamp] = 0;
  semi[timestamp] = best[timestamp] = timestamp;
  int sz = succ[now].size();
  for(int i = 0; i < sz; ++i) {
    if(dfn[succ[now][i]] == -1) {
      dfs(succ[now][i]);
      fa[dfn[succ[now][i]]] = dfn[now];
    }
    prod[dfn[succ[now][i]]].push_back(dfn[now]);
  }
} // hash-cpp-1 = 6412bfd6a0d21b66ddaa51ea79cbe7bd

void compress(int now) { // hash-cpp-2
  if(anc[anc[now]] != 0) {
    compress(anc[now]);
    if(semi[best[now]] > semi[best[anc[now]]])
      best[now] = best[anc[now]];
    anc[now] = anc[anc[now]];
  }
} // hash-cpp-2 = 1c9444eb3f768b7af8741fafbf3afb5a

inline int eval(int now) { // hash-cpp-3
  if(anc[now] == 0)
    return now;
  else {
    compress(now);
    return semi[best[anc[now]]] >= semi[best[now]] ? best[
      ↪now]
      : best[anc[now]];
  }
} // hash-cpp-3 = 4e235f39666315b46dcd3455d5f866d1

inline void link(int v, int w) { // hash-cpp-4
  int s = w;
  while(semi[best[w]] < semi[best[child[w]]]) {
    if(size[s] + size[child[child[s]]] >= 2*size[child[s]])
      ↪ {
      anc[child[s]] = s;
      child[s] = child[child[s]];
```

```cpp
    } else {
      size[child[s]] = size[s];
      s = anc[s] = child[s];
    }
  }
  best[s] = best[w];
  size[v] += size[w];
  if(size[v] < 2*size[w])
    swap(s, child[v]);
  while(s != 0) {
    anc[s] = v;
    s = child[s];
  }
} // hash-cpp-4 = 270548fd021351ae21e97878f367b6f9

// idom[n] and other vertices that cannot be reached from n
//   ↪ will be 0
void lengauer_tarjan(int n) { // n is the root's number //
  ↪hash-cpp-5
  memset(dfn, -1, sizeof dfn);
  memset(fa, -1, sizeof fa);
  timestamp = 0;
  dfs(n);
  fa[1] = 0;
  for(int w = timestamp; w > 1; --w) {
    int sz = prod[w].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(prod[w][i]);
      if(semi[w] > semi[u])
        semi[w] = semi[u];
    }
    bucket[semi[w]].push_back(w);
    //anc[w] = fa[w]; link operation for o(mlogm) version
                      link(fa[w], w);
    if(fa[w] == 0)
      continue;
    sz = bucket[fa[w]].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(bucket[fa[w]][i]);
      if(semi[u] < fa[w])
        idom[bucket[fa[w]][i]] = u;
      else
        idom[bucket[fa[w]][i]] = fa[w];
    }
    bucket[fa[w]].clear();
  }
  for(int w = 2; w <= timestamp; ++w) {
    if(idom[w] != semi[w])
      idom[w] = idom[idom[w]];
  }
  idom[1] = 0;
  for(int i = timestamp; i > 1; --i) {
    if(fa[i] == -1)
      continue;
    dom_t[idom[i]].push_back(i);
  }
  memset(tmp_idom, 0, sizeof tmp_idom);
  for (int i = 1; i <= timestamp; i++)
    tmp_idom[redfn[i]] = redfn[idom[i]];
  memcpy(idom, tmp_idom, sizeof idom);
} // hash-cpp-5 = f49c40461d92222d8d39b28b0de66828
```

Graph-Negative-Cycle.cpp
Description: negative cycle

33 lines

```cpp
double b[N][N];
```

```cpp
double dis[N];
int vis[N], pc[N];

bool dfs(int k) {
  vis[k] += 1; pc[k] = true;
  if (vis[k] > N)
    return true;
  for (int i = 0; i < N; i++)
    if (dis[k] + b[k][i] < dis[i]) {
      dis[i] = dis[k] + b[k][i];
      if (!pc[i]) {
        if (dfs(i))
          return true;
      } else return true;
    }
  pc[k] = false;
  return false;
}

bool chk(double d) {
  for (int i = 0; i < N; i ++)
    for (int j = 0; j < N; j ++) {
      b[i][j] = -a[i][j] + d;
    }
  for (int i = 0; i < N; i++)
    vis[i] = false, dis[i] = 0, pc[i] = false;
  for (int i = 0; i < N; i++)
    if (!vis[i] && dfs(i))
      return true;
  return false;
} // hash-cpp-all = ec5cf9bc61e058959ce8649f1e707b1b
```

TransitiveClosure.h
Description: Given a directed graph adjacency matrix, computes closure, where $closure[i][j] = 1$ if there is a path from i to j in the graph. Closure is computed in $O(N^3/64)$ due to bitset. Also supports adding an edge to the graph and updating the closure accordingly in $O(N^2/64)$.
Time: $\mathcal{O}\left(N^3/64\right)$

20 lines

```cpp
template<int sz>
struct TC {
  vector<bitset<sz>> closure;
  TC(vector<vector<int>> adj) : closure(sz) {
    for(int i = 0; i < sz; ++i)
      for(int j = 0; j < sz; ++j)
        closure[i][j] = adj[i][j];
    for(int i = 0; i < sz; ++i)
      for(int j = 0; j < sz; ++j)
        if (closure[j][i])
          closure[j] |= closure[i];
  }
  void addEdge(int a, int b) {
    if (closure[a][b]) return;
    closure[a].set(b);
    closure[a] |= closure[b];
    for (int i = 0; i < sz; ++i)
      if (closure[i][a]) closure[i] |= closure[a];
  }
}; // hash-cpp-all = eb5414544d683fe95d450ad4d8e805a0
```

7.10   Theorems and Facts

7.10.1   Vizing's Thereom

Dado um grafo G, seja δ o maior grau de um vértice. Então G tem número cromático de aresta δ ou $\delta + 1$.

7.10.2 Euler's theorem
Sendo V, A e F as quantidades de vértices, arestas e faces de um grafo planar conexo, $V - A + F = 2$.

7.10.3 Hall's Marriage theorem
Dado um grafo bipartido com classes V_1 e V_2, para $S \subset V_1$ seja $N(S)$ o conjunto de todos os vértices vizinhos a algum elemento de S. Um emparelhamento de V_1 em V_2 é um conjunto de arestas disjuntas cujas extremidades estão em classes diferentes. Então existe um emparelhamento completo de V_1 em V_2 sse $|N(S)| \geq |S| \ \forall \ S \subset V_1$.

7.10.4 Dilworth's theorem
Em todo conjunto parcialmente ordenado, a quantidade máxima de elementos de uma anticadeia é igual à quatidade mínima de cadeias disjuntas que cobrem o conjunto.

7.10.5 Conjunto Independente de Peso Máximo num Grafo Bipartido
É o mesmo que a cobertura de peso mínimo. Podemos resolver criando uma rede de fluxo com arestas $(S, u, w(u))$ para $u \in L$, $(v, T, w(v))$ para $v \in R$ e (u, v, ∞) para $(u, v) \in E$. O corte mínimo de S a T é a resposta. Vértices adjacentes a uma aresta de corte estão na cobertura de vértices.

7.10.6 Fecho de Peso Máximo
Dado um digrafo G com peso nos vértices. Transforme G numa rede de fluxo, colocando o peso de cada aresta como ∞. Adicione vértices S, T. Para cada vértice v de peso w, adicione uma aresta (S, v, w) se $w \geq 0$, ou a aresta $(v, T, -w)$ se $w < 0$. A soma de todos os pesos positivos menos o corte mínimo $c(S, T)$ é a resposta. Vértices que são alcançados a partir de S estão no fecho. O fecho de peso máximo é o mesmo que o complemento do fecho de peso mínimo num grafo com as arestas invertidas.

7.10.7 Menger's theorem
Para vértices: Um grafo é k-conexo sse todo par de vértices é conectado por pelo menos k caminhos sem vértices intermediários em comum.

Para arestas: Um grafo é dito k-aresta-conexo se a retirada de menos de k arestas do grafo o mantém conexo. Então um grafo é k-aresta-conexo sse para todo par de vértices u e v, existem k caminhos que ligam u a v sem arestas em comum.

7.10.8 Erdös-Gallai theorem
Existe um grafo simples com graus $d_1 \geq d_2 \geq \ldots \geq d_n$ sse:
- $d_1 + d_2 + \ldots + d_n$ é par
- $\sum\limits_{i=1}^{k} d_i \leq k(k-1) + \sum\limits_{i=k+1}^{n} \min(d_i, k) \quad \forall 1 \leq k \leq n$.

Para construir, ligamos 1 com $2, 3, \ldots, d_1 + 1$ e seguimos recursivamente.

7.10.9 Landau
Existe um torneio com graus de saída $d_1 \leq d_2 \leq \ldots \leq d_n$ sse:
- $d_1 + d_2 + \ldots + d_n = \binom{n}{2}$

- $d_1 + d_2 + \ldots + d_k \geq \binom{k}{2} \quad \forall 1 \leq k \leq n$.

Para construir, fazemos 1 apontar para $2, 3, \ldots, d_1 + 1$ e seguimos recursivamente.

7.10.10 Tutte-Berge formula
The theorem states that the size of a maximum matching of a graph $G = (V, E)$ equals $\frac{1}{2} \min_{U \subseteq V} \left(|U| - \mathrm{odd}(G - U) + |V| \right)$, where $\mathrm{odd}(H)$ counts how many of the connected components of the graph H have an odd number of vertices.

7.10.11 Tutte's theorem
Um grafo $G = (V, A)$ tem um emparelhamento perfeito sse para todo subconjunto U de V, o subgrafo induzido por $V \setminus U$ tem no máximo $|U|$ componentes conexas com uma quantidade ímpar de vértices.

7.10.12 Turán's theorem
Nenhum grafo com n vértices que é K_{r+1}-livre pode ter mais arestas do que o grafo de $Turán$: Um grafo completo k-partido com conjuntos de tamanho mais próximo possível.

7.10.13 Dirac's theorem
Seja G um grafo com n vértices, cada um com grau pelo menos $n/2$. Então G é hamiltoniano.

7.10.14 Ore's theorem
Seja G um grafo simples de ordem $n \geq 3$ tal que

$$g(u) + g(v) \geq n$$

para todo par u, v de vértices não adjacentes, então G é hamiltoniano.

7.10.15 Eulerian Cycles
A quantidade de ciclos $Eulerianos$ num digrafo G é:

$$t_w(G) \prod_{v \in G} (\deg v - 1)!,$$

onde $t_w(G)$ é a quantidade de arborescências (árvore geradora direcionada) enraizada em w:
$t_w(G) = \det (q_{ij})_{i, j \neq w}$, with
$q_{ij} = [i = j]\mathrm{indeg}(i) - \#(i, j) \in E$.

7.10.16 Matroid Intersection theorem
Sejam $M_1 = (E, I_1)$ e $M_2 = (E, I_2)$ matróides. Então $\max\limits_{S \in I_1 \cap I_2} |S| = \min\limits_{U \subseteq E} r_1(U) + r_2(E \setminus U)$.

7.10.17 König-Egervary theorem
Em todo grafo bipartido G, a quantidade de arestas no emparelhamento máximo é maior ou igual à quantidade de vértices na cobertura mínima. Ou seja, para todo G, $\alpha(G) \geq \beta(G)$. Note que isso prova que $\alpha(G) = \beta(G)$ para grafos bipartidos.

7.10.18 Number of Spanning Trees
Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.10.19 Fatos úteis
O número de vértices de um grafo é igual a sua cobertura mínima mais a cardinalidade do conjunto independente máximo.

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

26 lines

```cpp
template <class T> int sgn(T x) { return (x > 0) - (x < 0);
↪ }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
↪ ); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
↪ ); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this)
↪ ; }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90
↪ degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the
↪ origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
}; // hash-cpp-all = 4d90b59b170ae98f49395e2d118bddd9
```

LineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.

"Point.h"   4 lines

```cpp
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
} // hash-cpp-all = f6bf6b556d99b09f42b86d28d1eaa86d
```

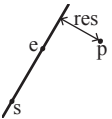### SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h"   6 lines

```cpp
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
↪ ;
```

```cpp
  return ((p-s)*d-(e-s)*t).dist()/d;
} // hash-cpp-all = 5c88f46fb14a05a4f47bbd23b8a9c427
```

SegmentClosestPoint.h

Description: Returns the closest point to p in the segment from point s to e as well as the distance between them

13 lines

```cpp
pair<P,double> SegmentClosestPoint(P &s, P &e, P &p){
  P ds=p-s, de=p-e;
  if(e==s)
    return {s, ds.dist()};
  P u=(e-s).unit();
  P proj=u*ds.dot(u);
  if(onSegment(s, e, proj+s))
    return {proj+s, (ds-proj).dist()};
  double dist_s=ds.dist(), dist_e=de.dist();
  if(cmp(dist_s, dist_e)==1)
    return {s, dist_s};
  return{e, dist_e};
} // hash-cpp-all = d4b82f64908a45c928d4451948ff0f60
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
"Point.h", "OnSegment.h"   13 lines

```cpp
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {s.begin(), s.end()};
} // hash-cpp-all = f6be1695014f7d839a498a46024031e2
```

SegmentIntersectionQ.h

Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

"Point.h"   16 lines

```cpp
template<class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
  if (e1 == s1) {
    if (e2 == s2) return e1 == e2;
    swap(s1,s2); swap(e1,e2);
  }
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = d.cross(v1), a2 = d.cross(v2)
↪ ;
  if (a == 0) { // parallel
```

```cpp
    auto b1 = s1.dot(v1), c1 = e1.dot(v1),
         b2 = s2.dot(v1), c2 = e2.dot(v1);
    return !a1 && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
  return (0 <= a1 && a1 <= a && 0 <= a2 && a2 <= a);
} // hash-cpp-all = 1ff4ba22bd0aefb04bf48cca4d6a7d8c
```

LineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
"Point.h"   8 lines

```cpp
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
} // hash-cpp-all = a01f815e2e60161e03879264c4826dd0
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point don't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h"   5 lines

```cpp
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
    P v = b - a;
    return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
} // hash-cpp-all = b5562d9ee2f720df36d24b4a7d427ea5
```

SideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: bool left = sideOf(p1,p2,q)==1;
"Point.h"   9 lines

```cpp
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
↪ {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
} // hash-cpp-all = 3af81cc4f24d9d9fb109d930f3b9764c
```

OnSegment.h
Description: Returns true iff p lies on the line segment from s to e.
Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.
`"Point.h"` 4 lines
```cpp
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
// hash-cpp-all = c597e8749250f940e4b0139f0dc3e8b9
```

LinearTransformation.h
Description:

Apply the linear transformation (translation, rotation and
scaling) which takes line p0-p1 to line q0-q1 to point r.
`"Point.h"` 6 lines
```cpp
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
      ↪dist2();
} // hash-cpp-all = 03a3061b3ef024b4e29ea06169932b21
```

Angle.h
Description: A class for ordering angles (as represented by int points
and a number of rotations around the origin). Useful for rotational
sweeping. Sometimes also represents points or vectors.
Usage: vector<Angle> v = {w[0], w[0].t360() ...}; //
sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of
positively oriented triangles with vertices at 0 and i 37 lines
```cpp
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
      ↪}; }
  int quad() const {
    assert(x || y);
    if (y < 0) return (x >= 0) + 2;
    if (y > 0) return (x <= 0);
    return (x <= 0) * 2;
  }
  Angle t90() const { return {-y, x, t + (quad() == 3)}; }
  Angle t180() const { return {-x, -y, t + (quad() >= 2)};
      ↪}
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.quad(), a.y * (ll)b.x) <
         make_tuple(b.t, b.quad(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
    ↪between
// them, i.e., the angle that covers the defined line
    ↪segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
```
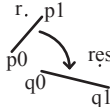
```cpp
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
      ↪)};
} // hash-cpp-all = 1856c5d371c2f8f342a22615fa92cd54
```

AngleCmp.h
Description: Useful utilities for dealing with angles of rays from origin.
OK for integers, only uses cross product. Doesn't support (0,0). 22 lines
```cpp
template <class P>
bool sameDir(P s, P t) {
  return s.cross(t) == 0 && s.dot(t) > 0;
}
// checks 180 <= s..t < 360?
template <class P>
bool isReflex(P s, P t) {
  auto c = s.cross(t);
  return c ? (c < 0) : (s.dot(t) < 0);
}
// operator < (s,t) for angles in [base,base+2pi]
template <class P>
bool angleCmp(P base, P s, P t) {
  int r = isReflex(base, s) - isReflex(base, t);
  return r ? (r < 0) : (0 < s.cross(t));
}
// is x in [s,t] taken ccw? 1/0/-1 for in/border/out
template <class P>
int angleBetween(P s, P t, P x) {
  if (sameDir(x, s) || sameDir(x, t)) return 0;
  return angleCmp(s, x, t) ? 1 : -1;
} // hash-cpp-all = 6edd25f30f9c69989bbd2115b4fdceda
```

Complex.h
Description: Exemple of geometry using complex numbers. Just to be
used as reference. std::complex has issues with integral data types, be
careful, you can't use polar or abs. 83 lines
```cpp
const double E = 1e-9;
typedef double T;
typedef complex<T> pt;
#define x real()
#define y imag()
// example of how to represent a line using complex numbers
struct line {
  pt p, v;
  line(pt a, pt b) {
    p = a;
    v = b - a;
  }
};

pt translate(pt v, pt p) {return p + v;}
//rotate point around origin by a
pt rotate(pt p, T a) { return p * polar(1.0, a); }
//around pivot
pt rotate(pt v, T a, pt pivot) { (a-pivot) * polar(1.0, a)
    ↪+ pivot; }
T dot(pt v, pt w) { return (conj(v)*w).x; }
T cross(pt v, pt w) { return (conj(v)*w).y; }
T cross(pt A, pt B, pt C) {
  return cross(B - A, C - A);
}
pt proj(pt a, pt v) {
```

```cpp
  return v * dot(a, v) / dot(v, v);
}

pt closest(pt p, line l) {
  return l.p + proj(p - l.p, l.v);
}

double dist(pt p, line l) {
  return fabs(p - closest(p, l));
}

pt proj(pt p, line l) {
  return
}

pt reflect(pt p, pt v, pt w) {
  pt z = p - v; pt q = w - v;
  return conj(z/q) * q + v;
}

pt intersection(line a, line b) { // undefined if parallel
  T d1 = cross(b.p - a.p, a.v - a.p);
  T d2 = cross(b.v - a.p, a.v - a.p);
  return (d1 * b.v - d2 * b.p)/(d1 - d2);
}

vector<pt> convex_hull(vector<pt> points) {
  if (points.size() <= 1) return points;
  sort(points.begin(), points.end(), [](pt a, pt b) {
      ↪return real(a) == real(b) ? imag(a) < imag(b) : real
      ↪(a) < real(b); });
  vector<pt> hull(points.size()+1);
  int s = 0, k = 0;
  for (int it = 2; it--; s = --k, reverse(points.begin(),
      ↪points.end())))
    for (pt p : points) {
      while(k >= s+2 && cross(hull[k-2], hull[k-1], p)
          ↪<= 0) k--;
      hull[k++] = p;
    }
  return {hull.begin(), hull.begin() + k - (k == 2 && hull
      ↪[0] == hull[1])};
}

pt p{4, 3};
// get the absolute value and angle in [-pi, pi]
cout << abs(p) << ' ' << arg(p) << '\n'; // 5 - 0.643501

// make a point in polar form
cout << polar(2.0, -M_PI/2) << '\n'; // (1.41421, -1.41421)
pt v{1, 0};
cout << rotate(v, -M_PI/2) << '\n';
// Projection of v onto Riemann sphere and norm of p
cout << proj(v) << ' ' << norm(p) << '\n';
// Distance between p and v and the squared distance
cout << abs(v-p) << ' ' << norm(v-p) << '\n';
// Angle of elevation of line vp and its slope
cout << arg(p-v) * (180/M_PI) << ' ' << tan(arg(p-v)) << '\
    ↪n';

// has trigonometric functions aswell (e.g. cos, sin, cosh,
    ↪ sinh, tan, tanh)
// and exp, pow, log
// hash-cpp-all = 2446aedc8bcd593691c082f59fae7479
```

LinearSolver.h

8.2   Circles

CircleIntersection.h

Description: Computes a pair of points at which two circles intersect. Returns false in case of no intersection.

`"Point.h"`   14 lines

```cpp
typedef Point<double> P;
bool circleIntersection(P a, P b, double r1, double r2,
    pair<P, P>* out) {
  P delta = b - a;
  assert(delta.x || delta.y || r1 != r2);
  if (!delta.x && !delta.y) return false;
  double r = r1 + r2, d2 = delta.dist2();
  double p = (d2 + r1*r1 - r2*r2) / (2.0 * d2);
  double h2 = r1*r1 - p*p*d2;
  if (d2 > r*r || h2 < 0) return false;
  P mid = a + delta*p, per = delta.perp() * sqrt(h2 / d2);
  *out = {mid + per, mid - per};
  return true;
} // hash-cpp-all = 828fbb1fff1469ed43b2284c8e07a06c
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

`"Point.h"`   13 lines

```cpp
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
} // hash-cpp-all = b0153d0ef1b8a6b1fa4d91480c4126e8
```

Circumcircle.h

Description:

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.
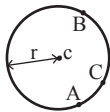
`"Point.h"`   9 lines

```cpp
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} // hash-cpp-all = 1caa3aea364671cb961900d4811f0282
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

`"circumcircle.h"`   19 lines

```cpp
pair<P, double> mec(vector<P> ps) {
  shuffle(ps.begin(),ps.end(), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  for(int i = 0; i < ps.size(); ++i)
    if ((o - ps[i]).dist() > r * EPS) {
      o = ps[i], r = 0;
      for(int j = 0; j < i; ++j) if ((o - ps[j]).dist() >
          r * EPS) {
        o = (ps[i] + ps[j]) / 2;
        r = (o - ps[i]).dist();
        for(int k = 0; k < j; ++k)
          if ((o - ps[k]).dist() > r * EPS) {
            o = ccCenter(ps[i], ps[j], ps[k]);
            r = (o - ps[i]).dist();
          }
      }
    }
  return {o, r};
} // hash-cpp-all = 8ab87fe7c0e622c4171e24dcad6bee01
```

CircleUnion.h

Description: Computes the circles union total area

101 lines

```cpp
struct CircleUnion {
    static const int maxn = 1e5 + 5;
    const double PI = acos((double) -1.0);
    int n;
    double x[maxn], y[maxn], r[maxn];
    int covered[maxn];
    vector<pair<double, double> > seg, cover;
    double arc, pol;
    inline int sign(double x) {return x < -EPS ? -1 : x >
        EPS;}
    inline int sign(double x, double y) {return sign(x - y)
        ;}
    inline double sqr(const double x) {return x * x;}
    inline double dist(double x1, double y1, double x2,
        double y2) {return sqrt(sqr(x1 - x2) + sqr(y1 - y2
        ));}
    inline double angle(double A, double B, double C) {
        double val = (sqr(A) + sqr(B) - sqr(C)) / (2 * A *
            B);
        if (val < -1) val = -1;
        if (val > +1) val = +1;
        return acos(val);
    }
    CircleUnion() {
        n = 0;
        seg.clear(), cover.clear();
        arc = pol = 0;
    }
    void init() {
        n = 0;
        seg.clear(), cover.clear();
        arc = pol = 0;
    }
    void add(double xx, double yy, double rr) {
        x[n] = xx, y[n] = yy, r[n] = rr, covered[n] = 0, n
            ++;
    }
    void getarea(int i, double lef, double rig) {
```

```cpp
        arc += 0.5 * r[i] * r[i] * (rig - lef - sin(rig -
            lef));
        double x1 = x[i] + r[i] * cos(lef), y1 = y[i] + r[i
            ] * sin(lef);
        double x2 = x[i] + r[i] * cos(rig), y2 = y[i] + r[i
            ] * sin(rig);
        pol += x1 * y2 - x2 * y1;
    }
    double calc() {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < i; j++) {
                if (!sign(x[i] - x[j]) && !sign(y[i] - y[j
                    ]) && !sign(r[i] - r[j])) {
                    r[i] = 0.0;
                    break;
                }
            }
        }
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (i != j && sign(r[j] - r[i]) >= 0 &&
                    sign(dist(x[i], y[i], x[j], y[j]) - (r
                    [j] - r[i])) <= 0) {
                    covered[i] = 1;
                    break;
                }
            }
        }
        for (int i = 0; i < n; i++) {
            if (sign(r[i]) && !covered[i]) {
                seg.clear();
                for (int j = 0; j < n; j++) {
                    if (i != j) {
                        double d = dist(x[i], y[i], x[j], y
                            [j]);
                        if (sign(d - (r[j] + r[i])) >= 0 ||
                            sign(d - abs(r[j] - r[i])) <=
                            0) {
                            continue;
                        }
                        double alpha = atan2(y[j] - y[i], x
                            [j] - x[i]);
                        double beta = angle(r[i], d, r[j]);
                        pair<double, double> tmp(alpha -
                            beta, alpha + beta);
                        if (sign(tmp.first) <= 0 && sign(
                            tmp.second) <= 0) {
                            seg.push_back(pair<double,
                                double>(2 * PI + tmp.first
                                , 2 * PI + tmp.second));
                        }
                        else if (sign(tmp.first) < 0) {
                            seg.push_back(pair<double,
                                double>(2 * PI + tmp.first
                                , 2 * PI));
                            seg.push_back(pair<double,
                                double>(0, tmp.second));
                        }
                        else {
                            seg.push_back(tmp);
                        }
                    }
                }
                sort(seg.begin(), seg.end());
                double rig = 0;
                for (vector<pair<double, double> >::
                    iterator iter = seg.begin(); iter !=
                    seg.end(); iter++) {
```

```cpp
          if (sign(rig - iter->first) >= 0) {
              rig = max(rig, iter->second);
          }
          else {
              getarea(i, rig, iter->first);
              rig = iter->second;
          }
        }
        if (!sign(rig)) {
            arc += r[i] * r[i] * PI;
        }
        else {
            getarea(i, rig, 2 * PI);
        }
      }
    }
    return pol / 2.0 + arc;
  }
} ccu;
// hash-cpp-all = 024a9290d20aec57c286f84dd8b35701
```

CircleLine.h
Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>

"Point.h", "lineDistance.h", "LineProjectionReflection.h"   8 lines
```cpp
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  double h2 = r*r - a.cross(b,c)*a.cross(b,c)/(b-a).dist2()
      ↪;
  if (h2 < 0) return {};
  P p = lineProj(a, b, c), h = (b-a).unit() * sqrt(h2);
  if (h2 == 0) return {p};
  return {p - h, p + h};
} // hash-cpp-all = debf8692dd3065ebd04e25a202df5a42
```

CircleCircleArea.h
Description: Calculates the area of the intersection of 2 circles   12 lines
```cpp
template<class P>
double circleCircleArea(P c, double cr, P d, double dr) {
    if (cr < dr) swap(c, d), swap(cr, dr);
    auto A = [&](double r, double h) {
        return r*r*acos(h/r)-h*sqrt(r*r-h*h);
    };
    auto l = (c - d).dist(), a = (l*l + cr*cr - dr*dr)/(2*l
        ↪);
    if (l - cr - dr >= 0) return 0; // far away
    if (l - cr + dr <= 0) return M_PI*dr*dr;
    if (l - cr >= 0) return A(cr, a) + A(dr, l-a);
    else return A(cr, a) + M_PI*dr*dr - A(dr, a-l);
} // hash-cpp-all = 8bf2b6afed06c7a4f47957f60986f58e
```

8.3   Polygons

InsidePolygon.h
Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h"   11 lines
```cpp
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = p.size();
```

```cpp
  for(int i = 0; i < n; ++i) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
        ↪0;
  }
  return cnt;
} // hash-cpp-all = f9442d2902bed2ba7b9bccd3adc59cf5
```

PolygonArea.h
Description: Returns the area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"   17 lines
```cpp
template<class T>
T polygonArea(vector<Point<T>> &v) {
  T a = v.back().cross(v[0]);
  for(int i = 0; i < v.size()-1; ++i)
      a += v[i].cross(v[i+1]);
  return abs(a)/2.0;
}

Point<T> polygonCentroid(vector<Point<T>> &v) { // not
    ↪tested
  Point<T> cent(0,0); T area = 0;
  for(int i = 0; i < v.size(); ++i) {
    int j = (i+1) % (v.size()); T a = cross(v[i], v[j]);
    cent += a * (v[i] + v[j]);
    area += a;
  }
  return cent/area/(T)3;
} // hash-cpp-all = 3794ee519cca1fca6c95078be8322d3a
```

PolygonCenter.h
Description: Returns the center of mass for a polygon.

"Point.h"   10 lines
```cpp
typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
  auto i = v.begin(), end = v.end(), j = end-1;
  Point<double> res{0,0}; double A = 0;
  for (; i != end; j=i++) {
    res = res + (*i + *j) * j->cross(*i);
    A += j->cross(*i);
  }
  return res / A / 3;
} // hash-cpp-all = d210bd2372832f7d074894d904e548ab
```

PolygonCut.h
Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h"   15 lines
```cpp
typedef Point<double> P;
vector<P> polygonCut(const vector<P> &poly, P s, P e) {
  vector<P> res;
  for(int i = 0; i < poly.size(); ++i) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0)) {
      res.emplace_back();
      lineIntersection(s, e, cur, prev, res.back());
    }
    if (side)
```
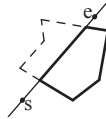
```cpp
      res.push_back(cur);
  }
  return res;
} // hash-cpp-all = 9494eaafe7195a30491957f5e29de37c
```

ConvexHull.h
Description:
Returns a vector of indices of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hulint.
Time: $\mathcal{O}(n \log n)$

"Point.h"   14 lines
```cpp
typedef Point<lint> P;
vector<P> convexHull(vector<P> pts) {
  if (pts.size() <= 1) return pts;
  sort(pts.begin(), pts.end());
  vector<P> h(pts.size()+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(pts.begin(), pts.
      ↪end()))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
          ↪--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h
      ↪[1])};
}
// hash-cpp-all = 55aed7c79f560700b3854be29895eeac
```

HullDiameter.h
Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/colinear points).   12 lines
```cpp
typedef Point<lint> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = S.size(), j = n < 2 ? 0 : 1;
  pair<lint, array<P, 2>> res({0, {S[0], S[0]}});
  for(int i = 0; i < j; ++i)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
          ↪;
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
          ↪ 0)
        break;
    }
  return res.second;
} // hash-cpp-all = 5d3363d31e941a4a0356469882ea89e1
```

PointInsideHull.h
Description: Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"   14 lines
```cpp
typedef Point<lint> P;

bool inHull(const vector<P> &l, P p, bool strict = true) {
  int a = 1, b = l.size() - 1, r = !strict;
  if (l.size() < 3) return r && onSegment(l[0], l.back(), p
      ↪);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
      ↪ -r)
    return false;
  while (abs(a - b) > 1) {
```

```cpp
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
} // hash-cpp-all = 13f9135bdca0b3cc782ea80b806ee99e
```

PolyUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. Guaranteed to be precise for integer coordinates up to 3e7. If epsilons are needed, add them in sideOf as well as the definition of sgn.
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

```
"Point.h", "sideOf.h"                                        34 lines
```
```cpp
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y;
 ↪ }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  for(int i = 0; i < poly.size(); ++i)
    for(int v = 0; v < poly[i].size(); ++v) {
      P A = poly[i][v], B = poly[i][(v + 1) % poly[i].size
       ↪ ()];
      vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
      for(int j = 0; j < poly.size(); ++j) if (i != j) {
        for(int u = 0; u < poly[j]; ++u) {
          P C = poly[j][u], D = poly[j][(u + 1) % poly[j].
           ↪ size()];
          int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
          if (sc != sd) {
            double sa = C.cross(D, A), sb = C.cross(D, B);
            if (min(sc, sd) < 0)
              segs.emplace_back(sa / (sa - sb), sgn(sc - sd
               ↪ ));
          } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C
           ↪ ))>0){
            segs.emplace_back(rat(C - A, B - A), 1);
            segs.emplace_back(rat(D - A, B - A), -1);
          }
        }
      }
      sort(segs.begin(), segs.end());
      for(auto& s : segs) s.first = min(max(s.first, 0.0),
       ↪ 1.0);
      double sum = 0;
      int cnt = segs[0].second;
      for(int j = 1; j < segs.size(); ++j) {
        if (!cnt) sum += segs[j].first - segs[j - 1].first;
        cnt += segs[j].second;
      }
      ret += A.cross(B) * sum;
    }
  return ret / 2;
} // hash-cpp-all = 7792a4559206ac7061afe751d69dcc24
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: ● $(-1,-1)$ if no collision, ● $(i,-1)$ if touching the corner i, ● (i,i) if along side $(i,i+1)$, ● (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}\left(N + Q\log n\right)$

```
"Point.h"                                                    39 lines
```
```cpp
typedef array<P, 2> Line;
```

```cpp
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%
 ↪ n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) { // hash-cpp-1
  int n = poly.size(), left = 0, right = n;
  if (extr(0)) return 0;
  while (left + 1 < right) {
    int m = (left + right) / 2;
    if (extr(m)) return m;
    int ls = cmp(left + 1, left), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(left, m)) ? right :
     ↪ left) = m;
  }
  return left;
} // hash-cpp-1 = 99da02a2645a6c072258fcdaf6294dc3

#define cmpL(i) sgn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) { // hash
 ↪ -cpp-2
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  for(int i = 0; i < 2; ++i) {
    int left = endB, right = endA, n = poly.size();
    while ((left + 1) % n != right) {
      int m = ((left + right + (left < right ? 0 : n)) / 2)
       ↪ % n;
      (cmpL(m) == cmpL(endB) ? left : right) = m;
    }
    res[i] = (left + !cmpL(right)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % poly.size())
     ↪ {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
} // hash-cpp-2 = 3e0265a348f4f3ff92f451fd599a582b
```

HalfPlane.h

Description: Halfplane intersection area

```
"Point.h", "lineIntersection.h"                              61 lines
```
```cpp
#define eps 1e-8
typedef Point<double> P;
struct Line { // hash-cpp-1
  P P1, P2;
  // Right hand side of the ray P1 -> P2
  explicit Line(P a = P(), P b = P()) : P1(a), P2(b) {};
  P intpo(Line y) {
    P r;
    assert(lineIntersection(P1, P2, y.P1, y.P2, r) == 1);
    return r;
  }
  P dir() { return P2 - P1; }
  bool contains(P x) { return (P2 - P1).cross(x - P1) < eps
   ↪ ; }
  bool out(P x) { return !contains(x); }
}; // hash-cpp-1 = 5bca174c3e03ed1b546e4ac3a5416d28

template<class T>
bool mycmp(Point<T> a, Point<T> b) { // hash-cpp-2
```

```cpp
  // return atan2(a.y, a.x) < atan2(b.y, b.x);
  if (a.x * b.x < 0) return a.x < 0;
  if (abs(a.x) < eps) {
    if (abs(b.x) < eps) return a.y > 0 && b.y < 0;
    if (b.x < 0) return a.y > 0;
    if (b.x > 0) return true;
  }
  if (abs(b.x) < eps) {
    if (a.x < 0) return b.y < 0;
    if (a.x > 0) return false;
  }
  return a.cross(b) > 0;
} // hash-cpp-2 = 5a80cc8032965e28a1894939bb91f3ec

bool cmp(Line a, Line b) { return mycmp(a.dir(), b.dir());
 ↪ }
double Intersection_Area(vector <Line> b) { // hash-cpp-3
  sort(b.begin(), b.end(), cmp);
  int n = b.size();
  int q = 1, h = 0, i;
  vector<Line> c(b.size() + 10);
  for (i = 0; i < n; i++) {
    while (q < h && b[i].out(c[h].intpo(c[h - 1]))) h--;
    while (q < h && b[i].out(c[q].intpo(c[q + 1]))) q++;
    c[++h] = b[i];
    if (q < h && abs(c[h].dir().cross(c[h - 1].dir())) <
     ↪ eps) {
      h--;
      if (b[i].out(c[h].P1))  c[h] = b[i];
    }
  }
  while (q < h - 1 && c[q].out(c[h].intpo(c[h - 1]))) h--;
  while (q < h - 1 && c[h].out(c[q].intpo(c[q + 1]))) q++;
  // Intersection is empty. This is sometimes different
   ↪ from the case when
  // the intersection area is 0.
  if (h - q <= 1) return 0;
  c[h + 1] = c[q];
  vector <P> s;
  for (i = q; i <= h; i++) s.push_back(c[i].intpo(c[i + 1])
   ↪ );
  s.push_back(s[0]);
  double ans = 0;
  for (i = 0; i < (int)s.size()-1; i++) ans += s[i].cross(s
   ↪ [i + 1]);
  return ans/2;
} // hash-cpp-3 = 42e408a367c0ed9cff988abd9b4b64ca
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: $i1$, $i2$ are the indices to the closest pair of points in the point vector p after the call. The distance is returned.
Time: $\mathcal{O}\left(n\log n\right)$

```
"Point.h"                                                    58 lines
```
```cpp
template<class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template<class It>
bool y_it_less(const It& i,const It& j) {return i->y < j->y
 ↪ ;}

template<class It, class IIt> /* IIt = vector<It>::iterator
 ↪ */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
  typedef typename iterator_traits<It>::value_type P;
  int n = yaend-ya, split = n/2;
  if(n <= 3) { // base case
```

```cpp
  double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
  if(n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).
      ↪dist();
  if(a <= b) { i1 = xa[1];
    if(a <= c) return i2 = xa[0], a;
    else return i2 = xa[2], c;
  } else { i1 = xa[2];
    if(b <= c) return i2 = xa[0], b;
    else return i2 = xa[1], c;
  } }
  vector<It> ly, ry, stripy;
  P splitp = *xa[split];
  double splitx = splitp.x;
  for(IIt i = ya; i != yaend; ++i) { // Divide
    if(*i != xa[split] && (**i-splitp).dist2() < 1e-12)
      return i1 = *i, i2 = xa[split], 0;// nasty special
          ↪case!
    if (**i < splitp) ly.push_back(*i);
    else ry.push_back(*i);
  } // assert((signed)lefty.size() == split)
  It j1, j2; // Conquer
  double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
  double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2)
      ↪;
  if(b < a) a = b, i1 = j1, i2 = j2;
  double a2 = a*a;
  for(IIt i = ya; i != yaend; ++i) { // Create strip (y-
      ↪sorted)
    double x = (*i)->x;
    if(x >= splitx-a && x <= splitx+a) stripy.push_back(*i)
        ↪;
  }
  for(IIt i = stripy.begin(); i != stripy.end(); ++i) {
    const P &p1 = **i;
    for(IIt j = i+1; j != stripy.end(); ++j) {
      const P &p2 = **j;
      if(p2.y-p1.y > a) break;
      double d2 = (p2-p1).dist2();
      if(d2 < a2) i1 = *i, i2 = *j, a2 = d2;
  } }
  return sqrt(a2);
}

template<class It> // It is random access iterators of
    ↪point<T>
double closestpair(It begin, It end, It &i1, It &i2 ) {
  vector<It> xa, ya;
  assert(end-begin >= 2);
  for (It i = begin; i != end; ++i)
    xa.push_back(i), ya.push_back(i);
  sort(xa.begin(), xa.end(), it_less<It>);
  sort(ya.begin(), ya.end(), y_it_less<It>);
  return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
} // hash-cpp-all = 42735b8e08701a3b73504ac0690e31df
```

KdTree.h
Description: KD-tree (2d, can be extended to 3d)
"Point.h"   63 lines

```cpp
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
```

```cpp
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a
      ↪point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if the box is wider than high (not best
          ↪ heuristic...)
      sort(vp.begin(),vp.end(), x1 - x0 >= y1 - y0 ? on_x :
          ↪ on_y);
      // divide by taking half the array for each child (
          ↪not
      // best performance with many duplicates in the
          ↪middle)
      int half = vp.size()/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({vp.begin(),
      ↪vp.end()})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared
      ↪distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
}; // hash-cpp-all = 915562277c057ca45f507138a826fa7d
```

DelaunayTriangulation.h
Description: Computes the Delaunay triangulation of a set of points.
Each circumcircle contains none of the input points. If any three points
are colinear or any four are on the same circle, behavior is undefined.

Time: $\mathcal{O}\left(n^2\right)$
"Point.h", "3dHull.h"   10 lines

```cpp
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (ps.size() == 3) { int d = (ps[0].cross(ps[1], ps[2])
      ↪< 0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  for(auto &p : ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (ps.size() > 3) for(auto &t: hull3d(p3)) if ((p3[t.b]-
      ↪p3[t.a]).
    cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
} // hash-cpp-all = f6175a3c9680ae285374fb369c3af995
```

FastDelaunay.h
Description: Fast Delaunay triangulation. There must be no dupli-
cate points. If all points are on a line, no triangles will be returned.
Should work for doubles as well, though there may be precision issues
in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...},
all counter-clockwise.
Time: $\mathcal{O}\left(n \log n\right)$
"Point.h"   90 lines

```cpp
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad { // hash-cpp-1
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return rot->r()->o->rot; }
}; // hash-cpp-1 = ae7c00e56c665d4b1231ab65e4a209f7
// hash-cpp-2
bool circ(P p, P a, P b, P c) { // is p in the circumcircle
    ↪?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
      ↪ 0;
} // hash-cpp-2 = 6aff7b12fbc9bf3e4cdc9425f5a62137
Q makeEdge(P orig, P dest) { // hash-cpp-3
  Q q0 = new Quad{0,0,0,orig}, q1 = new Quad{0,0,0,arb},
    q2 = new Quad{0,0,0,dest}, q3 = new Quad{0,0,0,arb};
  q0->o = q0; q2->o = q2; // 0-0, 2-2
  q1->o = q3; q3->o = q1; // 1-3, 3-1
  q0->rot = q1; q1->rot = q2;
  q2->rot = q3; q3->rot = q0;
  return q0;
} // hash-cpp-3 = 81016dffd34a695006075996590c4d6a
void splice(Q a, Q b) { // hash-cpp-4
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
} // hash-cpp-4 = 7e71f74a90f6e01fedeeb98e1fcb3d65

pair<Q,Q> rec(const vector<P>& s) { // hash-cpp-5
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
        ↪);
    if (sz(s) == 2) return { a, a->r() };
```

```
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = (sz(s) + 1) / 2;
    tie(ra, A) = rec({s.begin(), s.begin() + half});
    tie(B, rb) = rec({s.begin() + half, s.end()});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
           (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o);  DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
} // hash-cpp-5 = d3b6931a24cfd32c9af3573423c14605

vector<P> triangulate(vector<P> pts) { // hash-cpp-6
    sort(pts.begin(), pts.end());  assert(unique(pts.begin(),
        pts.end()) == pts.end());
    if (pts.size() < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p
        ); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
} // hash-cpp-6 = 4e0ca588db95eeafce87cd00038a4697
```

RectangleUnionArea.h

Description: Sweep line algorithm that calculates area of union of rectangles in the form [x1,x2) x [y1,y2)

Usage: Create vector with lower leftmost and upper rightmost coordinates of each rectangle.//vector<pair<int,int>,pair<int,int>> rectangles;// rectangles.push_back({{1, 3}, {2, 4}});// lint result = rectangle_union_area(rectangles);

<div style="text-align:right">63 lines</div>

```
pair<int,int> operator+(const pair<int,int>& l, const pair<
    int,int>& r) {
    if (l.first!= r.first) return min(l,r);
    return {l.first, l.second + r.second};
}
```

```
struct segtree_t { // stores min + # of mins
    int n;
    vector<int> lazy;
    vector<pair<int,int>> tree; // set n to a power of two
    segtree_t(int _n) : n(_n), tree(2*n, {0,0}), lazy(2*n,
        0) { }
    void build() {
        for(int i = n-1; i >= 1; --i)
            tree[i] = tree[2*i] + tree[2*i+1];
    }
    void push(int v, int lx, int rx) {
        tree[v].first += lazy[v];
        if (lx != rx) {
            lazy[2*v] += lazy[v];
            lazy[2*v+1] += lazy[v];
        }
        lazy[v] = 0;
    }
    void update(int a, int b, int delta) { update(1,0,n-1,a
        ,b,delta); }
    void update(int v, int lx, int rx, int a, int b, int
        delta) {
        push(v, lx, rx);
        if (b < lx || rx < a) return;
        if (a <= lx && rx <= b) {
            lazy[v] = delta;
            push(v, lx, rx);
        }
        else {
            int m = lx + (rx - lx)/2;
            update(2*v, lx, m, a, b, delta);
            update(2*v+1, m+1, rx, a, b, delta);
            tree[v] = (tree[2*v] + tree[2*v+1]);
        }
    }
};

lint rectangle_union_area(vector<pair<pair<int,int>,pair<
    int,int>>> v) { // area of union of rectangles
    segtree_t L(SZ);
    vector<int> y; for(auto &t : v) y.push_back(t.second.
        first), y.push_back(t.second.second);
    sort(y.begin(), y.end()); y.erase(unique(y.begin(), y.end
        ()),y.end());
    for(int i = 0; i < y.size()-1; i++) L.tree[SZ+i].second =
        y[i+1]-y[i]; // compress coordinates
    L.build();
    vector<array<int,4>> ev; // sweep line
    for(auto &t : v) {
        t.second.first= lower_bound(y.begin(), y.end(),t.second
            .first)-begin(y);
        t.second.second = lower_bound(y.begin(), y.end(),t.
            second.second)-begin(y)-1;
        ev.push_back({t.first.first,1,t.second.first,t.second.
            second});
        ev.push_back({t.first.second,-1,t.second.first,t.second
            .second});
    }
    sort(ev.begin(), ev.end());
    lint ans = 0;
    for(int i = 0; i < ev.size()-1; i++) {
        const auto& t = ev[i];
        L.update(t[2],t[3],t[1]);
        int len = y.back()-y.front()-L.tree[1].second; // L.mn
            [0].firstshould equal 0
        ans += (lint)(ev[i+1][0]-t[0])*len;
    }
```

```
    return ans;
} // hash-cpp-all = 1450ef44b416006a4c7cb39a7d3404ef
```

8.5   3D

PolyhedronVolume.h
Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

<div style="text-align:right">6 lines</div>

```
template<class V, class L>
double signed_poly_volume(const V &p, const L &trilist) {
    double v = 0;
    for(auto &i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.
        c]);
    return v / 6;
} // hash-cpp-all = 832599560d46de4dac772525327508df
```

Point3D.h
Description: Class to handle points in 3D space. T can be e.g. double or long long.

<div style="text-align:right">33 lines</div>

```
template<class T> struct Point3D { // hash-cpp-1
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z)
        {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    } // hash-cpp-1 = f914db739064a236fa80cdd6cb4a28da
// hash-cpp-2
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi,
        pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0,
        pi]
    double theta() const { return atan2(sqrt(x*x+y*y),z); }
    P unit() const { return *this/(T)dist(); } //makes dist()
        =1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit
            ();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
}; // hash-cpp-2 = c9d0298d203587721eca48adde037c27
```

3dHull.h
Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h"

<div style="text-align:right">49 lines</div>

```
typedef Point3D<double> P3;
```

```cpp
struct PR { // hash-cpp-1
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
}; // hash-cpp-1 = cf7c9e0e504697f2f68406fa666ee3e4

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) { // hash-cpp-2
  assert(A.size() >= 4);
  vector<vector<PR>> E(A.size(), vector<PR>(A.size(), {-1,
      -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  for(int i=0;i<4;i++) for(int j=i+1;j<4;j++) for(k=j+1;k
      <4;k++)
    mf(i, j, k, 6 - i - j - k);
// hash-cpp-2 = 795ac5f92c46fc81467bd587c2cbcfd5
  for(int i=4; i<A.size();++i) { // hash-cpp-3
    for(int j=0;j<FS.size();++j) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = FS.size();
    for(int j=0;j<nw;j++) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f
      .c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for(auto &it: FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
}; // hash-cpp-3 = 58a80c2b46187dcdf3c9db71c56711db
```

SphericalDistance.h
Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.
8 lines

```cpp
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
} // hash-cpp-all = 611f0797307c583c66413c2dd5b3ba28
```

Strings (9)

KMP.cpp
Description: failure[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a pattern in a text.
Time: $\mathcal{O}(n)$
29 lines

```cpp
template<typename T>
struct kmp_t {
  vector<T> word;
  vector<int> failure;
  kmp_t(const vector<T> &_word): word(_word) { // hash-
      cpp-1
    int n = word.size();
    failure.resize(n+1, 0);
    for (int s = 2; s <= n; ++s) {
      failure[s] = failure[s-1];
      while (failure[s] > 0 && word[failure[s]] !=
          word[s-1])
        failure[s] = failure[failure[s]];
      if (word[failure[s]] == word[s-1]) failure[s]
          += 1;
    }
  } // hash-cpp-1 = c66cf26827fd4607ce1cfa55401f3dea
  vector<int> matches_in(const vector<T> &text) { // hash
      -cpp-2
    vector<int> result;
    int s = 0;
    for (int i = 0; i < (int)text.size(); ++i) {
      while (s > 0 && word[s] != text[i])
        s = failure[s];
      if (word[s] == text[i]) s += 1;
      if (s == (int)word.size()) {
        result.push_back(i-(int)word.size()+1);
        s = failure[s];
      }
    }
    return result;
  } // hash-cpp-2 = 50ada13bcff4322771988e39d05fffe4
};
```

Extended-KMP.h
Description: extended KMP S[i] stores the maximum common prefix between s[i:] and t; T[i] stores the maximum common prefix between t[i:] and t for i>0;
24 lines

```cpp
int S[N], T[N];

void extKMP(const string &s, const string &t) {
  int m = t.size(), maT = 0, maS = 0;
  T[0] = 0;
  for (int i = 1; i < m; i++) {
    if (maT + T[maT] >= i)
      T[i] = min(T[i - maT], maT + T[maT] - i);
    else T[i] = 0;
    while (T[i] + i < m && t[T[i]] == t[T[i] + i])
      T[i]++;
    if (i + T[i] > maT + T[maT]) maT = i;
  }
  int n = s.size();
  for (int i = 0; i < n; i++) {
```

```cpp
    if (maS + S[maS] >= i)
      S[i] = min(T[i - maS], maS + S[maS] - i);
    else S[i] = 0;
    while (S[i] < m && i + S[i] < n && t[S[i]] == s[S[i
        ] + i])
      S[i]++;
    if (i + S[i] > maS + S[maS]) maS = i;
  }
}
// hash-cpp-all = 678f728e4713de687a5065ac74ae3d66
```

Duval.h
Description: A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes.
Time: $\mathcal{O}(N)$
28 lines

```cpp
template <typename T>
pair<int, vector<string>> duval(int n, const T &s) { //
    hash-cpp-1
  assert(n >= 1);
  // s += s  //uncomment if you need to know the min
      cyclic string
  vector<string> factors; // strings here are simple and
      in non-inc order
  int i = 0, ans = 0;
  while (i < n) { // until n/2 to find min cyclic string
    ans = i;
    int j = i + 1, k = i;
    while (j < n + n && !(s[j % n] < s[k % n])) {
      if (s[k % n] < s[j % n]) k = i;
      else k++;
      j++;
    }
    while (i <= k) {
      factors.push_back(s.substr(i, j-k));
      i += j - k;
    }
  }
  return {ans, factors};
  // returns 0-indexed position of the least cyclic shift
  // min cyclic string will be s.substr(ans, n/2)
} // hash-cpp-1 = cc666b9ac54cacdb7a4172ac1573d84b

template <typename T>
pair<int, vector<string>> duval(const T &s) {
  return duval((int) s.size(), s);
}
```

Z.h
Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$
18 lines

```cpp
vector<int> Z(string& S) {
  vector<int> z(S.size());
  int l = -1, r = -1;
  for(int i = 1; i < S.size(); ++i) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < S.size() && S[i + z[i]] == S[z[i
        ]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

```cpp
vector<int> get_prefix(string a, string b) { // hash-cpp-1
    string str = a + '@' + b;
    vector<int> k = z(str);
    return vector<int>(k.begin()+a.size()+1, k.end());
} // hash-cpp-1 = 6aa08403b9d47a6d0e421c570e0bf941
```

Manacher.h
Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

13 lines
```cpp
array<vector<int>, 2> manacher(const string &s) { // hash-
    ↪cpp-1
  int n = s.size();
  array<vector<int>, 2> p = {vector<int>(n+1), vector<int>(
    ↪n)};
  for(int z = 0; z < 2; ++z) for (int i=0,l=0,r=0; i < n; i
    ↪++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
} // hash-cpp-1 = 87e1f0950281807a59d4f6ef730e6943
```

MinRotation.h
Description: Finds the lexicographically smallest rotation of a string.
Usage: rotate(v.begin(), v.begin()+min_rotation(v), v.end());
Time: $\mathcal{O}(N)$

8 lines
```cpp
int min_rotation(string s) { // hash-cpp-1
  int a=0, N=s.size(); s += s;
  for(int b = 0; b < N; ++b) for(int i =0; i < N; ++i) {
    if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1);
      ↪break;}
    if (s[a+i] > s[b+i]) { a = b; break; }
  }
  return a;
} // hash-cpp-1 = 2a08fd228bd46d16ef7716c24c0a72ce
```

Trie.h
Description: Trie implementation.

68 lines
```cpp
struct Trie {
  int cnt, word;
  map<char, Trie> m;
  Trie() : word(0), cnt(0) { m.clear();}
  void add(const string &s, int i) {
    cnt++;
    if(i ==(int)s.size()) {
      word++;
      return;
    }
    if(!m.count(s[i])) m[s[i]] = Trie();
    m[s[i]].add(s, i + 1);
  }
  bool remove(const string &s, int i) {
    if(i ==(int)s.size()) {
      if(word) {
        cnt--;
        word--;
        return true;
```

```cpp
      }
      return false;
    }
    if(!m.count(s[i])) return false;
    if(m[s[i]].remove(s, i + 1) == true) {
      cnt--;
      return true;
    }
    return false;
  }
  bool count(const string &s, int i) {
    if(i ==(int)s.size()) return word;
    if(!m.count(s[i])) return false;
    return m[s[i]].count(s, i + 1);
  }
  bool count_prefix(const string &s, int i) {
    if (word) return true;
    if(i ==(int)s.size()) return false;
    if(!m.count(s[i])) return false;
    return m[s[i]].count_prefix(s, i + 1);
  }
  bool is_prefix(const string &s, int i) {
    if(i ==(int)s.size()) return cnt;
    if(!m.count(s[i])) return false;
    return m[s[i]].is_prefix(s, i + 1);
  }
  void add(const string &s) {
    add(s, 0);
  }
  bool remove(const string &s) {
    return remove(s, 0);
  }
  bool count(const string &s) {
    return count(s, 0);
  }
  // return if trie countains a string that is prefix os s
  // trie has 123, query 12345  returns true
  // trie has 12345, query 123  returns false
  bool count_prefix(const string &s) {
    return count_prefix(s, 0);
  }
  // return if s is prefix of some string countained in
    ↪trie
  // trie has 12345, query 123  returns true
  // trie has 123, query 12345  returns false
  bool is_prefix(const string &s) {
    return is_prefix(s, 0);
  }
} T; // hash-cpp-all = df13d0b71df32061e3bd54816152e3cf
```

TrieXOR.h
Description: Query max xor with some int in the Trie

30 lines
```cpp
template<int MX, int MXBIT> struct Trie { // hash-cpp-1
    vector<vector<int>> nex;// num is last node in trie
    vector<int> sz;
    int num = 0;
    // change 2 to 26 for lowercase letters
    Trie() {
        nex = vector<vector<int>>(MX, vector<int>(2));
        sz = vector<int>(MX);
    } // hash-cpp-1 = 171b2c3c86583019d3e96ea5c2fcfc4f
    // insert or delete
    void insert(lint x, int a = 1) { // hash-cpp-2
        int cur = 0; sz[cur] += a;
        for(int i = MXBIT-1; i >= 0; --i) {
```

```cpp
            int t = (x&(1lint<<i))>>i;
            if (!nex[cur][t]) nex[cur][t] = ++num;
            sz[cur = nex[cur][t]] += a;
        }
    } // hash-cpp-2 = c533ca7f6d0fcdf3a7011207856e065d
    // compute max xor
    lint test(lint x) { // hash-cpp-3
        if (sz[0] == 0) return -INF; // no elements in trie
        int cur = 0;
        for(int i = MXBIT-1; i >= 0; --i) {
            int t = ((x&(1lint<<i))>>i) ^ 1;
            if (!nex[cur][t] || !sz[nex[cur][t]]) t ^= 1;
            cur = nex[cur][t]; if (t) x ^= 1lint<<i;
        }
        return x;
    } // hash-cpp-3 = 3c8060e4c36b53d379b97008c71f1921
};
```

Hashing.h
Description: Various self-explanatory methods for string hashing.

44 lines
```cpp
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse,
    ↪where
// ABBA... and BAAB... of length 2^10 hash the same mod
    ↪2^64).
// "typedef ull H;" instead if you think test data is
    ↪random,
// or work mod 10^9+7 if the Birthday paradox is not a
    ↪problem.
struct H { // hash-cpp-1
    typedef uint64_t ull;
    ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
    (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) : B); return r
        ↪; }
    OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
    H operator-(H o) { return *this + ~o.x; }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
}; // hash-cpp-1 = 84fa0c42358c7eadadbb080c561c7211
static const H C = (lint)1e11+3; // (order ~ 3e9; random
    ↪also ok)

struct HashInterval { // hash-cpp-2
    vector<H> ha, pw;
    HashInterval(string& str) : ha(str.size()+1), pw(ha) {
        pw[0] = 1;
        for(int i = 0; i < str.size(); ++i)
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
}; // hash-cpp-2 = e34d1dce6f540fee1bacadab91d5a95d

vector<H> getHashes(string& str, int length) { // hash-cpp
    ↪-3
    if (str.size() < length) return {};
    H h = 0, pw = 1;
    for(int i = 0; i < length; ++i)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    for(int i = length; i < str.size(); ++i) {
        ret.push_back(h = h * C + str[i] - pw * str[i-length]);
```

```cpp
  }
  return ret;
} // hash-cpp-3 = 7487e28ee0ee7bdacd95dea6dffe122a

H hashString(string& s) { H h{}; for (auto &c : s) h=h*C+c;
  ↪ return h; }
```

HashGood.h

Description: Simple, short and efficient hashing using pairs to reduce load factor.

`<ModTemplate.h>`, `<PairNumTemplate.h>`   42 lines

```cpp
using num = modnum<int(1e9)+7>;
using hsh = pairnum<num, num>;
const hsh BASE(163, 311);
// uniform_int_distribution<int> MULT_DIST(0.1*MOD,0.9*MOD)
  ↪;
// constexpr hsh BASE(MULT_DIST(rng), MULT_DIST(rng));

struct hash_t {
    int n;
    string str;
    vector<hsh> hash, basePow;
    hash_t(const string& s) : n(s.size()), str(s), hash(n
        ↪+1), basePow(n) {
        basePow[0] = 1;
        for (int i = 1; i < n; ++i) basePow[i] = basePow[i
            ↪-1] * BASE;
        for (int i = 0; i < n; ++i)
            hash[i+1] = hash[i] * BASE + hsh(s[i]);
    }
    hsh get_hash(int left, int right) {
        assert(left <= right);
        return hash[right] - hash[left] * basePow[right -
            ↪left];
    }
    int lcp(hash_t &other) { // need some testing
        int left = 0, right = min(str.size(), other.str.
            ↪size());
        while (left < right) {
            int mid = (left + right + 1)/2;
            if (hash[mid] == other.hash[mid]) left = mid;
            else right = mid-1;
        }
        return left;
    }
};

vector<int> rabinkarp(string t, string p) {
    vector<int> matches;
    hsh h(0, 0);
    for (int i = 0; i < p.size(); ++i)
        h = BASE * h + hsh(p[i]);
    hash_t result(t);
    for (int i = 0; i + p.size() <= t.size(); ++i)
        if (result.get_hash(i, i + p.size()) == h)
            matches.push_back(i);
    return matches;
} // hash-cpp-all = 13369fd0b40b2a886e961bb6f6014dce
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$ 50 lines

```cpp
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,a.size());
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] =
       ↪ 0;
    for(int i = 0; i < a.size(); ++i) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA =
     ↪28)
  pair<int,int> best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) :
       ↪ 0;
    for(int c = 0; c < ALPHA; ++c) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pair<int,int> LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2)
       ↪);
    st.lcs(0, s.size(), s.size() + 1 + t.size(), 0);
    return st.best;
  }
}; // hash-cpp-all = 6c2a8bdd2a7412aab755d53b9d18fdc5
```

AhoCorasick.cpp

Description: String searching algorithm that matches all strings simultaneously. To use with stl string: (char *)stringname.c_str()   91 lines

```cpp
struct Node {
    int fail;
    vector<pair<int,int>> out; // num e tamanho do padrao
    //bool marc;  // p/ decisao
    map<char,int> link;
    int next; // aponta para o proximo sufixo que tenha out.
       ↪size > 0
```

```cpp
};
Node tree[1000003]; // quantida maxima de nos

struct AhoCorasick {
  //bool encontrado[1005]; // quantidade maxima de padroes,
      ↪ p/ decisao
  int qtdNos, qtdPadroes;
  vector<vector<int>> result;
  AhoCorasick() { // Construtor para inicializar
    result.resize(0);
    tree[0].fail = -1;
    tree[0].link.clear();
    tree[0].out.clear();
    tree[0].next = -1;
    qtdNos = 1;
    qtdPadroes = 0;
    //tree[0].marc = false; // p/ decisao
    //memset(encontrado, false, sizeof(encontrado)); // p
        ↪/ decisao
  }
  // Funcao para adicionar um padrao
  void add(string &pat) {
    vector<int> v;
    result.push_back(v);
    int no = 0, len = 0;
    for (int i = 0; i < pat.size(); i++, len++) {
        if (tree[no].link.find(pat[i]) == tree[no].link.
            ↪end()) {
            tree[qtdNos].link.clear(); tree[qtdNos].out.
                ↪clear();
            //tree[qtdNos].marc = false; // p/ decisao
            tree[no].link[pat[i]] = qtdNos;
            no = qtdNos++;
        } else no = tree[no].link[pat[i]];
    }
    tree[no].out.push_back({qtdPadroes++,len});
  }
  // Ativar Aho-corasick, ajustando funcoes de falha
  void activate() {
    int no,v,f,w;
    vector<int> bfs;
    for (auto  it = tree[0].link.begin();
      it != tree[0].link.end() ; it++) {
        tree[no = it->second].fail = 0;
        tree[no].next = tree[0].out.size() ? 0 : -1;
        bfs.push_back(no);
    }
    for(int i = 0; i < bfs.size(); ++i) {
        no = bfs[i];
        for (auto it = tree[no].link.begin();
            it != tree[no].link.end(); it++) {
            char c = it->first;
            v = it->second;
            bfs.push_back(v);
            f = tree[no].fail;
            while (tree[f].link.find(c) == tree[f].link.
                ↪end()) {
                if (f == 0) { tree[0].link[c] = 0; break;
                   ↪ }
                f = tree[f].fail;
            }
            w = tree[f].link[c];
            tree[v].fail = w;
            tree[v].next = tree[w].out.size() ? w : tree[
                ↪w].next;
        }
    }
  }
}
```

```
// Buscar padroes no aho-corasik
void search_all(string &text) {
    int v, no = 0;
    for (int i = 0; i < text.size(); ++i) {
        while (tree[no].link.find(text[i]) == tree[no].
            ↪link.end()) {
            if (no == 0) { tree[0].link[text[i]] = 0;
                ↪break; }
            no = tree[no].fail;
        }
        v = no = tree[no].link[text[i]];
        // marcar os encontrados
        while (v != -1 /* && !tree[v].marc */ ) { // p/
            ↪decisao
            //tree[v].marc = true; // p/ decisao: nao
                ↪continua a link
            for (int k = 0 ; k < tree[v].out.size() ; k
                ↪++) {
                //encontrado[tree[v].out[k].first] = true
                    ↪; // p/ decisao
                result[tree[v].out[k].first].push_back(i-
                    ↪tree[v].out[k].second+1);
                printf("Padrao %d na posicao %d\n", tree[
                    ↪v].out[k].first,
                    i-tree[v].out[k].second+1);
            }
            v = tree[v].next;
        }
    }
}
};
// hash-cpp-all = 1c53345cd6308673461388b1c17b8ddc
```

Suffix-Array.h
Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n + 1$, and `sa[0]` = n. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i]` = `lcp(sa[i], sa[i-1])`, `lcp[0]` = 0. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$
                             23 lines

```
struct SuffixArray {
    vector<int> sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<
        ↪int>
        int n = s.size()+1, k = 0, a, b;
        vector<int> x(s.begin(), s.end()+1), y(n), ws(max(n,
            ↪lim)), rank(n);
        sa = lcp = y, iota(sa.begin(), sa.end(), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
            ↪p) {
            p = j, iota(y.begin(), y.end(), n - j);
            for(int i=0;i<n;++i) if (sa[i] >= j) y[p++] = sa[i] -
                ↪ j;
            fill(ws.begin(), ws.end(), 0);
            for(int i=0;i<n;++i) ws[x[i]]++;
            for(int i=1;i<lim;++i) ws[i] += ws[i - 1];
            for (int i=n;i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            for(int i=1;i<n;++i) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p
                    ↪++;
        }
        for(int i=1;i<n;++i) rank[sa[i]] = i;
        for (int i=0,j;i<n-1;lcp[rank[i++]]=k)
            for (k && k--, j = sa[rank[i] - 1];
```

```
            s[i + k] == s[j + k]; k++);
    }
}; // hash-cpp-all = dc6caa155393cfe4a922768e1a0c851d
```

<u>Various</u> (10)

10.1   Intervals

IntervalContainer.h
Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$
                             23 lines

```
set<pair<int,int>>::iterator addInterval(set<pair<int,int>>
    ↪ &is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L,R});
}

void removeInterval(set<pair<int,int>> &is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
} // hash-cpp-all = f47dfb9edd525539da08472171658898
```

IntervalCover.h
Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$
                             19 lines

```
template<class T>
vector<int> cover(pair<T, T> G, vector<pair<T, T>> I) {
    vector<int> S(I.size()), R;
    iota(S.begin(), S.end(), 0);
    sort(S.begin(), S.end(), [&](int a, int b) { return I[a]
        ↪< I[b]; });
    T cur = G.first;
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = {cur, -1};
        while (at < I.size() && I[S[at]].first <= cur) {
            mx = max(mx, {I[S[at]].second, S[at]});
            at++;
        }
        if (mx.second == -1) return {};
        cur = mx.first;
        R.push_back(mx.second);
    }
    return R;
} // hash-cpp-all = 133eb4becbdaf3b99371a1e364b33a2b
```

ConstantIntervals.h
Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$
                             19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F f, G g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q;
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q);
    g(i, to, q);
} // hash-cpp-all = 792e7d94c54ab04f9efdb6834b12feca
```

10.2   Misc. algorithms

TernarySearch.h
Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: int ind = ternSearch(0,n-1,[&](int i){return a[i];});
Time: $\mathcal{O}(\log(b-a))$
                             11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    for(int i=a+1;i<b+1;i++) if (f(a) < f(i)) a = i; // (B)
    return a;
} // hash-cpp-all = 0b750a57790807d99a432f12841f1af2
```

LowerBound.h
                             11 lines

```
int LowerBound(vector<int> v, int n, int x){
    int l = 1, r = n, m;
    while(l <= r){
        m= (l+r)/2;
        if(v[m] >= x && (m == 1 || v[m-1] < x))
            return m;
        else if(v[m] >= x) r=m-1;
        else l=m+1;
    }
    return m;
} // hash-cpp-all = 7422d7a27dbb4142bd13b8cc1f0f3686
```

UpperBound.h

11 lines

```cpp
int UpperBound(vector<int> v, int n, int x){
    int l = 1, r = n, m;
    while(l <= r){
        m=(l+r)/2;
        if(v[m] > x && (m == 1 || v[m-1] <= x))
            return m;
        else if(v[m] > x) r=m-1;
        else l=m+1;
    }
    return m;
} // hash-cpp-all = 381d15e1acc45839a99189533b42d5eb
```

MergeSort.h
Time: $\mathcal{O}(N)$

18 lines

```cpp
int n, inv;
vector<int> v, result;
void merge_sort(int lx, int rx, vector<int> &v) {
    if (lx == rx) return;
    int m = lx + (rx - lx)/2;
    merge_sort(lx, m, v);
    merge_sort(m+1, rx, v);
    int i = lx, j = m+1, k = lx;
    while(i <= m || j <= rx) {
        if (i <= m && (j > rx || v[i] < v[j])) {
            result[k++] = v[i++];
            inv += (j - k);
        }
        else result[k++] = v[j++];
    }
    for (int i = lx; i <= rx; ++i)
        v[i] = result[i];
} // hash-cpp-all = 34a7b0c31ffe6abe903916da641d98b3
```

CoordCompression.h

9 lines

```cpp
vector<int> comp_coord(vector<int> &y, int N) {
    vector<int> result;
    for (int i = 0; i < N; ++i) result.emplace_back(y[i]);
    sort(result.begin(), result.end());
    result.resize(unique(result.begin(), result.end())-
        ↪result.begin());
    for (int i = 0; i < N; ++i)
        y[i] = lower_bound(result.begin(), result.end(), y[
            ↪i]) - result.begin();
    return result;
} // hash-cpp-all = 809d6ae9d2b00e4d11b3e8500c82eb70
```

CountTriangles.h
Description: Counts x, y >= 0 such that Ax + By <= C.

8 lines

```cpp
lint count_triangle(lint A, lint B, lint C) {
    if (C < 0) return 0;
    if (A > B) swap(A, B);
    lint p = C / B;
    lint k = B / A;
    lint d = (C - p * B) / A;
    return count_triangle(B - k * A, A, C - A * (k * p + d +
        ↪1)) + (p + 1) * (d + 1) + k * p * (p + 1) / 2;
} // hash-cpp-all = 8d67b384e4591dd4f0ba9538ad3bc5d9
```

sqrt.h

13 lines

```cpp
int64_t isqrt(int64_t n) {
    int64_t left = 0;
```

```cpp
    int64_t right = 10000000;
    while (right - left > 1) {
        int64_t middle = (left + right) / 2;
        if (middle * middle <= n) {
            left = middle;
        } else {
            right = middle;
        }
    }
    return left;
} // hash-cpp-all = fc5f42aa60261c39ccc263bfba494ef1
```

Karatsuba.h
Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length welint. See also FFT, under the Numerical chapter.
Time: $\mathcal{O}(N^{1.6})$

30 lines

```cpp
int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0;
    ↪ }

void karatsuba(lint *a, lint *b, lint *c, lint *t, int n) {
    int ca = 0, cb = 0;
    for(int i = 0; i < n; ++i) ca += !!a[i], cb += !!b[i];
    if (min(ca, cb) <= 1500/n) { // few numbers to multiply
        if (ca > cb) swap(a, b);
        for(int i = 0; i < n; ++i)
            if (a[i]) FOR(j,n) c[i+j] += a[i]*b[j];
    }
    else {
        int h = n >> 1;
        karatsuba(a, b, c, t, h); // a0*b0
        karatsuba(a+h, b+h, c+n, t, h); // a1*b1
        for(int i = 0; i < h; ++i) a[i] += a[i+h], b[i] +=
            ↪b[i+h];
        karatsuba(a, b, t, t+n, h); // (a0+a1)*(b0+b1)
        for(int i = 0; i < h; ++i) a[i] -= a[i+h], b[i] -=
            ↪b[i+h];
        for(int i = 0; i < n; ++i) t[i] -= c[i]+c[i+n];
        for(int i = 0; i < n; ++i) c[i+h] += t[i], t[i] =
            ↪0;
    }
}

vector<lint> conv(vector<lint> a, vector<lint> b) {
    int sa = a.size(), sb = b.size(); if (!sa || !sb)
        ↪return {};
    int n = 1<<size(max(sa,sb)); a.resize(n), b.resize(n);
    vector<lint> c(2*n), t(2*n);
    for(int i = 0; i < 2*n; ++i) t[i] = 0;
    karatsuba(&a[0], &b[0], &c[0], &t[0], n);
    c.resize(sa+sb-1); return c;
} // hash-cpp-all = 94626586a3d1b8e95703da4c97fb6c83
```

CountInversions.h
Description: Count the number of inversions to make an array sorted. Merge sort has another approach.
Time: $\mathcal{O}(n * log(n))$

<u>FenwickTree.h</u>   7 lines

```cpp
FT<int, 10010> bit;
int inv = 0;
for (int i = n-1; i >= 0; --i) {
    inv += bit.sum(values[i]); // careful with the interval
    bit.update(values[i], 1);    // [0, x) or [0, x] ?
}
// hash-cpp-all = ec29f15a711d07a68da5a50917037d64
```

Histogram.h
Description: Maximum area of a histogram.
Time: $\mathcal{O}(n)$

16 lines

```cpp
template<typename T = int>
T max_area(vector<int> v) {
    T ret = T();
    stack<int> s;
    v.insert(v.begin(), -1);
    v.insert(v.end(), -1);
    s.push(0);
    for(int i = 0; i < v.size(); ++i) {
        while (v[s.top()] > v[i]) {
            int h = v[s.top()]; s.pop();
            ret = max(ret, h * (i - s.top() - 1));
        }
        s.push(i);
    }
    return ret;
} // hash-cpp-all = 51da1acb56b7aba5750ea9568e40ba1b
```

DateManipulation.h

43 lines

```cpp
string week_day_str[7] = {"Sunday", "Monday", "Tuesday", "
    ↪Wednesday", "Thursday", "Friday", "Saturday"};
string month_str[13] = {"", "January", "February", "March",
    ↪ "April", "May", "June", "July", "August", "September"
    ↪, "October", "November", "December"};
map<string, int> week_day_int = {{"Sunday", 0}, {"Monday",
    ↪1}, {"Tuesday", 2}, {"Wednesday", 3}, {"Thursday", 4},
    ↪ {"Friday", 5}, {"Saturday", 6}};
map<string, int> month_int = {{"January", 1}, {"February",
    ↪2}, {"March", 3}, {"April", 4}, {"May", 5}, {"June",
    ↪6}, {"July", 7}, {"August", 8}, {"September", 9}, {"
    ↪October", 10}, {"November", 11}, {"December", 12}};
int month[2][13] = {{0, 31, 28, 31, 30, 31, 30, 31, 31, 30,
    ↪ 31, 30, 31}, {0, 31, 29, 31, 30, 31, 30, 31, 31, 30,
    ↪31, 30, 31}};

/* O(1) - Checks if year y is a leap year. */
bool leap_year(int y){
    return (y % 4 == 0 && y % 100 != 0) || y % 400 == 0;
}

/* O(1) - Increases the day by one. */
void update(int &d, int &m, int &y){
    if (d == month[leap_year(y)][m]){
        d = 1;
        if (m == 12) {
            m = 1;
            y++;
        }
        else m++;
    }
    else d++;
}

int intToDay(int jd) { return jd % 7; }
int dateToInt(int y, int m, int d) {
    return 1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075; }
void intToDate(int jd, int &y, int &m, int &d) {
    int x, n, i, j;
    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
```

```
i = (4000 * (x + 1)) / 1461001;
x -= 1461 * i / 4 - 31;
j = 80 * x / 2447;
d = x - 2447 * j / 80;
x = j / 11;
m = j + 2 - 12 * x;
y = 100 * (n - 49) + i + x; }
// hash-cpp-all = 0884598494c930f822d30e062be1cceb
```

NQueens.cpp
Description: NQueens
43 lines

```cpp
int ans;
bitset<30> rw, ld, rd; //2*MAX_N -1
bitset<30> iniqueens; //2*MAX_N -1
vector<int> col;
void init(int n){
    ans=0;
    rw.reset();
    ld.reset();
    rd.reset();
    col.assign(n,-1);
}
void init(int n, vector<pair<int,int>> initial_queens){
    //it does NOT check if initial queens are at valid
    ↪positions
    init(n);
    iniqueens.reset();
    for(pair<int,int> pos: initial_queens){
        int r=pos.first, c= pos.second;
        rw[r] = ld[r-c+n-1] = rd[r+c]=true;
        col[c]=r;
        iniqueens[c] = true;
    }
}
void backtracking(int c, int n){
    if(c==n){
        ans++;
        for(int r:col) cout<<r+1<<" ";
        cout<<"\n";
        return;
    }
    else if(iniqueens[c]){
        backtracking(c+1,n);
    }
    else for(int r=0;r<n;r++){
        if(!rw[r] && !ld[r-c+n-1] && !rd[r+c]){
            // if(board[r][c]!=blocked && !rw[r] && !ld[r-c+n
            ↪-1] && !rd[r+c]){ // if there are blocked
            ↪possitions
            rw[r] = ld[r-c+n-1] = rd[r+c]=true;
            col[c]=r;
            backtracking(c+1,n);
            col[c]=-1;
            rw[r] = ld[r-c+n-1] = rd[r+c]=false;
        }
    }
} // hash-cpp-all = e97e9e9198bfdafeb93f5b1021de2577
```

SudokuSolver.h
43 lines

```cpp
int N,m; // N = n*n, m = n; where n equal number of rows or
↪ columns
array<array<int, 10>, 10> grid;
struct SudokuSolver {
    bool UsedInRow(int row,int num){
        for(int col = 0; col < N; ++col)
```

```cpp
            if(grid[row][col] == num) return true;
        return false;
    }
    bool UsedInCol(int col,int num){
        for(int row = 0; row < N; ++row)
            if(grid[row][col] == num) return true;
        return false;
    }
    bool UsedInBox(int row_0,int col_0,int num){
        for(int row = 0; row < m; ++row)
            for(int col = 0; col < m; ++col)
                if(grid[row+row_0][col+col_0] == num)
                ↪return true;
        return false;
    }
    bool isSafe(int row,int col,int num){
        return !UsedInRow(row,num) && !UsedInCol(col,num)
        ↪&& !UsedInBox(row-row%m,col-col%m,num);
    }
    bool find(int &row,int &col){
        for(row = 0; row < N; ++row)
            for(col = 0; col < N; ++col)
                if(grid[row][col] == 0) return true;
        return false;
    }
    bool Solve(){
        int row, col;
        //cout<<row<<" "<<col<<endl;
        if(!find(row,col)) return true;
        for(int num = 1; num <= N; ++num){
            if(isSafe(row,col,num)){
                grid[row][col] = num;
                if(Solve()) return true;
                grid[row][col] = 0;
            }
        }
        return false;
    }
};
// hash-cpp-all = 6be9065d036cb0cb4f35ee043083f733
```

FloydCycle.h
Description: Detect loop in a list. Consider using mod template to avoid overflow.
Time: $\mathcal{O}(n)$
34 lines

```cpp
lint a, b, c;

lint f(lint x) {
    return (a * x + (x % b)) % c;
}

//mu -> first ocurrence
//lambda -> cycle length
lint mu, lambda;

void Floyd(lint x0) {
    //hare -> fast pointer
    //tortoise -> slow pointer
    lint hare, tortoise;
    tortoise = f(x0);
    hare = f(f(x0));
    while(hare != tortoise) {
        tortoise = f(tortoise);
        hare = f(f(hare));
    }
    hare = x0;
```

```cpp
    mu = 0;
    while(tortoise != hare) {
        tortoise = f(tortoise);
        hare = f(hare);
        mu++;
    }
    hare = f(tortoise);
    lambda = 1;
    while(t != h) {
        hare = f(hare);
        lambda++;
    }
} // hash-cpp-all = eb059fec84c1516c7f9a827c6c36ee4c
```

SlidingWindow.h
Description: Given an array v and an integer K, the problem boils down to computing for each index $i : min(v[i], v[i-1], ..., v[i-K+1])$. if $mx == true$, returns the maximum.
Time: $\mathcal{O}(N)$
18 lines

```cpp
vector<int> sliding_window_minmax(vector<int> &v, int K,
↪bool mx) {
    deque<pair<int, int>> window;
    vector<int> ans;
    for (int i = 0; i < v.size(); i++) {
        if (mx) {
            while (!window.empty() && window.back().first <= v[i
            ↪])
                window.pop_back();
        } else {
            while (!window.empty() && window.back().first >= v[i
            ↪])
                window.pop_back();
        }
        window.emplace_back(v[i], i);
        while(window.front().second <= i - K)
            window.pop_front();
        ans.push_back(window.front().first);
    }
    return ans;
} // hash-cpp-all = 5d8f400e1ad9debd0f3ed4b6b1504a50
```

10.3 Dynamic programming

DivideAndConquerDP.h
Description: Optimizes dp of the form (or similar) $dp[i][j] = min_{k<i}(dp[k][j-1] + f(k+1,i))$. The classical case is a partitioning dp, where k determines the break point for the next partition. In this case, i is the number of elements to partition and j is the number of partitions allowed.
Let $opt[i][j]$ be the values of k which minimize the function. (in case of tie, choose the smallest) To apply this optimization, you need $opt[i][j] \leq opt[i+1][j]$. That means the when you add an extra element $(i+1)$, your partitioning choice will not be to include more elements than before (e.g. will no go from choosing $[k,i]$ to $[k-1,i+1]$). This is usually intuitive by the problem details.
. To apply try to write the dp in the format above and verify if the property holds.
Time: Time goes from $\mathcal{O}(n^2m)$ to $\mathcal{O}(nm\log(n))$
54 lines

```cpp
const int INF = 1<<31;
int n, m;
template<typename MAXN, typename MAXM>
struct dp_task {
    array<array<int, MAXN>, MAXN> u;
    array<array<int, MAXN>, MAXM> dp;
    inline f(int i, int j) {
```

```cpp
        return (u[j][j] - u[j][i-1] - u[i-1][j] + u[i-1][i
            ↪-1]) / 2;
    }
    // This is responsible for computing tab[l...r][j],
        ↪knowing that opt[l...r][j] is in range [low_opt...
        ↪high_opt]
    void solve(int j, int l, int r, int low_opt, int
        ↪high_opt) {
        int mid = (l + r) / 2, opt = -1;
        dp[mid][j] = INF;
        for (int k = low_opt; k <= high_opt && k < mid; ++k
            ↪)
            if (dp[k][j-1] + f(k + 1, mid) < dp[mid][j]) {
                dp[mid][j] = dp[k][j-1] + f(k + 1, mid);
                opt = k;
            }
        // New bounds on opt for other pending computation.
        if (l <= mid - 1)
            solve(j, l, mid - 1, low_opt, opt);
        if (mid + 1 <= r)
            solve(j, mid + 1, r, opt, high_opt);
    }
};

int main() {
    dp_task<4123, 812> DP;
    cin >> n >> m;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++)
            cin >> DP.u[i][j];

    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++)
            DP.u[i][j] += DP.u[i - 1][j] + DP.u[i][j - 1] - DP.u[
                ↪i - 1][j - 1];

    for (int i = 1; i <= n; i++)
        DP.dp[i][0] = INF;

    // Original dp
    // for (int i = 1; i <= n; i++)
    //   for (int j = 1; j <= m; j++) {
    //     dp[i][j] = INF;
    //     for (int k = 0; k < i; k++)
    //       dp[i][j] = min(dp[i][j], dp[k][j-1] + f(k + 1,i);
    //   }

    for (int j = 1; j <= m; j++)
        DP.solve(j, 1, n, 0, n - 1);

    cout << DP.dp[n][m] << endl;
}
// hash-cpp-all = 9963fb5a8ba5aaa937eb85e7d242a141
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Generally, Optimizes dp of the form (or similar) $dp[i][j] = min_{i<=k<=j}(dp[i][k-1] + dp[k+1][j] + f(i,j))$. The classical case is building a optimal binary tree, where k determines the root. Let $opt[i][j]$ be the value of k which minimizes the function. (in case of tie, choose the smallest) To apply this optimization, you need $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$. That means the when you remove an element from the left $(i+1)$, you won't choose a breaking point more to the left than before. Also, when you remove an element from the right $(j-1)$, you won't choose a breking point more to the right than before. This is usually intuitive by the problem details. To apply try to write the dp in the format above and verify if the property holds. Be careful with edge cases for opt. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: from $\mathcal{O}\left(N^3\right)$ to $\mathcal{O}\left(N^2\right)$
30 lines

```cpp
array<array<lint, 1123>, 1123> dp;
array<array<int, 1123>, 1123> opt;
array<int, 1123> b;
int l, n;

inline f(int i, int j) {
    return b[j+1] - b[i-1];
}

int main() {
    while(cin >> l >> n) {
        for (int i = 1; i <= n; ++i) cin >> b[i];
        b[0] = 0;
        b[n + 1] = l;
        for (int i = 1; i <= n+1; ++i) {
            dp[i][i - 1] = 0
            opt[i][i - 1] = i;
        }
        for (int i = n; i > 0; --i)
            for (int j = i; j <= n; ++j) {
                dp[i][j] = LLONG_MAX; // INF
                for (int k = max(i, opt[i][j - 1]); k <= j
                    ↪&& k <= opt[i + 1][j]; ++k)
                    if (dp[i][k - 1] + dp[k + 1][j] + f(i,
                        ↪j) < dp[i][j]) {
                        dp[i][j] = dp[i][k - 1] + dp[k +
                            ↪1][j] + f(i, j);
                        opt[i][j] = k;
                    }
            }
        cout << dp[1][n] << '\n';
    }
} // hash-cpp-all = 134f0b098e66977bad949936d0b80a0b
```

ConvexHullTrick.h

Description: Transforms dp of the form (or similar) $dp[i] = min_{j<i}(dp[j] + b[j] * a[i])$. Time goes from $O(n^2)$ to $O(n \log n)$, if using online line container, or $O(n)$ if lines are inserted in order of slope and queried in order of x. To apply try to find a way to write the factor inside minimization as a linear function of a value related to i. Everything else related to j will become constant.
<LineContainer.h>
22 lines

```cpp
array<lint, 112345> dyn, a, b;

int main() {
    int n;
    cin >> n;
```

```cpp
    for (int i = 0; i < n; ++i) cin >> a[i];
    for (int i = 0; i < n; ++i) cin >> b[i];
    dyn[0] = 0;
    LineContainer cht;
    cht.add(-b[0], 0);
    for (int i = 1; i < n; ++i) {
        dyn[i] = cht.query(a[i]);
        cht.add(-b[i], dyn[i]);
    }
    // Original DP O(n^2).
    // for (int i = 1; i < n; i++) {
    //   dyn[i] = INF;
    //   for (int j = 0; j < i; j++)
    //     dyn[i] = min(dyn[i], dyn[j] + a[i] * b[j]);
    // }
    cout << -dyn[n-1] << '\n';
} // hash-cpp-all = 1e5a567f134332193437ca3ce8ce967d
```

Coin.h
Description: Number of ways to make value K with X coins
Time: $\mathcal{O}\left(N\right)$
8 lines

```cpp
int coin(vector<int> &c, int k) {
    vector<int> dp(k+1, 0); dp[0] = 1;
    for (int i = 0; i < c.size(); ++i)
        for (int j = c[i]; j <= k; ++j)
            dp[j] += dp[j-c[i]];
    return dp[k];
}
// hash-cpp-all = c38f010ad4252350bcc4fc8967fd1159
```

MinCoin.h
Description: minimum number of coins to make K
Time: $\mathcal{O}\left(kV\right)$
8 lines

```cpp
int coin(vector<int> &c, int k) {
    vector<int> dp(k+1, INF); dp[0] = 0;
    for (int i = 0; i < c.size(); ++i)
        for (int j = c[i]; j <= k; ++j)
            dp[j] = min(dp[j], 1 + dp[j-c[i]]);
    return dp[k];
}
// hash-cpp-all = 5fe4b1893507d900689285cdb60f4642
```

EditDistance.h
13 lines

```cpp
vector<vector<int>> dp(MAX_SIZE, vector<int>(MAX_SIZE));
int levDist(const string &s, const string &t) {
    for (int i = 0; i <= s.size(); ++i) dp[i][0] = i;
    for (int i = 0; i <= t.size(); ++i) dp[0][i] = i;
    for (int i = 1; i <= s.size(); ++i) {
        for (int j = 1; j <= t.size(); ++j) {
            dp[i][j] = min(1 + min(dp[i-1][j], dp[i][j-1]),
                ↪ dp[i-1][j-1]+(s[i-1] != t[i-1]));
        }
    }
    return dp[s.size()][t.size()];
}

// hash-cpp-all = bc7965e87ec60f5f908915db5495cf76
```

LIS.h
Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}\left(N \log N\right)$
17 lines

```cpp
template<class I> vector<int> lis(const vector<I>& S) {
    if (S.empty()) return {};
```

```cpp
    vector<int> prev(S.size());
    typedef pair<I, int> p;
    vector<p> res;
    for(int i = 0; i < (int)S.size(); i++) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(res.begin(), res.end(), p {S[i],
            ↪0});
        if (it == res.end()) res.emplace_back(), it = res.end()
            ↪-1;
        *it = {S[i], i};
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    }
    int L = res.size(), cur = res.back().second;
    vector<int> ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
} // hash-cpp-all = 0675f2d50356ddedf96d5db7d84ea048
```

LIS2.h
Description: Compute the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

9 lines
```cpp
template<typename T> int lis(const vector<T> &a) {
    vector<T> u;
    for (const T &x : a) {
        auto it = lower_bound(u.begin(), u.end(), x);
        if (it == u.end()) u.push_back(x);
        else *it = x;
    }
    return (int)u.size();
} // hash-cpp-all = 6182d9febfde6942e9eeaee00eec8bed
```

LCS.h
Description: Finds the longest common subsequence.
Memory: $\mathcal{O}(nm)$.
Time: $\mathcal{O}(nm)$ where n and m are the lengths of the sequences.

15 lines
```cpp
template<class T> T lcs(const T &X, const T &Y) {
    int a = X.size(), b = Y.size();
    vector<vvector<int>> dp(a+1, vector<int>(b+1));
    for(int i = 1; i <= a; ++i) for(int j = 1; j <= b; j++)
        dp[i][j] = X[i-1]==Y[j-1] ? dp[i-1][j-1]+1 :
            max(dp[i][j-1],dp[i-1][j]);
    int len = dp[a][b];
    T ans(len, 0);
    while (a && b)
        if (X[a-1] == Y[b-1]) ans[--len] = X[--a], --b;
        else if (dp[a][b-1] > dp[a-1][b]) --b;
        else --a;
    return ans;
}
// hash-cpp-all = 50f3641a242f9568be0038e955aa5cce
```

Knapsack.h
Description: Knapsack 01 problem, returns a vector that holds each chosen item.
Time: $\mathcal{O}(nW)$

16 lines
```cpp
vector<int> Knapsack(int limit, vector<int> &v, vector<int>
    ↪ &w) {
    vector<vector<int>> dp(v.size()+1);
    dp[0].resize(limit+1);
    for (int i = 0; i < v.size(); ++i) {
        dp[i+1] = dp[i];
        for (int j = 0; j <= limit - w[i]; ++j)
            dp[i+1][w[i]+j] = max(dp[i+1][w[i]+j], dp[i][j]
                ↪ + v[i]);
```

```cpp
    }
    vector<int> result;
    for (int i = v.size()-1; i >= 0; --i)
        if (dp[i][limit] != dp[i+1][limit]) {
            limit -= w[i];
            result.push_back(i);
        }
    return result;
} // hash-cpp-all = 2b2ab2ea31c6df6578f0e05563c4ea48
```

01Knapsack.h
Description: Bottom-up is faster in practice
Time: $\mathcal{O}(N \log N)$

35 lines
```cpp
// 1-indexed bottom-up, waaaay faster in practice
int knapsack(int limit, vector<int> &v, vector<int> &w) {
    vector<int> dp(limit+1, -1); int n = w.size();
    dp[0] = 0;
    for (int i = 0; i < n; ++i)
        for (int j = limit; j >= w[i]; --j)
            if (dp[j - w[i]] >= 0)
                dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
    int result = 0;
    for (int i = 0; i <= limit; ++i)
        result = max(result, dp[i]);
    return result;
}
// top-down
int n, c; // total of items and cost
array<int, MAXN> w, v; // weight, value
array<array<int, MAXN>, MAXN> dp; // filled -1

int get(int idx, int cap) {
    if (cap < 0) return -INT_MAX;
    if (idx == n) return 0;
    if (dp[idx][cap] != -1) return dp[idx][cap];
    return dp[idx][cap] = max(get(idx+1, cap), v[idx] + get
        ↪(idx+1, cap - w[idx]));
}

void recover(int idx, int cap) {
    if (idx == n) return;
    int grab = v[idx] + get(idx+1, cap - w[idx]);
    int change = get(idx+1, cap);
    if (grab >= change) {
        items.push_back(idx);
        recover(idx+1, cap - w[idx]);
    }
    else recover(idx+1, cap);
} // hash-cpp-all = bb18afa9ba816d9014e2a84a0f82aa8e
```

LargeKnapsack.h
Description: Knapsack with definition changed. Support large values because the weight isn't a dimension in our dp anymore.
Time: $\mathcal{O}(vW)$ where v is the sum of values.

9 lines
```cpp
constexpr int limit = (int)1e5+10;
int knapsack(int capacity, vector<lint> &v, vector<lint> &w
    ↪) {
    vector<lint> dp(limit, 1LL << 60); dp[0] = 0;
    for (int i = 0; i < v.size(); ++i)
        for (int j = limit - v[i] - 1; j >= 0; --j)
            dp[j + v[i]] = min(dp[j + v[i]], dp[j] + w[i]);
    for (int i = limit-1; i >= 0; --i)
        if (dp[i] <= capacity) return i;
} // hash-cpp-all = bfb05b5c660c8501005ed0788b35cdca
```

KnapsackUnbounded.h
Description: Knapsack problem but repetitions are allowed.
Time: $\mathcal{O}(N \log N)$

10 lines
```cpp
int unbounded_knapsack(vector<int> &v, vector<int> &w, int
    ↪total) {
    vector<int> dp(total+1, -1);
    int result = 0; dp[0] = 0;
    for (int i = 0; i <= total; ++i) for (int j = 0; j < n;
        ↪ ++j)
        if (w[j] <= i && dp[i - w[j]] >= 0)
            dp[i] = max(dp[i], dp[i - w[j]] + v[j]);
    int result = 0;
    for (int i = 0; i <= total; ++i) result = max(result,
        ↪dp[i]);
    return result;
} // hash-cpp-all = 390c2286ce88b58740d71dc5ba395434
```

KnapsackBounded.h
Description: You are given n types of items, you have $e[i]$ items of i-th type, and each item of i-th type weighs $w[i]$ and costs $c[i]$. What is the minimal cost you can get by picking some items weighing at most W in total?
Time: $\mathcal{O}(Wn)$

<MinQueue.h>

28 lines
```cpp
const int maxn = 1000;
const int maxm = 100000;
const int inf = 0x3f3f3f;

minQueue<int> q[maxm];

array<int, maxm> dyn; // the minimum cost dyn[i] I need to
    ↪pay in order to fill the knapsack with total weight i
int w[maxn], e[maxn], c[maxn]; // weight, number, cost

int main() {
    int n, m;
    cin >> n >> m;
    for (int i = 1; i <= n; i++) cin >> w[i] >> c[i] >> e[i];
    for (int i = 1; i <= m; i++) dyn[i] = inf;
    for (int i = 1; i <= n; i++) {
        for (int j = 0; j < w[i]; j++) q[j].clear();
        for (int j = 0; j <= m; j++) {
            minQueue<int> &mq = q[j % w[i]];
            if (mq.size() > e[i]) mq.pop();
            mq.add(c[i]);
            mq.push(dyn[j]);
            dyn[j] = mq.getMin();
        }
    }
    cout << "Minimum value i can pay putting a total weight "
        ↪ << m << " is " << dyn[m] << '\n';
    for (int i = 0; i <= m; i++) cout << dyn[i] << " " << i
        ↪<< '\n';
    cout << "\n";
} // hash-cpp-all = cac0faadab0e006a19e0104670f4b9ef
```

KnapsackBitset.h
Description: Find first value greater than m that cannot be formed by the sums of numbers from v.

12 lines
```cpp
bitset<int(1e7)> dp, dp1;

int knapsack(vector<int> &items, int n, int m) {
    dp[0] = dp1[0] = true;
    for (int i = 0; i < n; ++i) {
        dp1 <<= items[i];
```

```
        dp |= dp1;
        dp1 = dp;
    }
    dp.flip();
    return dp._Find_next(m);
} // hash-cpp-all = 25166e1e3548855c879386d1d513d579
```

TSP.h
Description: Solve the Travelling Salesman Problem.
Time: $\mathcal{O}\left(N^2 * 2^N\right)$
18 lines

```cpp
const int MX = 15;
array<array<int, MX>, 1<<N> dp;
array<array<int, MX>, MX> dist;
int N;

int TSP(int n) {
    dp[0][1] = 0;
    for (int j = 0; j < (1 << n); ++j)
        for (int i = 0; i < n; ++i)
            if (j & (1<<i))
                for (int k = 0; k < n; ++k)
                    if (!(j & (1<<k)))
                        dp[k][j^(1<<k)] = min(dp[k][j^(1<<k
                            ↪)], dp[i][j]+dist[i][k]);
    int ret = (1 << 31); // = INF
    for (int i = 1; i < n; ++i)
        ret = min(ret, dp[i][(1<<n)-1] + dist[i][0]);
    return ret;
} // hash-cpp-all = 9c40a0dd624797eaa12e7898a3960dfd
```

DistinctSubsequences.h
Description: DP eliminates overcounting. Number of different strings that can be generated by removing any number of characters, without changing the order of the remaining.
<ModTemplate.h>
7 lines

```cpp
num tot[30];
num distinct(const string &S) {
    num ans = 1; // tot[i] stands for number of distinct
        ↪strings ending with character 'a'+i
    for(auto &c : S) tie(ans, tot[c-'a']) = make_pair(2*ans
        ↪-tot[c-'a'],ans);
    return ans-1;
}
// hash-cpp-all = 7ec0c8d69757e755ccf5d3d3338a8f92
```

CircularLCS.h
Description: For strings a, b calculates LCS of a with all rotations of b
Time: $\mathcal{O}\left(N^2\right)$
48 lines

```cpp
pair<int,int> dp[2001][4001];
string A,B;

void init() {
    for(int i = 1; i <= A.size(); ++i)
        for(int j = 1; j <= B.size(); ++j) { // naive LCS,
            ↪store where value came from
            pair<int,int>& bes = dp[i][j]; bes = {-1,-1};
            bes = max(bes,{dp[i-1][j].first,0});
            bes = max(bes,{dp[i-1][j-1].first+(A[i-1] == B[j-1])
                ↪,-1});
            bes = max(bes,{dp[i][j-1].first,-2});
            bes.second *= -1;
        }
}
```

```cpp
void adjust(int col) { // remove col'th character of b,
    ↪adjust DP
    int x = 1;
    while (x <= A.size() && dp[x][col].second == 0) x ++;
    if (x > A.size()) return; // no adjustments to dp
    pair<int,int> cur = {x,col}; dp[cur.first][cur.second].
        ↪second = 0;
    while (cur.first <= A.size() && cur.second <= B.size()) {
        // essentially decrease every dp[cur.first][y >= cur.
            ↪second].first by 1
        if (cur.second < B.size() && dp[cur.first][cur.s+1].
            ↪second == 2) {
            cur.second ++;
            dp[cur.first][cur.second].second = 0;
        } else if (cur.first < A.size() && cur.second < B.size
            ↪()
            && dp[cur.first+1][cur.s+1].second == 1) {
            cur.first ++, cur.second ++;
            dp[cur.first][cur.second].second = 0;
        } else cur.first ++;
    }
}
int getAns(pair<int,int> x) {
    int lo = x.second-B.size()/2, ret = 0;
    while (x.first && x.second > lo) {
        if (dp[x.first][x.second].second == 0) x.first --;
        else if (dp[x.first][x.second].second == 1) ret ++, x.
            ↪first --, x.second --;
        else x.second --;
    }
    return ret;
}
int circLCS(str a, str b) {
    A = a, B = b+b; init();
    int ans = 0;
    for(int i = 0; i < B.size(); ++i) {
        ans = max(ans,getAns({A.size(),i+B.size()}));
        adjust(i+1);
    }
    return ans;
} // hash-cpp-all = a573993743cf9eb44b62bfd179cc65a4
```

MaxNonConsecutiveSum.h
Description: Computes the maximum sum of a non consecutive subsequence.
Time: $\mathcal{O}\left(N\right)$
8 lines

```cpp
const int MAXN = 100, MAXM = 100;
array<int, MAXN> A, dp;
int solve(int N) {
    dp[0] = A[0] > 0 ? A[0] : 0;
    if (N > 1) dp[1] = max(dp[0], A[1]);
    for (int i = 2; i < N; i++) dp[i] = max(dp[i - 2] + A[i
        ↪], dp[i - 1]);
    return dp[N - 1];
} // hash-cpp-all = a200bca981e5d321d717bb8cc022d03f
```

MaxSubarraySumSkip.h
Description: Computes the subarray with the maximum sum, removing at most one element from the array.
Time: $\mathcal{O}\left(N\right)$
10 lines

```cpp
const int MAXN = 100, MAXM = 100;
array<int, MAXN> A, fw, bw;
int solve(int N) {
    T curMax = fw[0] = A[0], maxSum = A[0];
```

```cpp
    for (int i = 1; i < N; i++) { fw[i] = curMax = max(A[i
        ↪], curMax + A[i]); maxSum = max(maxSum, curMax); }
    curMax = maxSum = bw[N - 1] = A[N - 1];
    for (int i = N - 2; i >= 0; i--) { bw[i] = curMax = max
        ↪(A[i], curMax + A[i]); maxSum = max(maxSum, curMax
        ↪); }
    for (int i = 1; i < N - 1; i++) maxSum = max(maxSum, fw
        ↪[i - 1] + bw[i + 1]);
    return maxSum;
} // hash-cpp-all = 5c42f35251ae84c29dc5ff6d7dd4ad13
```

MaxZeroSubmatrix.h
Description: Computes the area of the largest submatrix that contains only 0s
Time: $\mathcal{O}\left(NM\right)$
18 lines

```cpp
const int MAXN = 100, MAXM = 100;
array<array<int, MAXN>, MAXM> A, H;
int solve(int N, int M) {
    stack<int, vector<int>> s; int ret = 0;
    for (int j = 0; j < M; j++) for (int i = N - 1; i >= 0;
        ↪ i--) H[i][j] = A[i][j] ? 0 : 1 + (i == N - 1 ? 0
        ↪: H[i + 1][j]);
    for (int i = 0; i < N; i++) {
        for (int j = 0; j < M; j++) {
            int minInd = j;
            while (!s.empty() && H[i][s.top()] >= H[i][j])
                ↪{
                ret = max(ret, (j - s.top()) * (H[i][s.top
                    ↪()])));
                minInd = s.top(); s.pop(); H[i][minInd] = H
                    ↪[i][j];
            }
            s.push(minInd);
        }
        while (!s.empty()) ret = max(ret, (M - s.top()) * H
            ↪[i][s.top()]); s.pop();
    }
    return ret;
} // hash-cpp-all = d7bff28fbc9f249fa8daf1325f932613
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). _GLIBCXX_DEBUG violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).

- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks
10.5.1 Bit hacks

- `x & -x` is the least bit in x.

- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of m (except m itself).

- c = x&-x, r = x+c; (((r^x) >> 2)/c) | r is the next number after x with the same number of bits set.

- rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)]; computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).

- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.

- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastInput.h
Description: Returns an integer. Usage requires your program to pipe in input from file. Can replace calls to $gc()$ with

```cpp
struct GC {
    char buf[1 << 16 | 1];
    int bc = 0, be = 0;
    char operator()() {
        if (bc >= be) {
            be = fread(buf, 1, sizeof(buf) - 1, stdin);
            buf[be] = bc = 0;
        }
        return buf[bc++]; // return 0 on EOF
    }
} gc;

void read_int() {}
template <class T, class... S>
inline void read_int(T &a, S &... b) {
    char c, s = 1;
    while (isspace(c = gc()));
    if (c == '-') s = -1, c = gc();
    for (a = c - '0'; isdigit(c = gc()); a = a * 10 + c - '
        ↪0');
    a *= s;
    read_int(b...);
}

void read_float() {}
template <class T, class... S> inline void read_float(T &a,
    ↪ S &... b) {
    int c, s = 1, fp = 0, fpl = 1;
    while (isspace(c = gc()));
    if (c == '-') s = -1, c = gc();
    for (a = c - '0'; isdigit(c = gc()); a = a * 10 + c - '
        ↪0');
    a *= s;
    if (c == '.')
        for (; isdigit(c = gc()); fp = fp * 10 + c - '0',
            ↪fpl *= 10);
```

```cpp
    a += (double)fp / fpl;
    read_float(b...);
} // hash-cpp-all = de7573cedad7d78ab4967eb4c26e1fc0
```

BumpAllocator.h
Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.
<div align="right">9 lines</div>

```cpp
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf;
    assert(s < i);
    return (void*)&buf[i -= s];
}
void operator delete(void*) {}
// hash-cpp-all = 745db225903de8f3cdfa051660956100
```

SmallPtr.h
Description: A 32-bit pointer that points into BumpAllocator memory.
"BumpAllocator.h"
<div align="right">10 lines</div>

```cpp
template<class T> struct ptr {
    unsigned ind;
    ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
        assert(ind < sizeof buf);
    }
    T& operator*() const { return *(T*)(buf + ind); }
    T* operator->() const { return &**this; }
    T& operator[](int a) const { return (&**this)[a]; }
    explicit operator bool() const { return ind; }
}; // hash-cpp-all = 2dd6c9773f202bd47422e255099f4829
```

BumpAllocatorSTL.h
Description: BumpAllocator for STL containers.
Usage: vector<vector<int, small<int>>> ed(N);
<div align="right">14 lines</div>

```cpp
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
    typedef T value_type;
    small() {}
    template<class U> small(const U&) {}
    T* allocate(size_t n) {
        buf_ind -= n * sizeof(T);
        buf_ind &= 0 - alignof(T);
        return (T*)(buf + buf_ind);
    }
    void deallocate(T*, size_t) {}
}; // hash-cpp-all = bb66d4225a1941b85228ee92b9779d4b
```

Hashmap.h
Description: Faster/better hash maps, taken from CF
<div align="right">14 lines</div>

```cpp
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;

struct custom_hash {
    size_t operator()(uint64_t x) const {
        x += 48;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
```

```cpp
};
gp_hash_table<int, int, custom_hash> safe_table;
// hash-cpp-all = e62eb2668aee2263b6d72043f3652fb2
```

PQueue.h
Description: Efficient priority queue implementation. Initialize with highest possible value. Can obviously be extended to minheap/maxheap.
<div align="right">16 lines</div>

```cpp
template<typename T>
struct PQ {
    int sz;
    vector<T> q;
    T offset=0;
    PQ(int n) : sz(n+1), q(2*n, -n) {}
    T top() { return -q[0]+offset; }
    void push(T x) {
        q[sz++]=-(x-offset);
        push_heap(q.begin(),q.begin()+sz);
    }
    void shift(T x) { offset+=x; }
    void pop() {
        pop_heap(q.begin(),q.begin()+sz); --sz;
    }
}; // hash-cpp-all = ffd97e5129dcbb5cc4a7811cd07f7e37
```

OwnFunctions.h
<div align="right">18 lines</div>

```cpp
template <typename T>
T mabs(T v) {
    return v < 0 ? -v : v;
}


template <typename T>
T mceil(T v) {
    T x = ceil((long double)v) - 1.0;
    while (x < v) x += 1.0;
    return x;
}


template <typename T>
T mfloor(T v) {
    T x = floor((long double)v) + 1.0;
    while (x > v) x -= 1.0;
    return x;
} // hash-cpp-all = 786225192828898cbd2b5b423b2ec67b
```

10.6 Bit Twiddling Hack
Hacks.h
<div align="right">51 lines</div>

```cpp
// Returns one plus the index of the least significant 1-
    ↪bit of x, or if x is zero, returns zero.
__builtin_ffs(x)

// Returns the number of leading 0-bits in x, starting at
    ↪the most significant bit position. If x is 0, the
    ↪result is undefined.
__builtin_clz(x)

// Returns the number of trailing 0-bits in x, starting at
    ↪the least significant bit position. If x is 0, the
    ↪result is undefined.
__builtin_ctz(x)

// Returns the number of 1-bits in x.
__builtin_popcount(x)
```

```cpp
// For long long versions append ll (e.g.
    ↪__builtin_popcountll)

// Least significant bit in x.
x & -x

// Iterate on non-empty submasks of a bitmask.
for (int submask = mask; submask > 0; submask = (mask & (
    ↪submask - 1)))

// Iterate on non-zero bits of a bitset.
for (int j = btset._Find_next(0); j < MAXV; j = btset.
    ↪_Find_next(j))

int __builtin_clz(int x); // number of leading zero
int __builtin_ctz(int x); // number of trailing zero
int __builtin_clzll(lint x); // number of leading zero
int __builtin_ctzll(lint x); // number of trailing zero
int __builtin_popcount(int x); // number of 1-bits in x
int __builtin_popcountll(lint x); // number of 1-bits in x

// compute next perm. i.e. 00111, 01011, 01101, 10011, ...
lint next_perm(lint v) {
    lint t = v | (v-1);
    return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v)
        ↪ + 1));
}

template<typename F>  // All subsets of size k of {0..N-1}
void iterate_k_subset(ll N, ll k, F f){
  ll mask = (1ll << k) - 1;
  while (!(mask & 1ll<<N)) { f(mask);
    ll t = mask | (mask-1);
    mask = (t+1) | (((~t & -~t) - 1) >> (__builtin_ctzll(
        ↪mask)+1));
  }
}
template<typename F>  // All subsets of set
void iterate_mask_subset(ll set, F f){ ll mask = set;
  do  f(mask), mask = (mask-1) & set;
  while (mask != set);
} // hash-cpp-all = 59c333b5627ba2e7fea7f2a5da6d2881
```

Bitset.h
Description: Some bitset functions
<div align="right">18 lines</div>

```cpp
int main() {
    bitset<100> bt;
    cin >> bt;

    cout << bt[0] << "\n";
    cout << bt.count() << "\n"; // number of bits set
    cout << (~bt).none() << "\n"; // return true if has no
        ↪bits set
    cout << (~bt).any() << "\n"; // return true if has any
        ↪bit set
    cout << (~bt).all() << "\n"; // retun true if has all
        ↪bits set
    cout << bt._Find_first() << "\n"; // return first set
        ↪bit
    cout << bt._Find_next(10) << "\n";// returns first set
        ↪bit after index i
    cout << bt.flip() << '\n'; // flip the bitset
    cout << bt.test(3) << '\n'; // test if the ith bit of
        ↪bt is set
    cout << bt.reset(3) << '\n'; // reset the ith bit
    cout << bt.set() << '\n'; // turn all bits on
```

```cpp
    cout << bt.set(4, 1) << '\n'; // set the 4th bit to
        ↪value 1
    cout << bt << "\n";
} // hash-cpp-all = b9f55a20e426e6ea81485e438f9f3325
```

10.7   Random Numbers

RandomNumbers.h
Description: An example on the usage of generator and distribution.
<div align="right">5 lines</div>

```cpp
mt19937_64 mt (time (0));
uniform_int_distribution <int> uid (1, 100);
uniform_real_distribution <double> urd (1, 100);
cout << uid (mt) << " " << urd (mt) << "\n";
// hash-cpp-all = 63c591021510cd5bc0d42c6bb21c7c51
```

10.8   Other languages

Python3.py
<div align="right">50 lines</div>

```python
/**
 * Author: BenQ
 * Description: python3 (not pypy3) demo, solves
 * CF Good Bye 2018 Factorisation Collaboration
 * Source: own
 * Verification:
 * https://codeforces.com/contest/1091/problem/G
 * https://open.kattis.com/problems/catalansquare
 */

from math import *
import sys
import random

def nextInt():
  return int(input())
def nextStrs():
  return input().split()
def nextInts():
  return list(map(int,nextStrs()))

n = nextInt()
v = [n]
def process(x):
  global v
  x = abs(x)
  V = []
  for t in v: # print(type(t)) -> <class 'int'>
    g = gcd(t,x)
    if g != 1:
      V.append(g)
    if g != t:
      V.append(t//g)
  v = V
for i in range(50):
  x = random.randint(0,n-1)
  if gcd(x,n) != 1:
    process(x)
  else:
    sx = x*x%n # assert(gcd(sx,n) == 1)
    print(f"sqrt {sx}") # print value of var
    sys.stdout.flush()
    X = nextInt()
    process(x+X)
    process(x-X)
print(f'! {len(v)}',end='')
for i in v:
  print(f' {i}',end='')
```

```python
print()
sys.stdout.flush()
```

Main.java
Description: Basic template/info for Java
<div align="right">15 lines</div>

```java
import java.util.*;
import java.math.*;
import java.io.*;

public class Main {
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader(new
        ↪InputStreamReader(System.in));
    PrintStream out = System.out;
    StringTokenizer st = new StringTokenizer(br.readLine())
        ↪;
    assert st.hasMoreTokens(); // enable with java -ea main
    out.println("v=" + Integer.parseInt(st.nextToken()));
    ArrayList<Integer> a = new ArrayList<>();
    a.add(1234); a.get(0); a.remove(a.size()-1); a.clear();
  }
}
```

MiscJava.java
Description: Basic template/info for Java
<div align="right">47 lines</div>

```java
import java.math.BigInteger;
import java.util.*;

public class prob4 {
  void run() {
    Scanner scanner = new Scanner(System.in);
    while (scanner.hasNextBigInteger()) {
      BigInteger n = scanner.nextBigInteger();
      int k = scanner.nextInt();
      if (k == 0) {
        for (int p = 2; p <= 100000; p++) {
          BigInteger bp = BigInteger.valueOf(p);
          if (n.mod(bp).equals(BigInteger.ZERO)) {
            System.out.println(bp.toString() + " * " + n.
                ↪divide(bp).toString());
            break;
          }
        }
      } else {
        BigInteger ndivk = n.divide(BigInteger.valueOf(k));
        BigInteger sqndivk = sqrt(ndivk);
        BigInteger left = sqndivk.subtract(BigInteger.
            ↪valueOf(100000)).max(BigInteger.valueOf(2));
        BigInteger right = sqndivk.add(BigInteger.valueOf
            ↪(100000));
        for (BigInteger p = left; p.compareTo(right) != 1;
            ↪p = p.add(BigInteger.ONE)) {
          if (n.mod(p).equals(BigInteger.ZERO)) {
            BigInteger q = n.divide(p);
            System.out.println(p.toString() + " * " + q.
                ↪toString());
            break;
          }
        }
      }
    }
  }
  BigInteger sqrt(BigInteger n) {
    BigInteger left = BigInteger.ZERO;
    BigInteger right = n;
```

```
    while (left.compareTo(right) != 1) {
      BigInteger mid = left.add(right).divide(BigInteger.
        ↪valueOf(2));
      int s = n.compareTo(mid.multiply(mid));
      if (s == 0) return mid;
      if (s > 0) left = mid.add(BigInteger.ONE); else right
        ↪ = mid.subtract(BigInteger.ONE);
    }
    return right;
  }
  public static void main(String[] args) {
    (new prob4()).run();
  }
}
```

10.8.1 BigInteger

BigInteger   To convert to a `BigInteger`, use `BigInteger.valueOf (int)` or `BigInteger (String, radix)`.

To convert from a `BigInteger`, use `.intValue ()`, `.longValue ()`, `.toString (radix)`.

Common unary operations include `.abs ()`, `.negate ()`, `.not ()`.

Common binary operations include `.max`, `.min`, `.add`, `.subtract`, `.multiply`, `.divide`, `.remainder`, `.gcd`, `.modInverse`, `.and`, `.or`, `.xor`, `.shiftLeft (int)`, `.shiftRight (int)`, `.pow (int)`, `.compareTo`.

Divide and remainder: `Biginteger[] .divideAndRemainder (Biginteger val)`.

Power module: `.modPow (BigInteger exponent, module)`.

Primality check: `.isProbablePrime (int certainty)`.