Federal University of Rio de Janeiro

Sunflowers

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Contest (1)

template.cpp

17 lines

```
#include <bits/stdc++.h>
using namespace std;

using lint = long long;
using ldouble = long double;
const double PI = static_cast<double>(acosl(-1.0));

// Retorns −1 if a < b, 0 if a = b and 1 if a > b.
int cmp_double(double a, double b = 0, double eps = 1e-9) {
    return a + eps > b ? b + eps > a ? 0 : 1 : -1;
}

int main() {
    cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);

}
```

.bashrc

3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++14 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
```

.vimrc

6 lines

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on   |   im jk <esc>   |   im kj <esc>   |   no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by = e \\ cx + dy = f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Master theorem

Given a recurrence of the form $T(n) = aT(\frac{n}{b}) + f(n)$ where $a \geq 1$, $b > 1$.

1) If $f(n) = \mathcal{O}(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$, then

$$T(n) = \Theta(n^{\log_b a})$$

2) If $f(n) = \Theta(n^{\log_b a})$, then

$$T(n) = \Theta(n^{\log_b a} \log n)$$

3) If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$ (and $af(\frac{n}{b}) \leq cf(n)$ for some $c < 1$ for all n sufficiently large), then

$$T(n) = \Theta(f(n))$$

2.4 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.5 Geometry

2.5.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$s_a = \sqrt{bc\left[1 - \left(\dfrac{a}{b+c}\right)^2\right]}$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

Pick's: A polygon on an integer grid strictly containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$. (Nothing similar in higher dimensions)

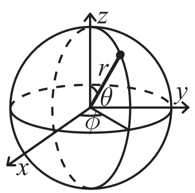
2.5.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.5.3 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \mathrm{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \mathrm{atan2}(y, x) \end{aligned}$$

2.5.4 Centroid of a polygon

The x coordinate of the centroid of a polygon is given by $\frac{1}{3A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$, where A is twice the signed area of the polygon.

2.6 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\mathrm{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6.1 XOR sum

$$\bigoplus_{x=0}^{n-1} x = \{0, n-1, 1, n\}[n \bmod 4]$$
$$\bigoplus_{x=l}^{r-1} x = \bigoplus_{a=0}^{r-1} a \oplus \bigoplus_{b=0}^{l-1} b$$

2.7 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.8 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.9 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.9.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.9.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a<b$.

$$f(x)=\begin{cases}\frac{1}{b-a} & a<x<b\\ 0 & \text{otherwise}\end{cases}$$

$$\mu=\frac{a+b}{2},\ \sigma^2=\frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda>0$.

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$$

$$\mu=\frac{1}{\lambda},\ \sigma^2=\frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu,\sigma^2)$, $\sigma>0$.

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1\sim\mathcal{N}(\mu_1,\sigma_1^2)$ and $X_2\sim\mathcal{N}(\mu_2,\sigma_2^2)$ then

$$aX_1+bX_2+c\sim\mathcal{N}(\mu_1+\mu_2+c,a^2\sigma_1^2+b^2\sigma_2^2)$$

2.10 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1,X_2,\ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P}=(p_{ij})$, with $p_{ij}=\Pr(X_n=i|X_{n-1}=j)$, and $\mathbf{p}^{(n)}=\mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)}=\Pr(X_n=i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi=\pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i=\frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty}\mathbf{P}^k=\mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii}=1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i\in\mathbf{A}$, when the initial state is j, is $a_{ij}=p_{ij}+\sum_{k\in\mathbf{G}}a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i=1+\sum_{k\in\mathbf{G}}p_{ki}t_k$.

A Markov chain is an absorbing chain if
1. there is at least one absorbing state and
2. it is possible to go from any state to at least one absorbing state in a finite number of steps.

Data Structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

UnionFind.h

Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$

7d5db8, 14 lines

```
struct UF {
  vector<int> e;
  UF(int n) : e(n, -1) {}
  bool same_set(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
  bool unite(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return 0;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;
    return 1;
  }
};
```

DSURoll.h

Description: Disjoint-set data structure with undo.
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}(\log(N))$

7ddf1d, 21 lines

```
struct RollbackUF {
    vector<int> e; vector<pair<int,int>> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return st.size(); }
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool unite(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
};
```

MinQueue.h

Description: Structure that supports all operations of a queue and get the minimum/maximum active value in the queue. Useful for sliding window 1D and 2D. For 2D problems, you will need to pre-compute another matrix, by making a row-wise traversal, and calculating the min/max value beginning in each cell. Then you just make a column-wise traverse as they were each an independent array.
Time: $\mathcal{O}(1)$

d40e77, 24 lines

```
template<typename T>
struct minQueue {
  int lx, rx, sum;
  deque<pair<T, T>> q;
  minQueue() { lx = 1; rx = 0; sum = 0; }
  void clear() { lx = 1, rx = 0, sum = 0; q.clear(); }
  void push(T delta) {
    // q.back().first + sum <= delta for a maxQueue
    while(!q.empty() && q.back().first + sum >= delta)
      q.pop_back();
    q.emplace_back(delta - sum, ++rx);
  }
  void pop() {
    if (!q.empty() && q.front().second == lx++)
      q.pop_front();
  }
  void add(T delta) {
    sum += delta;
  }
  T getMin() {
    return q.front().first + sum;
  }
  int size() { return rx-lx+1; }
};
```

LazySegmentTree.h

Description: Better SegTree. Range Sum, can be extended to max/min/product/gcd, pay attention to propagate, f and update functions when extending. Be careful with each initialization aswell.
Time: $\mathcal{O}(\lg(N))$

f2afda, 44 lines

```
template<typename T, typename Q> struct segtree_t {
    int n;
    vector<T> tree;
    vector<Q> lazy, og;
    segtree_t(int N) : n(N), tree(4*N), lazy(4*N) {}
    segtree_t(const vector<Q> &other) : n(other.size()), og(
        other),
    tree(4*n), lazy(4*n) { build(1, 0, n-1); }
    T f(const T &a, const T &b) { return (a + b); }
    T build(int v, int l, int r) {
        lazy[v] = 0;
```

```
        if (l == r) return tree[v] = og[l];
        int m = l + (r - l)/2;
        return tree[v] = f(build(v<<1, l, m), build(v<<1|1, m+1, r));
    }
    void propagate(int v, int l, int r) {
        if (!lazy[v]) return;
        int m = l + (r - l)/2;
        tree[v<<1] += lazy[v] * (m - l + 1);
        tree[v<<1|1] += lazy[v] * (r - (m + 1) + 1);
        for (int i = 0; i < 2; ++i) lazy[v<<1|i] += lazy[v];
        lazy[v] = 0;
    }
    T query(int a, int b) { return query(a, b, 1, 0, n-1); }
    T query(int a, int b, int v, int l, int r) {
        if (b < l || r < a) return 0;
        if (a <= l && r <= b) return tree[v];
        propagate(v, l, r);
        int m = l + (r - l)/2;
        return f(query(a, b, v<<1,l, m), query(a, b, v<<1|1, m+1, r));
    }
    T update(int a, int b, Q delta) { return update(a, b, delta, 1, 0, n-1); }
    T update(int a, int b, Q delta, int v, int l, int r) {
        if (b < l || r < a) return tree[v];
        if (a <= l && r <= b) {
            tree[v] += delta * (r - l + 1);
            lazy[v] += delta;
            return tree[v];
        }
        propagate(v, l, r);
        int m = l + (r - l)/2;
        return tree[v] = f(update(a, b, delta, v<<1, l, m),
            update(a, b, delta, v<<1|1, m+1, r));
    }
};
```

DynamicSegTree.h

Description: Dynamic Segment Tree with lazy propagation. Allows range query, range update (increment and assignment). For assignment change all += to = in push and update functions. If not using lazy, remove all push related function calls.

Usage: `node *segtree = build(0, n);`

Time: $\mathcal{O}(\lg(N))$

8a81e8, 71 lines

```
struct node {
  node *l, *r;
  int maxv, sumv, lazy;
  int lx, rx;
};
void push(node *v) {
  if(v != nullptr && v->lazy) {
    v->maxv += v->lazy;
    v->sumv += v->lazy * (v->rx - v->lx + 1);
    if(v->l) v->l->lazy += v->lazy;
    if(v->r) v->r->lazy += v->lazy;
    v->lazy = 0;
  }
}
void update(node *v, int lx, int rx, int delta) {
  push(v);
  if(rx < v->lx || v->rx < lx) return;
  if(lx <= v->lx && v->rx <= rx) {
    v->lazy += delta;
    push(v);
    return;
  }
  update(v->l, lx, rx, delta);
  update(v->r, lx, rx, delta);
  push(v->l);
  v->maxv = max(v->l->maxv, v->r->maxv);
  v->sumv = v->l->sumv + v->r->sumv;
}
int mquery(node *v, int lx, int rx) {
  push(v);
  if(rx < v->lx || v->rx < lx) return -1;
  if(lx <= v->lx && v->rx <= rx) return v->maxv;
  return max(mquery(v->l, lx, rx), mquery(v->r, lx, rx));
}
int squery(node *v, int lx, int rx) {
  push(v);
  if(rx < v->lx || v->rx < lx) return 0;
  if(lx <= v->lx && v->rx <= rx) return v->sumv;
  return squery(v->l, lx, rx) + squery(v->r, lx, rx);
}
int find_first(node *v, int lx, int rx, int delta) { // st pos >= delta
  if (rx < v->lx || v->rx < lx || v->maxv < delta) return -1;
  if (v->lx == v->rx) return v->lx;
  int x = find_first(v->l, lx, rx, delta);
  if (x != -1) return x;
  return find_first(v->r, lx, rx, delta);
}
int find_last(node *v, int lx, int rx, int delta) { // last pos >= delta
  if (rx < v->lx || v->rx < lx || v->maxv < delta) return -1;
  if (lx == rx) return lx;
  int x = find_last(v->r, lx, rx, delta);
  if (x != -1) return x;
  return find_last(v->l, lx, rx, delta);
}
node *build(int lx, int rx) {
  node *v = new node();
  v->lx = lx; v->rx = rx;
  if(lx == rx) {
    v->lazy = 0;
    v->l = v->r = nullptr;
    v->maxv = v->sumv = 0;
  }
  else {
    v->l = build(lx, (lx + rx)/2);
    v->r = build((lx + rx)/2 + 1, rx);
    v->maxv = max(v->l->maxv, v->r->maxv);
    v->sumv = v->l->sumv + v->r->sumv;
    v->lazy = 0;
  }
  return v;
}
```

SparseSegTree.h

Description: Sparse Segment Tree with point update. Doesnt allocate storage for nodes with no data. Use BumpAllocator for better performance!

014f68, 39 lines

```
const int SZ = 1<<19;
template<class T> struct node_t {
  T delta = 0; node_t<T>* c[2];
  node_t() { c[0] = c[1] = nullptr; }
  void upd(int pos, T v, int L = 0, int R = SZ-1) { // add v
    if (L == pos && R == pos) { delta += v; return; }
    int M = (L + R)>>1;
    if (pos <= M) {
      if (!c[0]) c[0] = new node_t();
      c[0]->upd(pos, v, L, M);
    } else {
      if (!c[1]) c[1] = new node_t();
      c[1]->upd(pos, v, M+1, R);
    }
    delta = 0;
    for (int i = 0; i < 2; ++i) if (c[i]) delta += c[i]->delta;
  }
  T query(int lx, int rx, int L = 0, int R = SZ-1) { // query sum of segment
    if (rx < L || R < lx) return 0;
    if (lx <= L && R <= rx) return delta;
    int M = (L + R)>>1; T res = 0;
    if (c[0]) res += c[0]->query(lx, rx, L, M);
    if (c[1]) res += c[1]->query(lx, rx, M+1, R);
    return res;
  }
  void upd(int pos, node_t *a, node_t *b, int L = 0, int R = SZ-1) {
    if (L != R) {
      int M = (L + R)>>1;
      if (pos <= M) {
        if (!c[0]) c[0] = new node_t();
        c[0]->upd(pos, a ? a->c[0] : nullptr, b ? b->c[0] : nullptr, L, M);
      } else {
        if (!c[1]) c[1] = new node_t();
        c[1]->upd(pos,a ? a->c[1] : nullptr, b ? b->c[1] : nullptr, M+1, R);
      }
    }
    delta = (a ? a->delta : 0)+(b ? b->delta : 0);
  }
};
```

SegTree2D.h

Description: 2D Segment Tree.

"SparseSegtree.h" 09098e, 25 lines

```
template<class T> struct Node {
    node_t<T> seg; Node* c[2];
    Node() { c[0] = c[1] = nullptr; }
    void upd(int x, int y, T v, int L = 0, int R = SZ-1) { // add v
        if (L == x && R == x) { seg.upd(y,v); return; }
        int M = (L+R)>>1;
        if (x <= M) {
            if (!c[0]) c[0] = new Node();
            c[0]->upd(x,y,v,L,M);
        } else {
            if (!c[1]) c[1] = new Node();
            c[1]->upd(x,y,v,M+1,R);
        }
        seg.upd(y,v); // only for addition
        // seg.upd(y,c[0]?&c[0]->seg:nullptr,c[1]?&c[1]->seg:nullptr);
    }
    T query(int x1, int x2, int y1, int y2, int L = 0, int R = SZ-1) { // query sum of rectangle
        if (x1 <= L && R <= x2) return seg.query(y1,y2);
        if (x2 < L || R < x1) return 0;
        int M = (L+R)>>1; T res = 0;
        if (c[0]) res += c[0]->query(x1, x2, y1, y2, L, M);
        if (c[1]) res += c[1]->query(x1, x2, y1, y2, M+1, R);
        return res;
    }
};
```

PersistentSegTree.h

Description: Persistent implementation of a segment tree. This one compute the kth smallest element in a subarray $[a, b]$.

b6eb05, 47 lines

```
struct segtree_t {
  struct snapshot {
    int cnt, linkl, linkr;
```

```
    snapshot() : cnt(0), linkl(0), linkr(0) {}
    snapshot(int cnt, int l, int r) : cnt(cnt), linkl(l), linkr(r) {}
  };
  int id;
  vector<snapshot> tree;
  segtree_t(int n) : id(1), tree(20*n) {}
  int update(int v, int l, int r, int x) {
    if (x < l || x > r) return v;
    if (l == r) {
      tree[id] = snapshot(1, 0, 0);
      return id++;
    }
    int m = l + (r - l)/2;
    int lx = update(tree[v].linkl, l, m, x);
    int rx = update(tree[v].linkr, m+1, r, x);
    tree[id] = snapshot(tree[lx].cnt + tree[rx].cnt, lx, rx);
    return id++;
  }
  int query(int a, int b, int l, int r, int k) {
    if (l == r) return l;
    int m = l + (r - l)/2;
    int cnt = tree[tree[b].linkl].cnt - tree[tree[a].linkl].cnt;
    if (k <= cnt)
      return query(tree[a].linkl, tree[b].linkl, l, m, k);
    return query(tree[a].linkr, tree[b].linkr, m+1, r, k-cnt);
  }
};
int main() {
  int n, q; cin >> n;
  segtree_t seg(n); vector<int> root(n+1), b(n), a(n);
  for (int i = 0; i < n; ++i) {
    cin >> a[i]; b[i] = a[i];
  }
  sort(b.begin(), b.end());
  for (int i = 0; i < n; ++i) {
    a[i] = lower_bound(b.begin(), b.end(), a[i]) - b.begin();
    root[i+1] = seg.update(root[i], 0, n-1, a[i]);
  }
  cin >> q;
  for (int i = 0; i < q; ++i) {
    int k, l, r; cin >> k >> l >> r;// kth smallest in range [l, r)
    cout << b[seg.query(root[l-1], root[r], 0, n-1, k)] << '\n';
  }
}
```

MergeSortTree.h

Description: Build segment tree where each node stores a sorted version of the underlying range.
Time: $\mathcal{O}(\log^2 N)$

342dec, 36 lines

```
struct MergeSortTree {
  vector<int> v, id;
  vector<vector<int>> tree;
  MergeSortTree(vector<int> &v) : v(v), tree(4*(v.size()+1)) {
    for(int i = 0; i < v.size(); ++i) id.push_back(i);
    sort(id.begin(), id.end(), [&v](int i, int j) { return v[i] < v[j]; });
    build(1, 0, v.size()-1);
  }
  void build(int id, int left, int right) {
    if (left == right) tree[id].push_back(id[left]);
    else {
      int mid = (left + right)>>1;
      build(id<<1, left, mid);
      build(id<<1|1, mid+1, right);
      tree[id] = vector<int>(right - left + 1);
      merge(tree[i<<1].begin(), tree[i<<1].end(),
        tree[id<<1|1].begin(), tree[id<<1|1].end(),
        tree[id].begin());
    }
  }
  // how many elements in this node have id in the range [a,b]
  int how_many(int id, int a, int b) {
    return (int)(upper_bound(tree[id].begin(), tree[id].end(), b)
      - lower_bound(tree[id].begin(), tree[id].end(), a));
  }
  int query(int id, int left, int right, int a, int b, int x) {
    if (left == right) return v[tree[id].back()];
    int mid = (left + right)>>1;
    int lcount = how_many(id<<1, a, b);
    if (lcount >= x) return query(id<<1, left, mid, a, b, x);
    else return query(id<<1|1, mid+1, right, a, b, x - lcount);
  }
  int kth(int a, int b, int k) {
    return query(1, 0, v.size()-1, a, b, k);
  }
};
```

RMQ.h

Description: Range Minimum/Maximum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time. Returns a pair that holds the answer, first element is the value and the second is the index.
Usage: `rmq_t<int> rmq(values);`
`rmq.query(inclusive, inclusive);`
`rmq_t<int, greater<pair<int,int>>> rmq(values) //max query`
Time: $\mathcal{O}(|V|\log|V| + Q)$

4fc7f7, 21 lines

```
// change cmp for max query or similar
template<typename T, typename Cmp=less<pair<T, int>>>
struct rmq_t {
  Cmp cmp; vector<vector<pair<T, int>>> table;
  rmq_t() {}
  rmq_t(const vector<T> &values) {
    int n = values.size();
    table.resize(__lg(n)+1);
    table[0].resize(n);
    for (int i = 0; i < n; ++i) table[0][i] = {values[i], i};
    for (int l = 1; l < (int)table.size(); ++l) {
      table[l].resize(n - (1<<l) + 1);
      for (int i = 0; i + (1<<l) <= n; ++i)
        table[l][i] = min(table[l-1][i], table[l-1][i+(1<<(l-1))], cmp);
    }
  }
  pair<T, int> query(int a, int b) {
    int l = __lg(b-a+1);
    return min(table[l][a], table[l][b-(1<<l)+1], cmp);
  }
};
```

RSQ.h

Description: Range Sum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: `rsq_t<int> rsq(values);`
`rsq.query(inclusive, inclusive);`
Time: $\mathcal{O}(|V|\log|V| + Q)$

74c891, 24 lines

```
template<typename T>
struct rsq_t {
  vector<vector<T>> table;
  rsq_t() {}
  rsq_t(const vector<T> &values) {
    int n = values.size();
    table.resize(__lg(n)+1); table[0].resize(n);
    for (int i = 0; i < n; ++i) table[0][i] = values[i];
    for (int l = 1; l < (int)table.size(); ++l) {
      table[l].resize(n - (1<<l) + 1);
      for (int i = 0; i + (1<<l) <= n; ++i)
        table[l][i] = table[l-1][i] + table[l-1][i+(1<<(l-1))];
    }
  }
  T query(int a, int b) {
    int l = b - a + 1; T ret{};
    for (int i = (int)table.size(); i >= 0; --i)
      if ((1 << i) <= l) {
        ret += table[i][a]; a += (1<<i);
        l = b - a + 1;
      }
    return ret;
  }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

d5645d, 22 lines

```
template<typename T> struct FT {
  vector<T> s;
  FT(int n) : s(n) {}
  void update(int pos, T dif) { // a[pos] += dif
    for (; pos < (int)s.size(); pos |= pos + 1) s[pos] += dif;
  }
  T query(int pos) { // sum of values in [0, pos)
    T res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(T sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= (int)s.size() && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 4b694a, 21 lines

```
template<typename T> struct FT2 {
  vector<vector<int>> ys; vector<FT<T>> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < (int)ys.size(); x |= x + 1) ys[x].push_back(y);
  }
```

```
  void init() {
    for(auto v : ys) sort(v.begin(), v.end()), ft.emplace_back(
         v.size());
  }
  int ind(int x, int y) {
    return (int)(lower_bound(ys[x].begin(), ys[x].end(), y) -
         ys[x].begin()); }
  void update(int x, int y, T dif) {
    for (; x < ys.size(); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  T query(int x, int y) {
    T sum = 0;
    for (; x; x &= x - 1) sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

Mo.h

Description: Mo's algorithm example problem: Count how many elements appear at least two times in given range $[l, r]$. For path queries on trees, flatten the tree by DFSing and pushing even-depth nodes at entry and odd-depth nodes at exit. If you need to squeeze Mo's in the TL and Q is greater than N, consider Hilbert Curves. Will work much faster.
Time: $(n+q)sqrt(n)$

33f45f, 29 lines

```
struct query_t { int l, r, id; };
int n, m, total = 0; // elements, queries, result.
const int sqn = sqrt(n), maxv = 1000000;
vector<int> values(n), freq(2*maxv), result(m);
vector<query_t> queries(m);
sort(queries.begin(), queries.end(), [sqn](const query_t &a,
     const query_t &b) {
  if (a.l/sqn != b.l/sqn) return a.l < b.l;
  return a.r < b.r;
});
int l = 0, r = -1;
for(query_t &q : queries) {
  auto add = [&](int i) {
    // Change if needed
    ++freq[values[i]];
    if (freq[values[i]] == 2) total += 2;
    else if (freq[values[i]] > 2) ++total;
  };
  auto del = [&](int i) {
    // Change if needed
    --freq[values[i]];
    if (freq[values[i]] == 1) total -= 2;
    else if (freq[values[i]] > 1) --total;
  };
  while(r < q.r) add(++r);
  while(l > q.l) add(--l);
  while(r > q.r) del(r--);
  while(l < q.l) del(l++);
  result[q.id] = total;
}
```

MisofTree.h

Description: A simple treedata structure for inserting, erasing, and querying the n^{th} largest element.
Time: $\mathcal{O}(\alpha(N))$

8c50f4, 15 lines

```
const int BITS = 15;
struct misof_tree{
    int cnt[BITS][1<<BITS];
    misof_tree() {memset(cnt, 0, sizeof cnt);}
    void add(int x, int dv) {
        for (int i = 0; i < BITS; cnt[i++][x] += dv, x >>= 1);
            }
    void del(int x, int dv) {
        for (int i = 0; i < BITS; cnt[i++][x] -= dv, x >>= 1);
            }
    int nth(int n) {
        int r = 0, i = BITS;
        while(i--) if (cnt[i][r <<= 1] <= n)
            n -= cnt[i][r], r |= 1;
        return r;
    }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

a7bd5b, 29 lines

```
struct Line {
  mutable lint k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(lint x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const lint inf = LLONG_MAX;
  lint div(lint a, lint b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(lint k, lint m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  lint query(lint x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

Matrix.h

Description: Basic operations on square matrices.
Usage: `Matrix<int, 3> A;`
`A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

ac7897, 28 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M &m) const {
    M a;
    for(int i = 0; i < N; ++i)
        for(int j = 0; j < N; ++j)
          for(int k = 0; k < N; ++k) a.d[i][j] += d[i][k]*m.d[k
              ][j];
    return a;
  }
  vector<T> operator*(const vector<T> &vec) const {
    vector<T> ret(N);
    for(int i = 0; i < N; ++i)
        for(int j = 0; j < N; ++j) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(T p) const {
    assert(p >= 0);
    M a, b(*this);
    for(int i = 0; i < N; ++i) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

SubMatrix.h

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
Usage: `SubMatrix<int> m(matrix);`
`m.sum(0, 0, 2, 2); // top left 4 elements`
Time: $\mathcal{O}(N^2+Q)$

cd3f87, 14 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = v.size(), C = v[0].size();
    p.assign(R+1, vector<T>(C+1));
    for (int r = 0; r < R; ++r)
        for (int c = 0; c < C; ++c)
          p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][
              c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

Wavelet.h

Description: Segment tree on values instead of indices. kth return the largest number in 0-indexed interval. count return the number of elements of a[i, j) that belong in [x, y].
Time: $\mathcal{O}(\log(n))$

ccafbc, 25 lines

```
template<int SZ> struct Wavelet {
  vector<int> L[SZ], R[SZ];
  void build(vector<int> &a, int v=1, int l=0, int r=SZ-1) {
    if (l == r) return;
    L[v] = R[v] = {0};
    vector<int> A[2]; int m = l + (r-l)/2;
    for(auto &t : a) {
      A[t>m].push_back(t);
      L[v].push_back(A[0].size()), R[v].push_back(A[1].size());
    }
    build(A[0],2*v,l,m), build(A[1],2*v+1,m+1,r);
  }
  int kth(int i,int j,int k,int v=1,int l=0,int r=SZ-1) { // [i
       , j)!!
    if (l == r) return l;
    int m = l + (r - l)/2, t = L[v][j]-L[v][i];
    if (t >= k) return kth(L[v][i], L[v][j],k,2*v,l,m);
    return kth(R[v][i], R[v][j],k-t,2*v+1,m+1,r);
  }
  int count(int i,int j,int x, int y, int v=1,int l=0,int r=SZ
       -1) {
    if (y < l || r < x) return 0; //count(i, j, x, y) retorna o
         numero de elementos
    if (x <= l && r <= y) return j - i; // de a[i, j) que
         pertencem a [x, y]
```

```
    int m = l + (r - l)/2;
    return count(L[v][i], L[v][j], x, y, 2*v, l, m) + count(i-L
        [v][i], j-L[v][j], x, y, 2*v+1, m+1, r);
  }
};
```

Numerical (4)

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

Polynomial.h

84593c, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = a.size(); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    for(int i = 1; i < a.size(); ++i) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i = a.size()-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
        b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h" 49396a, 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax) {
  if ((p.a).size() == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(dr.begin(), dr.end());
  for(int i = 0; i < dr.size()-1; ++i) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign^(p(h) > 0)) {
      for(int it = 0; it < 60; ++it) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}(n^2)$

97a266, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  for(int k = 0; k < n-1; ++k) for(int i = k+1; i < n; ++i)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  for(int k = 0; k < n; ++k) for(int i = 0; i < n; ++i) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

Lagrange.h

Description: Lagrange Polynomials.
Time: $\mathcal{O}(N)$

"ModPow.h", "ModInv.h", "Factorial.h" 31ad4a, 29 lines

```
template<typename T> struct Lagrange {
  const int n;
  vector<T> f, den;
  Lagrange(vector<T> other) : f(other), n(other.size()) {
    den.resize(n);
    for(int i = 0; i < n; ++i) {
      f[i] = (f[i] % mod + mod) % mod;
      den[i] = ifact[n-i-1] * ifact[i] % mod;
      if((n-i-1) % 2 == 1)
        den[i] = (mod - den[i]) % mod;
    }
  }
  T interpolate(T x) {
    x %= mod;
    vector<T> l, r;
    l.resize(n); r.resize(n);
    l[0] = r[n-1] = 1;
    for (int i = 1; i < n; ++i)
      l[i] = l[i-1] * (x - (i-1) + mod) % mod;
    for (int i = n-2; i >= 0; --i)
      r[i] = r[i+1] * (x - (i+1) + mod) % mod;
    T ans = 0;
    for (int i = 0; i < n; ++i) {
      T coef = l[i] * r[i] % mod;
      ans = (ans + coef * f[i] % mod * den[i]) % mod;
    }
    return ans;
  }
};
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}(N^2)$

"ModularArithmetic.h" 4c4a48, 19 lines

```
template <typename num>
vector<num> BerlekampMassey(const vector<num>& s) {
  int n = int(s.size()), L = 0, m = 0;
  vector<num> C(n), B(n), T;
  C[0] = B[0] = 1;
  num b = 1;
  for(int i = 0; i < n; i++) { ++m;
    num d = s[i];
    for (int j = 1; j <= L; j++) d += C[j] * s[i - j];
    if (d == 0) continue;
    T = C; num coef = d / b;
    for (int j = m; j < n; j++) C[j] -= coef * B[j - m];
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
  C.resize(L + 1); C.erase(C.begin());
  for (auto& x : C) x = -x;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}(n^2 \log k)$

"ModularArithmetic.h" 0baa7b, 22 lines

```
template <typename num>
num linearRec(const vector<num>& S, const vector<num>& tr, lint
    k) {
  int n = int(tr.size());
  assert(S.size() >= tr.size());
  auto combine = [&](vector<num> a, vector<num> b) {
    vector<num> res(n * 2 + 1);
    for (int i = 0; i <= n; i++) for (int j = 0; j <= n; j++)
        res[i + j] += a[i] * b[j];
    for (int i = 2 * n; i > n; --i) for (int j = 0; j < n; j++)
      res[i - 1 - j] += res[i] * tr[j];
    res.resize(n + 1);
    return res;
  };
  vector<num> pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  num res = 0;
  for (int i = 0; i < n; i++) res += pol[i + 1] * S[i];
  return res;
}
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

47a385, 14 lines

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
  pair<double, P> cur(f(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
```

```
    for(int j = 0; j < 100; ++j) for(int dx = -1; dx < 2; ++dx)
      for(int dy = -1; dy < 2; ++dy) {
        P p = cur.second;
        p[0] += dx*jmp;
        p[1] += dy*jmp;
        cur = min(cur, {f(p), p});
      }
  }
  return cur;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

7bb98e, 7 lines

```
template<class F>
double quad(double a, double b, F& f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  for(int i = 1; i < n*2; ++i)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.

Usage:
```
double sphereVolume = quad(-1, 1, [](double x) {
return quad(-1, 1, [&](double y) {
return quad(-1, 1, [&](double z) {
return x*x + y*y + z*z < 1; });});});
```

92dd79, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

5906bc, 15 lines

```
double det(vector<vector<double>> &a) {
  int n = a.size(); double res = 1;
  for(int i = 0; i < n; ++i) {
    int b = i;
    for(int j = i+1; j < n; ++j) if (fabs(a[j][i]) > fabs(a[b][
        i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    for(int j = i+1; j < n; ++j) {
      double v = a[j][i] / a[i][i];
      if (v != 0) for(int k = i+1; k < n; ++k) a[j][k] -= v * a
          [i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

6ddd70, 18 lines

```
const lint mod = 12345;
lint det(vector<vector<lint>>& a) {
  int n = a.size(); lint ans = 1;
  for(int i = 0; i < n; ++i) {
    for(int j = i+1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        lint t = a[i][i] / a[j][i];
        if (t) for(int k = i; k < n; ++k)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

Elimination.h

Description: Gauss-Jordan algorithm. Transform a matrix into its row echelon form. Returns a vector of pivots (for each variable) or -1 if free variable.

9c481f, 28 lines

```
vector<int> ToRowEchelon(vector<vector<double>> &M) {
    int cons = M.size(), vars = M[0].size() - 1;
    vector<int> pivot(vars, -1);
    int cur = 0;
    for (int var = 0; var < vars; ++var) {
        if (cur >= cons) continue;
        for (int con = cur + 1; con < cons; ++con)
            if(M[con][var] > M[cur][var])
                swap(M[con], M[cur]);
      if (abs(M[cur][var]) > kEps) {
            pivot[var] = cur;
            double aux = M[cur][var];
            for (int i = 0; i <= vars; ++i)
                M[cur][i] /= aux;
            for (int con = 0; con < cons; ++con) {
                if (con != cur) {
                    double mul = M[con][var];
                    for (int i = 0; i <= vars; ++i) {
                        M[con][i] -= mul * M[cur][i];
                    }
                    assert(M[con][var] < kEps);
                }
            }
            ++cur;
        }
    }
    return pivot;
}
```

Math-Simplex.cpp

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$.
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case. WARNING- segfaults on empty (size 0) max cx st Ax<=b, x>=0 do 2 phases; 1st check feasibility; 2nd check boundedness and ans

c3703c, 39 lines

```
vector<double> simplex(vector<vector<double>> A, vector<double>
      b, vector<double> c) {
    int n = A.size(), m = A[0].size() + 1, r = n, s = m-1;
    vector<vector<double>> D = vector<vector<double>>(n+2,
        vector<double>(m+1));
    vector<int> ix = vector<int>(n + m);
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m-1; ++j) D[i][j] = -A[i][j];
        D[i][m - 1] = 1;
        D[i][m] = b[i];
        if (D[r][m] > D[i][m]) r = i;
    }
    for (int j = 0; j < m-1; ++j) D[n][j] = c[j];
    D[n + 1][m - 1] = -1; int z = 0;
    for (double d;;) {
        if (r < n) {
            swap(ix[s], ix[r + m]);
            D[r][s] = 1.0/D[r][s];
            for (int j = 0; j <= m; ++j) if (j != s) D[r][j] *=
                -D[r][s];
            for (int i = 0; i <= n+1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s) D[i][j
                    ] += D[r][j] * D[i][s];
                D[i][s] *= D[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j) if (s < 0 || ix[s] > ix[j])
            if (D[n+1][j] > eps || D[n+1][j] > -eps && D[n][j]
                > eps) s = j;
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (D[i][s] < -eps) {
            if (r < 0 || (d = D[r][m]/D[r][s]-D[i][m]/D[i][s])
                < -eps
                || d < eps && ix[r+m] > ix[i+m]) r = i;
        }
        if (r < 0) return vector<double>(); // unbounded
    }
    if (D[n+1][m] < -eps) return vector<double>(); //
        infeasible
    vector<double> x(m-1);
    for (int i = m; i < n+m; ++i) if (ix[i] < m-1) x[ix[i]] = D
        [i-m][m];
    double result = D[n][m];
    return x; // ans: D[n][m]
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$

2654db, 36 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd> &A, vd &b, vd &x) {
  int n = A.size(), m = x.size(), rank = 0, br, bc;
  if (n) assert(A[0].size() == m);
  vector<int> col(m); iota(col.begin(), col.end(), 0);
  for(int i = 0; i < n; ++i) {
    double v, bv = 0;
    for(int r = i; r < n; ++r) for(int c = i; c < m; ++c)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      for(int j = i; j < n; ++j) if (fabs(b[j]) > eps) return
          -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
```

```
    swap(col[i], col[bc]);
    for(int j = 0; j < n; ++j) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    for(int j = i+1; j < n; ++j) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      for(int k = i+1; k < m; ++k) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }
  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    for(int j = 0; j < i; ++j) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" c8e85a, 7 lines

```
for(int j = 0; j < n; ++j) if (j != i) // instead of for(int j = i+1; j < n; ++j)
// ... then at the end:
x.assign(m, undefined);
for(int i = 0; i < rank; ++i) {
  for(int j = rank; j < m; ++j) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}\left(n^2 m\right)$

71d871, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs> &A, vector<int> &b, bs& x, int m) {
  int n = A.size(), rank = 0, br;
  assert(m <= x.size());
  vector<int> col(m); iota(col.begin(), col.end(), 0);
  for(int i = 0; i < n; ++i) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for(int j = 0; j < n; ++j) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    for(int j = i+1; j < n; ++j) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    for(int j = 0; j < i; ++j) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, foreatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}\left(n^3\right)$

4f2f15, 32 lines

```
int matInv(vector<vector<double>>& A) {
  int n = A.size(); vector<int> col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  for(int i = 0; i < n; ++i) tmp[i][i] = 1, col[i] = i;
  for(int i = 0; i < n; ++i) {
    int r = i, c = i;
    for(int j = i; j < n; ++j) for(int k = i; k < n; ++k)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    for(int j = 0; j < n; ++j)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    for(int j = i+1; j < n; ++j) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      for(int k = i+1; k < n; ++k) A[j][k] -= f*A[i][k];
      for(int k = 0; k < n; ++k) tmp[j][k] -= f*tmp[i][k];
    }
    for(int j = i+1; j < n; ++j) A[i][j] /= v;
    for(int j = 0; j < n; ++j) tmp[i][j] /= v;
    A[i][i] = 1;
  }
  for (int i = n-1; i > 0; --i) for(int j = 0; j < i; ++j) {
    double v = A[j][i];
    for(int k = 0; k < n; ++k) tmp[j][k] -= v*tmp[i][k];
  }
  for(int i = 0; i < n; ++i) for(int j = 0; j < n; ++j) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}\left(n^3\right)$

"../number-theory/ModPow.h" 594b17, 36 lines

```
int matInv(vector<vector<lint>>& A) {
  int n = A.size(); vi col(n);
  vector<vector<lint>> tmp(n, vector<lint>(n));
  for(int i = 0; i < n; ++i) tmp[i][i] = 1, col[i] = i;

  for(int i = 0; i < n; ++i) {
    int r = i, c = i;
    for(int j = i; j < n; ++j) for(int k = i; k < n; ++k) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    for(int j = 0; j < n; ++j) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    lint v = modpow(A[i][i], mod - 2);
    for(int i = i+1; i < n; ++i) {
      lint f = A[j][i] * v % mod;
      A[j][i] = 0;
      for(int k = i+1; k < n; ++k) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
      for(int k = 0; k < n; ++k) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    }
    for(int j = i+1; j < n; ++j) A[i][j] = A[i][j] * v % mod;
    for(int j = 0; j < n; ++j) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) for(int j = 0; j < i; ++j) {
    lint v = A[j][i];
    for(int k = 0; k < n; ++k) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  }

  for(int i = 0; i < n; ++i) for(int j = 0; j < n; ++j)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod : 0);
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \leq i \leq n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \\ \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$

d0855f, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T> &super,
    const vector<T> &sub, vector<T> b) {
  int n = b.size(); vector<int> tr(n);
  for(int i = 0; i < n-1; ++i) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
```

```
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

NewtonMethod.h

Description: Root find method

a8a5ba, 12 lines

```
double f(double x) { return (x*x) - 4; }
double df(double x) { return 2*x; }
double root(double x0) {
    const double eps = 1e-15;
    double x = x0;
    while (1) {
        double nx = x - (f(x)/df(x));
        if (abs(x - nx) < eps) break;
        x = nx;
    }
    return x;
}
```

NewtonSQRT.h

Description: Square root find method

25ff80, 21 lines

```
double sqrt_newton(double n) {
    const double eps = 1E-15;
    double x = 1;
    while (1) {
        double nx = (x + n / x) / 2;
        if (abs(x - nx) < eps) break;
        x = nx;
    }
    return x;
}
int isqrt_newton(int n) {
    int x = 1;
    bool decreased = false;
    while (1) {
        int nx = (x + n / x) >> 1;
        if (x == nx || nx > x && decreased) break;
        decreased = nx < x;
        x = nx;
    }
    return x;
}
```

Polyominoes.h

Description: Generate all free polyominoes with n squares. Takes less than a sec if $n < 10$, around 2s if $n = 10$ and around 6s if $n = 11$.

d63122, 34 lines

```
using pii = pair<int,int>;
vector<int> diri = {0, 1, 0, -1};
vector<int> dirj = {1, 0, -1, 0};
vector<vector<pii>> poly[LIM];
void generate(int n){
  poly[1] = {{{0 , 0}}};
  for(int i = 2 ; i <= n; i++) {
    set<vector<pii>> cur_om;
    for(auto &om : poly[i-1]) {
      pii mini = om[0];
      for(auto &p : om)
        for(int d = 0; d < 4; d++) {
          int x = p.st + diri[d], y = p.nd + dirj[d];
          if(!binary_search(om.begin(), om.end(), pii(x,y))) {
            pii m = min(mini, {x, y});
            pii new_cell(x - m.st, y - m.nd);
            bool new_in = false;
            vector<pii> norm;
            for(pii &pn : om) {
              pii cur(pn.st - m.st, pn.nd - m.nd);
              if(cur > new_cell && !new_in) {
                new_in = true;
                norm.push_back(new_cell);
              }
              norm.push_back(cur);
            }
            if(!new_in) norm.push_back(new_cell);
            if(!cur_om.count(norm)) cur_om.insert(norm);
          }
        }
    }
    poly[i].assign(cur_om.begin(),cur_om.end());
  }
}
```

4.1 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use long doubles/NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ (~1s for $N = 2^{22}$)

b9be02, 35 lines

```
using doublex = complex<long double>;
struct FFT {
    vector<doublex> fft(vector<doublex> y, bool invert = false)
        {
        const int N = y.size(); assert(N == (N&-N));
        vector<lint> rev(N);
        for (int i = 1; i < N; ++i) {
            rev[i] = (rev[i>>1]>>1) | (i&1 ? N>>1 : 0);
            if (rev[i] < i) swap(y[i], y[rev[i]]);
        }
        vector<doublex> rootni(N/2);
        for (lint n = 2; n <= N; n *= 2) {
            const doublex rootn = polar(1.0, (invert ? +1.0 :
                -1.0) * 2.0*acos(-1.0)/n);
            rootni[0] = 1.0;
            for (lint i = 1; i < n/2; ++i) rootni[i] = rootni[i
                -1] * rootn;
            for (lint left = 0; left != N; left += n) {
                const lint mid = left + n/2;
                for (lint i = 0; i < n/2; ++i) {
                    const doublex temp = rootni[i] * y[mid + i
                        ];
                    y[mid + i] = y[left + i] - temp; y[left + i
                        ] += temp;
                }
            }
        } if (invert) for (auto &v : y) v /= (doublex)N;
        return move(y);
    }
    uint nextpow2(uint v) { return v ? 1 << __lg(2*v-1) : 1; }
    vector<doublex> convolution(vector<doublex> a, vector<
        doublex> b) {
        const lint n = max((int)a.size()+(int)b.size()-1, 0),
            n2 = nextpow2(n);
        a.resize(n2); b.resize(n2);
        vector<doublex> fa = fft(move(a)), fb = fft(move(b)), &
            fc = fa;
        for (lint i = 0; i < n2; ++i) fc[i] = fc[i] * fb[i];
        vector<doublex> c = fft(move(fc), true);
        c.resize(n);
        return move(c);
    }
};
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

dbcea8, 85 lines

```
typedef unsigned int uint;
typedef long double ldouble;

template<typename T, typename U, typename B> struct ModularFFT
    {
    inline T ifmod(U v, T mod) { return v >= (U)mod ? v - mod :
        v; }
    T pow(T x, U y, T p) {
        T ret = 1, x2p = x;
        while (y) {
            if (y % 2) ret = (B)ret * x2p % p;
            y /= 2; x2p = (B)x2p * x2p % p;
        }
        return ret;
    }
    vector<T> fft(vector<T> y, T mod, T gen, bool invert =
        false) {
        int N = y.size(); assert(N == (N&-N));
        if (N == 0) return move(y);
        vector<int> rev(N);
        for (int i = 1; i < N; ++i) {
            rev[i] = (rev[i>>1]>>1) | (i&1 ? N>>1 : 0);
            if (rev[i] < i) swap(y[i], y[rev[i]]);
        }
        assert((mod-1)%N == 0);
        T rootN = pow(gen, (mod-1)/N, mod);
        if (invert) rootN = pow(rootN, mod-2, mod);
        vector<T> rootni(N/2);
        for (int n = 2; n <= N; n *= 2) {
            T rootn = pow(rootN, N/n, mod);
            rootni[0] = 1;
            for (int i = 1; i < n/2; ++i) rootni[i] = (B)rootni
                [i-1] * rootn % mod;
            for (int left = 0; left != N; left += n) {
                int mid = left + n/2;
                for (int i = 0; i < n/2; ++i) {
                    T temp = (B)rootni[i] * y[mid+i] % mod;
                    y[mid+i] = ifmod((U)y[left+i] + mod - temp,
                        mod);
                    y[left+i] = ifmod((U)y[left+i] + temp, mod)
                        ;
                }
            }
        }
        if (invert) {
            T invN = pow(N, mod-2, mod);
            for (T &v : y) v = (B)v * invN % mod;
        }
        return move(y);
    }
    vector<T> convolution(vector<T> a, vector<T> b, T mod, T
        gen) {
        int N = a.size()+b.size()-1, N2 = nextpow2(N);
        a.resize(N2); b.resize(N2);
        vector<T> fa = fft(move(a), mod, gen), fb = fft(move(b)
            , mod, gen), &fc = fa;
        for (int i = 0; i < N2; ++i) fc[i] = (B)fc[i] * fb[i] %
            mod;
        vector<T> c = fft(move(fc), mod, gen, true);
```

```
        c.resize(N); return move(c);
    }
    vector<T> self_convolution(vector<T> a, T mod, T gen) {
        int N = 2*a.size()-1, N2 = nextpow2(N);
        a.resize(N2);
        vector<T> fc = fft(move(a), mod, gen);
        for (int i = 0; i < N2; ++i) fc[i] = (B)fc[i] * fc[i] % mod;
        vector<T> c = fft(move(fc), mod, gen, true);
        c.resize(N); return move(c);
    }
    uint nextpow2(uint v) { return v ? 1 << __lg(2*v-1) : 1; }
};

const int mod = 998244353, mod_gen = 3;

vector<int> convolute(const vector<int> &a, const vector<int> & b) {
    if (a.empty() || b.empty()) return {};
    ModularFFT<int, uint, lint> modular_fft;
    return modular_fft.convolution(a, b, mod, mod_gen);
}

vector<int> convolute_all(const vector<vector<int>> &polys, int begin,
                          int end) {
    if (end - begin == 0) return {1};
    else if (end - begin == 1) return polys[begin];
    else {
        int mid = begin + (end - begin) / 2;
        return convolute(convolute_all(polys, begin, mid),
                         convolute_all(polys, mid, end));
    }
}

vector<int> convolute_all(const vector<vector<int>> &polys) {
  return convolute_all(polys, 0, (int)polys.size());
}
```

NumberTheoreticTransform.h

Description: Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. Inputs must be in [0, mod).

Time: $\mathcal{O}(N \log N)$

"../number-theory/modpow.h" 1f6be8, 32 lines

```
const lint mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.

typedef vector<lint> vl;
void ntt(vl& a, vl& rt, vl& rev, int n) {
  for(int i = 0; i < n; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) for(int j = 0; j < k; ++j) {
        lint z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
        a[i + j + k] = (z > ai ? ai - z + mod : ai - z);
        ai += (ai + z >= mod ? z - mod : z);
  }
}

vl conv(const vl& a, const vl& b) {
  if (a.empty() || b.empty())
    return {};
  int s = a.size()+b.size()-1, B = 32 - __builtin_clz(s), n = 1 << B;
  vl L(a), R(b), out(n), rt(n, 1), rev(n);
  L.resize(n), R.resize(n);
  for(int i = 0; i < n; ++i) rev[i] = (rev[i / 2] | (i & 1) << B) / 2;
  lint curL = mod / 2, inv = modpow(n, mod - 2);
  for (int k = 2; k < n; k *= 2) {
    lint z[] = {1, modpow(root, curL /= 2)};
    for(int i = k; i < 2*k; ++i) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  ntt(L, rt, rev, n); ntt(R, rt, rev, n);
  for(int i = 0; i <n; ++i) out[-i & (n-1)] = L[i] * R[i] % mod * inv % mod;
  ntt(out, rt, rev, n);
  return {out.begin(), out.begin() + s};
}
```

4.1.1 Duality

max $c^T x$ sjt to $Ax \leq b$. Dual problem is min $b^T x$ sjt to $A^T x \geq c$. By strong duality, min max value coincides.

4.1.2 Generating functions

A list of generating functions for useful sequences:

$(1, 1, 1, 1, 1, 1, \ldots)$	$\frac{1}{1-z}$
$(1, -1, 1, -1, 1, -1, \ldots)$	$\frac{1}{1+z}$
$(1, 0, 1, 0, 1, 0, \ldots)$	$\frac{1}{1-z^2}$
$(1, 0, \ldots, 0, 1, 0, 1, 0, \ldots, 0, 1, 0, \ldots)$	$\frac{1}{1-z^2}$
$(1, 2, 3, 4, 5, 6, \ldots)$	$\frac{1}{(1-z)^2}$
$(1, \binom{m+1}{m}, \binom{m+2}{m}, \binom{m+3}{m}, \ldots)$	$\frac{1}{(1-z)^{m+1}}$
$(1, c, \binom{c+1}{2}, \binom{c+2}{3}, \ldots)$	$\frac{1}{(1-z)^c}$
$(1, c, c^2, c^3, \ldots)$	$\frac{1}{1-cz}$
$(0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$	$\ln \frac{1}{1-z}$

A neat manipulation trick is:

$$\frac{1}{1-z} G(z) = \sum_n \sum_{k \leq n} g_k z^n$$

4.1.3 Polyominoes

How many free (rotation, reflection), one-sided (rotation) and fixed n-ominoes are there?

n	3	4	5	6	7	8	9	10
free	2	5	12	35	108	369	1.285	4.655
one-sided	2	7	18	60	196	704	2.500	9.189
fixed	6	19	63	216	760	2.725	9.910	36.446

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

565374, 55 lines

```
template <int MOD_>  struct modnum {
private:
  using lint = long long;
  lint v;
  static int modinv(int a, int m) {
    a %= m;
    assert(a);
    return a == 1 ? 1 : int(m - lint(modinv(m, a)) * lint(m) / a);
  }
public:
  static constexpr int MOD = MOD_;
  modnum() : v(0) {}
  modnum(lint v_) : v(int(v_ % MOD)) { if (v < 0) v += MOD; }
  explicit operator int() const { return v; }
  friend std::ostream &operator<<(std::ostream& out, const modnum& n) { return out << int(n); }
  friend std::istream &operator>>(std::istream& in, modnum& n) { lint v_; in >> v_; n = modnum(v_); return in; }
  friend bool operator==(const modnum& a, const modnum& b) {
      return a.v == b.v; }
  friend bool operator!=(const modnum& a, const modnum& b) {
      return a.v != b.v; }
  modnum inv() const {
    modnum res;
    res.v = modinv(v, MOD);
    return res;
  }
  modnum neg() const {
    modnum res;
    res.v = v ? MOD-v : 0;
    return res;
  }
  modnum operator-() const { return neg(); }
  modnum operator+() const { return modnum(*this); }
  modnum& operator+=(const modnum& o) {
    v += o.v;
    if (v >= MOD) v -= MOD;
    return *this;
  }
  modnum& operator-=(const modnum& o) {
    v -= o.v;
    if (v < 0) v += MOD;
    return *this;
  }
  modnum& operator*=(const modnum& o) {
    v = int(lint(v) * lint(o.v) % MOD);
    return *this;
  }
  modnum& operator/=(const modnum& o) { return *this *= o.inv(); }
  friend modnum operator+(const modnum& a, const modnum& b) {
      return modnum(a) += b; }
  friend modnum operator-(const modnum& a, const modnum& b) {
      return modnum(a) -= b; }
  friend modnum operator*(const modnum& a, const modnum& b) {
      return modnum(a) *= b; }
  friend modnum operator/(const modnum& a, const modnum& b) {
      return modnum(a) /= b; }
};
template <typename T> T pow(T a, lint b) {
  assert(b >= 0);
  T r = 1; while (b) { if (b & 1) r *= a; b >>= 1; a *= a; }
      return r;
}
using num = modnum<int(1e9)+7>;
```

PairNumTemplate.h

Description: Support pairs operations using modnum template. Pretty good for string hashing.

229a89, 42 lines

```cpp
template <typename T, typename U> struct pairnum {
  T t; U u;
  pairnum() : t(0), u(0) {}
  pairnum(long long v) : t(v), u(v) {}
  pairnum(const T& t_, const U& u_) : t(t_), u(u_) {}
  friend std::ostream& operator << (std::ostream& out, const
       pairnum& n) { return out << '(' << n.t << ',' << ' ' <<
       n.u << ')'; }
  friend std::istream& operator >> (std::istream& in, pairnum&
       n) { long long v; in >> v; n = pairnum(v); return in; }
  friend bool operator == (const pairnum& a, const pairnum& b)
       { return a.t == b.t && a.u == b.u; }
  friend bool operator != (const pairnum& a, const pairnum& b)
       { return a.t != b.t || a.u != b.u; }
  pairnum inv() const {
    return pairnum(t.inv(), u.inv());
  }
  pairnum neg() const {
    return pairnum(t.neg(), u.neg());
  }
  pairnum operator- () const {
    return pairnum(-t, -u);
  }
  pairnum operator+ () const {
    return pairnum(+t, +u);
  }
  pairnum& operator += (const pairnum& o) {
    t += o.t; u += o.u;
    return *this;
  }
  pairnum& operator -= (const pairnum& o) {
    t -= o.t; u -= o.u;
    return *this;
  }
  pairnum& operator *= (const pairnum& o) {
    t *= o.t; u *= o.u;
    return *this;
  }
  pairnum& operator /= (const pairnum& o) {
    t /= o.t; u /= o.u;
    return *this;
  }
  friend pairnum operator + (const pairnum& a, const pairnum& b
       ) { return pairnum(a) += b; }
  friend pairnum operator - (const pairnum& a, const pairnum& b
       ) { return pairnum(a) -= b; }
  friend pairnum operator * (const pairnum& a, const pairnum& b
       ) { return pairnum(a) *= b; }
  friend pairnum operator / (const pairnum& a, const pairnum& b
       ) { return pairnum(a) /= b; }
};
```

ModInv.h

Description: Find x such that $ax \equiv 1 (\text{mod } m)$. The inverse only exist if a and m are coprimes.

bf7b7c, 15 lines

```cpp
template<typename T>
T modinv(T a, T m) {
  assert(m > 0);
  if (m == 1) return 0;
  a %= m;
  if (a < 0) a += m;
  assert(a != 0);
  if (a == 1) return 1;
  return m - modinv(m, a) * m/a;
}
// Iff mod is prime
lint modinv(lint a) { return modpow(a % mod, mod-2); }
const lint mod = 1000000007, LIM = 200000;
lint* inv = new lint[LIM] - 1; inv[1] = 1;
for(int i = 2; i < LIM; ++i) inv[i] = mod - (mod/i) * inv[mod%i
     ] % mod;
```

Modpow.h

94d296, 11 lines

```cpp
lint modpow(lint a, lint e) {
  if (e == 0) return 1;
  lint x = modpow(a * a % mod, e >> 1);
  return e & 1 ? x * a % mod : x;
}
lint modpow(lint b, lint e) {
  lint ret = 1;
  for (int i = 1; i <= e; i *= 2, b = b * b % mod)
    if (i & e) ret = ret * b % mod;
  return ret;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

`modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c)\%m$. `divsum` is similar but for floored division.

Time: $\log(m)$, with a large constant.

decfb8, 17 lines

```cpp
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }
ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
  }
  return res;
}
lint modsum(ull to, lint c, lint k, lint m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMul.cpp

Description: Modular multiplication operation

ffdf54, 10 lines

```cpp
inline lint mul(lint a, lint b, lint m) {
  if (m <= 1000000000) return a * b % m;
  else if (m <= 1000000000000ll) return (((a*(b>>20)%m)<<20)+(a
       *(b&((1<<20)-1))))%m;
  else {
    lint x = (lint)floor(a*(long double)b/m+0.5);
    lint ret = (a*b - x*m) % m;
    if (ret < 0) ret += m;
    return ret;
  }
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b \le c \le 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

59afa8, 11 lines

```cpp
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  lint ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (lint)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).

Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 09107e, 23 lines

```cpp
lint sqrt(lint a, lint p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  lint s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  lint x = modpow(a, (s + 1) / 2, p);
  lint b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    lint t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    lint gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

MulOrder.h

Description: Find the smallest integer k such that $a^k \pmod m = 1$. $0 < k < m$.

Time: close to $\mathcal{O}(log(N))$

<Sieve.h>, <Divisors.h>, <PrimeFactors.h>, <Modpow.h> cb76aa, 16 lines

```cpp
template<typename T> T mulOrder(T a, T m) {
    auto pf = prime_factorize(m);
    T res = 1;
    for (auto &[p, e] : pf) {
      T k = 0, q = Pow(p, e);
      T t = q / p * (p - 1);
      auto factors = divisors(t);
      for (auto &pr : factors)
        if (modpow(a, pr, m) == 1) {
          k = pr;
          break;
        }
      res = res/__gcd(res, k) * k;
    }
    return res;
}
```

Quadratic.h

Description: Solve $x^2 \equiv n \mod p (0 \le a < p)$ where p is prime in $O(\log p)$. If $p > n$, factorize p and solve each of $x^2 \equiv n \mod p_i \forall i$.

<Modpow.h> 2d1b00, 20 lines

```cpp
void mul(lint &a1, lint &b1, lint a2, lint b2, lint w, lint p)
     {
  lint t1 = (a1*a2 + b1*b2 % p*w), t2 = (a1*b2 + a2*b1);
  a1 = t1 % p, b1 = t2 % p;
}
```

```
int Pow(lint a, lint w, lint b, lint p) {
  lint res1=1, res2=0, c1=a, c2=1;
  for (;b;b>>=1) { if (b&1) mul(res1,res2,c1,c2,w,p); mul(c1,c2
      ,c1,c2,w,p); }
  return res1;
}
int quadratic(lint n, int p) {
  lint a, r = 0; n %= p;
  if (p == 2) return -1;
  if (n == 0) return 0;
  if (modpow(n, p/2, p) != 1) return -1;
  do a = rng() % (p-1)+1; while((modpow(a*a-n, p/2, p)-1) % p
      == 0);
  r = Pow(a, (a*a-n) % p, (p+1)/2, p);
  if (r < 0) r += p;
  assert((r*r-n) % p == 0);
  return r;
}
```

5.2 Primality

5.2.1 Xudyh's Sieve

$F(n) = \sum_{d|n} f(d)$

$S(n) = \sum_{i \le n} f(i) = \sum_{i \le n} F(i) - \sum_{d=2}^{n} S\left(\left\lfloor \frac{n}{d} \right\rfloor\right)$

Preprocess $S(1)$ to $S(M)$ (Set $M = n^{\frac{2}{3}}$ for complexity)

$S(n) = \sum f(i) = \sum_{i \le n} \left[F(i) - \sum_{j|i, j \ne i} f(j)\right] = \sum F(i) - \sum_{i/j=d=2}^{n} \sum_{dj \le n} f(j)$

$S(n) = \sum i f(i) = \sum_{i \le n} i \left[F(i) - \sum_{j|i, j \ne i} f(j)\right] = \sum iF(i) - \sum_{i/j=d=2}^{n} \sum_{dj \le n} dj f(j)$

$\sum_{d|n} \varphi(d) = n$ $\sum_{d|n} \mu(d) =$ if $(n > 1)$ then 0 else 1 $\sum_{d|n} (\mu(\frac{n}{d}) \sum_{e|d} f(e)) = f(n)$

Sieve.h

Description: Prime sieve for generating all primes up to a certain limit. $lp[i]$ is the lowest prime factor of i. Also useful if you need to compute any multiplicative function (in this case Moebius..).
Time: $\mathcal{O}(n)$

cd705e, 14 lines

```
vector<int> lp, primes, mu;
void run_sieve(int n) {
  lp.resize(n+1); mu.assign(n+1,-1);
  mu[1] = 1; int p;
  iota(lp.begin(), lp.end(), 0);
  for (int i = 2; i <= n; ++i){
    if (lp[i] == i)  primes.push_back(i);
    for (int j = 0; j < primes.size() && (p = primes[j]*i) <= n
        ; ++j) {
      lp[p] = primes[j];
      mu[p] *= mu[i];
      if (i % primes[j] == 0) { mu[p] = 0; break; }
    }
  }
}
```

SegmentedSieve.h

Description: Prime sieve for generating all primes smaller than S.
Time: S=1e9 $\approx$ 1.5s

68455e, 20 lines

```
const int S = 1e6;
bitset<S> isPrime;
vector<int> eratosthenes() {
  const int S = round(sqrt(S)), R = S/2;
  vector<int> pr = {2}, sieve(S+1); pr.reserve(int(S/log(S)
      *1.1));
  vector<pair<int,int>> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i*i/2});
    for (int j = i*i; j <= S; j += 2*i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    for (int i = 0; i < min(S, R - L); ++i)
      if (!block[i]) pr.push_back((L + i)*2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

Mobius.h

Description: If g and f are arithmetic functions . Return 0 if divisible by any perfect square, 1 if has an even quantity of prime numbers and -1 if has an odd quantity of primes.
Time: $\mathcal{O}(sqrt(n))$

61b8ff, 10 lines

```
template<typename T> T mobius(T n) {
    T p = 0, aux = n;
    for (int i = 2; i*i <= n; ++i)
        if (n % i == 0) {
            n /= i;
            p += 1;
            if (n % i == 0) return 0;
        }
    return (p&1 || n == 1? 1 : -1);
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to 2^{64}; for larger numbers, extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" bbee97, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  vector<ull> A = {2, 325, 9375, 28178, 450775, 9780504,
      1795265022};
  ull s = __builtin_ctzll(n-1), d = n >> s;
  for(ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a % n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factorize.h

Description: Get all factors of n.
Time: $\mathcal{O}(sqrt(N)/log(N))$

46ea35, 17 lines

```
vector<pair<int, int>> factorize(int value) {
    vector<pair<int, int>> result;
    for (int p = 2; p*p <= value; ++p)
        if (value % p == 0) {
            int exp = 0;
            while (value % p == 0) {
                value /= p;
                ++exp;
            }
            result.emplace_back(p, exp);
        }
    if (value != 1) {
        result.emplace_back(value, 1);
        value = 1;
    }
    return result;
}
```

PollardRho.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" aeb78d, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return mod_mul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 0, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = mod_mul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n/x);
  l.insert(l.end(), r.begin(), r.end());
  return l;
}
```

5.3 Divisibility

ExtendedEuclidean.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

7bbefb, 10 lines

```
template<typename T>
T egcd(T a, T b, T &x, T &y) {
    if (a == 0) {
        x = 0, y = 1;
        return b;
    }
    T p = b/a, g = egcd(b - p * a, a, y, x);
    x -= y * p;
    return g;
}
```

DiophantineEquation.h

Description: Check if a the Diophantine Equation $ax + by = c$ has solution.

bea4d6, 34 lines

```
template<typename T>
bool diophantine(T a, T b, T c, T &x, T &y, T &g) {
    if (a == 0 && b == 0) {
        if (c == 0) {
            x = y = g = 0;
            return true;
        }
        return false;
    }
    if (a == 0) {
        if (c % b == 0) {
            x = 0; y = c / b; g = abs(b);
            return true;
        }
        return false;
    }
    if (b == 0) {
        if (c % a == 0) {
            x = c / a; y = 0; g = abs(a);
```

```
            return true;
        }
        return false;
    }
    g = egcd<lint>(a, b, x, y);
    if (c % g != 0) return false;
    T dx = c / a;
    c -= dx * a;
    T dy = c / b;
    c -= dy * b;
    x = dx + (T) ((__int128) x * (c / g) % b);
    y = dy + (T) ((__int128) y * (c / g) % a);
    g = abs(g);
    return true; // |x|, |y| <= max(|a|, |b|, |c|)
}
```

Divisors.h

Description: Get all divisors of n.

bcec3e, 10 lines

```
vector<int> divisors(int n) {
    vector<int> result, aux;
    for (int i = 1; i*i <= n; ++i)
        if (n % i == 0) {
            result.push_back(i);
            if (i*i != n) aux.push_back(n/i);
        }
    for (int i = aux.size()-1; i+1; --i) result.push_back(aux[i
        ]);
    return result;
}
```

Pell.h

Description: Find the smallest integer root of $x^2 - ny^2 = 1$ when n is not a square number, with the solution set $x_{k+1} = x_0x_k + ny_0y_k, y_{k+1} = x_0y_k + y_0x_k$.

21e64f, 16 lines

```
pair<int,int> Pell(int n) {
  int p0 = 0, p1 = 1, q0 = 1, q1 = 0;
  int a0 = (int)sqrt(n), a1 = a0, a2 = a0;
  if(a0 * a0 == n) return {-1, -1};
  int g1 = 0, h1 = 1;
  while (1) {
    int g2 = -g1 + a1 * h1;
    int h2 = (n - g2 * g2)/h1;
    a2 = (g2 + a0)/h2;
    int p2 = a1 * p1 + p0;
    int q2 = a1 * q1 + q0;
    if (p2*p2 - n*q2*q2 == 1) return {p2, q2};
    a1 = a2; g1 = g2; h1 = h2; p0 = p1;
    p1 = p2; q0 = q1; q1 = q2;
  }
}
```

PrimeFactors.h

Description: Find all prime factors of n.
Time: $\mathcal{O}(log(n))$

"Sieve.h" 6af45f, 25 lines

```
template<typename T>
vector<pair<T, int>> prime_factorize(T n) {
    vector<pair<T, int>> factors;
    while(n != 1) {
        T p = lp[n];
        int exp = 0;
        do {
            n /= p;
            ++exp;
        } while(n % p == 0);
        factors.push_back({p, exp});
    }
    for (T p : primes) {
        if (p * p > n) break;
        if (p * p == 0) {
            factors.push_back({p, 0});
            do {
                n /= p;
                ++factors.back().second;
            } while(n % p == 0);
        }
    }
    if (n > 1) factors.push_back({n, 1});
    return factors;
}
```

NumDiv.h

Description: Count the number of divisors of n. Requires having run Sieve up to at least sqrt(n).
Time: $\mathcal{O}(log(N))$

"Sieve.h" be1146, 15 lines

```
template<typename T> T numDiv(T n) {
    T how_many = 1, prime_factors = 0;
    while(n != 1) {
        T p = lp[n];
        int exp = 0;
        do {
            n /= p;
            ++exp;
            ++prime_factors; //count prime factors!
        } while(n % p == 0);
        how_many *= 1ll * (exp + 1);
    }
    if (n != 1) ++prime_factors;
    return how_many;
}
```

SumDiv.h

Description: Sum of all divisors of n.
Time: $\mathcal{O}(log(N))$

"Sieve.h", "Modpow.h" 1ebc7c, 13 lines

```
template<typename T> T divSum(T n) {
    T sum = 1;
    while (n > 1) {
        int exp = 0;
        T p = lp[n];
        do {
            n /= p;
            ++exp;
        } while(n % p == 0);
        sum *= (Pow(p, exp + 1) - 1)/(p - 1);
    }
    return sum;
}
```

Bezout.h

Description: Let $d := mdc(a, b)$. Then, there exist a pair x and y such that $ax + by = d$.

d5ea90, 5 lines

```
pair<int, int> find_bezout(int x, int y) {
    if (y == 0) return bezout(1, 0);
    pair<int, int> g = find_bezout(y, x % y);
    return {g.second, g.first - (x/y) * g.second};
}
```

phiFunction.h

Description: *Euler's totient* or *Euler's phi* function is defined as $\phi(n) :=$ # of positive integers $\leq n$ that are coprime with n. The *cototient* is $n - \phi(n)$. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$ then $\phi(n) = (p_1-1)p_1^{k_1-1}...(p_r-1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}\ \forall a$.

6984ee, 22 lines

```
const int n = int(1e5)*5;
vector<int> phi(n);
void calculatePhi() {
  for(int i = 0; i < n; ++i) phi[i] = i&1 ? i : i/2;
  for(int i = 3; i < n; i += 2) if (phi[i] == i)
    for(int j = i; j < n; j += i) phi[j] -= phi[j]/i;
}
template<typename T> T phi(T n){
    T aux, result;
    aux = result = n;
    for (T i = 2; i*i <= n; ++i)
        if (aux % i == 0) {
            while (aux % i == 0) aux /= i;
            result /= i;
            result *= (i-1);
        }
    if (aux > 1) {
      result /= aux;
      result *= (aux-1);
    }
    return result;
}
```

DiscreteLogarithm.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or -1 if no such x exists. modLog(a,1,m) can be used calculate the order of a.
Time: $\mathcal{O}(\sqrt{m})$

10faf5, 11 lines

```
lint modLog(lint a, lint b, lint m) {
  lint n = (lint)sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<lint, lint> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    for(int i = 2; i <= n+1; ++i) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

Legendre.h

Description: Given an integer n and a prime number p, find the largest x such that p^x divides $n!$.

81613f, 8 lines

```
int legendre(int n, int p){
    int ret = 0, prod = p;
    while (prod <= n) {
        ret += n/prod;
        prod *= p;
    }
    return ret;
}
```

GroupOrder.h

Description: Calculate the order of a in Z_n. A group Z_n is cyclic if, and only if n $= 1, 2, 4, p^k$ or $2p^k$, being p an odd prime number.
Time: $\mathcal{O}(sqrt(n)log(n))$

"Divisors.h" 82034c, 6 lines

```
template<typename T> T order(T a, T n) {
```

```
    vector<T> d = divisors(phi(n));
    for (int i : v)
        if (modpow(a, i, n) == 1) return i;
    return -1;
}
```

PrimitiveRoots.h

Description: a is a primitive root mod n if for every number x coprime to n there is an integer z s.t. $x \equiv g^z \pmod{n}$. The number of primitive roots mod n, if there are any, is equal to $phi(phi(N))$. If m isnt prime, replace $m-1$ by $phi(m)$.
Time: $\mathcal{O}(log(N))$

<Sieve.h>, <PrimeFactors.h>, <Modpow.h> 05729f, 6 lines

```
template<typename T> bool is_primitive(T a, T m) {
    vector<pair<T, T>> D = prime_factorize(m-1);
    for (auto p : D)
        if (modpow(a, (m-1)/p.first, m) == 1) return false;
    return true;
}
```

PrimeCounting.h

Description: Count the number of primes up to x. Also useful for sum of primes.
Time: $\mathcal{O}\left(n^{3/4}/\log n\right)$

<Sieve.h> cb2aae, 32 lines

```
const int N = 1e5, K = 50, T = 10000000; // T<= 1e17 is fine
     for N<= 10^11
vector<int> primes_until;
vector<vector<uint16_t>> dp(N+1, vector<uint16_t>(K+1)); // use
      32-bit integer if N>= 2^17

void fill_primes(int n) { // get # of primes up to i
  run_sieve(n);
  int walk = 0;
  for (int i = 0; i < n; ++i) {
    if (!i) primes_until.push_back(0);
    else primes_until.push_back(primes_until.back());
    if (primes[walk] == i) walk++, primes_until.back()++;
  }
}

int64_t solve(int64_t n, int k) { // how many numbers
  if (k == 0) return n; //  in [1, N] not divisible by
  int64_t p = primes[k]; // any of the first k primes
  if (n < p) return 1ll;
  if (n < min(int64_t(T), p*p)) return primes_until[n] - k + 1;
  bool mark = n < N && k < K;
  if (mark && dp[n][k]) return dp[n][k];
  p = primes[k-1];
  int64_t res = solve(n, k-1) - solve(n/p, k-1);
  if (mark) dp[n][k] = res;
  return res;
}

int64_t calc(int64_t x) {
  if (x < T) return primes_until[x];
  int k = primes_until[sqrt(x)];
  return solve(x, k) + k - 1;
}
```

5.4 Chinese remainder theorem

ChineseRemainder.h

Description: Chinese Remainder Theorem.
crt(a, m, b, n) computes x such that $x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m,n)$. Assumes $mn < 2^{62}$.
Time: $\mathcal{O}(\log(n)) - \mathcal{O}(n\log(LCM(m)))$

0d0b4b, 21 lines

```
template<typename T>
T crt(T a, T m, T b, T n, T &x, T &y) {
  if (n > m) swap(a, b), swap(m, n);
  T g = egcd(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
template<typename T> // Solve system up to n congruences
T crt_system(vector<T> &a, vector<T> &m, int n) {
  for (int i = 0; i < n; ++i)
    a[i] = (a[i] % m[i] + m[i]) % m[i];
  T ret = a.front(), lcm = m.front();
  for (int i = 1; i < n; ++i) {
    T x, y;
    ret = crt(ret, lcm, a[i], m[i], x, y);
    T d = egcd(lcm, m[i], x = 0, y = 0);
    lcm = lcm * m[i] / d;
  }
  return ret;
}
```

5.5 Fractions

Fractions.h

Description: Template that helps deal with fractions. f3ef3d, 31 lines

```
struct frac {
    lint n, d;
    frac() { n = 0, d = 1; }
    frac(lint _n, lint _d) {
        n = _n, d = _d;
        lint g = __gcd(n,d); n /= g, d /= g;
        if (d < 0) n *= -1, d *= -1;
    }
    frac(lint _n) : frac(_n,1) {}
    friend frac abs(frac F) { return frac(abs(F.n),F.d); }
    friend bool operator<(const frac& l, const frac& r) {
        return l.n*r.d < r.n*l.d; }
    friend bool operator==(const frac& l, const frac& r) {
        return l.n == r.n && l.d == r.d; }
    friend bool operator!=(const frac& l, const frac& r) {
        return !(l == r); }
    friend frac operator+(const frac& l, const frac& r) {
        return frac(l.n*r.d+r.n*l.d,l.d*r.d); }
    friend frac operator-(const frac& l, const frac& r) {
        return frac(l.n*r.d-r.n*l.d,l.d*r.d); }
    friend frac operator*(const frac& l, const frac& r) {
        return frac(l.n*r.n,l.d*r.d); }
    friend frac operator*(const frac& l, int r) { return l*frac
         (r,1); }
    friend frac operator*(int r, const frac& l) { return l*r; }
    friend frac operator/(const frac& l, const frac& r) {
        return l*frac(r.d,r.n); }
    friend frac operator/(const frac& l, const int& r) { return
         l/frac(r,1); }
    friend frac operator/(const int& l, const frac& r) { return
         frac(l,1)/r; }
    friend frac& operator+=(frac& l, const frac& r) { return l
        = l+r; }
    friend frac& operator-=(frac& l, const frac& r) { return l
        = l-r; }
    template<class T> friend frac& operator*=(frac& l, const T&
         r) { return l = l*r; }
    template<class T> friend frac& operator/=(frac& l, const T&
         r) { return l = l/r; }
    friend ostream& operator<<(ostream& strm, const frac& a) {
        strm << a.n;
        if (a.d != 1) strm << "/" << a.d;
        return strm;
    }
};
```

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

6c75b7, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<lint, lint> approximate(d x, lint N) {
  lint LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    lint lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (lint)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
      {NP, NQ} : {P, Q};
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0,1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

f83d46, 23 lines

```
struct Frac { lint p, q; };
template<class F>
Frac fracBS(F f, lint N) {
  bool dir = 1, A = 1, B = 1;
  Frac left{0, 1}, right{1, 1}; // Set right to 1/0 to search
       (0, N]
  assert(!f(left)); assert(f(right));
  while (A || B) {
    lint adv = 0, step = 1; // move right if dir, else left
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{left.p * adv + right.p, left.q * adv + right.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    right.p += left.p * adv;
    right.q += left.q * adv;
    dir = !dir;
    swap(left, right);
    A = B; B = !!adv;
  }
  return dir ? right : left;
}
```

5.5.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

5.5.2 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.5.3 Primitive Roots

It only exists when n is $2, 4, p^k, 2p^k$, where p odd prime. If g is a primitive root, all primitive roots are of the form g^k where $k, \phi(p)$ are coprime (hence there are $\phi(\phi(p))$ primitive roots).

5.5.4 Chicken McNugget theorem

Let x and y be two coprime integers, the greater integer that can't be written in the form of $ax + by$ is $\frac{(x-1)(y-1)}{2}$

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.6.1 Sum of primes

For any multiplicative f:

$$S(n, p) = S(n, p-1) - f(p) \cdot (S(n/p, p-1) - S(p-1, p-1))$$

5.6.2 Moebius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Moebius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

5.6.3 Dirichlet Convolution

Given a function $f(x)$, let $(f * g)(x) = \sum_{d|x} g(d) f(x/d)$. If the partial sums $s_{f*g}(n), s_g(n)$ can be computed in $O(1)$ and $s_f(1 \ldots n^{2/3})$ can be computed in $O\left(n^{2/3}\right)$ then $s_f(n)$ can as well. Use

$$s_{f*g}(n) = \sum_{d=1}^{n} g(d) s_f(n/d).$$

For example, if $f(x) = \phi(x)$ then $g(x) = 1, (f * g)(x) = n$, and $s_f(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s_f(n/i)$.

5.6.4 Wilson's theorem

Let $n > 1$. Then $n | (n-1)! + 1$ iff n is prime.

5.6.5 Wolstenholme's theorem

Let $p > 3$ be a prime number. Then its numerator $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{p-1}$ is divisible by p^2.

5.6.6 Estimates

$$\sum_{d|n} d = O(n \log \log n)$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Let $s(x) = \sum_{i=1}^{x} \phi(i)$. Then

$$s(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

5.6.7 Prime counting function $(\pi(x))$

The prime counting function is asymptotic to $\frac{x}{\log x}$, by the prime number theorem.

x	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$\pi(x)$	4	25	168	1.229	9.592	78.498	664.579	5.761.455

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1 2 3 4	5	6	7	8	9	10
$n!$	1 2 6 24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14
n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

Factorial.h

Description: Pre-compute all the factorial numbers until n. 0c5d87, 9 lines

```
void init(int n) {
    fact = {1};
    for (int i = 1; i <= n; ++i)
        fact[i] = (lint)i * fact[i-1] % mod;
    ifact.resize(n + 1);
    ifact.back() = modinv(fact.back(), mod);
    for (int i = n; i > 0; --i)
        ifact[i-1] = (lint)i * ifact[i] % mod;
}
```

numPerm.h

Description: Number of permutations 9063aa, 6 lines

```
lint num_perm(int n, int r) {
    if (r < 0 || n < r) return 0;
    lint ret = 1;
    for (int i = n; i > n-r; --i) ret *= i;
    return ret;
```

6.1.2 Binomials

PascalTriangle.h

Description: Pre-compute all the Binomial Coefficients until n.
Time: $\mathcal{O}(N^2)$ 8ccd69, 8 lines

```
void init() {
  c[0][0] = 1;
  for (int i = 0; i < n; ++i) {
      c[i][0] = c[i][i] = 1;
      for (int j = 1; j < i; ++j)
          c[i][j] = c[i-1][j-1] + c[i-1][j];
  }
}
```

- Sum of every element in the n-th row of pascal triangle is 2^n.
- The product of the elements in each row is $\frac{(n+1)^n}{n!}$
- $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$
- In a row p where p is a prime number, all the terms in that row except the 1s are multiples of p
- To count odd terms in row n, convert n to binary. Let x be the number of 1s in the binary representation. Then the number of odd terms will be 2^x
- Every entry in row $2^n - 1$ is odd

nCr.h

Time: $\mathcal{O}(min(k, n-k))$ e7126e, 8 lines

```
lint ncr(int n, int k) {
  lint res = 1, to = min(k, n-k);
  if (to < 0) return 0;
  for (int i = 0; i < to; ++i) {
    res = res * (n - i) / (i + 1);
  }
  return res;
}
```

RollingBinomial.h

Description: $\binom{n}{k} \pmod{m}$ in time proportional to the difference between (n, k) and the previous (n, k).
f84189, 14 lines

```
const int mod = int(1e9) + 7;
vector<lint> invs; // precomputed inverses up to n
struct Bin {
  int N = 0,  K = 0; lint r = 1;
  void m(lint a, lint b) { return r = r * a % mod * invs[b] %
      mod; }
  lint choose(int n, int k) {
    if (k > n || k < 0) return 0;
    while(N < n) ++N, m(N, N-K);
    while(K < k) ++K, m(N-K+1, K);
    while(K > k) m(K, N-K-1), --K;
    while(N > n) m(N, N-K), --N;
    return r;
  }
};
```

Multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$.
864cdb, 7 lines

```
lint multinomial(vector<int>& v) {
  lint c = 1, m = v.empty() ? 1 : v[0];
  for (int i = 1 < v.size(); ++i)
      for (int j = 0; j < v[i]; ++j)
        c = c * ++m / (j+1);
  return c;
}
```

6.1.3 Involutions

An involution is a permutation with maximum cycle length 2, and it is its own inverse.

$$a(n) = a(n-1) + (n-1)a(n-2)$$

$$a(0) = a(1) = 1$$

1, 1, 2, 4, 10, 26, 76, 232, 764, 2620, 9496, 35696, 140152

6.1.4 Cycles

Let the number of n-permutations whose cycle lenghts all belong to the set S be denoted by $g_S(n)$

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

6.1.5 Inclusion-Exclusion Principle

Let $A_1, A_2, \ldots, A_n$ be finite sets. Then $A_1 \cup A_2 \cup \ldots \cup A_n$ is

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{\substack{I\subseteq\{1,2,\ldots,n\}\\ I\neq\emptyset}} (-1)^{|I|+1}\left|\bigcap_{i\in I} A_i\right|$$

6.1.6 The twelvefold way (from Stanley)

How many functions $f\colon N \to X$ are there?

N	X	Any f	Injective	Surjective
dist.	dist.	x^n	$\frac{x!}{(x-n)!}$	$x!\left\{ n \atop x \right\}$
indist.	dist.	$\binom{x+n-1}{n}$	$\binom{x}{n}$	$\binom{n-1}{n-x}$
dist.	indist.	$\left\{ n \atop 1 \right\} + \ldots + \left\{ n \atop x \right\}$	$[n \le x]$	$\left\{ n \atop k \right\}$
indist.	indist.	$p_1(n) + \ldots p_x(n)$	$[n \le x]$	$p_x(n)$

Where $\binom{a}{b} = \frac{1}{b!}(a)_b$, $p_x(n)$ is the number of ways to partition the integer n using x summand and $\left\{ n \atop x \right\}$ is the number of ways to partition a set of n elements into x subsets (aka Stirling number of the second kind).

6.1.7 Burnside

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.1.8 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \ldots + n_1 p + n_0$ and $m = m_k p^k + \ldots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\, c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$.

$$\mathcal{B}_{n+1} = \sum_{k=0}^{n}\binom{n}{k}\mathcal{B}_k$$

Also possible to calculate using Stirling numbers of the second kind,

$$B_n = \sum_{k=0}^{n} S(n,k)$$

If p is prime:

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$ # forests with exactly k rooted trees:

$$\binom{n}{k} k \cdot n^{n-k-1}$$

.

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in a $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children) or $2n+1$ elements.
- ordered trees with $n+1$ vertices.
- # ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subsequence.

6.3.8 Super Catalan numbers

The number of monotonic lattice paths of a nxn grid that do not touch the diagonal.

$$S(n) = \frac{3(2n-3)S(n-1) - (n-3)S(n-2)}{n}$$

$$S(1) = S(2) = 1$$

1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049, 518859

6.3.9 Motzkin numbers

Number of ways of drawing any number of nonintersecting chords among n points on a circle. Number of lattice paths from (0, 0) to $(n, 0)$ never going below the x-axis, using only steps NE, E, SE.

$$M(n) = \frac{3(n-1)M(n-2) + (2n+1)M(n-1)}{n+2}$$

$$M(0) = M(1) = 1$$

1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835, 113634

6.3.10 Narayana numbers

Number of lattice paths from (0,0) to $(2n,0)$ never going below the x-axis, using only steps NE and SE, and with k peaks.

$$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k-1}$$

$$N(n,1) = N(n,n) = 1$$

$$\sum_{k=1}^{n} N(n,k) = C_n$$

1, 1, 1, 1, 3, 1, 1, 6, 6, 1, 1, 10, 20, 10, 1, 1, 15, 50

6.3.11 Schroder numbers

Number of lattice paths from (0, 0) to (n, n) using only steps N,NE,E, never going above the diagonal. Number of lattice paths from (0, 0) to $(2n, 0)$ using only steps NE, SE and double east EE, never going below the x-axis. Twice the Super Catalan number, except for the first term.
1, 2, 6, 22, 90, 394, 1806, 8558, 41586, 206098

6.3.12 Triangles

Given rods of length 1, ..., n,

$$T(n) = \frac{1}{24}\left\{ \begin{array}{ll} n(n-2)(2n-5) & n \text{ even} \\ (n-1)(n-3)(2n-1) & n \text{ odd} \end{array} \right\}$$

is the number of distinct triangles (positive are) that can be constructed, i.e., the # of 3-subsets of $[n]$ s.t. $x \le y \le z$ and $z \ne x + y$.

6.4 Fibonacci

$$Fib(x+y) = Fib(x+1)Fib(y) + Fib(x)Fib(y-1)$$

$$Fib(n+1)Fib(n-1) - Fib(n)^2 = (-1)^n$$

$$Fib(2n-1) = Fib(n)^2 - Fib(n-1)^2$$

$$\sum_{i=0}^{n} Fib(i) = Fib(n+2) - 1$$

$$\sum_{i=0}^{n} Fib(i)^2 = Fib(n)Fib(n+1)$$

$$\sum_{i=0}^{n} Fib(i)^3 = \frac{Fib(n)Fib(n+1)^2 - (-1)^n Fib(n-1) + 1}{2}$$

6.5 Game Theory

A game can be reduced to Nim if it is a finite impartial game. Nim and its variants include:

6.5.1 Nim

Let $X = \bigoplus_{i=1}^{n} x_i$, then $(x_i)_{i=1}^{n}$ is a winning position iff $X \ne 0$. Find a move by picking k such that $x_k > x_k \oplus X$.

6.5.2 Misère Nim

Regular Nim, except that the last player to move *loses*. Play regular Nim until there is only one pile of size larger than 1, reduce it to 0 or 1 such that there is an odd number of piles. The second player wins $(a_1, \ldots, a_n)$ if 1) there is a pile $a_i > 1$ and $\oplus_{i=1}^{n} a_i = 0$ or 2) all $a_i \le 1$ and $\oplus_{i=1}^{n} a_i = 1$.

6.5.3 Staircase Nim

Stones are moved down a staircase and only removed from the last pile. $(x_i)_{i=1}^{n}$ is an L-position if $(x_{2i-1})_{i=1}^{n/2}$ is (i.e. only look at odd-numbered piles).

6.5.4 Moore's Nim$_k$

The player may remove from at most k piles (Nim = Nim$_1$). Expand the piles in base 2, do a carry-less addition in base $k+1$ (i.e. the number of ones in each column should be divisible by $k+1$).

6.5.5 Dim$^+$

The number of removed stones must be a divisor of the pile size. The Sprague-Grundy function is $k+1$ where 2^k is the largest power of 2 dividing the pile size.

6.5.6 Aliquot Game

Same as above, except the divisor should be proper (hence 1 is also a terminal state, but watch out for size 0 piles). Now the Sprague-Grundy function is just k.

6.5.7 Nim (at most half)

Write $n+1 = 2^m y$ with m maximal, then the Sprague-Grundy function of n is $(y-1)/2$.

6.5.8 Lasker's Nim

Players may alternatively split a pile into two new non-empty piles. $g(4k+1) = 4k+1$, $g(4k+2) = 4k+2$, $g(4k+3) = 4k+4$, $g(4k+4) = 4k+3$ $(k \ge 0)$.

6.5.9 Hackenbush on Trees

A tree with stalks $(x_i)_{i=1}^{n}$ may be replaced with a single stalk with length $\bigoplus_{i=1}^{n} x_i$.

Nim-Product.cpp

Description: Product of nimbers is associative, commutative, and distributive over addition (xor). Forms finite field of size 2^{2^k}. Application: Given 1D coin turning games G_1, G_2 $G_1 \times G_2$ is the 2D coin turning game defined as follows. If turning coins at $x_1, x_2, \ldots, x_m$ is legal in G_1 and $y_1, y_2, \ldots, y_n$ is legal in G_2, then turning coins at all positions (x_i, y_j) is legal assuming that the coin at (x_m, y_n) goes from heads to tails. Then the grundy function $g(x, y)$ of $G_1 \times G_2$ is $g_1(x) \times g_2(y)$.
Time: 64^2 xors per multiplication, memorize to speed up. 8d57cb, 27 lines

```
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
    if (_nimProd2[i][j]) return _nimProd2[i][j];
    if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
    int a = (i&j) & -(i&j);
```

```
    return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i ^
        a) | (a-1), (j ^ a) | (i & (a-1)));
}
void allNimProd2() {
    for (int i = 0; i < 64; i++) {
        for (int j = 0; j < 64; j++) {
            if ((i & j) == 0) _nimProd2[i][j] = 1ull << (i|j);
            else {
                int a = (i&j) & -(i&j);
                _nimProd2[i][j] = _nimProd2[i ^ a][j] ^
                    _nimProd2[(i ^ a) | (a-1)][(j ^ a) | (i &
                    (a-1))];
            }
        }
    }
}
ull nimProd(ull x, ull y) {
    ull res = 0;
    for (int i = 0; (x >> i) && i < 64; ++i)
        if ((x >> i) & 1)
            for (int j = 0; (y >> j) && j < 64; ++j)
                if ((y >> j) & 1) res ^= nimProd2(i, j);
    return res;
}
```

Partitions.cpp

Description: Fills array with partition function $p(n)$ $\forall 0 \leq ilegn$

3b7c36, 17 lines

```
array<int, 122> part; // 121 is max partition that will fit
    into int
void partition(int n) {
    part[0] = 1;
    for (int i = 1; i <= n; ++i) {
        part[i] = 0;
        for (int k = 1, x; k <= i; ++k) {
            x = i - k * (3*k-1)/2;
            if (x < 0) break;
            if (k&1) part[i] += part[x];
            else part[i] -= part[x];
            x = i - k * (3*k+1)/2;
            if (x < 0) break;
            if (k&1) part[i] += part[x];
            else part[i] -= part[x];
        }
    }
}
```

Lucas.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
Time: $\mathcal{O}(\log_p n)$

5beb1c, 10 lines

```
lint chooseModP(lint n, lint m, int p, vi& fact, vi& invfact) {
  lint c = 1;
  while (n || m) {
    lint a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] % p;
    n /= p; m /= p;
  }
  return c;
}
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

5d9aa7, 19 lines

```
const lint inf = LLONG_MAX;
struct edge_t { int a, b, w, s() { return a < b ? a : -a; }};
struct node_t { lint dist = inf; int prev = -1; };
void bellmanFord(vector<node_t>& nodes, vector<edge_t>& eds,
    int s) {
  nodes[s].dist = 0;
  sort(eds.begin(), eds.end(), [](edge_t a, edge_t b) { return
      a.s() < b.s(); });
  int lim = nodes.size() / 2 + 2; // /3+100 with shuffled
      vertices
  for(int i = 0; i < lim; ++i) for(auto &ed : eds) {
    node_t cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    lint d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  for(int i = 0; i < lim; ++i) for(auto &e : eds)
    if (nodes[e.a].dist == -inf) nodes[e.b].dist = -inf;
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge distances. Input is an distance matrix m, where $m[i][j]$ = inf if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

578e31, 16 lines

```
const lint inf = 1LL << 62;
void floydWarshall(vector<vector<lint>>& m) {
  int n = m.size();
  for (int i = 0; i < n; ++i) m[i][i] = min(m[i][i], {});
  for (int k = 0; k < n; ++k)
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        if (m[i][k] != inf && m[k][j] != inf) {
          auto newDist = max(m[i][k] + m[k][j], -inf);
          m[i][j] = min(m[i][j], newDist);
        }
  for (int k = 0; k < n; ++k) if (m[k][k] < 0)
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -
            inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V| + |E|)$

d2ba1e, 12 lines

```
vector<int> topo_sort(const vector<vector<int>> &g) {
  vector<int> indeg(g.size()), ret;
  for(auto &li : g) for(auto &x : li) indeg[x]++;
  queue<int> q; // use priority queue for lexic. smallest ans.
  for(int i = 0; i < g.size(); ++i) if (indeg[i] == 0) q.push(-
      i);
  while (!q.empty()) {
    int i = -q.front(); // top() for priority queue
    ret.push_back(i); q.pop();
    for(auto &x : g[i]) if (--indeg[x] == 0) q.push(-x);
  }
  return ret;
}
```

Dijkstra.h

Description: n = vertices and s = starting point

95ff50, 19 lines

```
template<typename T>
vector<T> Dijkstra(vector<vector<pair<int,T>>> &edges, int s) {
    const int n = (int)edges.size();
  assert(0 <= s && s < n);
    vector<int> dist(n, numeric_limits<T>::max()/2), parent(n,
       -1);
    using Q = pair<T, int>;
    priority_queue<Q, vector<Q>, greater<Q>> q;
    q.push({dist[s] = 0, s});
    while(!q.empty()) {
        auto x = q.top(); q.pop();
        if (dist[x.second] < x.first) continue;
        for (auto u : edges[x.second])
            if (x.first + u.second < dist[u.first]) {
                q.push({dist[u.first] = x.first + u.second, u.
                    first});
                parent[u.first] = x.second;
            }
    }
    return dist;
}
```

Prim.h

Description: Find the minimum spanning tree. Better for dense graphs.
Time: $\mathcal{O}(E \log V)$

69c0c1, 17 lines

```
priority_queue<pair<int, int>> pq;
void process(int v) {
    seen[v] = true;
    for (auto u : edges[v])
        if (!seen[u.first])
            pq.push({-u.second, -u.first});
}
int mst() {
    int mst_cost = 0; process(0);
    while (!pq.empty()) {
        auto v = pq.top(); pq.pop();
        int u = -v.second, w = -v.first;
        if (!seen[u]) mst_cost += w;
        process(u);
    }
    return mst_cost;
}
```

Kruskal.h

Description: Find the minimum spanning tree. Better for sparse graphs.
Time: $\mathcal{O}(E \log E)$

3fba67, 12 lines

```
template<typename T>
T kruskal(int n, vector<pair<T, pair<int,int>>> &edges) {
    sort(edges.begin(), edges.end());
    T cost = 0;
    UF dsu(n);
    for (auto &e : edges)
```

```
        if (dsu.find(e.second.first) != dsu.find(e.second.second)) {
            dsu.unite(e.second.first, e.second.second);
            cost += e.first;
        }
    return cost;
}
```

SPFA.h

Description: Shortest Path Faster Algorithm.
Time: $\mathcal{O}(E)$

579f79, 22 lines

```
int d[100100], f[100100];
vector<pair<int,int>> edges[100100];
void spfa(int s = 0) {
    vector<int> q = {s};
    memset(d, 127, sizeof(d));
    memset(f, 0, sizeof(f));
    f[s] = 1, d[s] = 0;
    for (int i = 0; i < q.size(); ++i) {
        int now = q[i];
        f[now] = 0;
        for(auto u : edges[now]) {
            int cost = u.second;
            if (d[u.first] > d[now] + cost) {
                d[u.first] = d[now] + cost;
                if (!f[u.first]) {
                    f[u.first] = 1;
                    q.push_back(u.first);
                }
            }
        }
    }
}
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

400c6e, 16 lines

```
using pii = pair<int,int>;
vector<int> eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = gr.size();
    vector<int> D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = gr[x].size();
        if (it == end){ ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        }}
    for(auto &x : D) if (x < 0 || ret.size() != nedges+1) return {};
    return {ret.rbegin(), ret.rend()};
}
```

7.2 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only. id can be used to restore each edge and its amount of flow used.
Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ Better for dense graphs - Slower than Dinic (in practice)

968576, 42 lines

```
template<typename flow_t = lint> struct PushRelabel {
  struct edge_t { int dest, back; flow_t f, c; };
  vector<vector<edge_t>> g;
  vector<flow_t> ec;
  vector<edge_t*> cur;
  vector<vector<int>> hs; vector<int> H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}
  void addEdge(int s, int t, flow_t cap, flow_t rcap = 0) {
    if (s == t) return;
    g[s].push_back({t, (int)g[t].size(), 0, cap});
    g[t].push_back({s, (int)g[s].size()-1, 0, rcap});
  }
  void addFlow(edge_t& e, flow_t f) {
    edge_t &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  flow_t maxflow(int s, int t) {
    int v = g.size(); H[s] = v; ec[t] = 1;
    vector<int> co(2*v); co[0] = v-1;
    for(int i = 0; i < v; ++i) cur[i] = g[i].data();
    for(auto& e : g[s]) addFlow(e, e.c);
    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + g[u].size()) {
          H[u] = 1e9;
          for(auto &e : g[u]) if (e.c && H[u] > H[e.dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            for(int i = 0; i < v; ++i) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
          addFlow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    }
  }
  bool leftOfMinCut(int a) { return H[a] >= g.size(); }
};
```

Dinitz.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching. To obtain each partition A and B of the cut look at lvl, for $v \subset A$, $lvl[v] > 0$, for $u \subset B$, $lvl[u] = 0$.

8eed9f, 45 lines

```
template<typename T = lint> struct Dinitz {
  struct edge_t { int to, rev; T c, f; };
  vector<vector<edge_t>> adj;
  vector<int> lvl, ptr, q;
  Dinitz(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  inline void addEdge(int a, int b, T c, T rcap = 0) {
    adj[a].push_back({b, (int)adj[b].size(), c, 0});
    adj[b].push_back({a, (int)adj[a].size() - 1, rcap, 0});
  }
  T dfs(int v, int t, T f) {
    if (v == t || !f) return f;
    for (int &i = ptr[v]; i < adj[v].size(); ++i) {
      edge_t &e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (T p = dfs(e.to, t, min(f, e.c - e.f))) {
          e.f += p, adj[e.to][e.rev].f -= p;
          return p;
        }
    }
    return 0;
  }
  T maxflow(int s, int t) {
    T flow = 0; q[0] = s;
    for (int L = 0; L < 31; ++L) do { // 'int L=30' maybe faster for random data
      lvl = ptr = vector<int>(q.size());
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (edge_t &e : adj[v])
          if (!lvl[e.to] && (e.c - e.f) >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (T p = dfs(s, t, numeric_limits<T>::max()/4)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int v) { return lvl[v] != 0; }
  pair<T, vector<pair<int,int>>> minCut(int s, int t) {
    T cost = maxflow(s,t);
    vector<pair<int,int>> cut;
    for (int i = 0; i < adj.size(); i++) for(edge_t &e : adj[i])
      if (lvl[i] && !lvl[e.to]) cut.push_back({i, e.to});
    return {cost, cut};
  }
};
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.
Usage: unordered_map<int, T> graph;
graph[a][b] += c; //adds edge from a to b with capacity c, use "+=" NOT "="

61d890, 32 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T>> & graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vector<int> par(graph.size()), q = par;
  for (;;) {
    fill(par.begin(),par.end(), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;
    for (int i = 0; i < ptr; ++i) {
      int x = q[i];
      for (pair<int,int> e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);
    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
```

```
      graph[y][p] += inc;
    }
  }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not.
Time: Approximately $\mathcal{O}(E^2)$ faster than Kactl's on practice

<bits/extc++.h> // don't forget! 418972, 70 lines

```
template <typename flow_t = int, typename cost_t = long long>
struct MCMF_SSPA {
  int N;
  vector<vector<int>> adj;
  struct edge_t { int dest; flow_t cap; cost_t cost; };
  vector<edge_t> edges;
  vector<char> seen;
  vector<cost_t> pi;
  vector<int> prv;
  explicit MCMF_SSPA(int N_) : N(N_), adj(N), pi(N, 0), prv(N)
      {}
  void addEdge(int from, int to, flow_t cap, cost_t cost) {
    assert(cap >= 0);
    int e = int(edges.size());
    edges.emplace_back(edge_t{to, cap, cost});
    edges.emplace_back(edge_t{from, 0, -cost});
    adj[from].push_back(e);
    adj[to].push_back(e+1);
  }
  const cost_t INF_COST = numeric_limits<cost_t>::max() / 4;
  const flow_t INF_FLOW = numeric_limits<flow_t>::max() / 4;
  vector<cost_t> dist;
  __gnu_pbds::priority_queue<pair<cost_t, int>> q;
  vector<typename decltype(q)::point_iterator> its;
  void path(int s) {
    dist.assign(N, INF_COST);
    dist[s] = 0;
    its.assign(N, q.end());
    its[s] = q.push({0, s});
    while (!q.empty()) {
      int i = q.top().second; q.pop();
      cost_t d = dist[i];
      for (int e : adj[i]) {
        if (edges[e].cap) {
          int j = edges[e].dest;
          cost_t nd = d + edges[e].cost;
          if (nd < dist[j]) {
            dist[j] = nd;
            prv[j] = e;
            if (its[j] == q.end()) its[j] = q.push({-(dist[j] -
                pi[j]), j});
            else q.modify(its[j], {-(dist[j] - pi[j]), j});
          }
        }
      }
    }
    swap(pi, dist);
  }
  pair<flow_t, cost_t> maxflow(int s, int t) {
    assert(s != t);
    flow_t totFlow = 0; cost_t totCost = 0;
    while (path(s), pi[t] < INF_COST) {
      flow_t curFlow = numeric_limits<flow_t>::max();
      for (int cur = t; cur != s; ) {
        int e = prv[cur];
        int nxt = edges[e^1].dest;
        curFlow = min(curFlow, edges[e].cap);
        cur = nxt;
      }
      totFlow += curFlow;
      totCost += pi[t] * curFlow;
      for (int cur = t; cur != s; ) {
        int e = prv[cur];
        int nxt = edges[e^1].dest;
        edges[e].cap -= curFlow;
        edges[e^1].cap += curFlow;
        cur = nxt;
      }
    }
    return {totFlow, totCost};
  }
};
```

StoerWagner.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}(V^3)$

b7ae15, 30 lines

```
pair<int, vector<int>> GetMinCut(vector<vector<int>> &weights)
    {
  int N = weights.size();
  vector<int> used(N), cut, best_cut;
  int best_weight = -1;
  for (int phase = N-1; phase >= 0; phase--) {
    vector<int> w = weights[0], added = used;
    int prev, k = 0;
    for (int i = 0; i < phase; ++i){
      prev = k;
      k = -1;
      for (int j = 1; j < N; ++j)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase-1) {
        for (int j = 0; j < N; ++j) weights[prev][j] +=
            weights[k][j];
        for (int j = 0; j < N; ++j) weights[j][prev] =
            weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        for (int j = 0; j < N; ++j)
          w[j] += weights[k][j];
        added[k] = true;
      }
    }
  }
  return {best_weight, best_cut};
}
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}(V)$ Flow Computations

"PushRelabel.h" ce9ee3, 13 lines

```
typedef array<lint, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vector<int> par(N);
    for(int i = 1; i < N; ++i) {
        PushRelabel D(N); // Dinitz/HLPP also works
        for(auto &t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        for(int j = i+1; j < N; ++j)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] =
                i;
    }
    return tree;
}
```

7.3 Matching

HopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vector<int> btoa(m, -1); hopcroftKarp(g, btoa);`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

d4d272, 35 lines

```
using vi = vector<int>;
bool dfs(int a, int L, const vector<vi> &g, vi &btoa, vi &A, vi
    &B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for(auto &b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L+1, g, btoa, A, B))
            return btoa[b] = a, 1;
    }
    return 0;
}
int hopcroftKarp(const vector<vi> &g, vi &btoa) {
    int res = 0;
    vector<int> A(g.size()), B(btoa.size()), cur, next;
    for (;;) {
        fill(A.begin(), A.end(), 0), fill(B.begin(), B.end(),
            0);
        cur.clear();
        for(auto &a : btoa) if (a != -1) A[a] = -1;
        for (int a = 0; a < g.size(); ++a) if (A[a] == 0) cur.
            push_back(a);
        for (int lay = 1;; ++lay) {
            bool islast = 0; next.clear();
            for(auto &a : cur) for(auto &b : g[a]) {
                if (btoa[b] == -1) B[b] = lay, islast = 1;
                else if (btoa[b] != a && !B[b])
                    B[b] = lay, next.push_back(btoa[b]);
            }
            if (islast) break;
            if (next.empty()) return res;
            for(auto &a : next) A[a] = lay;
            cur.swap(next);
        }
        for(int a = 0; a < g.size(); ++a)
            res += dfs(a, 0, g, btoa, A, B);
    }
}
```

MaxBipartiteMatching.h

Description: Fast Kuhn! Simple maximum cardinality bipartite matching algorithm. Fast and reliable maximum cardinality matching solver, better than DFSMatching and sometimes even faster than hopcroftKarp (Crazy heuristic huh). This implementation has got an $O(n^2)$ worst case on a sparse graph. Shuffling the edges and vertices ordering might fix it. Good Luck. $R[i]$ will be the match for vertex i on the right side, or -1 if it's not matched. $L[i]$ will be the match for vertex i on the left side.

Time: $\mathcal{O}(VE)$ worst case with shuffling I guess

624aae, 38 lines

```
struct BipartiteMatcher {
    vector<vector<int>> edges;
    vector<int> L, R, seen;
    BipartiteMatcher(int n, int m) : edges(n), L(n, -1), R(m,
        -1), seen(n) {}
    void addEdge(int a, int b) { edges[a].push_back(b); }
    void improve() {
        mt19937 rng(chrono::steady_clock::now().
            time_since_epoch().count());
        for (int i = 0; i < edges.size(); ++i)
            shuffle(edges[i].begin(), edges[i].end(), rng);
    }
    bool find(int v) {
        if (seen[v]) return false;
        seen[v] = true;
        for (int u : edges[v])
            if (R[u] == -1) {
                L[v] = u, R[u] = v;
                return true;
            }
        for (int u : edges[v])
            if (find(R[u])) {
                L[v] = u, R[u] = v;
                return true;
            }
        return false;
    }
    int maxMatching() {
        int ok = true;
        while (ok--) {
            fill(seen.begin(), seen.end(), 0);
            for (int i = 0; i < (int)L.size(); ++i)
                if (L[i] == -1) ok |= find(i);
        }
        int ret = 0;
        for (int i = 0; i < L.size(); ++i)
            ret += (L[i] != -1);
        return ret;
    }
};
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.
Time: $\mathcal{O}(N^2M)$

7a2392, 31 lines

```
pair<int, vector<int>> hungarian(const vector<vector<int>> &a)
    {
  if (a.empty()) return {0, {}};
  int n = a.size() + 1, m = a[0].size() + 1;
  vector<int> u(n), v(m), p(m), ans(n - 1);
  for(int i = 1; i < n; ++i) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vector<int> dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do {
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      for(int j = 1; j < m; ++j) if (!done[j]) {
        auto cur = a[i0-1][j-1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      for(int j = 0; j < m; ++j) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  for(int j = 1; j < m; ++j) if (p[j]) ans[p[j]-1] = j-1;
  return {-v[0], ans}; // min cost
}
```

GeneralMatching.h

Description: Maximum Matching for general graphs (undirected and non bipartite) using Edmond's Blossom Algorithm.
Time: $\mathcal{O}(EV^2)$

0b82ee, 68 lines

```
struct blossom_t {
    int t, n; // 1-based indexing!!
    vector<vector<int>> edges;
    vector<int> seen, parent, og, match, aux, Q;
    blossom_t(int _n) : n(_n), edges(n+1), seen(n+1),
        parent(n+1), og(n+1), match(n+1), aux(n+10), t(0) {}
    void addEdge(int u, int v) {
        edges[u].push_back(v);
        edges[v].push_back(u);
    }
    void augment(int u, int v) {
        int pv = v, nv; // flip states of edges on u-v path
        do {
            pv = parent[v]; nv = match[pv];
            match[v] = pv; match[pv] = v;
            v = nv;
        } while(u != pv);
    }
    int lca(int v, int w) { // find LCA in O(dist)
        ++t;
        while (1) {
            if (v) {
                if (aux[v] == t) return v; aux[v] = t;
                v = og[parent[match[v]]];
            }
            swap(v, w);
        }
    }
    void blossom(int v, int w, int a) {
        while (og[v] != a) {
            parent[v] = w; w = match[v]; // go other way around
                cycle
            if(seen[w] == 1) Q.push_back(w), seen[w] = 0;
            og[v] = og[w] = a;      // merge into supernode
            v = parent[w];
        }
    }
    bool bfs(int u) {
        for (int i = 1; i <= n; ++i) seen[i] = -1, og[i] = i;
        Q = vector<int>(); Q.push_back(u); seen[u] = 0;
        for(int i = 0; i < Q.size(); ++i) {
            int v = Q[i];
            for(auto &x : edges[v]) {
                if (seen[x] == -1) {
                    parent[x] = v; seen[x] = 1;
                    if (!match[x]) return augment(u, x), true;
                    Q.push_back(match[x]); seen[match[x]] = 0;
                } else if (seen[x] == 0 && og[v] != og[x]) {
                    int a = lca(og[v], og[x]);
                    blossom(x, v, a); blossom(v, x, a);
                }
            }
        }
        return false;
    }
    int solve() {
        int ans = 0; // find random matching (not necessary,
            constant improvement)
        vector<int> V(n-1); iota(V.begin(), V.end(), 1);//
        shuffle(V.begin(), V.end(), mt19937(0x94949));
        for(auto &x : V) if(!match[x])
            for(auto &y : edges[x]) if (!match[y]) {
                match[x] = y, match[y] = x;
                ++ans; break;
            }
        for (int i = 1; i <= n; ++i)
            if (!match[i] && bfs(i)) ++ans;
        return ans;
    }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"MaxBipartiteMatching.h" fbfea3, 19 lines

```
vector<int> cover(BipartiteMatcher& B, int n, int m) {
    int res = B.maxMatching();
    vector<bool> lfound(n, true), seen(m);
    for(int &it : B.R) if (it != -1) lfound[it] = false;
    vector<int> q, cover;
    for(int i = 0; i < n; ++i) if (lfound[i]) q.push_back(i);
    for(int i = 0; i < q.size(); ++i) {
        int v = q[i];
        lfound[v] = true;
        for (int e : B.edges[v]) if (!seen[e] && B.R[e] != -1)
            {
            seen[e] = true;
            q.push_back(B.R[e]);
        }
    }
    for(int i = 0; i < n; ++i) if (!lfound[i]) cover.push_back(
        i);
    for(int i = 0; i < m; ++i) if (seen[i]) cover.push_back(n+i
        );
    assert(cover.size() == res);
    return cover;
}
```

MinimumEdgeCover.h

Description: Finds a minimum edge cover in a bipartite graph. The size is the same as the number of vertices minus the size of a maximum matching. The mark vector represents who the vertices of set B has an edge to.
Usage: `vector<int> mark(n+m, -1);`
`auto cover = minEdgeCover(g, mark);`

"MaxBipartiteMatching.h" 6cb7da, 12 lines

```
vector<pair<int,int>> minEdgeCover(BipartiteMatcher &g, vector<
    int> &mark) {
    int maxMatching = g.maxMatching();
    vector<pair<int,int>> cover;
    for (int i = 0; i < g.L.size(); ++i) {
        if (g.L[i] >= 0) cover.push_back({i, g.L[i]});
```

```
        else if (g.edges[i].size()) cover.push_back({i, g.edges
            [i][0]});
    }
    for (int i = 0; i < g.R.size(); ++i)
        if (g.R[i] == -1 && mark[i] >= 0)
            cover.push_back({mark[i], i});
    return cover;
}
```

MinimumPathCover.h

Description: Finds a minimum vertex-disjoint path cover in a dag. The size is the same as the number of vertices minus the size of a maximum matching.

"MaxBipartiteMatching.h" dbe138, 15 lines

```
vector<vector<int>> minPathCover(BipartiteMatcher &g) {
    int how_many = g.edges.size() - g.maxMatching();
    vector<vector<int>> paths;
    for (int i = 0; i < g.edges.size(); ++i)
        if (g.R[i] == -1) {
            vector<int> path = {i};
            int cur = i;
            while (g.L[cur] >= 0) {
                cur = g.L[cur];
                path.push_back(cur);
            }
            paths.push_back(path);
        }
    return paths;
}
```

7.4 DFS algorithms

DFSTree.h

Description: Builds dfs tree. Find cut vertices and bridges.
Usage: Call solve right after build the graph

dbc136, 51 lines

```
struct tree_t {
    int timer, n;
    vector<vector<int>> edges;
    vector<pair<int,int>> bridges;
    vector<int> depth, mindepth, parent, st, cut, children;
    tree_t(int n) : n(n), timer(0), edges(n), parent(n,-1),
        mindepth(n,-1), depth(n,-1), st(n,-1) {}
    void addEdge(int a, int b) {
        edges[a].push_back(b); edges[b].push_back(a);
    }
    void dfs(int v) {
        st[v] = timer;
        mindepth[v] = depth[v];
        for (int u : edges[v]) {
            if (u == parent[v]) continue;
            if (st[u] == timer) {
                mindepth[v] = min(mindepth[v], depth[u]);
                continue;
            }
            depth[u] = 1 + depth[v];
            parent[u] = v;
            dfs(u);
            mindepth[v] = min(mindepth[v], mindepth[u]);
        }
    }
    vector<pair<int,int>> find_bridges() {
        for (int i = 0; i < n; ++i)
            if (parent[i] != -1 && mindepth[i] == depth[i])
                bridges.push_back({parent[i], i});c
        return bridges;
    }
    vector<bool> find_cut() {
        cut.resize(n), children.resize(n);
        for (int i = 0; i < n; ++i)
            if (parent[i] != -1 && mindepth[i] >= depth[parent[
                i]])
                cut[parent[i]] = 1;
        for (int i = 0; i < n; ++i)
            if (parent[i] != -1) child[parent[i]]++;
        for (int i = 0; i < n; ++i)
            if (parent[i] == -1 && child[i] < 2) cut[i] = 0;
        return cut;
    }
    void solve() {
        for (int i = 0; i < n; ++i)
            if (depth[i] == -1) {
                depth[i] = 0; parent[i] = -1;
                ++timer;
                dfs(i);
            }
    }
};
```

CentroidDecomposition.h

Description: Divide and Conquer on Trees.

f347ec, 57 lines

```
struct centroid_t {
    int n;
    vector<vector<int>> edges;
    vector<vector<int>> dist; // dist to all ancestors
    vector<bool> blocked; // processed centroid
    vector<int> sz, depth, parent; // centroid parent
    centroid_t(int _n) : n(_n), edges(n), blocked(n), sz(n),
        depth(n),
        parent(n), dist(32 - __builtin_clz(n), vector<int>(n))
            {}
    void addEdge(int a, int b) {
        edges[a].push_back(b);
        edges[b].push_back(a);
    }
    void dfs_sz(int v, int p) {
        sz[v] = 1;
        for (int u : edges[v]) {
            if (u == p || blocked[u]) continue;
            dfs_sz(u, v);
            sz[v] += sz[u];
        }
    }

    int find(int v, int p, int tsz) { // find a centroid
        for (int u : edges[v])
            if (!blocked[u] && u != p && 2*sz[u] > tsz)
                return find(u, v, tsz);
        return v;
    }

    void dfs_dist(int v, int p, int layer, int d) {
        dist[layer][v] = d;
        for (int u : edges[v]) {
            if (blocked[u] || u == p) continue;
            dfs_dist(u, v, layer, d + 1);
        }
    }

    int solve(int v, int p) {
        // solve the problem for each subtree here xD
    }

    int decompose(int v, int layer) {
        dfs_sz(v, -1);
        int x = find(v, v, sz[v]);
        blocked[x] = true;
        depth[x] = layer;
        parent[x] = v;
        dfs_dist(x, x, layer, 0);

        int res = solve(x, v); // solving for each subtree

        for (int u : edges[x]) {
            if (blocked[u]) continue;
            res += decompose(u, layer + 1);
        }
        return res;
    }
};
```

Tarjan.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: cnt_of[i] holds the component index of a node (a component only has edges to components with lower index). ncnt will contain the number of components.
Time: $\mathcal{O}(E+V)$

3d87ef, 27 lines

```
struct tarjan_t {
    int n, ncnt = 0, time = 0;
    vector<vector<int>> edges;
    vector<int> preorder, cnt_of, order, stack_t;
    tarjan_t(int n): n(n), edges(n), preorder(n), cnt_of(n, -1)
        {}
    int dfs(int v) {
        int reach = preorder[v] = ++time, u;
        stack_t.push_back(v);
        for (int u : edges[v]) if (cnt_of[u] == -1)
            reach = min(reach, preorder[u]?:dfs(u));
        if (reach == preorder[v]) {
            do {
                u = stack_t.back();
                stack_t.pop_back();
                order.push_back(v);
                cnt_of[u] = ncnt;
            } while (v != u);
            ++ncnt;
        }
        return preorder[v] = reach;
    }
    void scc() {
        time = ncnt = 0;
        for (int i = 0; i < int(edges.size()); ++i)
            if (cnt_of[i] == -1) dfs(i);
    }
};
```

Kosaraju.h

Description: Kosaraju's Algorithm, DFS twice to generate strongly connected components in topological order. a, b in same component if both $a \to b$ and $b \to a$ exist.
Time: $\mathcal{O}(V+E)$

25be07, 35 lines

```
struct Kosaraju_t {
  int n;
  vector<vector<int>> edges, redges;
  vector<bool> seen;
  vector<int> cnt_of, cnts;
  Kosaraju_t(const int &N) : n(N), edges(N), redges(N), seen(N)
      , cnt_of(N, -1) {}
  void addEdge(int a, int b) {
    edges[a].push_back(b);
    redges[b].push_back(a);
  }
```

```
  void dfs(int v) {
    seen[v] = true;
    for (int u : edges[v]) {
      if (seen[u]) continue;
      dfs(u);
    }
    toposort.push_back(v);
  }
  void dfs_fix(int v, int w) {
    cnt_of[v] = x;
    for (int u : redges[v]) {
      if (cnt_of[u] == -1) dfs_fix(u, w);
    }
  }
  void solve() {
    for (int i = 0; i < n; ++i)
      if (seen[i] == false) dfs(i);
    reverse(toposort.begin(), toposort.end());
    for (int u : toposort) {
      if (cnt_of[u] != -1) continue;
      dfs_fix(u, u);
      cnts.push_back(u);
    }
  }
};
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

Usage: `int eid = 0; ed.resize(N);`
```
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++);
}
```
Time: $\mathcal{O}(E+V)$

0dcf46, 36 lines

```
typedef vector<vector<pair<int,int>>> vii;
vector<int> num, st;
vii ed;
int Time;
int dfs(int at, int par,vector<vector<int>> &comps) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto &pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me) st.push_back(e);
    } else {
      int si = st.size();
      int up = dfs(y, e, comps);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        comps.push_back(vector<int>());
        for(int i = st.size()-1;i >= si; --i)
          comps[comps.size()-1].push_back(st[i]);
        st.resize(si);
        cont_comp++;
      }
      else if (up < me){ st.push_back(e);}
      else { cont_comp++; comps.push_back({e});/* e is a bridge
           */ }
    }
  }
  return top;
}
vector<vector<int>> solve() { // returns components
  vector<vector<int>> comps; // and its edges ids
  num.assign(ed.size(), 0);
  for (int i = 0; i < ed.size(); ++i)
    if (!num[i]) dfs(i, -1, comps);
  return comps;
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: `TwoSat ts(number of boolean variables);`
```
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

ad8fb0, 49 lines

```
struct TwoSat {
  int N;
  vector<vector<int>> gr;
  vector<int> values; // 0 = false, 1 = true
  TwoSat(int n = 0) : N(n), gr(2*n) {}
  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }
  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void set_value(int x) { either(x, x); }
  void at_most_one(const vector<int>& li) { // (optional)
    if (li.size() <= 1) return;
    int cur = ~li[0];
    for (int i = 2; i < li.size(); ++i) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }
  vector<int> val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    trav(e, gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }
  bool solve() {
    values.assign(N, -1);val.assign(2*N, 0); comp = val;
    for (int i = 0; i < 2*N; ++i) if (!comp[i]) dfs(i);
    for (int i = 0; i < N; ++i) if (comp[2*i] == comp[2*i+1])
        return 0;
    return 1;
  }
};
```

Cycles.h

Description: Cycle Detection (Detects a cycle in a directed or undirected graph.)

Time: $\mathcal{O}(V)$

7ff93a, 24 lines

```
bool detectCycle(vector<vector<int>> &edges, bool undirected) {
    vector<int> seen(n, 0), parent(n), stack_t;
    for (int i = 0; i < edges.size(); ++i) {
        if (seen[i] == 2) continue;
        stack_t.push_back(i);
        while(!stack_t.empty()) {
            int u = stack_t.back();
            stack_t.pop_back();
            if (seen[u] == 1) seen[u] = 2;
            else {
                stack_t.push_back(u);
                seen[u] = 1;
                for (int w : edges[u]) {
                    if (seen[w] == 0) {
                        parent[w] = u;
                        stack_t.push_back(w);
                    }
                    else if (seen[w] == 1 && (!undirected || w
                         != parent[u]))
                        return true;
                }
            }
        }
    }
}
```

7.5 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Possible optimization: on the top-most recursion level, ignore 'cands', and go through nodes in order of increasing degree, where degrees go down as nodes are removed.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

57e107, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B> &eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  for(int i = 0; i < eds.size(); ++i) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

0fb921, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit = 0.025, pk = 0;
  struct Vertex { int i, d = 0; };
  typedef vector<Vertex> vv;
```

```
  vb e;
  vv V;
  vector<vector<int>> C;
  vector<int> qmax, q, S, old;
  void init(vv& r) {
    for(auto& v : r) v.d = 0;
    for(auto& v : r) for(auto& j : r) v.d += e[v.i][j.i];
    sort(r.begin(), r.end(), [](auto a, auto b) { return a.d >
        b.d; });
    int mxD = r[0].d;
    for(int i = 0; i < r.size(); ++i) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (R.size()) {
      if (q.size() + R.back().d <= qmax.size()) return;
      q.push_back(R.back().i);
      vv T;
      for(auto& v : R) if (e[R.back().i][v.i]) T.push_back({v.i
          });
      if (T.size()) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(qmax.size() - q.size() +
            1, 1);
        C[1].clear(), C[2].clear();
        for(auto& v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(C[k].begin(), C[k].end(), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        for(int k = mnk; k <= mxk; ++k) for(auto& i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (q.size() > qmax.size()) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vector<int> maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(C.size()), old(S)
      {
    for(int i = 0; i < e.size(); ++i) V.push_back({i});
  }
};
```

CycleCounting.cpp

Description: Counts 3 and 4 cycles

132662, 49 lines

```
#define P 1000000007
#define N 110000
int n, m;
vector<int> go[N], lk[N];
int w[N], deg[N], pos[N], id[N];
int circle3(){
  int ans=0;
  for (int i = 1; i <= n; i++) w[i] = 0;
  for (int x = 1; x <= n; x++) {
    for(int y : lk[x]) w[y] = 1;
    for(int y:lk[x]) for(int z:lk[y]) if(w[z]){
      ans=(ans+go[x].size()+go[y].size()+go[z].size()-6)%P;
    }
    for(int y:lk[x])w[y]=0;
  }
  return ans;
}
int circle4(){
  for (int i = 1; i <= n; i++) w[i]=0;
  int ans=0;
  for (int x = 1; x <= n; x++) {
    for(int y:go[x])for(int z:lk[y])if(pos[z]>pos[x]){
      ans=(ans+w[z])%P;
      w[z]++;
    }
    for(int y:go[x])for(int z:lk[y])w[z]=0;
  }
  return ans;
}
inline bool cmp(const int &x,const int &y){
  return deg[x]<deg[y];
}
void init() {
  scanf("%d%d", &n, &m);
  for (int i = 1; i <= n; i++)
    deg[i] = 0, go[i].clear(), lk[i].clear();;
  while (m--) {
    int a,b;
    scanf("%d%d",&a,&b);
    deg[a]++;deg[b]++;
    go[a].push_back(b);go[b].push_back(a);
  }
  for (int i = 1; i <= n; i++) id[i] = i;
  sort(id+1,id+1+n,cmp);
  for (int i = 1; i <= n; i++) pos[id[i]]=i;
  for (int x = 1; x <= n; x++)
    for(int y:go[x])
      if(pos[y]>pos[x])lk[x].push_back(y);
}
```

7.6 Trees

Tree.h

Description: Structure that handles tree's, can find its diameter points, diameter length, center vertices, etc. Consider using two BFS's if constraints are too tight.

efc11e, 41 lines

```
struct tree_t {
    int n;
    vector<vector<int>> edges;
    vector<int> parent, dist;
    pair<int, int> center, diameter;
    tree_t(vector<vector<int>> g) : n(g.size()), parent(n),
        dist(n) {
        edges = g;
        diameter = {1, 1};
    }
    void dfs(int v, int p) {
        for (int u : edges[v]) {
            if (u == p) continue;
            parent[u] = v;
            dist[u] = dist[v] + 1;
            dfs(u, v);
        }
    }
    pair<int,int> find_diameter() { // diameter start->finish
        point
        parent[0] = -1; dist[0] = 0;
        dfs(0, 0);
        for (int i = 0; i < n; ++i)
            if (dist[i] > dist[diameter.first]) diameter.first
                = i;
        parent[diameter.first] = -1;
        dist[diameter.first] = 0;
        dfs(diameter.first, diameter.first);
        for (int i = 0; i < n; ++i)
            if (dist[i] > dist[diameter.second]) diameter.
                second = i;
        return diameter;
    }
    int get_diameter() { // length of diameter
        diameter = find_diameter();
        return dist[diameter.second];
    }
    pair<int,int> find_center() {
        diameter = find_diameter();
        int k = diameter.second, length = dist[diameter.second
            ];
        for (int i = 0; i < length/2; ++i) k = parent[k];
        if (length%2) return center = {k, parent[k]}; // two
            centers
        else return center = {k, -1}; // k is the only center
            of the tree
    }
};
```

LCA.cpp

Description: Solve lowest common ancestor queries using binary jumps. Can also find the distance between two nodes.

Time: $\mathcal{O}(N \log N + Q \log N)$

b4e1b3, 53 lines

```
struct lca_t {
    int logn{0}, preorderpos{0};
    vector<int> invpreorder, height;
    vector<vector<int>> jump, edges;
    lca_t(int n, vector<vector<int>>& adj) :
    edges(adj), height(n), invpreorder(n) {
        while((1<<(logn+1)) <= n) ++logn;
        jump.assign(n+1, vector<int>(logn+1, 0));
        dfs(0, -1, 0);
    }
    void dfs(int v, int p, int h) {
        invpreorder[v] = preorderpos++;
        height[v] = h;
        jump[v][0] = p < 0 ? v : p;
        for (int l = 1; l <= logn; ++l)
            jump[v][l] = jump[jump[v][l-1]][l-1];
        for (int u : edges[v]) {
            if (u == p) continue;
            dfs(u, v, h + 1);
        }
    }
    int climb(int v, int dist) {
        for (int l = 0; l <= logn; ++l)
            if (dist&(1<<l)) v = jump[v][l];
        return v;
    }
    int query(int a, int b) {
        if (height[a] < height[b]) swap(a, b);
        a = climb(a, height[a] - height[b]);
        if (a == b) return a;
        for (int l = logn; l >= 0; --l)
            if (jump[a][l] != jump[b][l])
                a = jump[a][l], b = jump[b][l];
        return jump[a][0];
    }
    T dist(int a, int b) {
        return height[a] + height[b] - 2 * height[query(a,b)];
    }
    bool is_parent(int p, int v) {
        if (height[p] > height[v]) return false;
        return p == climb(v, height[v] - height[p]);
    }
    bool on_path(int x, int a, int b) {
        int v = query(a, b);
```

```
        return is_parent(v, x) && (is_parent(x, a) || is_parent
            (x, b));
    }
    int get_kth_on_path(int a, int b, int k) {
        int v = query(a, b);
        int x = height[a] - height[v], y = height[b] - height[v
            ];
        if (k < x) return climb(a, k);
        else return climb(b, x + y - k);
    }
};
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). edges should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" ff2c92, 21 lines

```
struct lca_t {
    int T = 0;
    vector<int> time, path, walk, depth;
    rmq_t<int> rmq;
    lca_t(vector<vector<int>> &edges) : time(edges.size()),
    depth(edges.size()), rmq((dfs(edges,0,-1), walk)) {}
    void dfs(vector<vector<int>> &edges, int v, int p) {
        time[v] = T++;
        for(int u : edges[v]) {
            if (u == p) continue;
            depth[u] = depth[v] + 1;
            path.push_back(v), walk.push_back(time[v]);
            dfs(edges, u, v);
        }
    }
    int lca(int a, int b) {
        if (a == b) return a;
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b-1).first];
    }
};
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 1543dc, 20 lines

```
vector<pair<int,int>> compressTree(lca_t &lca, const vector<int
    >& subset) {
  static vector<int> rev; rev.resize(lca.time.size());
  vector<int> li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(li.begin(), li.end(), cmp);
  int m = li.size()-1;
  for (int i = 0; i < m; ++i) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  }
  sort(li.begin(), li.end(), cmp);
  li.erase(unique(li.begin(), li.end()), li.end());
  for (int i = 0; i < li.size(); ++i) rev[li[i]] = i;
  vector<pair<int,int>> ret = {{0, li[0]}};
  for (int i = 0; i < li.size()-1; ++i) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}
```

Heavylight.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code supports commutative segtree modifications/queries on paths, edges and subtrees. Takes as input the full adjacency list with pairs of (vertex, value). USE_EDGES being true means that values are stored in the edges and are initialized with the adjacency list, otherwise values are stored in the nodes and are initialized to the T defaults value.

Time: $\mathcal{O}\left((\log N)^2\right)$

"../data-structures/LazySegmentTree.h" 16f0e2, 94 lines

```
using G = vector<vector<pair<int,int>>>;
template<typename T, bool USE_EDGES> struct heavylight_t {
  int t, n;
  vector<int> timer, preorder;
  vector<int> chain, par;
  vector<int> dep, sz;
  vector<T> val;
  heavylight_t() {}
  heavylight_t(G &g, int r = 0) : t(0), n(g.size()), par(n, -1)
      , chain(n, -1),
  dep(n), timer(n), sz(n),  val(n), preorder(n) { par[r] =
      chain[r] = r;
    dfs_sz(g, r), dfs_hld(g, r);
  }
  int dfs_sz(G &g, int u) {
    int subtree = 1;
    for(auto &e : g[u]) {
      int v = e.first;
      if (par[v] != -1) continue;
      par[v] = u; dep[v] = dep[u] + 1;
      subtree += dfs_sz(g, v);
      if (sz[v] > sz[g[u][0].first]) swap(g[u][0], e);
    }
    return sz[u] = subtree;
  }
  void dfs_hld(G &g, int u) {
    preorder[timer[u] = t++] = u;
    for (auto &e : g[u]) {
      int v = e.first;
      if (chain[v] != -1) continue;
      chain[v] = (e == g[u][0] ? chain[u] : v);
      dfs_hld(g, v);
      if (USE_EDGES) val[timer[v]] = e.second;
    }
  }
  template<class F> void path(int u, int v, F op) {
    if (u == v) return op(timer[u], timer[u]);
    for(int e, p; chain[u] != chain[v]; u = p) {
      if (dep[chain[u]] < dep[chain[v]]) swap(u,v);
      u == (p = chain[u]) ? e = 0, p = par[u] : e = 1;
      op(timer[chain[u]] + e, timer[u]);
    }
    if (timer[u] > timer[v]) swap(u, v);
    op(timer[u] + USE_EDGES, timer[v]);
  }
};

template<typename T, bool USE_EDGES> struct hld_solver {
  heavylight_t<T, USE_EDGES> h;
  segtree_t<T, int> seg;
  hld_solver(const HLD<T, USE_EDGES> &g) : h(g), seg(h.val) {}
  void updatePath(int u, int v, T value) {
    h.path(u, v, [&](int a,int b) { seg.update(a, b, value); })
        ;
  }
  T queryPath(int u, int v) {
    T ans = 0;
    h.path(u, v, [&](int a,int b) { ans = max(ans, seg.query(a,
         b)); });
    return ans;
  }
  void updateEdge(int u, int v, T value) {
    int pos = h.timer[h.dep[u] < h.dep[v] ? v : u];
    seg.update(pos, pos, value);
  }
  T querySubtree(int v) {
    return seg.query(h.timer[v] + USE_EDGES, h.timer[v] + h.sz[
        v] - 1);
  }
  void updateSubtree(int v, T value) {
    seg.update(h.timer[v] + USE_EDGES, h.timer[v] + h.sz[v] -
        1, value);
  }
};

template<typename T, bool USE_EDGES> struct lca_t { // lca
    operations using hld
  HLD<T, USE_EDGES> h;
  lca_t(const HLD<T, USE_EDGES> &g) : h(g) {}
  int kth_ancestor(int u, int k) const {
    int kth = u;
    for(int p = h.chain[kth]; k && h.timer[kth]; kth = p, p = h
        .chain[kth]) {
      if (p == kth) p = h.par[kth];
      if (h.dep[kth] - h.dep[p] >= k) p = h.preorder[h.timer[
          kth]-k];
      k -= (h.dep[kth] - h.dep[p]);
    }
    return (k ? -1 : kth);
  }
  int lca(int u, int v) {
    if (u == v) return u;
    int x = h.timer[u];
    h.path(u, v, [&](int a,int b) { x = a - USE_EDGES; });
    return h.preorder[x];
  }
  int kth_on_path(int u, int v, int k) { //k 0-indexed
    int x = lca(u,v);
    if (k > h.dep[u] + h.dep[v] - 2 * h.dep[x]) return -1;
    if (h.dep[u] - h.dep[x] > k) return kth_ancestor(u, k);
    return kth_ancestor(v, h.dep[u] + h.dep[v] - 2 * h.dep[x] -
         k );
  }
};
```

TreeIsomorphism.h

Time: $\mathcal{O}(N \log(N))$

92e59f, 51 lines

```
map<vector<int>, int> delta;
struct tree_t {
  int n;
  pair<int,int> centroid;
  vector<vector<int>> edges;
  vector<int> sz;
  tree_t(vector<vector<int>>& graph) :
    edges(graph), sz(edges.size()) {}
  int dfs_sz(int v, int p) {
    sz[v] = 1;
    for (int u : edges[v]) {
      if (u == p) continue;
      sz[v] += dfs_sz(u, v);
    }
    return sz[v];
  }
  int dfs(int tsz, int v, int p) {
    for (int u : edges[v]) {
      if (u == p) continue;
      if (2*sz[u] <= tsz) continue;
```

```
      return dfs(tsz, u, v);
    }
    return centroid.first = v;
  }
  pair<int,int> find_centroid(int v) {
    int tsz = dfs_sz(v, -1);
    centroid.second = dfs(tsz, v, -1);
    for (int u : edges[centroid.first]) {
      if (2*sz[u] == tsz)
        centroid.second = u;
    }
    return centroid;
  }
  int hash_it(int v, int p) {
    vector<int> offset;
    for (int u : edges[v]) {
      if (u == p) continue;
      offset.push_back(hash_it(u, v));
    }
    sort(offset.begin(), offset.end());
    if (!delta.count(offset))
      delta[offset] = int(delta.size());
    return delta[offset];
  }
  lint get_hash(int v = 0) {
    pair<int,int> cent = find_centroid(v);
    lint x = hash_it(cent.first, -1), y = hash_it(cent.second,
        -1);
    if (x > y) swap(x, y);
    return (x << 30) + y;
  }
};
```

LineTree.h

Description: Performs a preprocessing to enable querying the maximum/minimum edge weight on any path in a tree in constant time.
Time: $\mathcal{O}(n \log(n))$

<RMQ.h> 8c170d, 73 lines

```
struct UF {
    vector<int> parent, size, left, right;
    UF(int n) : parent(n), size(n, 1), left(n), right(n) {
        for (int i = 0; i < n; i++)
            parent[i] = left[i] = right[i] = i;
    }
    int find(int x) {
        return x == parent[x] ? x : parent[x] = find(parent[x])
            ;
    }
    pair<int, int> unite(int x, int y) {
        x = find(x);
        y = find(y);
        assert(x != y);
        if (size[x] < size[y]) swap(x, y);
        parent[y] = x;
        size[x] += size[y];
        pair<int,int> result = {right[x], left[y]};
        right[x] = right[y];
        return result;
    }
};
template<typename T> struct linetree_t {
  struct edge_t {
    int u, v; T w;
    edge_t() {}
    edge_t(int a, int b, T c) : u(a), v(b), w(c) {}
    bool operator<(const edge_t &other) const {
      return w < other.w;
    }
  };
  int n;
  const T limit = numeric_limits<T>::min();
  vector<int> index, line;
  vector<edge_t> edges; vector<T> line_w;
  unique_ptr<rmq_t<T>> rmq;
  linetree_t(int _n) : n(_n), index(n) {}
  void addEdge(int from, int to, T weight) {
    edges.emplace_back(from, to, weight);
  }
  void make_tree() {
    sort(edges.begin(), edges.end());
    UF dsu(n);
    vector<int> next_v(n, -1), has_prev(n);
    vector<T> next_w(n, limit);
    for (edge_t& e : edges) {
      pair<int, int> united = dsu.unite(e.u, e.v);
      next_v[united.first] = united.second;
      has_prev[united.second] = 1;
      next_w[united.first] = e.w;
    }
    int start = -1;
    for (int i = 0; i < n; ++i)
      if (!has_prev[i]) {
        start = i;
        break;
      }
    while(start >= 0) {
      line.push_back(start);
      if (next_v[start] >= 0)
        line_w.push_back(next_w[start]);
      start = next_v[start];
    }
    for (int i = 0; i < n; ++i)
      index[line[i]] = i;
    rmq.reset(new RMQ<T>(line_w));
  }
  T query(int a, int b) {
    if (a == b) return limit;
    a = index[a], b = index[b];
    if (a > b) swap(a, b);
    return rmq->query(a-1, b-1).first;
  }
};
```

7.6.1 Sqrt Decomposition

HLD generally suffices. If not, here are some common strategies:

- Rebuild the tree after every $\sqrt{N}$ queries.
- Consider vertices with $>$ or $<$ $\sqrt{N}$ degree separately.
- For subtree updates, note that there are $O(\sqrt{N})$ distinct sizes among child subtrees of any vertex.

Block Tree: Use a DFS to split edges into contiguous groups of size $\sqrt{N}$ to $2\sqrt{N}$.

Mo's Algorithm for Tree Paths: Maintain an array of vertices where each one appears twice, once when a DFS enters the vertex (`st`) and one when the DFS exists (`en`). For a tree path $u \leftrightarrow v$ such that `st[u]<st[v]`,

- If u is an ancestor of v, query `[st[u],st[v]]`.
- Otherwise, query `[en[u],st[v]]` and consider $lca(u, v)$ separately.

7.7 Functional Graphs

Lumberjack.h

Description: Called lumberjack technique, solve functional graphs problems for digraphs, it's also pretty good for dp on trees. Consists in go cutting the leaves until there is no leaves, only cycles. For that we keep a processing queue of the leaves, note that during this processing time we go through all the childrens of v before reaching a vertex v, therefore we can compute some infos about the children, like subtree of a given vertex
Usage: `Lumberjack<10010> g; g.init(N); (Be careful with the size of cyles when declared locally!)`

7211bc, 70 lines

```
template<int T> struct Lumberjack {
  int n, numcycle;
  vector<int> subtree, order, par, cycle;
  vector<int> parincycles, idxcycle, sz, st;
  vector<int> depth, indeg, cycles[T];
  vector<bool> seen, incycle, leaf;
  void init(vector<int>& _par, vector<int>& _indeg){
    init(_par.size());
    par = _par; indeg = _indeg;
  }
  void init(int N) {
    n = N;
    order.resize(0);
    subtree.assign(n, 0);
    seen.assign(n, false);
    sz = st = subtree;
    parincycles = par = cycle = sz;
    idxcycle = depth = indeg = sz;
    incycle = leaf = seen;
  }
  void find_cycle(int u){
    int idx= ++numcycle, cur = 0, p = u;
    st[idx] = u;
    sz[idx] = 0;
    cycles[idx].clear();
    while (!seen[u]) {
      seen[u] = incycle[u] = 1;
      parincycles[u] = u;
      cycle[u] = idx;
      idxcycle[u] = cur;
      cycles[idx].push_back(u);
      ++sz[idx];
      depth[u] = 0;
      ++subtree[u];
      u = par[u];
      ++cur;
    }
  }
  void bfs() {
    queue<int> q;
    for (int i = 0; i < n; ++i)
      if (!indeg[i]){
        seen[i] = leaf[i] = true;
        q.push(i);
      }
    while(!q.empty()){
      int v = q.front(); q.pop();
      order.push_back(v);
      ++subtree[v];
      int curpar = par[v];
      indeg[curpar]--;
      subtree[curpar] += subtree[v];
      if(!indeg[curpar]){
        q.push(curpar);
        seen[curpar] = true;
```

```
      }
    }
    numcycle = 0;
    for (int i = 0; i < n; ++i)
      if (!seen[i]) find_cycle(i);
    for(int i = order.size()-1; i >= 0; --i){
      int v = order[i], curpar = par[v];
      parincycles[v] = parincycles[curpar];
      cycle[v] = cycle[curpar];
      incycle[v] = false;
      idxcycle[v] = -1;
      depth[v] = 1 + depth[curpar];
    }
  }
};
```

Lumberjack2.h

Description: Called lumberjack technique, solve functional graphs problems for graphs, it's also pretty good for dp on trees. Consists in go cutting the leaves until there is no leaves, only cycles. For that we keep a processing queue of the leaves, note that during this processing time we go through all the childrens of v before reaching a vertex v, therefore we can compute some infos about the children, like subtree of a given vertex

1c72ef, 79 lines

```
template<int T> struct Lumberjack {
  int n, numcycle;
  vector<int> subtree, order, par, cycle;
  vector<int> parincycles, idxcycle, sz, st;
  vector<int> depth, deg, cycles[T];
  vector<bool> seen, incycle, leaf;
  vector<vector<int>> graph;
  void init(vector<vector<int>>& _graph, vector<int>& _deg){
    init(_graph.size());
    graph = _graph; deg = _deg;
  }
  void init(int N) {
    n = N;
    order.resize(0);
    subtree.assign(n, 0);
    seen.assign(n, false);
    sz = st = subtree;
    parincycles = par = cycle = sz;
    idxcycle = depth = deg = sz;
    incycle = leaf = seen;
    vector<int> adj; graph.assign(n, adj);
  }
  int find_par(int u) {
      for (int v : graph[u]) if (!seen[v])
        return v;
      return -1;
  }
  void find_cycle(int u){
    int idx= ++numcycle, cur = 0, p = u;
    st[idx] = u;
    sz[idx] = 0;
    cycles[idx].clear();
    while (!seen[u]) {
      seen[u] = incycle[u] = true;
      par[u] = find_par(u);
      if(par[u] == -1) par[u] = p;
      parincycles[u] = u;
      cycle[u] = idx;
      idxcycle[u] = cur;
      cycles[idx].push_back(u);
      ++sz[idx];
      depth[u] = 0;
      ++subtree[u];
      u = par[u];
      ++cur;
    }
  }
  void bfs() {
    queue<int> q;
    for (int i = 0; i < n; ++i)
      if (deg[i] == 1){
        seen[i] = leaf[i] = true;
        q.push(i);
      }
    while(!q.empty()){
      int v = q.front(); q.pop();
      order.push_back(v);
      ++subtree[v];
      int curpar = find_par(v);
      deg[curpar]--;
      subtree[curpar] += subtree[v];
      if(deg[curpar] == 1){
        q.push(curpar);
        seen[curpar] = true;
      }
    }
    numcycle = 0;
    for (int i = 0; i < n; ++i)
      if (!seen[i]) find_cycle(i);
    for(int i = order.size()-1; i >= 0; --i){
      int v = order[i], curpar = par[v];
      parincycles[v] = parincycles[curpar];
      cycle[v] = cycle[curpar];
      incycle[v] = false;
      idxcycle[v] = -1;
      depth[v] = 1 + depth[curpar];
    }
  }
};
```

7.8 Other

kthShortestPath.h

Description: Find Kth shortest path from s to t.
Time: $\mathcal{O}((V+E)lg(V)*k)$

b61179, 21 lines

```
int getCost(vector<vector<pair<int,int>>> &G, int s, int t, int
      k) {
    int n = G.size();
    vector<int> dist(n, INF), count(n, 0);
    priority_queue<pair<int,int>, vector<pair<int,int>>,
         greater<pair<int,int>>> Q;
  Q.push({0, s});
  while (!Q.empty() && (count[t] < k)) {
    pair<int,int> v = Q.top();
    int u = v.second, w = v.first;
    Q.pop();
    if ((dist[u] == INF) || (w > dist[u])) { // remove equal
         paths
      count[u] += 1;
      dist[u] = w;
    }
    if (count[u] <= k)
        for (int x : G[u]) {
        int v = x.first, w = x.second;
        Q.push({dist[u] + w, v});
        }
  }
    return dist[t];
}
```

MatrixTreeMST.h

Description: Returns the number of msts in undirected weighted graph using the Matrix Tree theorem.
Time: $\mathcal{O}(N^3)$

"DSU.h" d2ed36, 85 lines

```
lint det(vector<vector<lint>> a, int n, int p) {
  lint ans = 1;
  for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) a[i][
       j] %= p;
  for(int i = 1; i < n; ++i) {
    for(int j = i+1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        lint t = a[i][i] / a[j][i];
        if (t) for(int k = i; k < n; ++k) {
          a[i][k] = (a[i][k] - a[j][k] * t) % p;
          a[i][k] %= p;
        }
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % p;
    if (!ans) return 0;
  }
  return (ans + p) % p;
}
struct edge_t {
  int u, v, w;
  bool operator<(const edge_t& o) const {
    return w < o.w;
  }
};
const int N = 101;
int edgenum = 0;
vector<edge_t> edge;
vector<bool> seen;
vector<int> g[N];
vector<vector<lint>> p, deg;
void addEdge(int u, int v, int d){
  edge_t E = { u, v, d };
  edge[++edgenum] = E;
}
lint MST_count(int n, lint MOD) {
  sort(edge.begin()+1, edge.begin()+edgenum+1);
  int pre = edge[1].w;
  lint ans = 1;
  UF a(n+1), b(n+1);
  seen = vector<bool>(n+1, false);
  deg = vector<vector<lint>>(n+1, vector<lint>(n+1));
  for (int i = 0; i <= n; i++) g[i].clear();
  for (int t = 1; t <= edgenum+1; ++t) {
    if (edge[t].w != pre || t == edgenum + 1) {
      for (int i = 1, k; i <= n; i++) if (seen[i]) {
        k = b.find(i);
        g[k].push_back(i);
        seen[i] = false;
      }
      for (int i = 1; i <= n; ++i)
        if (g[i].size()) {
          p = vector<vector<lint>>(n+1, vector<lint>(n+1));
          for (int j = 0; j < g[i].size(); j++)
          for (int k = j+1, x, y; k < g[i].size(); ++k) {
            x = g[i][j];
            y = g[i][k];
            p[j][k] = p[k][j] = -deg[x][y];
            p[j][j] += deg[x][y];
            p[k][k] += deg[x][y];
          }
          ans = ans*det(p, g[i].size(), MOD) % MOD;
          for (int j = 0; j < g[i].size(); ++j) a.par[g[i][j]]
               = i;
        }
```

```
      deg = vector<vector<lint>>(n+1, vector<lint>(n+1));
      for (int i = 1; i <= n; ++i) {
        b.par[i] = a.find(i);
        g[i].clear();
      }
      if (t == edgenum+1) break;
      pre = edge[t].w;
    }
    int x = a.find(edge[t].u);
    int y = a.find(edge[t].v);
    if (x == y) continue;
    seen[x] = seen[y] = true;
    b.unite(x, y);
    deg[x][y]++; deg[y][x]++;
  }
  if (!edgenum) return 0;
  for (int i = 2; i <= n; i++)
    if (b.find(i) != b.find(1)) return 0;
  return ans;
}
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights $w(p,q) = |p.x - q.x| + |p.y - q.y|$. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(NlogN)$

<UnionFind.h> de8170, 28 lines

```
typedef Point<int> P;
pair<vector<array<int, 3>>, int> manhattanMST(vector<P> ps) {
    vector<int> id(ps.size());
    iota(id.begin(), id.end(), 0);
    vector<array<int, 3>> edges;
    for(int k = 0; k < 4; ++k) {
        sort(id.begin(), id.end(), [&](int i, int j) {
             return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
        map<int, int> sweep;
        for(auto& i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y);
                        it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                P d = ps[i] - ps[j];
                if (d.y > d.x) break;
                edges.push_back({d.y + d.x, i, j});
            }
            sweep[-ps[i].y] = i;
        }
        if (k & 1) for(auto& p : ps) p.x = -p.x;
        else for(auto& p : ps) swap(p.x, p.y);
    }
    sort(edges.begin(), edges.end());
    UF uf(ps.size());
    int cost = 0;
    for (auto e: edges) if (uf.unite(e[1], e[2])) cost += e[0];
    return {edges, cost};
}
```

Pruefer.cpp

Description: Given a tree, construct its pruefer sequence. The Pruefer code is a way of encoding a labeled tree with n vertices using a sequence of (n2) integers in the interval from 0 to n-1. This encoding also acts as a bijection between all spanning trees of a complete graph and the numerical sequences.

961713, 37 lines

```
struct pruefer_t {
    vector<vector<int>> adj;
    vector<int> parent;
    pruefer_t(int _n) : adj(n), parent(n) {}
    void dfs (int u) {
        for (int i = 0; i < adj[u].size(); ++i) {
            if (i != parent[u]) {
                parent[i] = v;
                dfs(i);
            }
        }
    }
    vector<int> pruefer() {
        int n = adj.size();
        parent.resize(n);
        parent[n-1] = -1;
        dfs(n-1);
        int one_leaf = -1;
        vector<int> degree(n), ret(n-2);
        for (int i = 0; i < n; ++i) {
            degree[i] = adj[i].size();
            if (degree[i] == 1 && one_leaf == -1) one_leaf = 1;
        }
        int leaf = one_leaf;
        for (int i = 0; i < n-2; ++i) {
            int next = parent[leaf];
            ret[i] = next;
            if (--degree[next] == 1 && next < one_leaf) leaf =
                 next;
            else {
                ++one_leaf;
                while (degree[one_leaf] != 1) ++one_leaf;
                leaf = one_leaf;
            }
        }
        return ret;
    }
};
```

ErdosGallai.h

Description: Check if an array of degrees can represent a graph
Time: if sorted $\mathcal{O}(n)$, otherwise $\mathcal{O}(nlog(n))$

56391b, 15 lines

```
bool EG(vector<int> deg) {
    sort(deg.begin(), deg.end(), greater<int>());
    vector<long long> dp(deg.size());
    int n = deg.size(), p = n-1;
    for(int i = 0; i < n; i++)
        dp[i] = deg[i] + (i > 0 ? dp[i-1] : 0);
    for(int k = 1; k <= n; k++) {
        while(p >= 0 && deg[p] < k) p--;
        long long sum;
        if (p >= k-1) sum = (p-k+1)*1ll*k + dp[n-1] - dp[p];
        else sum = dp[n-1] - dp[k-1];
        if (dp[k-1] > k*(k-1LL) + sum) return 0;
    }
    return dp[n-1] % 2 == 0;
}
```

MisraGries.h

Description: Finds a $\max_i \deg(i) + 1$-edge coloring where there all incident edges have distinct colors. Finding a D-edge coloring is NP-hard.

b27b0c, 47 lines

```
struct edge {int to, color, rev; };
struct MisraGries {
    int N, K = 0;
    vector<vector<int>> F;
    vector<vector<edge>> graph;
    MisraGries(int n) : N(n), graph(n) {}
    // add an undirected edge, NO DUPLICATES ALLOWED
  void addEdge(int u, int v) {
    graph[u].push_back({v, -1, (int) graph[v].size()});
    graph[v].push_back({u, -1, (int) graph[u].size()-1});
  }
  void color(int v, int i) {
    vector<int> fan = { i };
    vector<bool> used(graph[v].size());
    used[i] = true;
    for (int j = 0; j < (int) graph[v].size(); j++)
      if (!used[j] && graph[v][j].col >= 0 && F[graph[v][fan.
           back()].to][graph[v][j].col] < 0)
        used[j] = true, fan.push_back(j), j = -1;
    int c = 0; while (F[v][c] >= 0) c++;
    int d = 0; while (F[graph[v][fan.back()].to][d] >= 0) d++;
    int w = v, a = d, k = 0, ccol;
    while (true) {
      swap(F[w][c], F[w][d]);
      if (F[w][c] >= 0) graph[w][F[w][c]].col = c;
      if (F[w][d] >= 0) graph[w][F[w][d]].col = d;
      if (F[w][a^=c^d] < 0) break;
      w = graph[w][F[w][a]].to;
    }
    do {
      Edge &e = graph[v][fan[k]];
      ccol = F[e.to][d] < 0 ? d : graph[v][fan[k+1]].col;
      if (e.col >= 0) F[e.to][e.col] = -1;
      F[e.to][ccol] = e.rev;
      F[v][ccol] = fan[k];
      e.col = graph[e.to][e.rev].col = ccol;
      k++;
    } while (ccol != d);
  }
  // finds a K-edge-coloringraph
  void color() {
    for(int v = 0; v < N; ++v)
        K = max(K, (int)graph[v].size() + 1);
    F = vector<vector<int>>(N, vector<int>(K, -1));
    for(int v = 0; v < N; ++v) for (int i = graph[v].size(); i
         --; )
      if (graph[v][i].col < 0) color(v, i);
  }
};
```

DirectedMST.cpp

Description: Edmonds' algorithm for finding the weight of the minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFind.h" e6517a, 47 lines

```
struct edge_t { int a, b; lint w; };
struct node_t {
    edge_t key;
    node_t *l, *r;
    lint delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    edge_t top() { prop(); return key; }
};
node_t *merge(node_t *a, node_t *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(node_t*& a) { a->prop(); a = merge(a->l, a->r); }
lint dmst(int n, int r, vector<edge_t>& g) {
    UF uf(n);
    vector<node_t*> heap(n);
```

```
    for(auto &e : g) heap[e.b] = merge(heap[e.b], new node_t{e
        });
    lint res = 0;
    vector<int> seen(n, -1), path(n);
    seen[r] = r;
    for(int s = 0; s < n; ++s) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            path[qi++] = u, seen[u] = s;
            if (!heap[u]) return -1;
            edge_t e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                node_t* cyc = 0;
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.unite(u, w));
                u = uf.find(u);
                heap[u] = cyc, seen[u] = -1;
            }
        }
    }
    return res;
}
```

7.9.2 Dirac's theorem

Let G be a graph with n vertices, each one with degree at least $n/2$. Then G is hamiltonian.

7.9.3 Ore's theorem

Let G be a simple graph of order $n \geq 3$ st

$$g(u) + g(v) \geq n$$

for all pair u,v of non adjacent vertices, then G is hamiltonian.

7.9.4 Eulerian Cycles

The number of Eulerian cycles in a *directed* graph G is:

$$t_w(G) \prod_{v \in G} (\deg v - 1)!,$$

where $t_w(G)$ is the number of arborescences ("directed spanning" tree) rooted at w: $t_w(G) = \det(q_{ij})_{i,j \neq w}$, with $q_{ij} = [i = j]\text{indeg}(i) - \#(i,j) \in E$.

7.9.5 Landau

There are a tournament with outdegree $d_1 \leq d_2 \leq \ldots \leq d_n$ iff:

- $d_1 + d_2 + \ldots + d_n = \binom{n}{2}$
- $d_1 + d_2 + \ldots + d_k \geq \binom{k}{2} \quad \forall 1 \leq k \leq n.$

7.9.6 Dilworth's theorem

For any partially ordered set, the sizes of the max antichain and of the min chain decomposition are equal. Equivalent to Konig's theorem on the bipartite graph (U, V, E) where $U = V = S$ and (u, v) is an edge when $u < v$. Those vertices outside the min vertex cover in both U and V form a max antichain.

7.9.7 König-Egervary theorem

For bipartite graphs, the number of edges in the maximum matching is greater than or equal the number of vertices in the minimum cover.

Maximum Weight Closure

Given a vertex-weighted directed graph G. Turn the graph into a flow network, adding weight ∞ to each edge. Add vertices S, T. For each vertex v of weight w, add edge (S, v, w) if $w \geq 0$, or edge $(v, T, -w)$ if $w < 0$. Sum of positive weights minus minimum $S - T$ cut is the answer. Vertices reachable from S are in the closure. The maximum-weight closure is the same as the complement of the minimum-weight closure on the graph with edges reversed.

7.9.8 Maximum Weighted Independent Set in a Bipartite Graph

This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S, u, w(u))$ for $u \in L$, $(v, T, w(v))$ for $v \in R$ and (u, v, ∞) for $(u, v) \in E$. The minimum S, T-cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

7.9.9 Tutte-Berge formula

The theorem states that the size of a maximum matching of a graph $G = (V, E)$ equals $\frac{1}{2} \min_{U \subseteq V} \left(|U| - \texttt{odd}(G - U) + |V|\right)$, where $\texttt{odd}$(H) counts how many of the connected components of the graph H have an odd number of vertices.

7.9.10 Tutte's theorem

A graph $G = (V, A)$ has a perfect matching iff for all subset U of V, the induced subgraph by $V \setminus U$ has at most $|U|$ connected components with odd number of vertices.

7.9.11 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

7.9.12 Number of Spanning Trees

Define Laplacian Matrix as $L = D - A$, D being a Diagonal Matrix with $D_{i,i} = deg(i)$ and A an Adjacency Matrix. Create an $N \times N$ Laplacian matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.9.13 Tutte Matrix

- A graph has a perfect matching iff the *Tutte* matrix has a non-zero determinant.
- The rank of the *Tutte* matrix is equal to twice the size of the maximum matching. The maximum cost matching can be found by polynomial interpolation.

7.9.14 Menger's theorem

Vertices: A graph is k-connected iff all pairwise vertices are connected to at least k internally disjoint paths.

Edges: A graph is called k-edge-connected if the removal of at least k edges of the graph keeps it connected. A graph is k-edge-connected iff for all pairwise vertices u and v, exist k paths which link u to v without sharing an edge.

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

f90ade, 27 lines

```cpp
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
};
using P = Point<double>;
```

Complex.h

Description: Example of geometry using complex numbers. Just to be used as reference. std::complex has issues with integral data types, be careful, you can't use polar or abs.

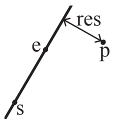
145247, 63 lines

```cpp
const double E = 1e-9;
typedef double T;
using P = complex<T>;
#define x real()
#define y imag()
// example of how to represent a line using complex numbers
struct line {
  P p, v;
  line(P a, P b) {
    p = a;
    v = b - a;
  }
};
P dir(T angle) { return polar((T)1, angle); }
P unit(P p) { return p/abs(p); }
P translate(P v, P p) {return p + v;}
//rotate point around origin by a
P rotate(P p, T a) { return p * polar(1.0, a); }
//around pivot
P rotate(P v, T a, P pivot) { (a-pivot) * polar(1.0, a) + pivot
    ; }
T dot(P v, P w) { return (conj(v)*w).x; }
T cross(P v, P w) { return (conj(v)*w).y; }
T cross(P A, P B, P C) { return cross(B - A, C - A); }
P proj(P a, P v) { return v * dot(a, v) / dot(v, v); }
P closest(P p, line l) { return l.p + proj(p - l.p, l.v); }
double dist(P p, line l) { return fabs(p - closest(p, l)); }
P reflect(P p, P v, P w) {
    P z = p - v; P q = w - v;
    return conj(z/q) * q + v;
}
P intersection(line a, line b) { // undefined if parallel
    T d1 = cross(b.p - a.p, a.v - a.p);
    T d2 = cross(b.v - a.p, a.v - a.p);
    return (d1 * b.v - d2 * b.p)/(d1 - d2);
}
vector<P> convex_hull(vector<P> points) {
    if (points.size() <= 1) return points;
  sort(points.begin(), points.end(), [](P a, P b) {
    return real(a)==real(b) ? imag(a)<imag(b) : real(a)<real(b)
        ; });
  vector<P> hull(points.size()+1);
  int s = 0, k = 0;
  for (int it = 2; it--; s = --k, reverse(points.begin(),
      points.end()))
      for (P p : points) {
          while(k >= s+2 && cross(hull[k-2], hull[k-1], p) <=
              0) k--;
          hull[k++] = p;
      }
  return {hull.begin(), hull.begin() + k - (k == 2 && hull[0]
      == hull[1])};
}
P p{4, 3};
// get the absolute value and angle in [-pi, pi]
cout << abs(p) << ' ' << arg(p) << '\n'; // 5 - 0.643501
// make a point in polar form
cout << polar(2.0, -M_PI/2) << '\n'; // (1.41421, -1.41421)
P v{1, 0};
cout << rotate(v, -M_PI/2) << '\n';
// Projection of v onto Riemann sphere and norm of p
cout << proj(v) << ' ' << norm(p) << '\n';
// Distance between p and v and the squared distance
cout << abs(v-p) << ' ' << norm(v-p) << '\n';
// Angle of elevation of line vp and its slope
cout << arg(p-v) * (180/M_PI) << ' ' << tan(arg(p-v)) << '\n';
// has trigonometric functions aswell (e.g. cos, sin, cosh,
    sinh, tan, tanh)
// and exp, pow, log
```

LineDistance.h

Description:

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.



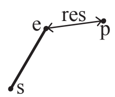
"Point.h" f6bf6b, 4 lines

```cpp
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:

Returns the shortest distance between point p and the line segment from point s to e.



Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" ae751a, 5 lines

```cpp
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentClosestPoint.h

Description: Returns the closest point to p in the segment from point s to e as well as the distance between them

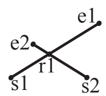
d4b82f, 13 lines

```cpp
pair<P,double> SegmentClosestPoint(P &s, P &e, P &p){
  P ds=p-s, de=p-e;
  if(e==s)
    return {s, ds.dist()};
  P u=(e-s).unit();
  P proj=u*ds.dot(u);
  if(onSegment(s, e, proj+s))
    return {proj+s, (ds-proj).dist()};
  double dist_s=ds.dist(), dist_e=de.dist();
  if(cmp(dist_s, dist_e)==1)
    return {s, dist_s};
  return{e, dist_e};
}
```

SegmentIntersection.h

Description:

If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" f6be16, 13 lines

```cpp
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {s.begin(), s.end()};
}
```

SegmentIntersectionQ.h

Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

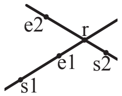
"Point.h" 1ff4ba, 16 lines

```cpp
template<class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
  if (e1 == s1) {
    if (e2 == s2) return e1 == e2;
    swap(s1,s2); swap(e1,e2);
  }
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = d.cross(v1), a2 = d.cross(v2);
  if (a == 0) { // parallel
    auto b1 = s1.dot(v1), c1 = e1.dot(v1),
         b2 = s2.dot(v1), c2 = e2.dot(v1);
    return !a1 && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
```

```
  return (0 <= a1 && a1 <= a && 0 <= a2 && a2 <= a);
}
```

LineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
    P v = b - a;
    return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

SideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

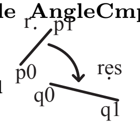
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

LinearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

0f0602, 34 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

AngleCmp.h

Description: Useful utilities for dealing with angles of rays from origin. OK for integers, only uses cross product. Doesn't support (0,0).

6edd25, 22 lines

```
template <class P>
bool sameDir(P s, P t) {
  return s.cross(t) == 0 && s.dot(t) > 0;
}
// checks 180 <= s..t < 360?
template <class P>
bool isReflex(P s, P t) {
  auto c = s.cross(t);
  return c ? (c < 0) : (s.dot(t) < 0);
}
// operator < (s,t) for angles in [base,base+2pi)
template <class P>
bool angleCmp(P base, P s, P t) {
  int r = isReflex(base, s) - isReflex(base, t);
  return r ? (r < 0) : (0 < s.cross(t));
}
// is x in [s,t] taken ccw? 1/0/-1 for in/border/out
template <class P>
int angleBetween(P s, P t, P x) {
  if (sameDir(x, s) || sameDir(x, t)) return 0;
  return angleCmp(s, x, t) ? 1 : -1;
}
```

LinearSolver.h

Description: Solves the linear system (a * x + b * y = e) and (c * x + d * y = f) Returns tuple (1, Point(x, y)) if solution is unique, (0, Point(0,0)) if no solution and (-1, Point(0,0)) if infinite solutions. If using integer function type, this will give wrong answer if answer is not integer.

e093c1, 12 lines

```
template<class T>
pair<int, Point<T>> solve_linear(T a, T b, T c, T d, T e, T f)
     {
    Point<T> ret;
    T det = a * d - b * c;
    if (det == 0) {
        if (b * f == d * e && a * f == c * e) return {-1, Point
            <T>()};
        return {0, Point<T>()};
    }
     //In case solution needs to be integer, use something like
         the line below.
   //assert((e * d - f * b) % det == 0 && (a * f - c * e) % det
        == 0);
    return {1, Point<T>((e*d - f*b) / det, (a*f - c*e) / det)};
}
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" c64785, 10 lines

```
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

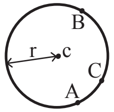
```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
```

```
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

Circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" c98102, 8 lines

```
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 8ab87f, 19 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(ps.begin(),ps.end(), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  for(int i = 0; i < ps.size(); ++i)
      if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        for(int j = 0; j < i; ++j) if ((o - ps[j]).dist() > r *
             EPS) {
          o = (ps[i] + ps[j]) / 2;
          r = (o - ps[i]).dist();
          for(int k = 0; k < j; ++k)
              if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).dist();
              }
        }
      }
  return {o, r};
}
```

CircleUnion.h

Description: Computes the circles union total area

fd65da, 86 lines

```
struct CircleUnion {
    static const int maxn = 1e5 + 5;
    const double PI = acos((double)-1.0);
    int n;
    double x[maxn], y[maxn], r[maxn];
    int covered[maxn];
    vector<pair<double, double>> seg, cover;
    double arc, pol;
    inline int sign(double x) {return x < -EPS ? -1 : x > EPS;}
    inline int sign(double x, double y) {return sign(x - y);}
    inline double sqr(const double x) {return x * x;}
    inline double dist(double x1, double y1, double x2, double
         y2) {return sqrt(sqr(x1 - x2) + sqr(y1 - y2));}
    inline double angle(double A, double B, double C) {
        double val = (sqr(A) + sqr(B) - sqr(C)) / (2 * A * B);
        if (val < -1) val = -1;
        if (val > +1) val = +1;
        return acos(val);
    }
    CircleUnion() {
        n = 0;
        seg.clear(), cover.clear();
        arc = pol = 0;
    }
    void init() {
        n = 0;
        seg.clear(), cover.clear();
        arc = pol = 0;
    }
    void add(double xx, double yy, double rr) {
        x[n] = xx, y[n] = yy, r[n] = rr, covered[n] = 0, n++;
    }
    void getarea(int i, double lef, double rig) {
        arc += 0.5 * r[i] * r[i] * (rig - lef - sin(rig - lef))
            ;
        double x1 = x[i] + r[i] * cos(lef), y1 = y[i] + r[i] *
             sin(lef);
        double x2 = x[i] + r[i] * cos(rig), y2 = y[i] + r[i] *
             sin(rig);
        pol += x1 * y2 - x2 * y1;
    }
    double calc() {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < i; j++)
                if (!sign(x[i] - x[j]) && !sign(y[i] - y[j]) &&
                     !sign(r[i] - r[j])) {
                    r[i] = 0.0;
                    break;
                }
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                if (i != j && sign(r[j] - r[i]) >= 0 && sign(
                     dist(x[i], y[i], x[j], y[j]) - (r[j] - r[i
                     ])) <= 0) {
                    covered[i] = 1;
                    break;
                }
        for (int i = 0; i < n; i++) {
            if (sign(r[i]) && !covered[i]) {
                seg.clear();
                for (int j = 0; j < n; j++)
                    if (i != j) {
                        double d = dist(x[i], y[i], x[j], y[j])
                            ;
                        if (sign(d - (r[j] + r[i])) >= 0 ||
                            sign(d - abs(r[j] - r[i])) <= 0)
                          continue;
                        double alpha = atan2(y[j] - y[i], x[j]
                            - x[i]);
                        double beta = angle(r[i], d, r[j]);
                        pair<double, double> tmp(alpha - beta,
                            alpha + beta);
                        if (sign(tmp.first) <= 0 && sign(tmp.
                            second) <= 0)
                            seg.push_back(pair<double, double
                                >(2 * PI + tmp.first, 2 * PI +
                                 tmp.second));
                        else if (sign(tmp.first) < 0) {
                            seg.push_back(pair<double, double
                                >(2 * PI + tmp.first, 2 * PI))
                                ;
                            seg.push_back(pair<double, double
                                >(0, tmp.second));
                        }
                        else seg.push_back(tmp);
                    }
                sort(seg.begin(), seg.end());
                double rig = 0;
                for (vector<pair<double, double> >::iterator
                     iter = seg.begin(); iter != seg.end();
                     iter++) {
                    if (sign(rig - iter->first) >= 0)
                        rig = max(rig, iter->second);
                    else {
                        getarea(i, rig, iter->first);
                        rig = iter->second;
                    }
                }
                if (!sign(rig)) arc += r[i] * r[i] * PI;
                else getarea(i, rig, 2 * PI);
            }
        }
        return pol / 2.0 + arc;
    }
} ccu;
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>

"Point.h", "lineDistance.h", "LineProjectionReflection.h" debf86, 8 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  double h2 = r*r - a.cross(b,c)*a.cross(b,c)/(b-a).dist2();
  if (h2 < 0) return {};
  P p = lineProj(a, b, c), h = (b-a).unit() * sqrt(h2);
  if (h2 == 0) return {p};
  return {p - h, p + h};
}
```

CircleCircleArea.h

Description: Calculates the area of the intersection of 2 circles

8bf2b6, 12 lines

```
template<class P>
double circleCircleArea(P c, double cr, P d, double dr) {
    if (cr < dr) swap(c, d), swap(cr, dr);
    auto A = [&](double r, double h) {
        return r*r*acos(h/r)-h*sqrt(r*r-h*h);
    };
    auto l = (c - d).dist(), a = (l*l + cr*cr - dr*dr)/(2*l);
    if (l - cr - dr >= 0) return 0; // far away
    if (l - cr + dr <= 0) return M_PI*dr*dr;
    if (l - cr >= 0) return A(cr, a) + A(dr, l-a);
    else return A(cr, a) + M_PI*dr*dr - A(dr, a-l);
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"Point.h" cf9deb, 18 lines

```
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
```

```
  };
  auto sum = 0.0;
  for (int i = 0; i < ps.size(); ++i)
    sum += tri(ps[i] - c, ps[(i + 1) % ps.size()] - c);
  return sum;
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" f9442d, 12 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = p.size();
  for(int i = 0; i < n; ++i) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict; // change to
        // -1 if u need to detect points in the boundary
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns the area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 3794ee, 17 lines

```
template<class T>
T polygonArea(vector<Point<T>> &v) {
  T a = v.back().cross(v[0]);
  for(int i = 0; i < v.size()-1; ++i)
      a += v[i].cross(v[i+1]);
  return abs(a)/2.0;
}

Point<T> polygonCentroid(vector<Point<T>> &v) { // not tested
  Point<T> cent(0,0); T area = 0;
  for(int i = 0; i < v.size(); ++i) {
    int j = (i+1) % (v.size()); T a = cross(v[i], v[j]);
    cent += a * (v[i] + v[j]);
    area += a;
  }
  return cent/area/(T)3;
}
```

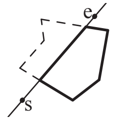
PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 26a00f, 8 lines

```
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = v.size() - 1; i < v.size(); j = ++i) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

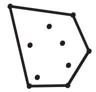


Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" 7df36f, 11 lines

```
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  for(int i = 0; i < poly.size(); ++i) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side) res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h



Description:
Returns a vector of indices of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

"Point.h" 3612d7, 12 lines

```
vector<P> convexHull(vector<P> pts) {
  if (pts.size() <= 1) return pts;
  sort(pts.begin(), pts.end());
  vector<P> h(pts.size()+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(pts.begin(), pts.end
      ()))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/colinear points).

0e0c1f, 11 lines

```
array<P, 2> hullDiameter(vector<P> S) {
  int n = S.size(), j = n < 2 ? 0 : 1;
  pair<lint, array<P, 2>> res({0, {S[0], S[0]}});
  for(int i = 0; i < j; ++i)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 7b8514, 12 lines

```
bool inHull(const vector<P> &l, P p, bool strict = true) {
  int a = 1, b = l.size() - 1, r = !strict;
  if (l.size() < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

PolyUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. Guaranteed to be precise for integer coordinates up to 3e7. If epsilons are needed, add them in sideOf as well as the definition of sgn.
Time: $\mathcal{O}(N^2)$, where N is the total number of points

"Point.h", "sideOf.h" a45bd4, 33 lines

```
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  for(int i = 0; i < poly.size(); ++i)
    for(int v = 0; v < poly[i].size(); ++v) {
      P A = poly[i][v], B = poly[i][(v + 1) % poly[i].size()];
      vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
      for(int j = 0; j < poly.size(); ++j) if (i != j) {
        for(int u = 0; u < poly[j]; ++u) {
          P C = poly[j][u], D = poly[j][(u + 1) % poly[j].size
            ()];
          int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
          if (sc != sd) {
            double sa = C.cross(D, A), sb = C.cross(D, B);
            if (min(sc, sd) < 0)
              segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
          } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))
              >0){
            segs.emplace_back(rat(C - A, B - A), 1);
            segs.emplace_back(rat(D - A, B - A), -1);
          }
        }
      }
      sort(segs.begin(), segs.end());
      for(auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
      double sum = 0;
      int cnt = segs[0].second;
      for(int j = 1; j < segs.size(); ++j) {
        if (!cnt) sum += segs[j].first - segs[j - 1].first;
        cnt += segs[j].second;
      }
      ret += A.cross(B) * sum;
    }
  return ret / 2;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(N + Q \log n)$

"Point.h" 691449, 39 lines

```
typedef array<P, 2> Line;
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
  int n = poly.size(), left = 0, right = n;
  if (extr(0)) return 0;
  while (left + 1 < right) {
```

```
    int m = (left + right) / 2;
    if (extr(m)) return m;
    int ls = cmp(left + 1, left), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(left, m)) ? right : left
        ) = m;
  }
  return left;
}

#define cmpL(i) sgn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) {
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  for(int i = 0; i < 2; ++i) {
    int left = endB, right = endA, n = poly.size();
    while ((left + 1) % n != right) {
      int m = ((left + right + (left < right ? 0 : n)) / 2) % n
          ;
      (cmpL(m) == cmpL(endB) ? left : right) = m;
    }
    res[i] = (left + !cmpL(right)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % poly.size()) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

HalfPlane.h

Description: Halfplane intersection area

"Point.h", "lineIntersection.h" e8e2d4, 59 lines

```
#define eps 1e-8
struct Line {
  P P1, P2;
  // Right hand side of the ray P1 -> P2
  explicit Line(P a = P(), P b = P()) : P1(a), P2(b) {};
  P intpo(Line y) {
    P r;
    assert(lineIntersection(P1, P2, y.P1, y.P2, r) == 1);
    return r;
  }
  P dir() { return P2 - P1; }
  bool contains(P x) { return (P2 - P1).cross(x - P1) < eps; }
  bool out(P x) { return !contains(x); }
};

template<class T>
bool mycmp(Point<T> a, Point<T> b) {
  // return atan2(a.y, a.x) < atan2(b.y, b.x);
  if (a.x * b.x < 0) return a.x < 0;
  if (abs(a.x) < eps) {
    if (abs(b.x) < eps) return a.y > 0 && b.y < 0;
    if (b.x < 0) return a.y > 0;
    if (b.x > 0) return true;
  }
  if (abs(b.x) < eps) {
    if (a.x < 0) return b.y < 0;
    if (a.x > 0) return false;
  }
  return a.cross(b) > 0;
}
bool cmp(Line a, Line b) { return mycmp(a.dir(), b.dir()); }
double Intersection_Area(vector <Line> b) {
  sort(b.begin(), b.end(), cmp);
  int n = b.size();
  int q = 1, h = 0, i;
  vector<Line> c(b.size() + 10);
  for (i = 0; i < n; i++) {
    while (q < h && b[i].out(c[h].intpo(c[h - 1]))) h--;
    while (q < h && b[i].out(c[q].intpo(c[q + 1]))) q++;
    c[++h] = b[i];
    if (q < h && abs(c[h].dir().cross(c[h - 1].dir())) < eps) {
      h--;
      if (b[i].out(c[h].P1))  c[h] = b[i];
    }
  }
  while (q < h - 1 && c[q].out(c[h].intpo(c[h - 1]))) h--;
  while (q < h - 1 && c[h].out(c[q].intpo(c[q + 1]))) q++;
  // Intersection is empty. This is sometimes different from
      the case when
  // the intersection area is 0.
  if (h - q <= 1) return 0;
  c[h + 1] = c[q];
  vector <P> s;
  for (i = q; i <= h; i++) s.push_back(c[i].intpo(c[i + 1]));
  s.push_back(s[0]);
  double ans = 0;
  for (i = 0; i < (int)s.size()-1; i++) ans += s[i].cross(s[i +
      1]);
  return ans/2;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

"Point.h" 32b14f, 16 lines

```
pair<P, P> closest(vector<P> v) {
  assert(v.size() > 1);
  set<P> S;
  sort(v.begin(), v.end(), [](P a, P b) { return a.y < b.y; });
  pair<int64_t, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for(P &p : v) {
    P d{1 + (int64_t)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

KdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" 915562, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if the box is wider than high (not best
          heuristic...)
      sort(vp.begin(),vp.end(), x1 - x0 >= y1 - y0 ? on_x :
          on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = vp.size()/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({vp.begin(), vp.
      end()})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

DelaunayTriangulation.h

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are colinear or any four are on the same circle, behavior is undefined.

Time: $\mathcal{O}(n^2)$

"Point.h", "3dHull.h" f6175a, 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (ps.size() == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0)
      ;
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  for(auto &p : ps) p3.emplace_back(p.x, p.y, p.dist2());
```

```
  if (ps.size() > 3) for(auto &t: hull3d(p3)) if ((p3[t.b]-p3[t
      .a]).
    cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

FastDelaunay.h

Description: Fast Delaunay triangulation. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order $\{t[0][0], t[0][1], t[0][2], t[1][0], \dots\}$, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" a1f392, 89 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return rot->r()->o->rot; }
};
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q q0 = new Quad{0,0,0,orig}, q1 = new Quad{0,0,0,arb},
    q2 = new Quad{0,0,0,dest}, q3 = new Quad{0,0,0,arb};
  q0->o = q0; q2->o = q2; // 0-0, 2-2
  q1->o = q3; q3->o = q1; // 1-3, 3-1
  q0->rot = q1; q1->rot = q2;
  q2->rot = q3; q3->rot = q0;
  return q0;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = (sz(s) + 1) / 2;
  tie(ra, A) = rec({s.begin(), s.begin() + half});
  tie(B, rb) = rec({s.begin() + half, s.end()});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(pts.begin(), pts.end());  assert(unique(pts.begin(), pts
      .end()) == pts.end());
  if (pts.size() < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

RectangleUnionArea.h

Description: Sweep line algorithm that calculates area of union of rectangles in the form $[x1, x2) \times [y1, y2)$
Usage: Create vector with both x coordinates and y coordinates of each rectangle.//vector<pair<int,int>,pair<int,int>> rectangles;// rectangles.push_back({{x1, x2}, {y1, y2}});// lint result = area(rectangles);

73f787, 59 lines

```
pair<int,int> operator+(const pair<int,int>& l, const pair<int,
    int>& r) {
    if (l.first != r.first) return min(l,r);
    return {l.first, l.second + r.second};
}
struct segtree_t { // stores min + # of mins
    int n;
    vector<int> lazy;
    vector<pair<int,int>> tree; // set n to a power of two
    segtree_t(int _n) : n(_n), tree(2*n, {0,0}), lazy(2*n, 0) {
        }
    void build() {
        for(int i = n-1; i >= 1; --i)
            tree[i] = tree[i<<1] + tree[i<<1|1]; }
    void push(int v, int lx, int rx) {
        tree[v].first += lazy[v];
        if (lx != rx) {
            lazy[v<<1] += lazy[v];
            lazy[v<<1|1] += lazy[v];
        }
        lazy[v] = 0;
    }
    void update(int a, int b, int delta) { update(1,0,n-1,a,b,
        delta); }
    void update(int v, int lx, int rx, int a, int b, int delta)
        {
        push(v, lx, rx);
        if (b < lx || rx < a) return;
        if (a <= lx && rx <= b) {
            lazy[v] = delta; push(v, lx, rx);
        }
        else {
            int m = lx + (rx - lx)/2;
            update(v<<1, lx, m, a, b, delta);
            update(v<<1|1, m+1, rx, a, b, delta);
            tree[v] = (tree[v<<1] + tree[v<<1|1]);
        }
    }
};
lint area(vector<pair<pair<int,int>,pair<int,int>>> v) { //
    area of union of rectangles
    const int n = 1<<18;
    segtree_t tree(n);
    vector<int> y; for(auto &t : v) y.push_back(t.second.first)
        , y.push_back(t.second.second);
    sort(y.begin(), y.end()); y.erase(unique(y.begin(), y.end()
        ),y.end());
    for(int i = 0; i < y.size()-1; ++i) tree.tree[n+i].second =
        y[i+1]-y[i]; // compress coordinates
    tree.build();
    vector<array<int,4>> ev; // sweep line
    for(auto &t : v) {
        t.second.first = lower_bound(y.begin(), y.end(),t.
            second.first)-begin(y);
        t.second.second = lower_bound(y.begin(), y.end(),t.
            second.second)-begin(y)-1;
        ev.push_back({t.first.first,1,t.second.first,t.second.
            second});
        ev.push_back({t.first.second,-1,t.second.first,t.second
            .second});
    }
    sort(ev.begin(), ev.end());
    lint ans = 0;
    for(int i = 0; i < ev.size()-1; ++i) {
        const auto& t = ev[i];
        tree.update(t[2],t[3],t[1]);
        int len = y.back()-y.front()-tree.tree[1].second; //
            tree.mn[0].firstshould equal 0
        ans += (lint)(ev[i+1][0]-t[0])*len;
    }
    return ans;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

832599, 6 lines

```
template<class V, class L>
double signed_poly_volume(const V &p, const L &trilist) {
  double v = 0;
  for(auto &i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
```

```
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 3ed613, 48 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(A.size() >= 4);
  vector<vector<PR>> E(A.size(), vector<PR>(A.size(), {-1, -1})
      );
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  for(int i=0;i<4;i++) for(int j=i+1;j<4;j++) for(k=j+1;k<4;k
      ++)
    mf(i, j, k, 6 - i - j - k);
  for(int i=4; i<A.size();++i) {
    for(int j=0;j<FS.size();++j) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = FS.size();
    for(int j=0;j<nw;j++) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for(auto &it: FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

SphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.cpp

Description: failure[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a pattern in a text.
Time: $\mathcal{O}(n)$

469044, 28 lines

```
template<typename T> struct kmp_t {
    vector<T> word;
    vector<int> failure;
    kmp_t(const vector<T> &_word): word(_word) {
        int n = word.size();
        failure.resize(n+1, 0);
        for (int s = 2; s <= n; ++s) {
            failure[s] = failure[s-1];
            while (failure[s] > 0 && word[failure[s]] != word[s
                -1])
                failure[s] = failure[failure[s]];
            if (word[failure[s]] == word[s-1]) failure[s] += 1;
        }
    }
    vector<int> matches_in(const vector<T> &text) {
        vector<int> result;
        int s = 0;
        for (int i = 0; i < (int)text.size(); ++i) {
            while (s > 0 && word[s] != text[i])
                s = failure[s];
            if (word[s] == text[i]) s += 1;
            if (s == (int)word.size()) {
                result.push_back(i-(int)word.size()+1);
                s = failure[s];
            }
        }
        return result;
    }
};
```

Extended-KMP.h

Description: extended KMP S[i] stores the maximum common prefix between s[i:] and t; T[i] stores the maximum common prefix between t[i:] and t for i>0;

678f72, 22 lines

```
int S[N], T[N];
void extKMP(const string &s, const string &t) {
    int m = t.size(), maT = 0, maS = 0;
    T[0] = 0;
    for (int i = 1; i < m; i++) {
        if (maT + T[maT] >= i)
            T[i] = min(T[i - maT], maT + T[maT] - i);
        else T[i] = 0;
        while (T[i] + i < m && t[T[i]] == t[T[i] + i])
            T[i]++;
        if (i + T[i] > maT + T[maT]) maT = i;
    }
    int n = s.size();
    for (int i = 0; i < n; i++) {
        if (maS + S[maS] >= i)
            S[i] = min(T[i - maS], maS + S[maS] - i);
        else S[i] = 0;
        while (S[i] < m && i + S[i] < n && t[S[i]] == s[S[i] +
            i])
            S[i]++;
        if (i + S[i] > maS + S[maS]) maS = i;
    }
}
```

Duval.h

Description: A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes.
Time: $\mathcal{O}(N)$

d9b2cb, 27 lines

```
template <typename T>
pair<int, vector<string>> duval(int n, const T &s) {
    assert(n >= 1);
    // s += s  //uncomment if you need to know the min cyclic
        string
    vector<string> factors; // strings here are simple and in
        non-inc order
    int i = 0, ans = 0;
    while (i < n) { // until n/2 to find min cyclic string
        ans = i;
        int j = i + 1, k = i;
        while (j < n + n && !(s[j % n] < s[k % n])) {
            if (s[k % n] < s[j % n]) k = i;
            else k++;
            j++;
        }
        while (i <= k) {
            factors.push_back(s.substr(i, j-k));
            i += j - k;
        }
    }
    return {ans, factors};
    // returns 0-indexed position of the least cyclic shift
    // min cyclic string will be s.substr(ans, n/2)
}
template <typename T>
pair<int, vector<string>> duval(const T &s) {
    return duval((int) s.size(), s);
}
```

Z.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

7b625b, 16 lines

```
vector<int> Z(string& S) {
    vector<int> z(S.size());
    int l = -1, r = -1;
    for(int i = 1; i < S.size(); ++i) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < S.size() && S[i + z[i]] == S[z[i]])
            z[i]++;
        if (i + z[i] > r) l = i, r = i + z[i];
    }
    return z;
}
vector<int> get_prefix(string a, string b) {
    string str = a + '@' + b;
    vector<int> k = z(str);
    return vector<int>(k.begin()+a.size()+1, k.end());
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

87e1f0, 13 lines

```
array<vector<int>, 2> manacher(const string &s) {
  int n = s.size();
  array<vector<int>, 2> p = {vector<int>(n+1), vector<int>(n)};
  for(int z = 0; z < 2; ++z) for (int i=0,l=0,r=0; i < n; i++)
      {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R > r) l = L, r = R;
    }
    return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+min_rotation(v), v.end());`
Time: $\mathcal{O}(N)$

2a08fd, 8 lines

```
int min_rotation(string s) {
  int a=0, N=s.size(); s += s;
  for(int b = 0; b < N; ++b) for(int i =0; i < N; ++i) {
    if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
    if (s[a+i] > s[b+i]) { a = b; break; }
  }
  return a;
}
```

Trie.h

Description: Trie implementation.

b6b21a, 37 lines

```
struct Trie {
    struct node_t {
      unordered_map<char, node_t*> child;
      int cnt = 0, prefixCnt = 0;
    } *root = new node_t();
    void add(node_t *v, const string &s) {
        node_t *cur = v;
        for (char c : s) {
            if (cur->child.count(c)) cur = cur->child[c];
            else cur = cur->child[c] = new node_t();
            cur->prefixCnt++;
        }
        cur->cnt++;
    }
    int count(node_t *v, const string &s) {
        node_t *cur = v;
        for (char c : s) {
            if (cur->child.count(c)) cur = cur->child[c];
            else return 0;
        }
        return cur->cnt;
    }
    int prefixCount(node_t *v, const string &s) {
        node_t *cur = v;
        for (char c : s) {
            if (cur->child.count(c)) cur = cur->child[c];
            else return 0;
        }
        return cur->prefixCnt;
    }
    Trie() {}
    void add(const string &s) { add(root, s); }
    bool contains(const string &s) { return count(root, s) >=
        1; }
    bool hasPrefix(const string &s) { return prefixCount(root,
        s) >= 1; }
    int count(const string &s) { return count(root, s); }
    int prefixCount(const string &s) { return prefixCount(root,
         s); }
};
```

TrieXOR.h

Description: Query max xor with some int in the xor trie

d66338, 27 lines

```
template<int MX, int MXBIT>
struct xorTrie {
    int nxt[MX][2], sz[MX];// num is last node in trie
    int num = 0;
    // change 2 to 26 for lowercase letters
    xorTrie() { memset(nxt, 0, sizeof nxt), memset(sz, 0,
         sizeof sz); }
    // add or delete
    void add(lint x, int a = 1) {
        int cur = 0; sz[cur] += a;
        for(int i = MXBIT-1; i >= 0; --i) {
            int t = (x & (1 << i)) >> i;
            if (!nxt[cur][t]) nxt[cur][t] = ++num;
            sz[cur = nxt[cur][t]] += a;
        }
    }
    // compute max xor
    lint query(lint x) {
        if (sz[0] == 0) return INT_MIN; // no elements in trie
        int cur = 0;
        for(int i = MXBIT-1; i >= 0; --i) {
            int t = ((x & (1 << i)) >> i) ^ 1;
            if (!nxt[cur][t] || !sz[nxt[cur][t]]) t ^= 1;
            cur = nxt[cur][t]; if (t) x ^= 1lint<<i;
        }
        return x;
    }
};
```

Hashing.h

Description: Simple, short and efficient hashing using pairs to reduce load factor.

<ModTemplate.h>, <PairNumTemplate.h> 13369f, 40 lines

```
using num = modnum<int(1e9)+7>;
using hsh = pairnum<num, num>;
const hsh BASE(163, 311);
// uniform_int_distribution<int> MULT_DIST(0.1*MOD,0.9*MOD);
// constexpr hsh BASE(MULT_DIST(rng), MULT_DIST(rng));
struct hash_t {
    int n;
    string str;
    vector<hsh> hash, basePow;
    hash_t(const string& s) : n(s.size()), str(s), hash(n+1),
         basePow(n) {
        basePow[0] = 1;
        for (int i = 1; i < n; ++i) basePow[i] = basePow[i-1] *
             BASE;
        for (int i = 0; i < n; ++i)
            hash[i+1] = hash[i] * BASE + hsh(s[i]);
    }
    hsh get_hash(int left, int right) {
        assert(left <= right);
        return hash[right] - hash[left] * basePow[right - left
            ];
    }
    int lcp(hash_t &other) { // need some testing
        int left = 0, right = min(str.size(), other.str.size())
            ;
        while (left < right) {
            int mid = (left + right + 1)/2;
            if (hash[mid] == other.hash[mid]) left = mid;
            else right = mid-1;
        }
        return left;
    }
};
vector<int> rabinkarp(string t, string p) {
    vector<int> matches;
    hsh h(0, 0);
    for (int i = 0; i < p.size(); ++i)
        h = BASE * h + hsh(p[i]);
    hash_t result(t);
    for (int i = 0; i + p.size() <= t.size(); ++i)
        if (result.get_hash(i, i + p.size()) == h)
            matches.push_back(i);
    return matches;
}
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

6c2a8b, 47 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;
  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) {  v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
```

```
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }
  SuffixTree(string a) : a(a) {
    fill(r,r+N,a.size());
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    for(int i = 0; i < a.size(); ++i) ukkadd(i, toi(a[i]));
  }
  // example: find longest common substring (uses ALPHA = 28)
  pair<int,int> best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    for(int c = 0; c < ALPHA; ++c) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pair<int,int> LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, s.size(), s.size() + 1 + t.size(), 0);
    return st.best;
  }
};
```

AhoCorasick.h

Description: Aho-Corasick tree is used for multiple pattern matching. Initialize the tree with create(patterns). find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(_, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input.
Time: create is $\mathcal{O}(26N)$ where N is the sum of length of patterns. find is $\mathcal{O}(M)$ where M is the length of the word. findAll is $\mathcal{O}(NM)$. 154a8c, 64 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha]{}, pat = -1, t = -1, nmatches = 0;
    void p(int y, vector<int>& L) { t = (pat == -1 ? pat : L[t
        ]) = y; }
  };
  vector<Node> N;
  vector<int> backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 1;
    for(char c : s) {
      int& m = N[n].next[c - first];
      if (m) n = m;
      else { n = m = N.size(); N.emplace_back(); }
    }
    backp.push_back(0);
    N[n].p(j, backp);
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(2) {
    for(int i = 0; i < alpha; ++i) N[0].next[i] = 1;
    N[1].back = 0;
    for(int i = 0; i < pat.size(); ++i) insert(pat[i], i);

    vector<int> q(N.size()); int qe = q[0] = 1;
    for(int qi = 0; qi < qe; ++qi) {
      int n = q[qi], prev = N[n].back;
      for(int i = 0; i < alpha; ++i) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (!ed) ed = y;
        else {
          N[ed].back = y;
          N[ed].p(N[y].pat, backp);
          N[ed].nmatches += N[y].nmatches;
          q[qe++] = ed;
        }
      }
    }
  }
  vector<int> find(string word) {
    int n = 1;
    vector<int> res; // ll count = 0;
    for(char &c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].pat);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vector<int>> findAll(vector<string>& pat, string word)
      {
    vector<int> r = find(word);
    vector<vector<int>> res(word.size());
    for(int i = 0; i < word.size(); ++i) {
      int ind = r[i];
      while (ind != -1) {
        res[i - pat[ind].size() + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

SuffixArray.h

Description: Builds suffix array for a string. The `lcp` function calculates longest common prefixes for neighbouring strings in suffix array. The returned vector is of size $n+1$, and $ret[0]=0$.
Time: $\mathcal{O}(N\log N)$ where N is the length of the string for creation of the SA. $\mathcal{O}(N)$ for longest common prefixes. 62b51f, 51 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());
struct suffix_array_t {
    vector<vector<pair<int, int>>> rmq;
    int n, h; vector<int> sa, invsa, lcp;
    bool cmp(int a, int b) { return invsa[a+h] < invsa[b+h]; }
    void ternary_sort(int a, int b) {
        if (a == b) return;
        int pivot = sa[a+rng()%(b-a)];
        int left = a, right = b;
        for (int i = a; i < b; ++i) if (cmp(sa[i], pivot)) swap
            (sa[i], sa[left++]);
        for (int i = b-1; i >= left; --i) if (cmp(pivot, sa[i])
            ) swap(sa[i], sa[--right]);
        ternary_sort(a, left);
        for (int i = left; i < right; ++i) invsa[sa[i]] = right
            -1;
        if (right-left == 1) sa[left] = -1;
        ternary_sort(right, b);
    }
    suffix_array_t() {}
    suffix_array_t(vector<int> v): n(v.size()), sa(n) {
        v.push_back(INT_MIN);
        invsa = v; iota(sa.begin(), sa.end(), 0);
        h = 0; ternary_sort(0, n);
        for (h = 1; h <= n; h *= 2)
            for (int j = 0, i = j; i != n; i = j)
                if (sa[i] < 0) {
                    while (j < n && sa[j] < 0) j += -sa[j];
                    sa[i] = -(j-i);
                }
                else { j = invsa[sa[i]]+1; ternary_sort(i, j);
                }
        for (int i = 0; i < n; ++i) sa[invsa[i]] = i;
        lcp.resize(n); int res = 0;
        for (int i = 0; i < n; ++i) {
            if (invsa[i] > 0) while (v[i+res] == v[sa[invsa[i
                ]-1]+res]) ++res;
            lcp[invsa[i]] = res; res = max(res-1, 0);
        }
        int logn = 0; while ((1<<(logn+1)) <= n) ++logn;
        rmq.resize(logn+1, vector<pair<int, int>>(n));
        for (int i = 0; i < n; ++i) rmq[0][i] = {lcp[i], i};
        for (int l = 1; l <= logn; ++l)
            for (int i = 0; i+(1<<l) <= n; ++i)
                rmq[l][i] = min(rmq[l-1][i], rmq[l-1][i+(1<<(l
                    -1))]);
    }
    pair<int, int> rmq_query(int a, int b) {
        int size = b-a+1, l = __lg(size);
        return min(rmq[l][a], rmq[l][b-(1<<l)+1]);
    }
    int get_lcp(int a, int b) {
        if (a == b) return n-a;
        int ia = invsa[a], ib = invsa[b];
        return rmq_query(min(ia, ib)+1, max(ia, ib)).first;
    }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$ f47dfb, 23 lines

```
set<pair<int,int>>::iterator addInterval(set<pair<int,int>> &is
    , int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pair<int,int>> &is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
```

```
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

133eb4, 19 lines

```
template<class T>
vector<int> cover(pair<T, T> G, vector<pair<T, T>> I) {
  vector<int> S(I.size()), R;
  iota(S.begin(), S.end(), 0);
  sort(S.begin(), S.end(), [&](int a, int b) { return I[a] < I[
       b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = {cur, -1};
    while (at < I.size() && I[S[at]].first <= cur) {
      mx = max(mx, {I[S[at]].second, S[at]});
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

e2f9fb, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F& f, G& g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

35ef73, 12 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  for(int i = a+1; i <= b; ++i)
    if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LowerBound.h

7422d7, 11 lines

```
int LowerBound(vector<int> v, int n, int x){
    int l = 1, r = n, m;
    while(l <= r){
        m= (l+r)/2;
        if(v[m] >= x && (m == 1 || v[m-1] < x))
            return m;
        else if(v[m] >= x) r=m-1;
        else l=m+1;
    }
    return m;
}
```

UpperBound.h

381d15, 11 lines

```
int UpperBound(vector<int> v, int n, int x){
    int l = 1, r = n, m;
    while(l <= r){
        m=(l+r)/2;
        if(v[m] > x && (m == 1 || v[m-1] <= x))
            return m;
        else if(v[m] > x) r=m-1;
        else l=m+1;
    }
    return m;
}
```

MergeSort.h

Time: $\mathcal{O}(N \log(N))$

fac159, 25 lines

```
vector<int> merge(vector<int> &values, int l, int r) {
    static vector<int> result(values.size());
    int i = l, j = l + (r - l)/2;
    int mid = j, k = i, inversions = 0;
    while (i < mid && j < r) {
        if (values[i] < values[j]) result[k++] = values[i++];
        else {
            result[k++] = values[j++];
            inversions += (mid - i);
        }
    }
    while (i < mid) result[k++] = values[i++];
    while (j < r) result[k++] = values[j++];
    for (k = l; k < r; ++k) values[k] = result[k];
    return result;
}

vector<int> msort(vector<int> &values, int l, int r) {
    if (r - l > 1) {
        int mid = l + (r - l)/2;
        msort(values, l, mid); msort(values, mid, r);
        return merge(values, l, r);
    }
    return {};
}
```

RadixSort.h

Description: Radix Sort Algorithm.
Time: $\mathcal{O}(NK)$ where K is the number of bits in the largest element of the array to be sorted.

889884, 54 lines

```
struct identity {
    template<typename T>
    T operator()(const T &x) const {
        return x;
    }
};
template<typename T, typename T_extract_key = identity>
void radix_sort(vector<T> &data, int bits_per_pass = 10, const
     T_extract_key &extract_key = identity()) {
    if (data.size() < 256) {
        sort(data.begin(), data.end(), [&](const T &a, const T
             &b) {
            return extract_key(a) < extract_key(b);
        });
        return;
    }
    using T_key = decltype(extract_key(data.front()));
    T_key minimum = numeric_limits<T_key>::max();
    for (T &x : data) minimum = min(minimum, extract_key(x));
    int max_bits = 0;
    for (T &x : data) {
        T_key key = extract_key(x);
        max_bits = max(max_bits, key == minimum ? 0 : 64 -
             __builtin_clzll(key - minimum));
    }
    int passes = max((max_bits + bits_per_pass / 2) /
         bits_per_pass, 1);
    if (32 - __builtin_clz(data.size()) <= 1.5 * passes) {
        sort(data.begin(), data.end(), [&](const T &a, const T
             &b) {
            return extract_key(a) < extract_key(b);
        });
        return;
    }
    vector<T> buffer(data.size());
    vector<int> counts;
    int bits_so_far = 0;
    for (int p = 0; p < passes; p++) {
        int bits = (max_bits + p) / passes;
        counts.assign(1 << bits, 0);
        for (T &x : data) {
            T_key key = extract_key(x) - minimum;
            counts[(key >> bits_so_far) & ((1 << bits) - 1)]++;
        }
        int count_sum = 0;
        for (int &count : counts) {
            int current = count;
            count = count_sum;
            count_sum += current;
        }
        for (T &x : data) {
            T_key key = extract_key(x) - minimum;
            int key_section = (key >> bits_so_far) & ((1 <<
                bits) - 1);
            buffer[counts[key_section]++] = x;
        }
        swap(data, buffer);
        bits_so_far += bits;
    }
}
```

CoordCompression.h

809d6a, 9 lines

```
vector<int> comp_coord(vector<int> &y, int N) {
    vector<int> result;
```

```
    for (int i = 0; i < N; ++i) result.emplace_back(y[i]);
    sort(result.begin(), result.end());
    result.resize(unique(result.begin(), result.end())- result.
        begin());
    for (int i = 0; i < N; ++i)
        y[i] = lower_bound(result.begin(), result.end(), y[i])
            - result.begin();
    return result;
}
```

CountTriangles.h

Description: Counts x, y >= 0 such that Ax + By <= C.

8d67b3, 8 lines

```
lint count_triangle(lint A, lint B, lint C) {
  if (C < 0) return 0;
  if (A > B) swap(A, B);
  lint p = C / B;
  lint k = B / A;
  lint d = (C - p * B) / A;
  return count_triangle(B - k * A, A, C - A * (k * p + d + 1))
      + (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
```

Karatsuba.h

Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length welint. See also FFT, under the Numerical chapter.
Time: $\mathcal{O}(N^{1.6})$

946265, 30 lines

```
int size(int s) { return s > 1 ? 32-__builtin_clz(s-1) : 0; }

void karatsuba(lint *a, lint *b, lint *c, lint *t, int n) {
    int ca = 0, cb = 0;
    for(int i = 0; i < n; ++i) ca += !!a[i], cb += !!b[i];
    if (min(ca, cb) <= 1500/n) { // few numbers to multiply
        if (ca > cb) swap(a, b);
        for(int i = 0; i < n; ++i)
            if (a[i]) FOR(j,n) c[i+j] += a[i]*b[j];
    }
    else {
        int h = n >> 1;
        karatsuba(a, b, c, t, h); // a0*b0
        karatsuba(a+h, b+h, c+n, t, h); // a1*b1
        for(int i = 0; i < h; ++i) a[i] += a[i+h], b[i] += b[i+
            h];
        karatsuba(a, b, t, t+n, h); // (a0+a1)*(b0+b1)
        for(int i = 0; i < h; ++i) a[i] -= a[i+h], b[i] -= b[i+
            h];
        for(int i = 0; i < n; ++i) t[i] -= c[i]+c[i+n];
        for(int i = 0; i < n; ++i) c[i+h] += t[i], t[i] = 0;
    }
}

vector<lint> conv(vector<lint> a, vector<lint> b) {
    int sa = a.size(), sb = b.size(); if (!sa || !sb) return
        {};
    int n = 1<<size(max(sa,sb)); a.resize(n), b.resize(n);
    vector<lint> c(2*n), t(2*n);
    for(int i = 0; i < 2*n; ++i) t[i] = 0;
    karatsuba(&a[0], &b[0], &c[0], &t[0], n);
    c.resize(sa+sb-1); return c;
}
```

CountInversions.h

Description: Count the number of inversions to make an array sorted. Merge sort has another approach.
Time: $\mathcal{O}(nlog(n))$

<FenwickTree.h> 0002df, 22 lines

```
FT<lint> bit(n);
lint inv = 0;
for (int i = n-1; i >= 0; --i) {
    inv += bit.query(values[i]); // careful with the interval
    bit.update(values[i], 1);    // [0, x) or [0, x] ?
}

// using D&C, the constant is quite high but still nlogn
lint msort(vector<int> &values, int left, int right) {
  if ((right - left) <= 1) return 0;
  int mid = left + (right - left)/2;
  lint result = msort(values, left, mid) + msort(values, mid,
      right);
  auto cmp = [](int i, int j) { return i > j; };
  sort(values.begin() + left, values.begin() + mid, cmp);
  sort(values.begin() + mid, values.begin() + right, cmp);
  int pos = left;
  for (int i = mid; i < right; ++i) {
    while (pos != mid && values[pos] > values[i]) ++pos;
    result += (pos - left);
  }
  return result;
}
```

Histogram.h

Description: Maximum area of a histogram.
Time: $\mathcal{O}(n)$

51da1a, 16 lines

```
template<typename T = int>
T max_area(vector<int> v) {
    T ret = T();
    stack<int> s;
    v.insert(v.begin(), -1);
    v.insert(v.end(), -1);
    s.push(0);
    for(int i = 0; i < v.size(); ++i) {
        while (v[s.top()] > v[i]) {
            int h = v[s.top()]; s.pop();
            ret = max(ret, h * (i - s.top() - 1));
        }
        s.push(i);
    }
    return ret;
}
```

DateManipulation.h

088459, 42 lines

```
string week_day_str[7] = {"Sunday", "Monday", "Tuesday", "
    Wednesday", "Thursday", "Friday", "Saturday"};
string month_str[13] = {"", "January", "February", "March", "
    April", "May", "June", "July", "August", "September", "
    October", "November", "December"};
map<string, int> week_day_int = {{"Sunday", 0}, {"Monday", 1},
    {"Tuesday", 2}, {"Wednesday", 3}, {"Thursday", 4}, {"
    Friday", 5}, {"Saturday", 6}};
map<string, int> month_int = {{"January", 1}, {"February", 2},
    {"March", 3}, {"April", 4}, {"May", 5}, {"June", 6}, {"
    July", 7}, {"August", 8}, {"September", 9}, {"October",
    10}, {"November", 11}, {"December", 12}};
int month[2][13] = {{0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31,
     30, 31}, {0, 31, 29, 31, 30, 31, 30, 31, 31, 30, 31, 30,
    31}};

/* O(1) - Checks if year y is a leap year. */
bool leap_year(int y){
  return (y % 4 == 0 && y % 100 != 0) || y % 400 == 0;
}

/* O(1) - Increases the day by one. */
void update(int &d, int &m, int &y){
  if (d == month[leap_year(y)][m]){
    d = 1;
    if (m == 12) {
      m = 1;
      y++;
    }
    else m++;
  }
  else d++;
}

int intToDay(int jd) { return jd % 7; }
int dateToInt(int y, int m, int d) {
  return 1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075; }
void intToDate(int jd, int &y, int &m, int &d) {
  int x, n, i, j;
  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x; }
```

NQueens.cpp

Description: NQueens

e97e9e, 43 lines

```
int ans;
bitset<30> rw, ld, rd; //2*MAX_N -1
bitset<30> iniqueens; //2*MAX_N -1
vector<int> col;
void init(int n){
  ans=0;
    rw.reset();
    ld.reset();
    rd.reset();
    col.assign(n,-1);
}
void init(int n, vector<pair<int,int>> initial_queens){
    //it does NOT check if initial queens are at valid
        positions
    init(n);
    iniqueens.reset();
    for(pair<int,int> pos: initial_queens){
        int r=pos.first, c= pos.second;
        rw[r] = ld[r-c+n-1] = rd[r+c]=true;
        col[c]=r;
        iniqueens[c] = true;
    }
}
void backtracking(int c, int n){
    if(c==n){
      ans++;
        for(int r:col) cout<<r+1<<" ";
        cout<<"\n";
        return;
    }
    else if(iniqueens[c]){
        backtracking(c+1,n);
    }
    else for(int r=0;r<n;r++){
        if(!rw[r] && !ld[r-c+n-1] && !rd[r+c]){
```

```
        // if(board[r][c]!=blocked && !rw[r] && !ld[r-c+n-1] &&
            !rd[r+c]){ // if there are blocked possitions
            rw[r] = ld[r-c+n-1] = rd[r+c]=true;
            col[c]=r;
            backtracking(c+1,n);
            col[c]=-1;
            rw[r] = ld[r-c+n-1] = rd[r+c]=false;
        }
    }
}
```

SudokuSolver.h

6be906, 41 lines

```
int N,m; // N = n*n, m = n; where n equal number of rows or
    columns
array<array<int, 10>, 10> grid;
struct SudokuSolver {
    bool UsedInRow(int row,int num){
        for(int col = 0; col < N; ++col)
            if(grid[row][col] == num) return true;
        return false;
    }
    bool UsedInCol(int col,int num){
        for(int row = 0; row < N; ++row)
            if(grid[row][col] == num) return true;
        return false;
    }
    bool UsedInBox(int row_0,int col_0,int num){
        for(int row = 0; row < m; ++row)
            for(int col = 0; col < m; ++col)
                if(grid[row+row_0][col+col_0] == num) return
                    true;
        return false;
    }
    bool isSafe(int row,int col,int num){
        return !UsedInRow(row,num) && !UsedInCol(col,num) && !
            UsedInBox(row-row%m,col-col%m,num);
    }
    bool find(int &row,int &col){
        for(row = 0; row < N; ++row)
            for(col = 0; col < N; ++col)
                if(grid[row][col] == 0) return true;
        return false;
    }
    bool Solve(){
        int row, col;
        if(!find(row,col)) return true;
        for(int num = 1; num <= N; ++num){
            if(isSafe(row,col,num)){
                grid[row][col] = num;
                if(Solve()) return true;
                grid[row][col] = 0;
            }
        }
        return false;
    }
};
```

FloydCycle.h

Description: Detect loop in a list. Consider using mod template to avoid overflow.
Time: $\mathcal{O}(n)$

b456ab, 10 lines

```
template<class F>
pair<int,int> find(int x0, F f) {
    int t = f(x0), h = f(t), mu = 0, lam = 1;
    while (t != h) t = f(t), h = f(f(h));
    h = x0;
    while (t != h) t = f(t), h = f(h), ++mu;
    h = f(t);
    while (t != h) h = f(h), ++lam;
    return {mu, lam};
}
```

SubsetXOR.h

Description: Given an array compute the maximum/minimum subset xor.

a72252, 21 lines

```
template<typename T> struct XorGauss {
    int n; vector<T> a;
    XorGauss(int bits) : n(bits), a(bits) {}
    T reduce(T x) {
        for(int i = n-1; i >= 0; i--)
            x = max(x, x ^ a[i]);
        return x;
    }
    T augment(T x) { return ~reduce(~x); }
    bool add(T x) {
        for(int i = n-1; i >= 0; i--) {
            if (!(x & (1ll << i))) continue;
            if (a[i]) x ^= a[i];
            else {
                a[i] = x;
                return true;
            }
        }
        return false;
    }
};
```

10.3 Dynamic programming

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i)\leq k<hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N+(hi-lo))\log N)$

c9b6d0, 17 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  lint f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, lint v) { res[ind] = {k, v}; }
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<lint, int> best(LLONG_MAX, LO);
    for(int k = max(LO,lo(mid)); k < min(HI,hi(mid)); ++k)
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

ConvexHullTrick.h

Description: Transforms dp of the form (or similar) $dp[i] = min_{j<i}(dp[j] + b[j]*a[i])$. Time goes from $O(n^2)$ to $O(n\log n)$, if using online line container, or $O(n)$ if lines are inserted in order of slope and queried in order of x. To apply try to find a way to write the factor inside minimization as a linear function of a value related to i. Everything else related to j will become constant.

<LineContainer.h> 1e5a56, 22 lines

```
array<lint, 112345> dyn, a, b;

int main() {
    int n;
    cin >> n;
    for (int i = 0; i < n; ++i) cin >> a[i];
    for (int i = 0; i < n; ++i) cin >> b[i];
    dyn[0] = 0;
    LineContainer cht;
    cht.add(-b[0], 0);
    for (int i = 1; i < n; ++i) {
        dyn[i] = cht.query(a[i]);
        cht.add(-b[i], dyn[i]);
    }
    // Original DP O(n^2).
  // for (int i = 1; i < n; i++) {
  //   dyn[i] = INF;
  //   for (int j = 0; j < i; j++)
  //     dyn[i] = min(dyn[i], dyn[j] + a[i] * b[j]);
  // }
  cout << -dyn[n-1] << '\n';
}
```

Coin.h

Description: Number of ways to make value K with X coins
Time: $\mathcal{O}(NC)$

208759, 3 lines

```
for (int i = 0; i < n; ++i)
  for (int j = coins[i]; j <= k; ++j)
    dp[j] += dp[j - coins[i]];
```

MinCoin.h

Description: minimum number of coins to make K
Time: $\mathcal{O}(kV)$

5fe4b1, 7 lines

```
int coin(vector<int> &c, int k) {
    vector<int> dp(k+1, INF); dp[0] = 0;
    for (int i = 0; i < c.size(); ++i)
        for (int j = c[i]; j <= k; ++j)
            dp[j] = min(dp[j], 1 + dp[j-c[i]]);
    return dp[k];
}
```

EditDistance.h

Description: Find the minimum numbers of edits required to convert string s into t. Only insertion, removal and replace operations are allowed.

f0f083, 32 lines

```
int edit_dist(string &s, string &t) {
    const int n = int(s.size()), m = int(t.size());
    vector<vector<int>> dp(n+1, vector<int>(m+1, n+m+2));
    vector<vector<int>> prv(n+1, vector<int>(m+1, 0));
    dp[0][0] = 0;
    for (int i = 0; i <= n; i++) {
        for (int j = 0; j <= m; j++) {
            if (i < n) { // remove
                int cnd = dp[i][j] + 1;
                if (cnd < dp[i+1][j]) {
                    dp[i+1][j] = cnd;
                    prv[i+1][j] = 1;
                }
            }
            if (j < m) { // insert
```

```
            int cnd = dp[i][j] + 1;
            if (cnd < dp[i][j+1]) {
                dp[i][j+1] = cnd;
                prv[i][j+1] = 2;
            }
        }
        if (i < n && j < m) { // modify
            int cnd = dp[i][j] + (s[i] != t[j]);
            if (cnd < dp[i+1][j+1]) {
                dp[i+1][j+1] = cnd;
                prv[i+1][j+1] = 3;
            }
        }
    }
  }
  return dp[n][m];
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

0675f2, 17 lines

```
template<class I> vector<int> lis(const vector<I>& S) {
  if (S.empty()) return {};
  vector<int> prev(S.size());
  typedef pair<I, int> p;
  vector<p> res;
  for(int i = 0; i < (int)S.size(); i++) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(res.begin(), res.end(), p {S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = res.size(), cur = res.back().second;
  vector<int> ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

LIS2.h

Description: Compute the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

6182d9, 9 lines

```
template<typename T> int lis(const vector<T> &a) {
    vector<T> u;
    for (const T &x : a) {
        auto it = lower_bound(u.begin(), u.end(), x);
        if (it == u.end()) u.push_back(x);
        else *it = x;
    }
    return (int)u.size();
}
```

LCS.h

Description: Finds the longest common subsequence.
Memory: $\mathcal{O}(nm)$.
Time: $\mathcal{O}(nm)$ where n and m are the lengths of the sequences.

463080, 14 lines

```
template<class T> T lcs(const T &X, const T &Y) {
  int a = X.size(), b = Y.size();
  vector<vector<int>> dp(a+1, vector<int>(b+1));
  for(int i = 1; i <= a; ++i) for(int j = 1; j <= b; j++)
    dp[i][j] = X[i-1]==Y[j-1] ? dp[i-1][j-1]+1 :
      max(dp[i][j-1],dp[i-1][j]);
  int len = dp[a][b];
  T ans(len, 0);
  while (a && b)
    if (X[a-1] == Y[b-1]) ans[--len] = X[--a], --b;
    else if (dp[a][b-1] > dp[a-1][b]) --b;
    else --a;
  return ans;
}
```

Knapsack.h

Description: Same 0-1 Knapsack problem, but returns a vector that holds each chosen item.
Time: $\mathcal{O}(nW)$

2b2ab2, 16 lines

```
vector<int> Knapsack(int limit, vector<int> &v, vector<int> &w)
      {
    vector<vector<int>> dp(v.size()+1);
    dp[0].resize(limit+1);
    for (int i = 0; i < v.size(); ++i) {
        dp[i+1] = dp[i];
        for (int j = 0; j <= limit - w[i]; ++j)
            dp[i+1][w[i]+j] = max(dp[i+1][w[i]+j], dp[i][j] + v
                [i]);
    }
    vector<int> result;
    for (int i = v.size()-1; i >= 0; --i)
        if (dp[i][limit] != dp[i+1][limit]) {
            limit -= w[i];
            result.push_back(i);
        }
    return result;
}
```

01Knapsack.h

Description: Given weights and values of n items, put these items in a knapsack of capacity W to get the maximum total value possible. More efficient space-wise since we work in only one row.
Time: $\mathcal{O}(NW)$

e3f233, 12 lines

```
int knapsack(int limit, vector<int> &v, vector<int> &w) {
    vector<int> dp(limit+1, -1); int n = w.size();
    dp[0] = 0;
    for (int i = 0; i < n; ++i)
        for (int j = limit; j >= w[i]; --j)
            if (dp[j - w[i]] >= 0)
                dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
    int result = 0;
    for (int i = 0; i <= limit; ++i)
        result = max(result, dp[i]);
    return result;
}
```

LargeKnapsack.h

Description: Knapsack with definition changed. Support large values because the weight isn't a dimension in our dp anymore.
Time: $\mathcal{O}(vW)$ where v is the sum of values.

47c6d1, 9 lines

```
constexpr int limit = (int)1e5+10;
int knapsack(int capacity, vector<lint> &v, vector<lint> &w) {
    vector<lint> dp(limit, 1ll << 60); dp[0] = 0;
    for (int i = 0; i < v.size(); ++i)
        for (int j = limit-v[i]-1; j >= 0; --j)
            dp[j + v[i]] = min(dp[j + v[i]], dp[j] + w[i]);
    for (int i = limit-1; i >= 0; --i)
        if (dp[i] <= capacity) return i;
}
```

KnapsackUnbounded.h

Description: Knapsack problem but now take the same item multiple items is allowed.
Time: $\mathcal{O}(N \log N)$

1acfe0, 10 lines

```
int knapsack(vector<int> &v, vector<int> &w, int total) {
    vector<int> dp(total+1, -1);
    int result = 0; dp[0] = 0;
    for (int i = 0; i <= total; ++i) for (int j = 0; j < n; ++j
        )
        if (w[j] <= i && dp[i - w[j]] >= 0)
            dp[i] = max(dp[i], dp[i - w[j]] + v[j]);
    int result = 0;
    for (int i = 0; i <= total; ++i) result = max(result, dp[i
        ]);
    return result;
}
```

KnapsackBounded.h

Description: You are given n types of items, each items has a weight and a quantity. Is possible to fill a knapsack with capacity k using any subset of items?
Time: $\mathcal{O}(Wn)$

9bddad, 15 lines

```
vector<int> how_many(n+1), dp(k+1);
dp[0] = 1;
for (int i = 1; i <= n; ++i) cin >> how_many[i];
for (int i = 1; i <= n; ++i) {
  for (int j = k-items[i]; j >= 0; --j) {
    if (dp[j]) {
      int x = 1;
      while (x <= how_many[i] &&
        j + x*items[i] <= k && !dp[j + x*items[i]]) {
        dp[j + x*items[i]] = 1;
        ++x;
      }
    }
  }
}
```

KnapsackBoundedCosts.h

Description: You are given n types of items, you have $e[i]$ items of i-th type, and each item of i-th type weight $w[i]$ and cost $c[i]$. What is the minimal cost you can get by picking some items weighing at most W in total?
Time: $\mathcal{O}(Wn)$

<MinQueue.h> 3ade3c, 28 lines

```
const int maxn = 1000;
const int maxm = 100000;
const int inf = 0x3f3f3f;

minQueue<int> q[maxm];

array<int, maxm> dp; // the minimum cost dp[i] I need to pay in
      order to fill the knapsack with total weight i
int w[maxn], e[maxn], c[maxn]; // weight, number, cost

int main() {
  int n, m;
  cin >> n >> m;
  for (int i = 1; i <= n; i++) cin >> w[i] >> c[i] >> e[i];
  for (int i = 1; i <= m; i++) dp[i] = inf;
  for (int i = 1; i <= n; i++) {
    for (int j = 0; j < w[i]; j++) q[j].clear();
    for (int j = 0; j <= m; j++) {
      minQueue<int> &mq = q[j % w[i]];
      if (mq.size() > e[i]) mq.pop();
      mq.add(c[i]);
      mq.push(dp[j]);
      dp[j] = mq.getMin();
    }
  }
  cout << "Minimum value i can pay putting a total weight " <<
      m << " is " << dp[m] << '\n';
  for (int i = 0; i <= m; i++) cout << dp[i] << " " << i << '\n
      ';
```

```
  cout << "\n";
}
```

KnapsackBitset.h

Description: Find first value greater than m that cannot be formed by the sums of numbers from v.

25166e, 11 lines

```
bitset<int(1e7)> dp, dp1;
int knapsack(vector<int> &items, int n, int m) {
    dp[0] = dp1[0] = true;
    for (int i = 0; i < n; ++i) {
        dp1 <<= items[i];
        dp |= dp1;
        dp1 = dp;
    }
    dp.flip();
    return dp._Find_next(m);
}
```

TSP.h

Description: Solve the Travelling Salesman Problem.
Time: $\mathcal{O}\left(N^2 * 2^N\right)$

9c40a0, 17 lines

```
const int MX = 15;
array<array<int, MX>, 1<<N> dp;
array<array<int, MX>, MX> dist;
int N;
int TSP(int n) {
    dp[0][1] = 0;
    for (int j = 0; j < (1 << n); ++j)
        for (int i = 0; i < n; ++i)
            if (j & (1<<i))
                for (int k = 0; k < n; ++k)
                    if (!(j & (1<<k)))
                        dp[k][j^(1<<k)] = min(dp[k][j^(1<<k)],
                            dp[i][j]+dist[i][k]);
    int ret = (1 << 31); // = INF
    for (int i = 1; i < n; ++i)
        ret = min(ret, dp[i][(1<<n)-1] + dist[i][0]);
    return ret;
}
```

DistinctSubsequences.h

Description: DP eliminates overcounting. Number of different strings that can be generated by removing any number of characters, without changing the order of the remaining.

<ModTemplate.h> 6fa2c5, 7 lines

```
num tot[30];
num distinct(const string &str) {
    num ans = 1; // tot[i] stands for number of distinct
        strings ending with character 'a'+i
    for(auto &c : str)
      tie(ans, tot[c-'a']) = {2*ans-tot[c-'a'], ans};
    return ans-1;
}
```

CircularLCS.h

Description: For strings a, b calculates LCS of a with all rotations of b
Time: $\mathcal{O}\left(N^2\right)$

a57399, 48 lines

```
pair<int,int> dp[2001][4001];
string A,B;

void init() {
  for(int i = 1; i <= A.size(); ++i)
    for(int j = 1; j <= B.size(); ++j) { // naive LCS, store
        where value came from
      pair<int,int>& bes = dp[i][j]; bes = {-1,-1};
      bes = max(bes,{dp[i-1][j].first,0});
      bes = max(bes,{dp[i-1][j-1].first+(A[i-1] == B[j-1]),-1})
        ;
      bes = mex(bes,{dp[i][j-1].first,-2});
      bes.second *= -1;
    }
}
void adjust(int col) { // remove col'th character of b, adjust
    DP
  int x = 1;
  while (x <= A.size() && dp[x][col].second == 0) x ++;
  if (x > A.size()) return; // no adjustments to dp
  pair<int,int> cur = {x,col}; dp[cur.first][cur.second].second
      = 0;
  while (cur.first <= A.size() && cur.second <= B.size()) {
    // essentially decrease every dp[cur.first][y >= cur.second
        ].first by 1
    if (cur.second < B.size() && dp[cur.first][cur.s+1].second
        == 2) {
      cur.second ++;
      dp[cur.first][cur.second].second = 0;
    } else if (cur.first < A.size() && cur.second < B.size()
      && dp[cur.first+1][cur.s+1].second == 1) {
      cur.first ++, cur.second ++;
      dp[cur.first][cur.second].second = 0;
    } else cur.first ++;
  }
}
int getAns(pair<int,int> x) {
  int lo = x.second-B.size()/2, ret = 0;
  while (x.first && x.second > lo) {
    if (dp[x.first][x.second].second == 0) x.first --;
    else if (dp[x.first][x.second].second == 1) ret ++, x.first
        --, x.second --;
    else x.second --;
  }
  return ret;
}
int circLCS(str a, str b) {
  A = a, B = b+b; init();
  int ans = 0;
  for(int i = 0; i < B.size(); ++i) {
    ans = max(ans,getAns({A.size(),i+B.size()}));
    adjust(i+1);
  }
  return ans;
}
```

MaxZeroSubmatrix.h

Description: Computes the area of the largest submatrix that contains only 0s
Time: $\mathcal{O}(NM)$

d7bff2, 18 lines

```
const int MAXN = 100, MAXM = 100;
array<array<int, MAXN>, MAXM> A, H;
int solve(int N, int M) {
    stack<int, vector<int>> s; int ret = 0;
    for (int j = 0; j < M; j++) for (int i = N - 1; i >= 0; i
        --) H[i][j] = A[i][j] ? 0 : 1 + (i == N - 1 ? 0 : H[i
        + 1][j]);
    for (int i = 0; i < N; i++) {
        for (int j = 0; j < M; j++) {
            int minInd = j;
            while (!s.empty() && H[i][s.top()] >= H[i][j]) {
                ret = max(ret, (j - s.top()) * (H[i][s.top()]))
                    ;
                minInd = s.top(); s.pop(); H[i][minInd] = H[i][
                    j];
            }
            s.push(minInd);
        }
        while (!s.empty()) ret = max(ret, (M - s.top()) * H[i][
            s.top()]); s.pop();
    }
    return ret;
}
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

10.5 Optimization tricks

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastInput.h

Description: Returns an integer. Usage requires your program to pipe in input from file. Can replace calls to gc() with getchar_unlocked() if extra speed isn't necessary (60% slowdown).
Usage: `./a.out < input.txt`
Time: About 5x as fast as cin/scanf.

b31afb, 18 lines

```
struct GC {
    char buf[1 << 16];
    size_t bc = 0, be = 0;
    char operator()() {
        if (bc >= be) {
            buf[0] = 0, bc = 0;
            be = fread(buf, 1, sizeof(buf), stdin);
        }
        return buf[bc++]; // returns 0 on EOF
    }
```

```
} gc;
int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt();
    while ((c = gc()) >= 48) a = a * 10 + c - 480;
    return a - 48;
}
```

Pragmas.h

Description: Be careful.

375d37, 6 lines

```
#pragma GCC optimize("Ofast") // enable all O3 optimizations
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx,avx
    ,avx2,fma")
#pragma GCC optimize("unroll-loops")
#pragma GCC optimize("Ofast","unroll-loops","omit-frame-pointer
    ","inline") //Optimization flags
#pragma GCC option("arch=native","tune=native","no-zero-upper")
     //Enable AVX
#pragma GCC target("avx2")  //Enable AVX
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 2dd6c9, 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.

Usage: `vector<vector<int, small<int>>> ed(N);`

bb66d4, 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```

Hashmap.h

Description: Faster/better hash maps, taken from CF

09a72f, 19 lines

```
#include<bits/extc++.h>
struct splitmix64_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x^(x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x^(x >> 27)) * 0x94d049bb133111eb;
        return x^(x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = std::chrono::
             steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

template <typename K, typename V, typename Hash =
     splitmix64_hash>
using hash_map = __gnu_pbds::gp_hash_table<K, V, Hash>;

template <typename K, typename Hash = splitmix64_hash>
using hash_set = hash_map<K, __gnu_pbds::null_type, Hash>;
```

Unrolling.h

520e76, 5 lines

```
#define F {...; ++i;}
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern `"_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)"`. Not all are described here; grep for _mm_ in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define __SSE__ and __MMX__ before including it. For aligned memory use _mm_malloc`(size, 32)` or `int buf[N] alignas(32)`, but prefer loadu/storeu.

b75e03, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x
// sad_epu8: sum of absolute differences of u8, outputs 4xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g. _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));
  }
  union {ll v[4]; mi m;} u; u.m = acc; for(int i=0;i<4;i++) r
      += u.v[i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <- equiv
  return r;
}
```

PQueue.h

Description: Efficient priority queue implementation. Initialize with highest possible value. Can obviously be extended to minheap/maxheap.

ffd97e, 15 lines

```
template<typename T> struct PQ {
    int sz;
    vector<T> q;
    T offset = 0;
    PQ(int n) : sz(n+1), q(2*n, -n) {}
    T top() { return -q[0]+offset; }
    void push(T x) {
        q[sz++] = -(x-offset);
        push_heap(q.begin(),q.begin()+sz);
    }
    void shift(T x) { offset+=x; }
    void pop() {
        pop_heap(q.begin(),q.begin()+sz); --sz;
    }
};
```

OwnFunctions.h

786225, 18 lines

```
template <typename T>
T mabs(T v) {
  return v < 0 ? -v : v;
}

template <typename T>
T mceil(T v) {
  T x = ceil((long double)v) - 1.0;
  while (x < v) x += 1.0;
  return x;
}

template <typename T>
T mfloor(T v) {
  T x = floor((long double)v) + 1.0;
  while (x > v) x -= 1.0;
  return x;
}
```

FastMod.h

Description: Compute $a\%b$ about 4 times faster than usual, where b is constant but not known at compile time. Fails for $b = 1$.

c977c5, 10 lines

```
typedef unsigned long long ull;
typedef __uint128_t L;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(ull((L(1) << 64) / b)) {}
```

```
  ull reduce(ull a) {
    ull q = (ull)((L(m) * a) >> 64), r = a - q * b;
    return r >= b ? r - b : r;
  }
};
```

10.6 Bit Twiddling Hack

Hacks.h

59c333, 51 lines

```
// Returns one plus the index of the least significant 1-bit of
     x, or if x is zero, returns zero.
__builtin_ffs(x)

// Returns the number of leading 0-bits in x, starting at the
     most significant bit position. If x is 0, the result is
     undefined.
__builtin_clz(x)

// Returns the number of trailing 0-bits in x, starting at the
     least significant bit position. If x is 0, the result is
     undefined.
__builtin_ctz(x)

// Returns the number of 1-bits in x.
__builtin_popcount(x)

// For long long versions append ll (e.g. __builtin_popcountll)

// Least significant bit in x.
x & -x

// Iterate on non-empty submasks of a bitmask.
for (int submask = mask; submask > 0; submask = (mask & (
     submask - 1)))

// Iterate on non-zero bits of a bitset.
for (int j = btset._Find_next(0); j < MAXV; j = btset.
     _Find_next(j))

int __builtin_clz(int x); // number of leading zero
int __builtin_ctz(int x); // number of trailing zero
int __builtin_clzll(lint x); // number of leading zero
int __builtin_ctzll(lint x); // number of trailing zero
int __builtin_popcount(int x); // number of 1-bits in x
int __builtin_popcountll(lint x); // number of 1-bits in x

// compute next perm. i.e. 00111, 01011, 01101, 10011, ...
lint next_perm(lint v) {
    lint t = v | (v-1);
    return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) +
         1));
}

template<typename F>  // All subsets of size k of {0..N-1}
void iterate_k_subset(ll N, ll k, F f){
  ll mask = (1ll << k) - 1;
  while (!(mask & 1ll<<N)) { f(mask);
    ll t = mask | (mask-1);
    mask = (t+1) | (((~t & -~t) - 1) >> (__builtin_ctzll(mask)
         +1));
  }
}
template<typename F>  // All subsets of set
void iterate_mask_subset(ll set, F f){ ll mask = set;
  do  f(mask), mask = (mask-1) & set;
  while (mask != set);
}
```

Bitset.h

Description: Some bitset functions

b9f55a, 17 lines

```
int main() {
    bitset<100> bt;
    cin >> bt;
    cout << bt[0] << "\n";
    cout << bt.count() << "\n"; // number of bits set
    cout << (~bt).none() << "\n"; // return true if has no bits
        set
    cout << (~bt).any() << "\n"; // return true if has any bit
        set
    cout << (~bt).all() << "\n"; // retun true if has all bits
        set
    cout << bt._Find_first() << "\n"; // return first set bit
    cout << bt._Find_next(10) << "\n";// returns first set bit
        after index i
    cout << bt.flip() << '\n'; // flip the bitset
    cout << bt.test(3) << '\n'; // test if the ith bit of bt is
        set
    cout << bt.reset(3) << '\n'; // reset the ith bit
    cout << bt.set() << '\n'; // turn all bits on
    cout << bt.set(4, 1) << '\n'; // set the 4th bit to value 1
    cout << bt << "\n";
}
```

10.7 Random Numbers

RandomNumbers.h

Description: An example on the usage of generator and distribution. Use shuffle instead of random shuffle.

2859c6, 5 lines

```
mt19937 rng(random_device{}());
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().
     count());
shuffle(permutation.begin(), permutation.end(), rng);
uniform_int_distribution<int> uid(1, 100); //  [1, 100]
     inclusive!
uniform_real_distribution<double> urd(1, 100);
```

10.8 Other languages

Main.java

Description: Basic template/info for Java

11488d, 15 lines

```
import java.util.*;
import java.math.*;
import java.io.*;

public class Main {
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader(new
         InputStreamReader(System.in));
    PrintStream out = System.out;
    StringTokenizer st = new StringTokenizer(br.readLine());
    assert st.hasMoreTokens(); // enable with java -ea main
    out.println("v=" + Integer.parseInt(st.nextToken()));
    ArrayList<Integer> a = new ArrayList<>();
    a.add(1234); a.get(0); a.remove(a.size()-1); a.clear();
  }
}
```

MiscJava.java

Description: Basic template/info for Java

186de5, 47 lines

```
import java.math.BigInteger;
import java.util.*;

public class prob4 {
  void run() {
    Scanner scanner = new Scanner(System.in);
    while (scanner.hasNextBigInteger()) {
      BigInteger n = scanner.nextBigInteger();
      int k = scanner.nextInt();
      if (k == 0) {
        for (int p = 2; p <= 100000; p++) {
          BigInteger bp = BigInteger.valueOf(p);
          if (n.mod(bp).equals(BigInteger.ZERO)) {
            System.out.println(bp.toString() + " * " + n.divide
                 (bp).toString());
            break;
          }
        }
      } else {
        BigInteger ndivk = n.divide(BigInteger.valueOf(k));
        BigInteger sqndivk = sqrt(ndivk);
        BigInteger left = sqndivk.subtract(BigInteger.valueOf
             (100000)).max(BigInteger.valueOf(2));
        BigInteger right = sqndivk.add(BigInteger.valueOf
             (100000));
        for (BigInteger p = left; p.compareTo(right) != 1; p =
             p.add(BigInteger.ONE)) {
          if (n.mod(p).equals(BigInteger.ZERO)) {
            BigInteger q = n.divide(p);
            System.out.println(p.toString() + " * " + q.
                 toString());
            break;
          }
        }
      }
    }
  }
  BigInteger sqrt(BigInteger n) {
    BigInteger left = BigInteger.ZERO;
    BigInteger right = n;
    while (left.compareTo(right) != 1) {
      BigInteger mid = left.add(right).divide(BigInteger.
           valueOf(2));
      int s = n.compareTo(mid.multiply(mid));
      if (s == 0) return mid;
      if (s > 0) left = mid.add(BigInteger.ONE); else right =
           mid.subtract(BigInteger.ONE);
    }
    return right;
  }
  public static void main(String[] args) {
    (new prob4()).run();
  }
}
```

10.8.1 BigInteger

BigInteger To convert to a `BigInteger`, use `BigInteger.valueOf (int)` or `BigInteger (String, radix)`.

To convert from a `BigInteger`, use `.intValue ()`, `.longValue ()`, `.toString (radix)`.

Common unary operations include `.abs ()`, `.negate ()`, `.not ()`.

Common binary operations include `.max`, `.min`, `.add`, `.subtract`, `.multiply`, `.divide`, `.remainder`, `.gcd`, `.modInverse`, `.and`, `.or`, `.xor`, `.shiftLeft (int)`, `.shiftRight (int)`, `.pow (int)`, `.compareTo`.

Divide and remainder: `Biginteger[] .divideAndRemainder (Biginteger val)`.

Power module: `.modPow (BigInteger exponent, module)`.

Primality check: `.isProbablePrime (int certainty)`.

Techniques (A)

techniques.txt 159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```