

COMP90054 – Week 12 tutorial

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Normal form games

Definition

- n agents playing moves at the same time
- A normal form game is a tuple $G = (N, A, u)$
 - N is a set of n players
 - $A = A_1 \times \dots \times A_n$ is an *action profile*, where A_i is the action for player i
 - $a = (a_1, \dots, a_n) \in A$ describes simultaneous moves
 - $u: A \rightarrow \mathbb{R}^n$ is the *utility* function that describes each player's reward
- Assumptions
 - All agents are *rational*: agents try to maximise their utility
 - All agents are *self-interested*: agents do not care about other's utility
 - *Perfect information* game: agents know all rules, actions and utilities of everyone

Representation

- Normal form games can be visualised as matrices
- In two player games, usually player 1 = row player = receive utility on left, player 2 = column player = receive utility on right
- e.g., Prisoner's dilemma

\downarrow P1 P2 \rightarrow	Deny	Confess
Deny	-3, -3	-15, -1
Confess	-1, -15	-10, -10

Strategy

- Pure strategy: The agent selects a single action and plays it
- Mixed strategy: The agent selects an action based on some probability distribution
- Dominant strategy: strategy that gives higher (or equal) utility than others
 - s_i *weakly dominates* s_i' if utility received by the agent for playing $s_i \geq$ utility received by playing s_i'
 - s_i *strongly dominates* s_i' if utility received by the agent for playing $s_i >$ utility received by playing s_i'
- e.g., in prisoner's dilemma, both prisoner has a strongly dominant strategy of playing "Confess"

Nash equilibrium

- Best response: the best strategy that an agent could select if it knows how all other agents are going to play
- Nash equilibrium: a stable strategy profile for all agents such that no agent has incentive to change their strategy, i.e., all agents play strategy that is best response to others
- For any game in which all agents have a dominant strategy, it will form a unique Nash equilibrium
- e.g., in prisoner's dilemma, the strategy profile (Confess, Confess) is a pure-strategy nash equilibrium

Mixed strategy games

- In some games no agent has a dominant strategy, and there are no pure-strategy Nash equilibria
- Need to randomise to maximise overall utility by picking the pure strategy with the highest expected utility, i.e., mixed-strategies
- To reach a mixed-strategy Nash equilibrium, each agent should choose a mixed strategy such that it makes their opponents *indifferent* to their own actions
- e.g., P1, P2 show a coin together

↓P1 P2→	Head	Tail
Head	1, -1	-1, 1
Tail	-1, 1	1, -1

- Assuming playing as P1 in the previous example
 - Let $Y, 1 - Y$ be the probability of P1 playing head and tail respectively
 - To make P2 indifferent, their expected utility shall be the same $U_{P2}(H) = U_{P2}(T)$
 - $Y \times -1 + (1 - Y) \times 1 = Y \times 1 + (1 - Y) \times -1$
 - $\Rightarrow Y = \frac{1}{2}$