# COMP90054 – Week 7 tutorial

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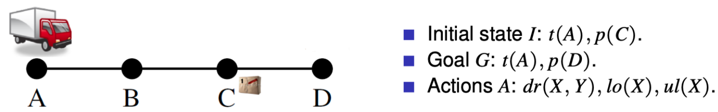
## Best-Supporters and Relaxed plan heuristics

### Best-supporter functions $bs$

**Definition**

- Best-supporter function $bs$: For every fact $p \in (F \backslash s)$, returns an action that can achieve $p$ with the cheapest cost within relaxation
- In this subject we use the two following closed well-founded best-supporter functions
  - $bs_s^{\text{add}}(p) \coloneqq \underset{a \in A, p \in add_a}{\operatorname{argmin}} \left(c(a) + h^{\text{add}}(s, pre_a)\right)$
  - $bs_s^{\max}(p) \coloneqq \underset{a \in A, p \in add_a}{\operatorname{argmin}} \left(c(a) + h^{\max}(s, pre_a)\right)$

**Example**

- "Logistics": 


| | $t(A)$ | $t(B)$ | $t(C)$ | $t(D)$ | $p(T)$ | $p(A)$ | $p(B)$ | $p(C)$ | $p(D)$ |
|---|---|---|---|---|---|---|---|---|---|
| $h^{\text{add}}$ | 0 | 1 | 2 | 3 | 3 | 4 | 5 | 0 | 7 |
| $bs_s^{\text{add}}$ | - | $dr(A,B)$ | $dr(B,C)$ | $dr(C,D)$ | $lo(C)$ | $ul(A)$ | $ul(B)$ | - | $ul(D)$ |

- For $p = \text{t(B)}$, $bs_s^{\text{add}}(p) = \underset{a \in \{dr(A,B), dr(C,B)\}}{\operatorname{argmin}} \left(c(a) + h^{\text{add}}(s, pre_a)\right)$
  - $a = dr(A,B) \rightarrow 1 + h^{\text{add}}\big(t(A)\big) = 1$
  - $a = dr(C,B) \rightarrow 1 + h^{\text{add}}\big(t(C)\big) = 3$

## Relaxed plan heuristics $h^{\mathrm{FF}}$

**Idea**

1. Compute a best-supporter function $bs$
2. Extract a relaxed plan $RPlan$ by applying $bs$ to each $g \in G$ and collecting all actions
3. Use the relaxed plan heuristics $h^{\mathrm{FF}} = \Sigma_{a \in RPlan} c(a)$ to estimate $h^*$

**Algorithm**

**Relaxed Plan Extraction for state $s$ and best-supporter function $bs$**

$Open := G \setminus s$; $Closed := \emptyset$; $RPlan := \emptyset$
**while** $Open \neq \emptyset$ **do**:
  select $g \in Open$
  $Open := Open \setminus \{g\}$; $Closed := Closed \cup \{g\}$;
  $RPlan := RPlan \cup \{bs(g)\}$; $Open := Open \cup (pre_{bs(g)} \setminus (s \cup Closed))$
**endwhile**
**return** $RPlan$

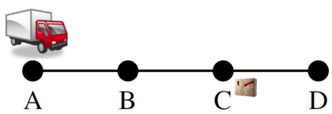
- Requires a closed well-founded best-supporter function to make sense

**Properties of $h^{\mathrm{FF}}$**

- $h^{\mathrm{FF}}$ is pessimistic: $h^{\mathrm{FF}} \geq h^+$ so $\exists \Pi, s$ such that $h^{\mathrm{FF}}(s) > h^*(s)$
- $h^{\mathrm{FF}}$ agrees with $h^+$ on $\infty$: $h^{\mathrm{FF}}(s) = \infty \Leftrightarrow h^+(s) = \infty$
- $h^{\mathrm{FF}}$ does not over-count like $h^{\mathrm{add}}$

**Example**

- Logistics":




- Initial state $I$: $t(A), p(C)$.
- Goal $G$: $t(A), p(D)$.
- Actions $A$: $dr(X,Y), lo(X), ul(X)$.

| | $t(A)$ | $t(B)$ | $t(C)$ | $t(D)$ | $p(T)$ | $p(A)$ | $p(B)$ | $p(C)$ | $p(D)$ |
|---|---|---|---|---|---|---|---|---|---|
| $bs_s^{\mathrm{add}}$ | - | $dr(A,B)$ | $dr(B,C)$ | $dr(C,D)$ | $lo(C)$ | $ul(A)$ | $ul(B)$ | - | $ul(D)$ |

- Relaxed plan extraction

| g | Open | Closed | RPlan |
|---|---|---|---|
| - | {p(D)} | {} | {} |
| p(D) | {t(D),p(T)} | {p(D)} | {ul(D)} |
| t(D) | {p(T),t(C)} | {p(D),t(D)} | {dr(C,D),ul(D)} |
| t(C) | {p(T),t(B)} | {p(D),t(D),t(C)} | {dr(B,C),dr(C,D),ul(D)} |
| t(B) | {p(T)} | {p(D),t(D),t(C),t(B)} | {dr(A,B),dr(B,C),dr(C,D),ul(D)} |
| p(T) | {} | {p(D),t(D),t(C),t(B),p(T)} | {lo(C),dr(A,B),dr(B,C),dr(C,D),ul(D)} |

- Relaxed plan heuristics $h^{\mathrm{FF}} = 1 + 1 + 1 + 1 + 1 = 5$

## Width-based Planning

**Iterated Width $IW$**

- Uses the idea of novelty $w$: $w(s)$ is the size of the smallest subset of atoms in $s$ that is true for the first time in the search
- $IW(k)$ is a breadth-first search that prunes generated states $s$ with $w(s) > k$
- $IW$ is an iterative/sequence of calls to $IW(k)$ for $k = 0,1,2, \ldots$ over problem $P$ until problem solved or $i$ exceeds number of variables in problem

**Properties of $IW$**

- The order of search node expansion can affect which nodes are pruned
- $IW(k)$ expands at most $O(n^k)$ states

**Example**

Left->right order

- IW(1): {1,2,3}
- IW(2): {1,2,3,4}
- IW(3): {1,2,3,4}

Right->left order

- IW(1): {1,3}
- IW(2): {1,3,2,5}
- IW(3): {1,3,2,5,4}