# COMP90054 – Week 12 tutorial

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## Normal form games

**Definition**

- $n$ agents playing moves at the same time
- A normal form game is a tuple $G = (N, A, u)$
  - $N$ is a set of $n$ players
  - $A = A_1 \times \cdots \times A_n$ is an *action profile*, where $A_i$ is the action for player $i$
    - $a = (a_1, \cdots, a_n) \in A$ describes simultaneous moves
  - $u: A \to \mathbb{R}^n$ is the *utility* function that describes each player's reward
- Assumptions
  - All agents are *rational*: agents try to maximise their utility
  - All agents are *self-interested*: agents do not care about other's utility
  - *Perfect information* game: agents know all rules, actions and utilities of everyone

**Representation**

- Normal form games can be visualised as matrices
- In two player games, usually player 1 = row player -> receive utility on left, player 2 = column player -> receive utility on right
- e.g., Prisoner's dilemma

| ↓P1 P2→ | Deny | Confess |
|---|---|---|
| Deny | -3, -3 | -15, -1 |
| Confess | -1, -15 | -10, -10 |

**Strategy**

- Pure strategy: The agent selects a single action and plays it
- Mixed strategy: The agent selects an action based on some probability distribution
- Dominant strategy: strategy that gives higher (or equal) utility than others
  - $s_i$ *weakly dominates* $s_i'$ if utility received by the agent for playing $s_i \geq$ utility received by playing $s_i'$
  - $s_i$ strongly dominates ${s_i}'$ if utility received by the agent for playing $s_i >$ utility received by playing ${s_i}'$
- e.g., in prisoner's dilemma, both prisoner has a strongly dominant strategy of playing "Confess"

**Nash equilibrium**

- Best response: the best strategy that an agent could select if it knows how all other agents are going to play
- Nash equilibrium: a stable strategy profile for all agents such that no agent has incentive to change their strategy, i.e., all agents play strategy that is best response to others
- For any game in which all agents have a dominant strategy, it will form a unique Nash equilibrium
- e.g., in prisoner's dilemma, the strategy profile (Confess, Confess) is a pure-strategy nash equilibrium

**Mixed strategy games**

- In some games no agent has a dominant strategy, and there are no pure-strategy Nash equilibria
- Need to randomise to maximise overall utility by picking the pure strategy with the highest expected utility, i.e., mixed-strategies
- To reach a mixed-strategy Nash equilibrium, each agent should choose a mixed strategy such that it makes their opponents *indifferent* to their own actions
- e.g., P1, P2 show a coin together

| ↓P1 P2→ | Head | Tail |
|---|---|---|
| Head | 1, -1 | -1, 1 |
| Tail | -1, 1 | 1, -1 |

- Assuming playing as P1 in the previous example
    - Let $Y$,$1 - Y$ be the probability of P1 playing head and tail respectively
    - To make P2 indifferent, their expected utility shall be the same $U_{P2}(H) = U_{P2}(T)$
    - $Y \times -1 + (1 - Y) \times 1 = Y \times 1 + (1 - Y) \times -1$
    - $\Rightarrow Y = \frac{1}{2}$