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Instructions

1. Print this problem set and use the space provided under the statement of each question to write your own solution. Then scan and submit your hand-written solutions to Assignments on iCollege.
2. Clearly indicate the final answer in each of your solutions. You should work alone. You are permitted to use your textbook, notes, and a calculator.
3. Each of problems 1-5 counts for 1 point. Problems 6-10 count for 2 points each.

1. An insurance company sells a policy with a linearly disappearing deductible such that no payment is made on a claim of 250 or less and full payment is made on a claim of 1000 or more. Calculate the payment made by the insurance company for a loss of 700.

Components

- Linearly disappearing deductible
- No payment for claim 250k or less
- Full payment is made on a claim of 1000 or more.
- What is the insurer's payment on a loss of 700?

Disappearing Deductible — a formula deductible that decreases as the amount of loss increases and disappears entirely to provide full coverage when the loss reaches a specified amount. Disappearing deductibles were once commonly used in property insurance policies.
https://www.irmi.com/term/insurance-definitions/disappearing-deductible

Notes: So between 250 and 1000 there is a linearly disappearing deductible, where by 250 represents 0 and 1000 100%. So there is a linear relationship here. How do we calculate values across this line?

Interpolation:
If the two known points are given by the coordinates (x0, y0, and (x1, y1), the linear interpolant is the straight line between these points. For a value x in the interval (x0, x1), the value y along the straight line is given from the equation of slopes
Y – y0 / x – x0 = (y1 – y0) / (x1 – x0)
https://en.wikipedia.org/wiki/Linear_interpolation

Solution: Linearly interpolate to get the solution.

(700-250) / (1000 – 250) = 600.

2. The random variable X represents the random loss, before any deductible is applied, covered by an insurance policy.

a. The probability density function of X is
$f(x) = 2x$, $0 < x < 1$.

Payments are made subject to a deductible, d, where $0 < d < 1$. The probability that a claim payment is less than 0.5 is equal to 0.64. Calculate the value of d.

I found this problem difficult to understanding.

Solution: The probability of a loss paid above the deductible is some function of 0.5 + d.

3. Calculate the percentage reduction in loss costs by moving from a 100 deductible to a 250 deductible.

Size of Loss	Number of Claims	Ground-up Total Losses	Reduction 100k Deductible	Net Total Losses	Reduction 250k Deductible	Net Losses	Pct Diff
0-99	110	$ 58,500	$ (58,500)	$ -	$ (58,500)	$ -	
100-249	400	$ 70,000	$ (40,000)	$ 30,000	$ (70,000)	$ -	
250-499	300	$ 120,000	$ (30,000)	$ 90,000	-75000	$ 45,000	
500-999	200	$ 150,000	$ (20,000)	$ 130,000	-50000	$ 100,000	
>999	100	$ 200,000	$ (10,000)	$ 190,000	-25000	$ 175,000	
Total	**1,110**	**$ 598,500**	**$ (158,500)**	**$440,000**	**$ (278,500)**	**$320,000**	**-0.27273**

Solution: -27%

4. Mr. Orfanos purchases a homeowners policy with an 80% coinsurance clause. The home is insured for 150,000. The home was worth 180,000 on the day the policy was purchased. Lightning causes 20,000 worth of damage. On the day of the storm the home is worth 250,000. Calculate the benefit payment Mr. Orfanos receives from his policy.

Coinsurance	80%
Limit (amount insured)	$ 150,000
Home Value Start	$ 180,000
Damage	$ 20,000
Home Value End	$ 250,000
Implied Limit	$ 200,000
Ratio 80% Value to Limit	75%
Times Loss	$ 15,000

Solution: Implied limit is the coinsurance times the value of the home. Since 200,000 is greater than 150,000, the homeowner is underinsured. Therefore, the solution is the ratio of the limit to the value of the home times the loss (150,000 / 200,000) * 20,000 = 15,000.

5. A company purchases a commercial insurance policy with a property policy limit of 70,000. The actual value of the property at the time of a loss is 100,000. The insurance policy has a coinsurance provision of 80% and a 200 deductible, which is applied to the loss before the limit or coinsurance are applied. A storm causes damage in the amount of 20,000. Calculate the insurance company's payment.

Property Limit	$ 70,000
Actual Value @ Time of Loss	$ 100,000
Coinsurance	80%
Deductible	$ 200
Damage	$ 20,000
Damage less deductible	$ 19,800
Value implied by coinsurance	$ 80,000
Limit divided by implied value	88%
Loss Payout (88% * (loss - deductible))	$ 17,325

Solution: 17,325. I applied the same approach as in question #4, just discounted the loss amount by the $200 deductible before making the final loss calculation.

6. Find the end-of-1999 estimated loss reserve using:
   a. The expected loss ratio technique
   b. The chain ladder technique with arithmetic average loss development factors

AY	1996	1997	1998	1999	Earned Premium	Expected Loss Ratio	Cumulative Paid Losses	Est Expected Losses	Loss Reserve
1996	$ 10,000	$ 5,000	$ 2,000	$ -	$ 25,000	0.68	$ 17,000	$17,000	$ -
1997		$ 12,050	$ 6,025	$ 2,400	$ 29,750	0.688	$ 20,475	$20,468	$ (7)
1998			$ 14,500	$ 7,250	$ 33,000	0.7	$ 21,750	$23,100	$ 1,350
1999				$ 17,465	$ 38,000	0.7	$ 17,465	$26,600	$ 9,135
						Totals	**$ 76,690**	**$87,168**	**$10,478**

Chain Ladder Technique

	Development Periods			
AY	1	2	3	4
1996	$ 10,000	$ 15,000	$ 17,000	$ 17,000
1997	$ 12,050	$ 18,075	$ 20,475	
1998	$ 14,500	$ 21,750		
1999	$ 17,465			

Age to Age Factors

Development Periods				
AY	0/1	1/2	2/3	
1996	1.50	1.13	1.00	
1997	1.50	1.13		
1998	1.50			
1999	-			

Arithmetic Average

Factor	1.5	1.13	1.00

Age to Ultimate

	0 to inf	1 to inf	2 to inf
Factor	1.695	1.130	1.000

Reserve Calculation

Development Periods							
AY	1	2	3	4	**Ultimate**	**Loss Paid**	**Reserve**
1996	$ 10,000	$ 15,000	$ 17,000	$ 17,000	$ 17,000	$ 17,000	$ -
1997	$ 12,050	$ 18,075	$ 20,475		$ 20,475	$ 20,475	$ -
1998	$ 14,500	$ 21,750			$ 24,578	$ 21,750	$ 2,828
1999	$ 17,465				$ 29,603	$ 17,465	$ 12,138
				Sum	**$ 91,656**	**$ 76,690**	**$ 14,966**

Solutions:
   a. $10,478
   b. $14,966

7. Find the end-of-1999 estimated loss reserve using the Bornhuetter-Ferguson technique with geometric average loss development factors.

Bornhuetter-Ferguson Method (Steps)

Technique	Utilizes both the expected loss ratio and chain ladder techniques.
	Utilizes CDF
Formula	Ultimate Claims = Reported Claims + Unreported Claims
	Equals Reported Claims + (Expected Claims) * (1- % Reported)
	Pct Reported is the recipricol of the CDF of the development factors.

Step1	Expected Claims:	Calculate from the expected loss ratio * premiums earned.
Step2	CDF:	Age to ultimate.
Step3	Pct Unreported:	1-(1/CDF)
Step4	Expected Claims "Unreported":	Pct Unreported * Expected Claims
Step5	Ultimate Claims	Reported Claims (From triangle) + Unreported
Step6	Case Outstanding	Reported - Paid
Step7	IBNR	Projected Ultimate Cliams - Reported
Step8	Total Reserve	Case Outstanding + IBNR

Geometric Average: Multiply all age factors together for a given development period (column) and take the nth root

AY	**1996**	**1997**	**1998**	**1999**	**Earned Premium**	**Expected Loss Ratio**	**Cumulative Paid Losses**
1996	$ 10,000	$ 5,000	$ 2,000	$ -	$ 25,000	0.68	$ 17,000
1997		$ 12,050	$ 6,025	$ 2,400	$ 29,750	0.688	$ 20,475
1998			$ 14,500	$ 7,250	$ 33,000	0.7	$ 21,750
1999				$ 17,465	$ 38,000	0.7	$ 17,465
						Totals	**$ 76,690**

	Development Periods			
AY	1	2	3	4
1996	$ 10,000	$ 15,000	$ 17,000	$ 17,000
1997	$ 12,050	$ 18,075	$ 20,475	
1998	$ 14,500	$ 21,750		
1999	$ 17,465			

Age To Age Factors

AY	**0/1**	**1/2**	**2/3**
1996	1.5	1.133	1.00
1997	1.5	1.133	
1998	1.5	0.000	
1999	0		

Geometric Average

	1.5	1.133	1.00

(CDF) Age to Ultimate

	1.70	1.13	1.00

Pct Reported (1/CDF)

	0.59	0.88	1.00

Pct Unreported (1 - Pct Reported)

	0.41	0.12	-

Expected Loss Ratio Technique

AY	1996	1997	1998	1999	Earned Premium	Expected Loss Ratio	Cumulative (Reported) Claims	Expected Losses	Unpaid Claim Estimates	Pc Reported	Unreported Claims	Ultimate Claims	IBNR
1996	$ 10,000	$ 5,000	$ 2,000	$ -	$ 25,000	0.68	$ 17,000	$ 17,000	$ -	1.00	$ -	$ 17,000	$ -
1997		$ 12,050	$ 6,025	$ 2,400	$ 29,750	0.688	$ 20,475	$ 20,468	$ (7)	0.88	$ 2,404	$ 22,879	$ 2,404
1998			$ 14,500	$ 7,250	$ 33,000	0.7	$ 21,750	$ 23,100	$ 1,350	0.59	$ 8,953	$ 30,703	$ 8,953
1999				$ 17,465	$ 38,000	0.7	$ 17,465	$ 26,600	$ 9,135	0%	$ 17,465	$ 34,930	$ 17,465
						Totals	**$ 76,690**	**$ 87,168**	$ 10,478		**$ 28,822**	**$ 105,512**	$ 28,822

Chain Ladder Technique

	Development Periods				
AY	1	2	3	4	**Ultimate Losses**
1996	$ 10,000	$ 15,000	$ 17,000	$ 17,000	$ 17,000
1997	$ 12,050	$ 18,075	$ 20,475		$ 20,475
1998	$ 14,500	$ 21,750			$ 24,644
1999	$ 17,465				$ 29,683
				Totals	**$ 91,802**

8. Calculate the indicated actuarial reserve using the Bornhuetter-Ferguson method and volume-weighted average loss development factor.

Data

AY	12 months	24 months	36 months	48 months	Earned Premium
2005	$ 4,850	$ 9,700	$ 14,100	$ 16,200	$ 19,000.00
2006	$ 5,150	$ 10,300	$ 14,900		$ 20,000.00
2007	$ 5,400	$ 10,800			$ 21,000.00
2008	$ 7,200				$ 22,000.00

* No development after 48 months

AY	Expected Loss Ratio	Expected Claims	CDF	Pct Unreported	Expected Claims Unreported	Claims Reported	Projected Ultimate Claims	IBNR	Reserve (Solution)
2005	0.9	$ 17,100	1.00	0%	$ -	$ 16,200	$ 16,200	$ -	$ -
2006	0.85	$ 17,000	1.15	13%	$ 2,204	$ 14,900	$ 17,104	$ 2,204	$ 4,407
2007	0.91	$ 19,110	1.67	40%	$ 7,639	$ 10,800	$ 18,439	$ 7,639	$ 15,278
2008	0.88	$ 19,360	3.33	70%	$ 13,550	$ 7,200	$ 20,750	$ 13,550	$ 27,099
Totals		**$ 72,570**			**$ 23,392**	**$ 49,100**	**$ 72,492**	**$ 23,392**	**$ 46,785**

Chain Ladder Technique

Expected Losses Expected Loss Ratio * Earned Premium

Volume Weighted Average

	0to1	1to2	2to3
Age 2 Age Factors	2.00	1.45	1.15

Age to Ultimate	0 to inf	1 to inf	2 to inf
CDF	3.33	1.67	1.15

Pct Reported

	30%	60%	87%

Pct Unreported

	70%	40%	13%

9. Calculate the ratio discounted reserves to undiscounted reserves as of December 31, 2008.

AY	12 months	24 months	36 months	48 months	Age to Ultimate	Est Ultimage Claims	Undiscounted Reserves
2005	$ 27,000	$ 49,000	$ 65,000	$ 72,000	1.00	$ 72,000	$ -
2006	$ 28,000	$ 57,000	$ 71,000	$ 81,650	1.15	$ 81,650	$ 10,650
2007	$ 33,000	$ 65,000	$ 78,000	$ 89,700	1.38	$ 89,700	$ 24,700
2008	$ 35,000	$ 70,000	$ 84,000	$ 96,600	2.76	$ 96,600	$ 61,600
					Totals	**$ 339,950**	**$ 96,950**

Selected Age-to-Age Paid Loss Development Factors are: 2.00 for 12/24 months, 1.20 for 24/36 months, 1.15 for 36/48 months, and 1.00 for 48/1months.
The interest rate is 5.0% per annum effective.

Age to Age Factors

	12/24	24/36	36/48	48 >
Factor	2.00	1.20	1.15	1.00

Age to Ultimate	12 to inf	24 to inf	36 to inf	48 to inf
	2.76	1.38	1.15	1.00

Differences By Period (Est Amount Incremental Increase - Known)

AY	12 months	24 months	36 months	48 months	Undiscounted Reserves
2005					$ -
2006				$ 10,650	$ 10,650
2007			$ 13,000	$ 11,700	$ 24,700
2008		$ 35,000	$ 14,000	$ 12,600	$ 61,600
				Total	**$ 96,950**

Apply Interest Rate

AY	12 months	24 months	36 months	48 months	Discounted Reserves	Ratio Discounted to Undiscounted (Solution)
2005					$ -	
2006				$ 10,143	$ 10,143	95.24%
2007			$ 12,381	$ 10,612	$ 22,993	93.09%
2008		$ 33,333	$ 12,698	$ 10,884	$ 56,916	92.40%
				Total	**$ 90,052**	92.89%

* I assume we discount the premiums based on a factor from right (most recent) to left
* Also assuming that payments are made at the beginning of each period such that each period = 1

10. Calculate the total loss reserve using the Bornhuetter-Ferguson method and three year arithmetic average paid loss development factors

Cumulative Loss Payments						
	Development Periods					
AY	**Year 0**	**Year 1**	**Year2**	**Year3**	**Year4**	**Year5**
2004	$ 1,400	$ 5,200	$ 7,300	$ 8,800	$ 9,800	$ 9,800
2005	$ 2,200	$ 6,400	$ 8,800	$ 10,200	$ 11,500	
2006	$ 2,500	$ 7,500	$ 10,700	$ 12,600		
2007	$ 2,800	$ 8,700	$ 12,900			
2008	$ 2,500	$ 7,900				
2009	$ 2,600					

AY	Earned Premium	Expected Loss Ratio	Expected Claims	Total Reported	Pct Unreported	Expected Claims Unreported	Ultimate Claims	Case Outstanding	IBNR	Total Loss Reserver (Solution)
2004	$ 18,000	0.55	$ 9,900	$ 9,800	0.00	$ -	$ 9,800	$ -	$ -	**$ -**
2005	$ 20,000	0.55	$ 11,000	$ 11,500	0.00	$ -	$ 11,500	$ -	$ -	**$ -**
2006	$ 25,000	0.55	$ 13,750	$ 12,600	0.11	$ 1,479	$ 14,079	$ 1,479	$ 2,958	**$ 4,438**
2007	$ 26,000	0.55	$ 14,300	$ 12,900	0.24	$ 3,492	$ 16,392	$ 3,492	$ 6,983	**$ 10,475**
2008	$ 27,000	0.55	$ 14,850	$ 7,900	0.47	$ 6,991	$ 14,891	$ 6,991	$13,981	**$ 20,972**
2009	$ 28,000	0.55	$ 15,400	$ 2,600	0.83	$ 12,762	$ 15,362	$12,762	$25,523	**$ 38,285**
Totals	**$ 144,000**		**$ 79,200**	**$ 57,300**		**$ 24,723**	**$ 82,023**	**$24,723**	**$49,446**	**$ 74,169**

Age to Age Factors

AY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5
2004	3.71	1.40	1.21	1.11	1.00
2005	2.91	1.38	1.16	1.13	
2006	3.00	1.43	1.18		
2007	3.11	1.48			
2008	3.16				
2009	-				

Arithmetic Average (3 years)

Factor	3.09	1.43	1.18	1.12	1.00

Age to Ultimate	**0 to 5**	**1 to 5**	**2 to 5**	**3 to 5**	**4 to 5**
Factor	5.84	1.89	1.32	1.12	1.00

Pct Reported

	0.17	0.53	0.76	0.89	1.00

Pct Unreported

	0.83	0.47	0.24	0.11	-