

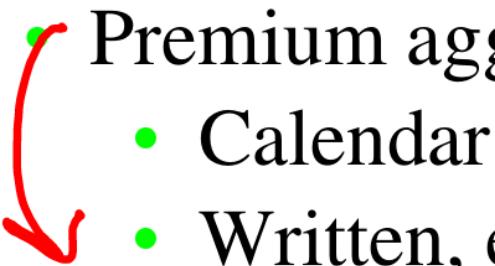
# Lecture 7: On-level Premium and Losses

*AS 8360: Insurance Ratemaking*

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# Overview



- Premium aggregation
  - Calendar vs policy year
  - Written, earned, unearned, in-force
- Premium at current rate level
- Premium development
- Premium and loss trending
  - One-step
  - Two step

# Premium aggregation

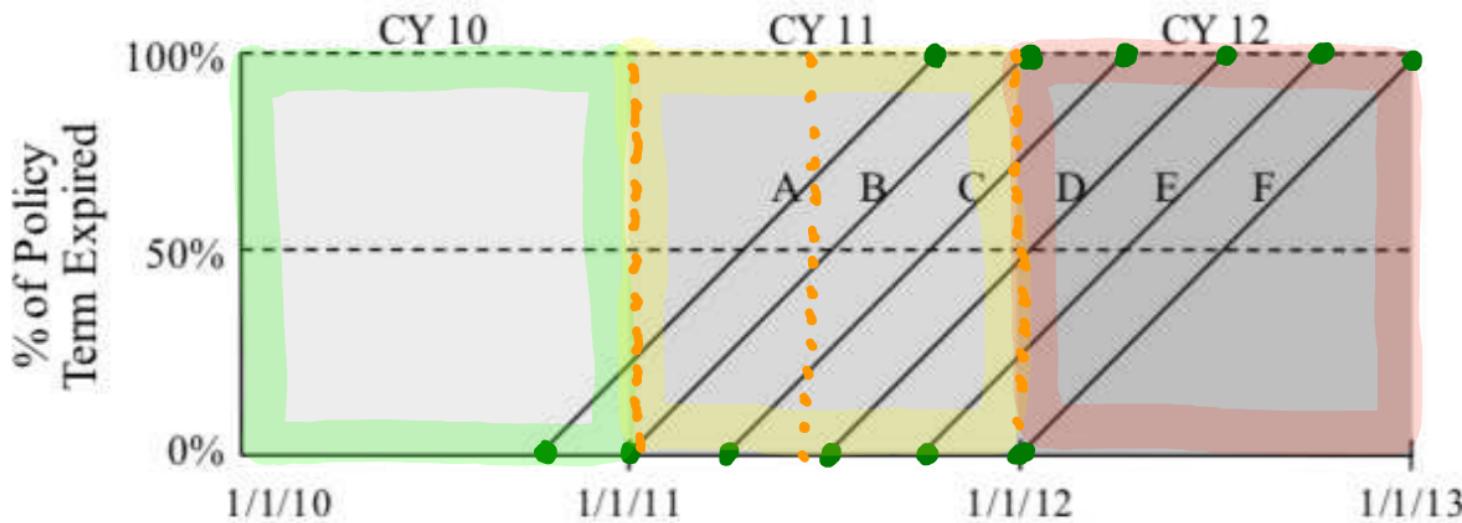
The goal of ratemaking is to determine rates that will produce premium for a future policy period.

In regards to aggregating premiums, there are also only two methods applicable: *calendar year* and *policy year*.

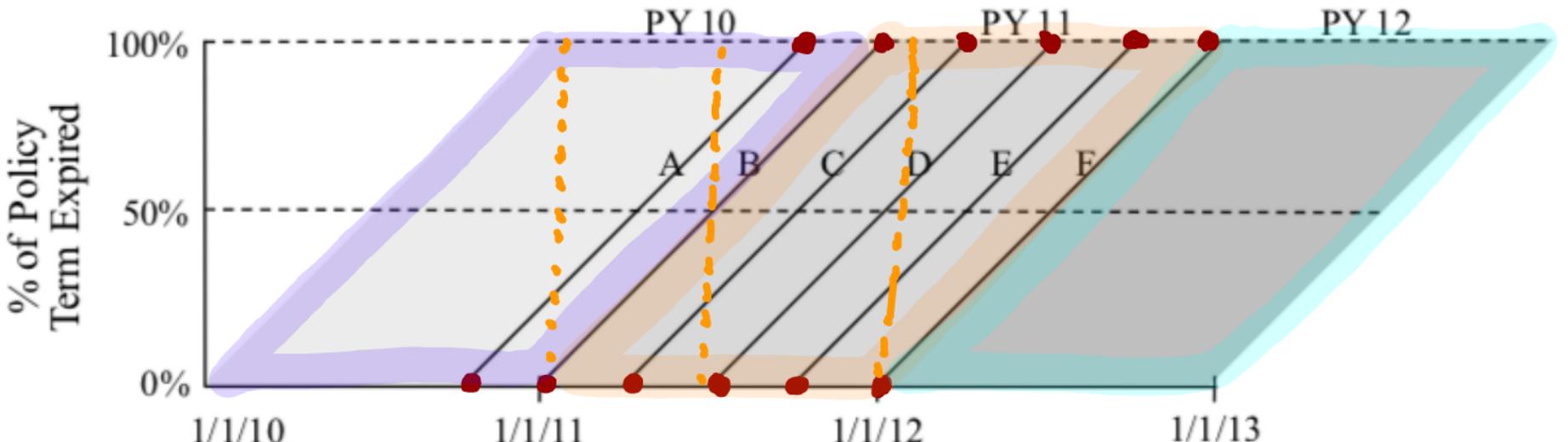
The treatment of these cases is similar to the discussion of exposures. Start with:

Policy	Effective Date	Expiration Date	Premium
A	10/01/10	09/30/11	\$200
B	01/01/11	12/31/11	\$250
C	04/01/11	03/31/12	\$300
D	07/01/11	06/30/12	\$400
E	10/01/11	09/30/12	\$350
F	01/01/12	12/31/12	\$225

Calendar year considers all premiums during the year without regard to the date of policy issuance:



Policy year considers all premiums on policies with effective dates during the year. It can be represented as:



We can further compute the written, earned, unearned, and in-force premiums for each aggregation method:

### Written premiums:

	CY method	PY method
2010	200	200
2011	1300	1300
2012	225	225

### Earned premiums:

	CY method	PY method
2010	50	200
2011	912.50	1300
2012	762.50	225

Recall that Unearned = Written – Earned.

## Unearned premiums:

	CY method	PY method
2010	150	0
2011	387.50	0
2012	-537.50	0

## In-force premiums:

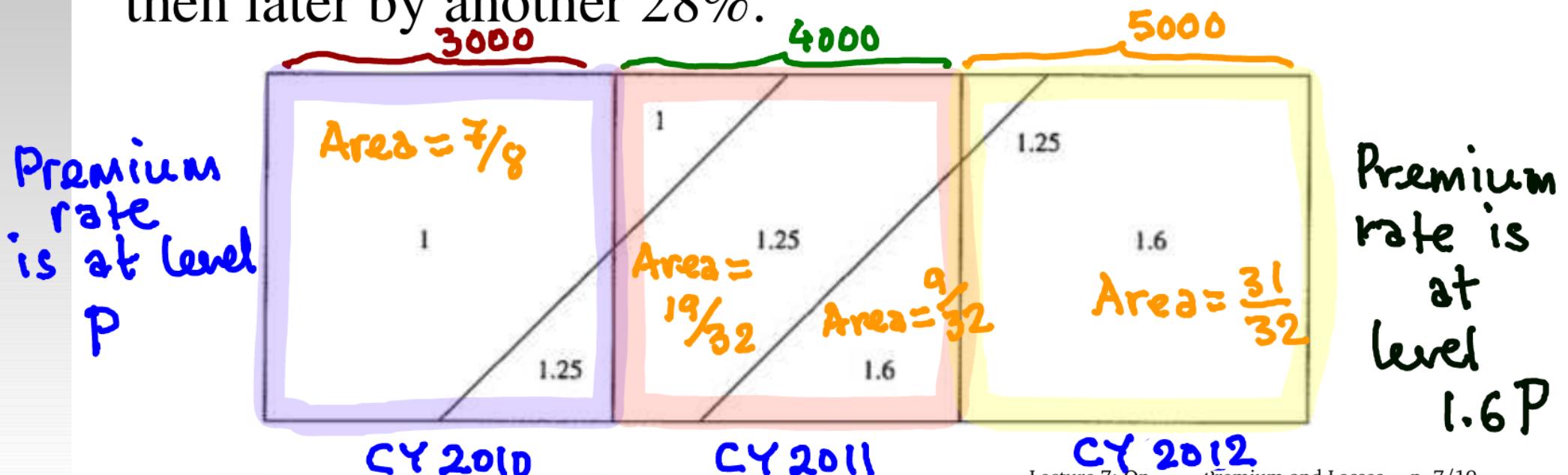
	CY method	PY method
01/01/11	450	450
06/15/11	750	750
01/01/12	1275	1275

# Premium at current rate level

We now take a closer look to future premiums.

Forecasting what the premium would be under current rates is part of deciding whether rates are sufficient, or if they should be raised.

The parallelogram method allows computing the premium at current rates using geometry. Let the original rate be  $P$  which is then increased by 25%, and then later by another 28%.



Current rate level is at 1.6 P.

$$\begin{aligned}\text{CY 2010: } \frac{7}{8} \cdot P + \frac{1}{8} \cdot 1.25P &= 3000 \Rightarrow P = 2909.09 \\ &\Rightarrow 1.6P = 4654.55\end{aligned}$$

$$\begin{aligned}\text{CY 2011: } \frac{1}{8} \cdot P + \frac{19}{32} \cdot 1.25P + \frac{9}{32} \cdot 1.6P &= 4000 \Rightarrow \\ &\Rightarrow 1.6P = 4858.84\end{aligned}$$

$$\begin{aligned}\text{CY 2012: } \frac{1}{32} \cdot 1.25P + \frac{31}{32} \cdot 1.6P &= 5000 \Rightarrow \\ &\Rightarrow 1.6P = 5034.41\end{aligned}$$

Given dollar amounts of premiums per calendar year, we can adjust them to reflect the most current rate.

A more accurate way to compute premiums at current rates is to **extend exposures**. In this method, each individual exposure (think of a separate policy) is given the updated rate that is appropriate to its risk classification, and all are then aggregated.

This method requires that very granular data are available, which is not always the case.

	Territory 1	Territory 2	Territory 3
Risk Class A	$X_{A1}, CR_{A1}$	$X_{A2}, CR_{A2}$	$X_{A3}, CR_{A3}$
Risk Class B	$X_{B1}, CR_{B1}$	$X_{B2}, CR_{B2}$	$X_{B3}, CR_{B3}$
Risk Class C	$X_{C1}, CR_{C1}$	$X_{C2}, CR_{C2}$	$X_{C3}, CR_{C3}$
Risk Class D	$X_{D1}, CR_{D1}$	$X_{D2}, CR_{D2}$	$X_{D3}, CR_{D3}$

Total premium at current level

$$\sum_r \sum_t X_{rt} CR_{rt}$$

↑      ↑

risk class      territory

# Example

assumption: annual policies

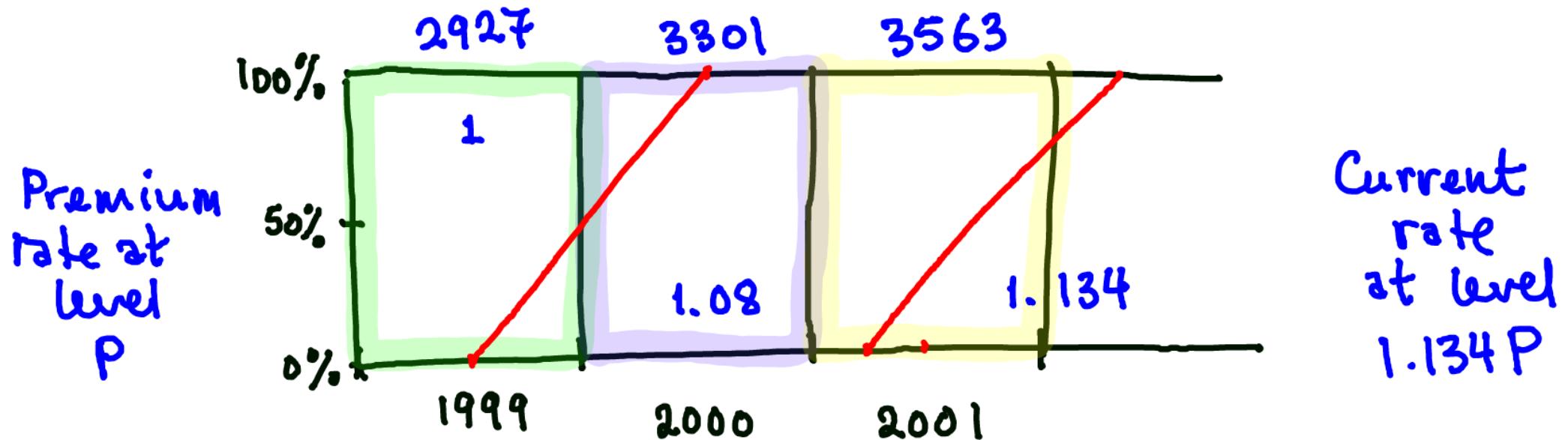
You are given the following calendar year earned premium:

Year	Earned premium
1999	2927
2000	3301
2001	3563

You are also given the following rate changes:

Date	Average rate change
7/1/1997	10%
7/1/1999	8%
4/1/2001	5%

Determine the approximate earned premium at current rates for each of these years.



$$\text{CY 99 : } \frac{7}{8}P + \frac{1}{8}1.08P = 2927 \Rightarrow 1.134P = 3286.35$$

$$\text{CY 00 : } \frac{1}{8}P + \frac{7}{8} \cdot 1.08P = 3301 \Rightarrow 1.134P = 3498.44$$

$$\text{CY 01 : } \frac{23}{32} \cdot 1.08P + \frac{9}{32} \cdot 1.134P = 3563 \Rightarrow 1.134P = 3689.90$$

# Example

You are given the following information:

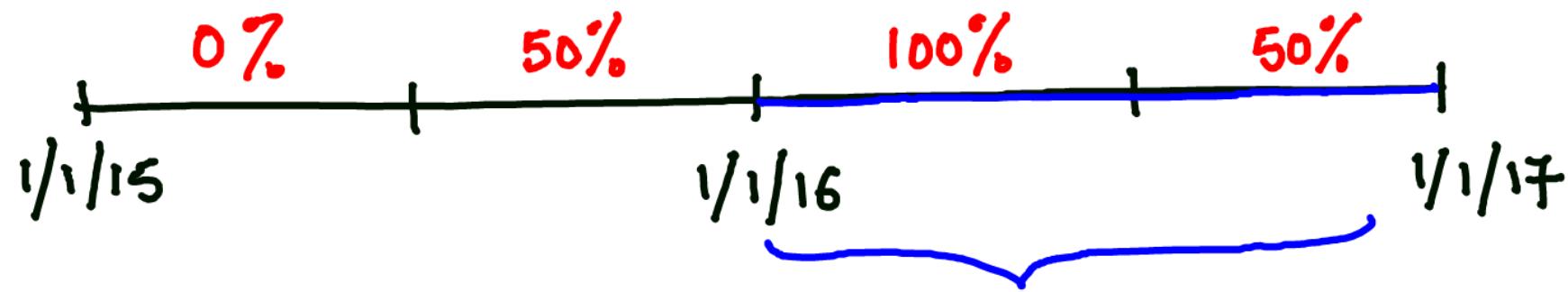
- (i) Policies have six-month terms and can't be cancelled
- (ii) Policies are written uniformly during the six months
- (iii) The rating algorithm is: (base rate)  $\times$  (class factor) + (expense fee)

Effective Date	Rate per Exposure	Class A	Class B	Expense per Exposure
1/1/15	\$400	1.00	1.30	\$40
1/1/16	\$420	1.00	1.25	\$45
1/1/17	\$450	1.00	1.20	\$50

Using the extension of exposures method, calculate the calendar year 2016 earned premium at current rate level, given that

written exposures are:

Policy Effective Date	Class A	Class B
1/1/15 – 6/30/15	200	100
7/1/15 – 12/31/15	250	150
1/1/16 – 6/30/16	300	200
7/1/16 – 12/31/16	400	300



Earned Premium  
in CY 2016

(at current levels)

$$= \left( 50\% \cdot 250 + 300 + 50\% \cdot 400 \right) (420 + 45) + \left( 50\% \cdot 150 + 200 + 50\% \cdot 300 \right) (420 \cdot 1.25)$$

# Premium development <sup>290,625</sup> +

<sup>+45)</sup>  
242,250 = 532,875

In some cases, the actuary **may not know the ultimate amount of premium** for the experience period at the time the analysis is being performed.

When this occurs, the actuary must estimate how the premium will develop to ultimate.

- when an actuary has an **incomplete year of data**
- when the line of business uses **premium audits**

In the latter case, the pattern of **premium development** can depend on several factors, including:

- The type of plan and the company operations
- The historical relationship between the original premium estimate and the final audited premium

# Premium and loss trending

We will estimate premium trends under the assumption of constant exposures. Premiums may be changing overtime because:

- Rating characteristics change across a number of policies
- Policies change (e.g. moving to a higher deductible)
- Companies merge or sell portfolios of risks to another company

In all of the above, it is important to decide whether we are dealing with a one-off event or a persistent pattern.

The following considerations are important:

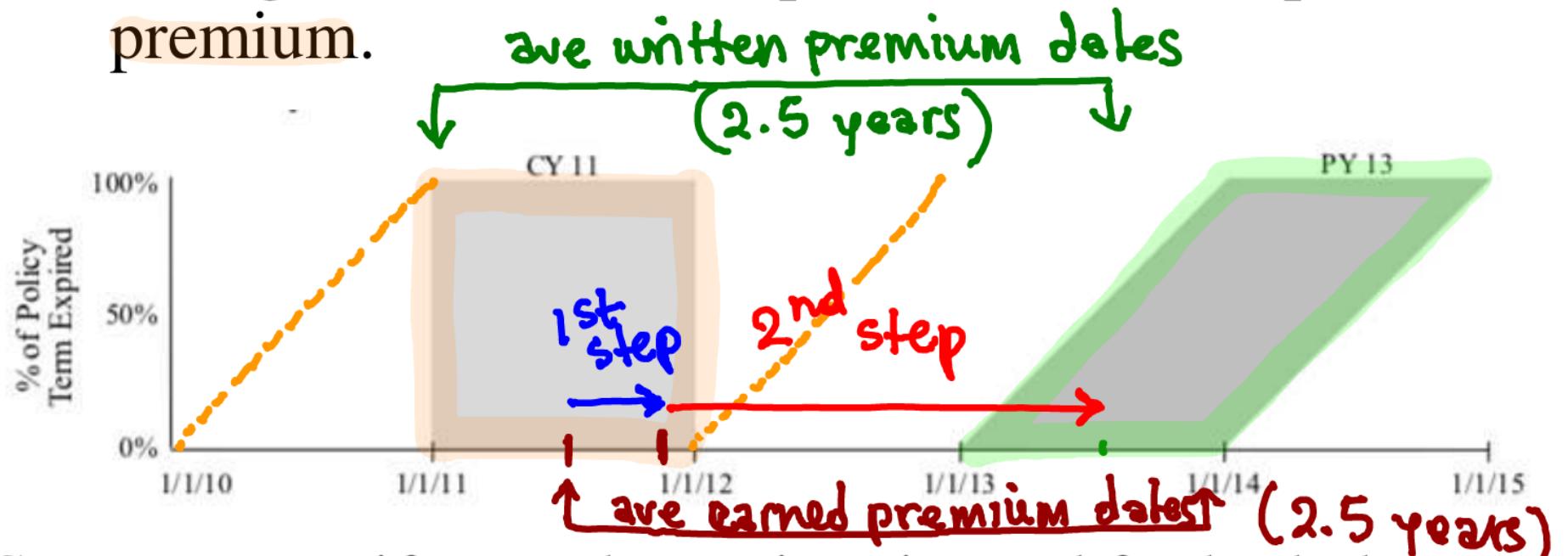
- ① Use average premium per exposure rather than total premium, or premium per policy
- ② Written premium is a leading indicator of trends that will later appear in earned premium
- ③ Quarterly data is more responsive than yearly

Quarter	(1) Written Premium at Current Rate Level	(2) Written Exposures	(4) Average Written Premium at Current Rate Level	(5) Annual Change
1Q09	\$ 323,189.17	453	\$ 713.44	--
2Q09	\$ 328,324.81	458	\$ 716.87	--
3Q09	\$ 333,502.30	463	\$ 720.31	--
4Q09	\$ 338,721.94	468	\$ 723.76	--
1Q10	\$ 343,666.70	472	\$ 728.11	2.1%
2Q10	\$ 348,696.47	477	\$ 731.02	2.0%
3Q10	\$ 353,027.03	481	\$ 733.94	1.9%
4Q10	\$ 358,098.58	485	\$ 738.35	2.0%
1Q11	\$ 361,754.88	488	\$ 741.30	1.8%
2Q11	\$ 367,654.15	493	\$ 745.75	2.0%
3Q11	\$ 372,305.01	497	\$ 749.10	2.1%
4Q11	\$ 377,253.00	501	\$ 753.00	2.0%

} on average  
trend = 2%  
annual

## One-step trending: Consists of the following:

1. Fit a linear or exponential curve to the data. In the example above, an annual rate of 2% fits the data well.
2. Compute the length of time from the average written date corresponding to the data to the average written date for policies with the updated premium.



Same answer if earned premium is used for both, but not if we are dealing with six-month policies.

**Two-step trending:** More appropriate when trends vary over time and/or one-time events have occurred.

Consists of the following:

1. Fit a linear or exponential curve to the data. In the example above, an annual rate of 2% fits the data well.
2. Compute the length of time to the most recent point in the trend data, and adjust each year's historical premium.
3. Apply a separate adjustment to project this premium into the future policy period.

Using the above example, the first adjustment is to 11/15/11 (the midpoint of 4Q11), after which a separate adjustment takes place from 11/15/11 to 6/30/13.

See the calculations below:

### 5.29 Two-Step Trending

(1)	Calendar Year 2011 Earned Premium at Current Rate Level	\$ 1,440,788
(2)	Calendar Year 2011 Earned Exposures	1,947
(3)	Calendar Year 2011 Average Earned Premium at Current Rate Level	\$ 740.00
(4)	4th Quarter of 2011 Average Written Premium at Current Rate Level	\$ 753.00
(5)	Step 1 (Current) Trend Factor	1.0176
(6)	Selected Projected Premium Trend	-1.0%
(7)	Projected Trend Period	1.6250
(8)	Step 2 (Projected) Trend Factor	0.9838
(9)	Total Premium Trend Factor	1.0011
(10)	Projected Premium at Current Rate Level	\$ 1,442,373

$$(3) = (1) / (2)$$

$$(5) = (4) / (3)$$

$$(8) = (1.0 + (6))^7$$

$$(9) = (5) \times (8)$$

$$(10) = (1) \times (9)$$

$$\textcircled{1} \quad \frac{1}{8} 343,666 + \frac{3}{8} 348,696 + \frac{5}{8} 353,027 \\ + \frac{7}{8} 358,098 + \frac{7}{8} 361,754 + \frac{5}{8} 367,654 \\ + \frac{3}{8} 372,305 + \frac{1}{8} 377,253$$

$$\textcircled{2} \quad \frac{1}{8} 472 + \frac{3}{8} 477 + \frac{5}{8} 481 + \frac{7}{8} 485 + \frac{7}{8} 488 + \frac{5}{8} 493 + \frac{3}{8} 497 \\ + \frac{1}{8} 501$$

$$\textcircled{3} \quad \text{Division. CY2011 earned premium is on } \frac{8}{8}$$

6/30/2011

Loss trending is similar to that done with premiums.

The following factors should be taken into consideration:

- Inflation
- Increasing medical costs
- Advancements in safety technology
- Social influences
- Distributional changes

The actuary must also calculate the applicable loss trend period.

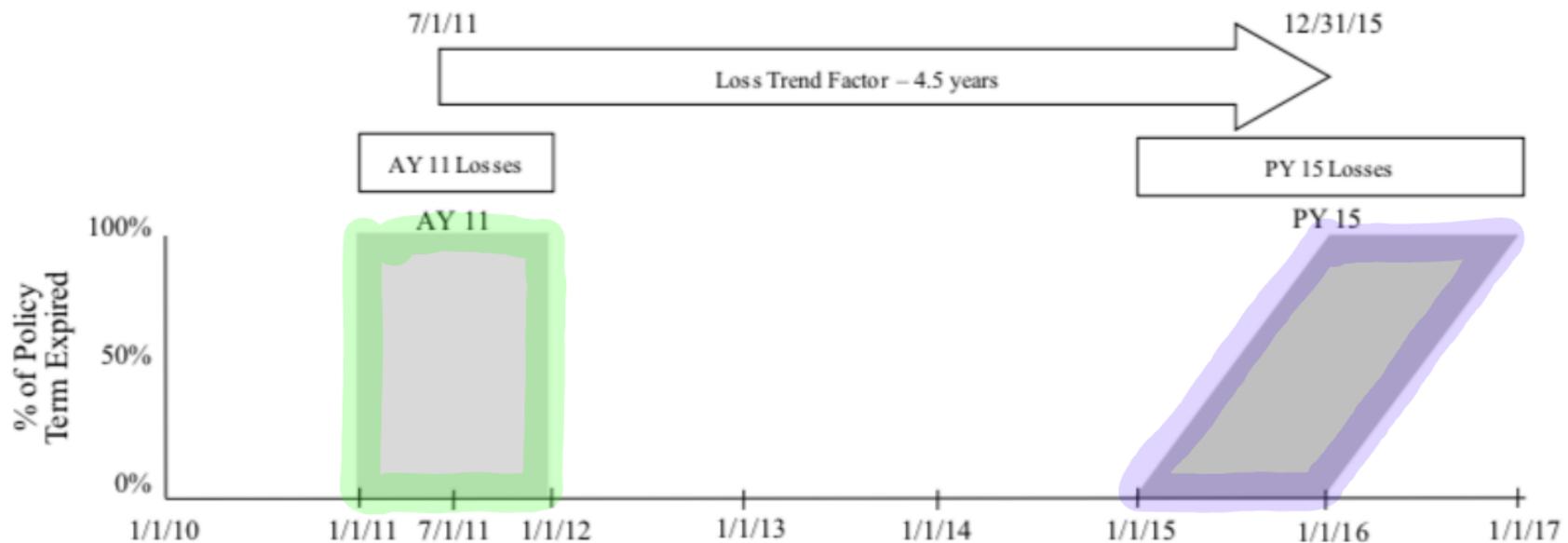
This is the period of time from the average loss occurrence date of each experience period to the average loss occurrence date for the period in which the rates will be in effect.

(4), (7) 11/15/2011

from 11/15/2011

to 6/30/2013

is 1.625 years



The term of the policy obviously affects the trend period.

