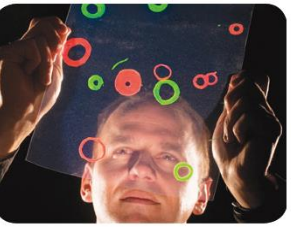
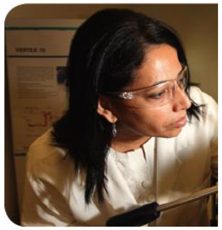




3. Pyomo Fundamentals

Sandia National Laboratories

Exceptional service in the national interest

U.S. DEPARTMENT OF ENERGY

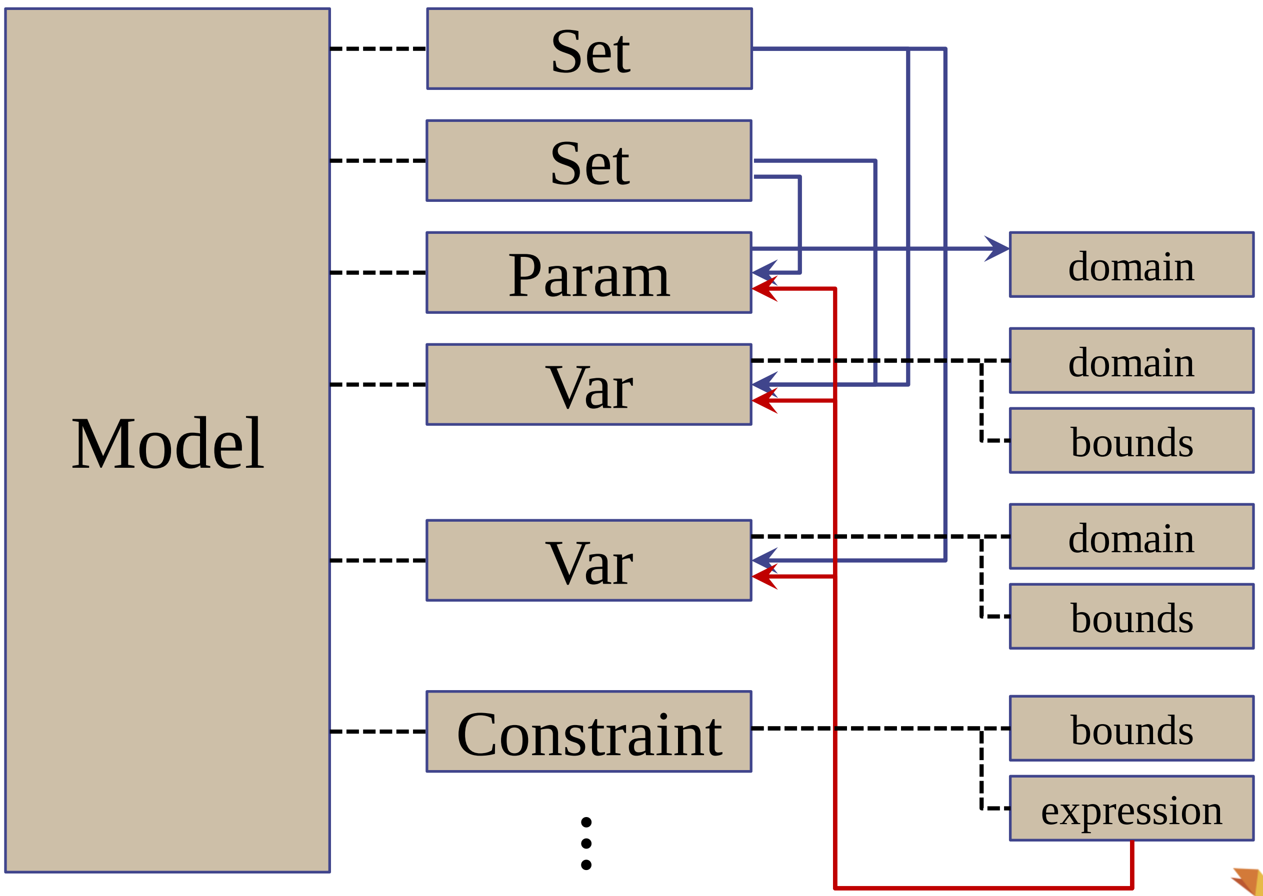
NNSA National Nuclear Security Administration

CCR Center for Computing Research

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

3. Fundamental Pyomo Components

- Pyomo is an *object model* for describing optimization problems
  - The fundamental objects used to build models are *Components*



Cutting to the chase: a simple Pyomo model

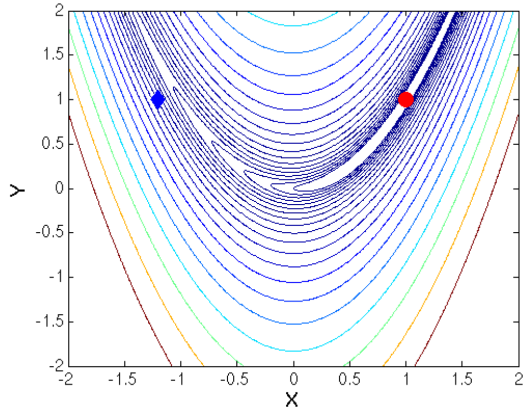
- `rosenbrock.py`:

```
from pyomo.environ import *

model = ConcreteModel()

model.x = Var( initialize=-1.2, bounds=(-2, 2) )
model.y = Var( initialize= 1.0, bounds=(-2, 2) )

model.obj = Objective(
    expr= (1-model.x)**2 + 100*(model.y-model.x**2)**2,
    sense= minimize )
```



Cutting to the chase: a simple Pyomo model

- Solve the model:
  - The pyomo command

```
% pyomo solve rosenbrock.py --solver=ipopt --summary
[    0.00] Setting up Pyomo environment
[    0.00] Applying Pyomo preprocessing actions
[    0.00] Creating model
[    0.00] Applying solver
[    0.03] Processing results
    Number of solutions: 1
    Solution Information
      Gap: <undefined>
      Status: optimal
      Function Value: 2.98956421871e-17
    Solver results file: results.json

=====================================================
Solution Summary
=====================================================

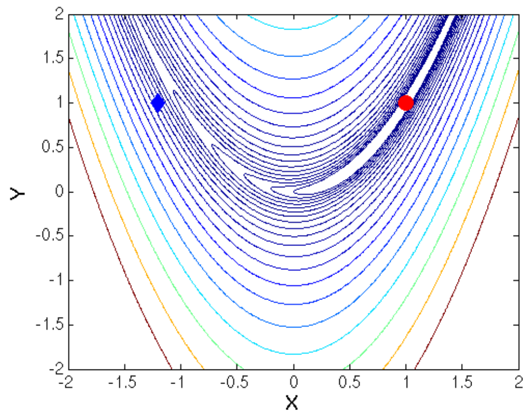
Model unknown

  Variables:
    Variable x : Size=1 Domain=Reals
      Value=0.999999994543
    Variable y : Size=1 Domain=Reals
      Value=0.999999989052

  Objectives:
    Objective obj : Size=1
      Value=2.98956421871e-17

  Constraints:
    None

[    0.03] Applying Pyomo postprocessing actions
[    0.03] Pyomo Finished
```



Regarding *namespaces*

- Pyomo objects exist within the `pyomo.environ` namespace:

```
import pyomo.environ
model = pyomo.environ.ConcreteModel()
```

- …but this gets verbose. To save typing, we will import the core Pyomo classes into the main namespace:

```
from pyomo.environ import *
model = ConcreteModel()
```

- To clarify Pyomo-specific syntax in this tutorial, we will highlight Pyomo symbols in green

Getting Started: the *Model*

```
from pyomo.environ import *

model = ConcreteModel()
```

Every Pyomo model starts with this; it tells Python to load the Pyomo Modeling Environment

Create an instance of a *Concrete* model

- Concrete models are immediately constructed
- Data must be present *at the time* components are defined

Local variable to hold the model we are about to construct

- While not required, by convention we use "`model`"
- If you choose to name your model something else, you will need to tell the Pyomo script the object name through the command line

Populating the Model: *Variables*

```
model.a_variable = Var(within = NonNegativeReals)
```

The name you assign the object to becomes the object's name, and must be unique in any given model.

"`within`" is optional and sets the variable domain ("`domain`" is an alias for "`within`")

Several pre-defined domains, e.g., "`Binary`"

```
model.a_variable = Var(bounds = (0, None))
```

Same as above: "`domain`" is assumed to be `Reals` if missing

Defining the *Objective*

```
model.x = Var( initialize=-1.2, bounds=(-2, 2) )
model.y = Var( initialize= 1.0, bounds=(-2, 2) )

model.obj = Objective(
    expr= (1-model.x)**2 + 100*(model.y-model.x**2)**2,
    sense= minimize )
```

If "sense" is omitted, Pyomo assumes minimization

Note that the Objective expression is not a *relational expression*

"expr" can be an expression, or any function-like object that returns an expression

Defining the Problem: *Constraints*

```
model.a = Var()
model.b = Var()
model.c = Var()
model.c1 = Constraint(
    expr = model.b + 5 * model.c <= model.a )
```

"expr" can be an expression, or any function-like object that returns an expression

```
model.c2 = Constraint(expr = (None, model.a + model.b, 1))
```

"expr" can also be a tuple:

- 3-tuple specifies (LB, expr, UB)
- 2-tuple specifies an equality constraint.

In general, we do not recommend this notation

Lists of Constraints

```
model.a = Var()
model.b = Var()
model.c = Var()
model.limits = ConstraintList()

model.limits.add(30*model.a + 15*model.b + 10*model.c <= 100)
model.limits.add(10*model.a + 25*model.b +  5*model.c <= 75)
model.limits.add(6*model.a  + 11*model.b +  3*model.c <= 30)
```

"add" adds a single new constraint to the list.
The constraints need not be related.

Higher-dimensional components

- (Almost) All Pyomo *components* can be *indexed*
  - *All* non-keyword arguments are assumed to be *indices*
  - Individual indices may be multi-dimensional (e.g., a list of pairs)

```
<Type>(<IDX1>, <IDX2>, […] <keyword>=<value>, …)
```

- Indexed variables

```
model.a_vector = Var(IDX)
model.a_matrix = Var(IDX_A, IDX_B)
```

The indexes are any iteratable object, e.g., list or Set

- ConstraintList is a special case with an implicit index

- *Note:* while indexed variables look like matrices, they are not.
  - In particular, we do not support matrix algebra (*yet…)*

Manipulating indices: list comprehensions

```
model.IDX = range(10)
model.a = Var()
model.b = Var(model.IDX)
model.c1 = Constraint(
    expr = sum(model.b[i] for i in model.IDX) <= model.a )
```

Python *list comprehensions* are very common for working over indexed variables and nicely parallel mathematical notation:

$$\sum_{i \in IDX} b_i \leq a$$

Concrete Modeling

Putting It All Together: *Concrete p-Median*

- Determine the set of P warehouses chosen from N candidates that minimizes the total cost of serving all customers M where $d_{n,m}$ is the cost of serving customer m from warehouse location n.

min	$\sum_{n \in N, m \in M} d_{n,m} x_{n,m}$		(minimize total cost)
$s.t.$	$\sum_{n} x_{n,m} = 1$	$\forall \; m \in M$	(guarantee all customers served)
	$x_{n,m} \leq y_n$	$\forall \; n \in N, m \in M$	(customer n can only be served from warehouse m if warehouse m is selected)
	$\sum_{n \in N} y_n = P$		(select P warehouses)
	$0 \leq x \leq 1$	$y \in \{0,1\}^N$	

Concrete p-Median (1)

```
from pyomo.environ import *

N = 3
M = 4
P = 3
d = {(1, 1): 1.7, (1, 2): 7.2, (1, 3): 9.0, (1, 4): 8.3,
     (2, 1): 2.9, (2, 2): 6.3, (2, 3): 9.8, (2, 4): 0.7,
     (3, 1): 4.5, (3, 2): 4.8, (3, 3): 4.2, (3, 4): 9.3}

model = ConcreteModel()
model.Locations = range(N)
model.Customers = range(M)

model.x = Var( model.Locations, model.Customers,
               bounds=(0.0,1.0) )

model.y = Var( model.Locations, within=Binary )
```

Concrete p-Median (2)

```
model.obj = Objective( expr = sum( d[n,m]*model.x[n,m]
    for n in model.Locations for m in model.Customers ) )

model.single_x = ConstraintList()
for m in model.Customers:
    model.single_x.add(
        sum( model.x[n,m] for n in model.Locations ) == 1.0 )

model.bound_y = ConstraintList()
for n in model.Locations:
    for m in model.Customers:
        model.bound_y.add( model.x[n,m] <= model.y[n] )

model.num_facilities = Constraint(
    expr=sum( model.y[n] for n in model.Locations ) == P )
```

Solving models: *the* pyomo *command*

- pyomo (`pyomo.exe` on Windows):
  - Constructs model and passes it to an (external) solver

```
pyomo solve <model_file> [<data_file> …] [options]
```

  - Installed to:
    - `[PYTHONHOME]\Scripts` [Windows; `C:\Python27\Scripts`]
    - `[PYTHONHOME]/bin` [Linux; `/usr/bin`]

- Key options (*many* others; see `--help`)

`--help`	Get list of all options
`--help-solvers`	Get the list of all recognized solvers
`--solver=<solver_name>`	Set the solver that Pyomo will invoke
`--solver-options="key=value[ …]"`	Specify options to pass to the solver as a space-separated list of keyword-value pairs
`--stream-solver`	Display the solver output during the solve
`--summary`	Display a summary of the optimization result
`--report-timing`	Report additional timing information, including construction time for each model component

In Class Exercise: Concrete Knapsack

$$\max \quad \sum_{i=1}^{N} v_i x_i$$

$$s.t. \quad \sum_{i=1}^{N} w_i x_i \le W_{\max}$$

$$x_i \in \{0,1\}$$

Item	Weight	Value
`hammer`	5	8
`wrench`	7	3
`screwdriver`	4	6
`towel`	3	11

`Max weight:` 14

Syntax reminders:

```
from pyomo.environ import *
ConcreteModel()
Var( [index, …], [within=domain], [bounds=(lower,upper)] )
ConstraintList()
    c.add( expression )
Objective( sense={maximize|minimize},
           expr=expression )
```

Concrete Knapsack: *Solution*

```
from pyomo.environ import *

v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
w = {'hammer':5, 'wrench':7, 'screwdriver':4, 'towel':3}
W_max = 14

model = ConcreteModel()
model.ITEMS = v.keys()
model.x = Var( model.ITEMS, within=Binary )

model.value = Objective(
   expr = sum( v[i]*model.x[i] for i in model.ITEMS ),
   sense = maximize )

model.weight = Constraint(
   expr = sum( w[i]*model.x[i] for i in model.ITEMS ) <= W_max )
```

Abstract Modeling

Concrete *vs.* Abstract Models

- Concrete Models: data first, then model
  - 1-pass construction
  - All data must be present before Python starts processing the model
  - Pyomo will construct each component in order at the time it is declared
  - Straightforward logical process; easy to script.
  - Familiar to modelers with experience with GAMS

- Abstract Models: model first, then data
  - 2-pass construction
  - Pyomo stores the basic model declarations, but does not construct the actual objects
    - Details on how to construct the component hidden in functions, or *rules*
    - e.g., it will declare an indexed variable “x”, but will not expand the indices or populate any of the individual variable values.
  - At “creation time”, data is applied to the abstract declaration to create a concrete instance (components are still constructed in declaration order)
  - Encourages generic modeling and model reuse
    - e.g., model can be used for arbitrary-sized inputs
  - Familiar to modelers with experience with AMPL

Generating and Managing Indices: *Sets*

- Any iterable object can be an index, e.g., lists:
  - `IDX_a = [1,2,5]`
  - ```
    DATA = {1: 10, 2: 21, 5:42};
    IDX_b = DATA.keys()
    ```

- Sets: objects for managing multidimensional indices
  - `model.IDX = Set( initialize = [1,2,5] )`

Note: capitalization matters:
  `Set` = Pyomo class
  `set` = native Python set

Like indices, Sets can be initialized from any iterable

  - `model.IDX = Set( [1,2,5] )`

Note: This doesn't do what you want.
  This creates a 3-member *indexed set*, where each set is *empty.*
    ```

Sequential Indices: *RangeSet*

- Sets of sequential integers are common
  - `model.IDX = Set( initialize=range(5) )`
  - `model.IDX = RangeSet( 5 )`

Note: RangeSet is 1-based.
This gives [1, 2, 3, 4, 5]

Note: Python range is 0-based.
This gives [0, 1, 2, 3, 4]

- You can provide lower and upper bounds to RangeSet
  - `model.IDX = RangeSet( 0, 4 )`

This gives [0, 1, 2, 3, 4]

Manipulating Sets

- Sets support efficient higher-dimensional indices

```
model.IDX = Set( initialize=[1,2,5] )
model.IDX2 = model.IDX * model.IDX
```

This creates a *virtual* 2-D “matrix” Set

Sets also support union (&), intersection (|), difference (-), symmetric difference (^)

- Creating sparse sets

```
model.IDX = Set( initialize=[1,2,5] )
def lower_tri_filter(model, i, j):
     return j <= i
model.LTRI = Set( initialize = model.IDX * model.IDX,
                  filter = lower_tri_filter )
```

The `filter` should return *True* if the element is in the set; *False* otherwise.

Deferred construction: *Rules*

- Abstract modeling constructs the model in two passes:
  - Python parses the model declaration
    - creating “empty” Pyomo components in the model
  - Pyomo loads and parses external data
- Components are constructed in declaration order
  - The instructions for *how* to construct the object are provided through a function, or *rule*
  - Pyomo calls the rule for each component index
  - *Rules* can be provided to virtually all Pyomo components (even when using Concrete models)
- Naming conventions
  - the component name prepended with “`_`” (`c4` → `_c4`)
  - the component name with “`_rule`” appended (`c4` → `c4_rule`)
  - each rule is called “`rule`” (Python implicitly overrides each declaration)

Indexed Constraints

```
model.IDX = Set( initialize=range(5) )
model.a = Var( model.IDX )
model.b = Var()
def c4_rule(model, i):
     return model.a[i] + model.b <= 1
model.c4 = Constraint( model.IDX, rule=c4_rule )
```

For indexed constraints, you provide a "`rule`" (function) that returns an expression (or tuple) for each index.

```
model.IDX2 = model.IDX * model.IDX
def c5_rule(model, i, j, k):
     return model.a[i] + model.a[j] + model.a[k] <= 1
model.c5 = Constraint( model.IDX2, model.IDX, rule=c5_rule )
```

Each dimension of each index is a separate argument to the rule

Importing Data: *Parameters*

- Scalar numeric values

```
model.a_parameter = Param( initialize = 42 )
```

Provide an (initial) value of 42 for the parameter

- Indexed numeric values

```
model.a_param_vec = Param( IDX,
                           initialize = data,
                           default = 0 )
```

Providing "`default`" allows the initialization data to only specify the "unusual" values

"`data`" *must* be a dictionary(*) of index keys to values because all sets are assumed to be *unordered*

(*) – actually, it must define __getitem__(), but that only really matters to Python geeks

Data Sources

- Data can be imported from "`.dat`" file
  - Format similar to AMPL style
  - Explicit data from "`param`" declarations
  - External data through "`load`" declarations:
    - Excel

```
load ABCD.xls range=ABCD : Z=[A, B, C] Y=D ;
```

    - Databases

```
load "DBQ=diet.mdb" using=pyodbc query="SELECT FOOD, cost,
  f_min, f_max from Food" : [FOOD] cost f_min f_max ;
```

- External data overrides "`initialize=`" declarations

Abstract p-Median (pmedian.py, 1)

```
from pyomo.environ import *

model = AbstractModel()

model.N = Param( within=PositiveIntegers )
model.P = Param( within=RangeSet( model.N ) )
model.M = Param( within=PositiveIntegers )

model.Locations = RangeSet( model.N )
model.Customers = RangeSet( model.M )

model.d = Param( model.Locations, model.Customers )

model.x = Var( model.Locations, model.Customers, bounds=(0.0, 1.0) )
model.y = Var( model.Locations, within=Binary )
```

Abstract p-Median (pmedian.py, 2)

```
def obj_rule(model):
    return sum( model.d[n,m]*model.x[n,m]
       for n in model.Locations for m in model.Customers )
model.obj = Objective( rule=obj_rule )

def single_x_rule(model, m):
    return sum( model.x[n,m] for n in model.Locations ) == 1.0
model.single_x = Constraint( model.Customers, rule=single_x_rule )

def bound_y_rule(model, n,m):
    return model.x[n,m] - model.y[n] <= 0.0
model.bound_y = Constraint( model.Locations, model.Customers,
                            rule=bound_y_rule )

def num_facilities_rule(model):
    return sum( model.y[n] for n in model.Locations ) == model.P
model.num_facilities = Constraint( rule=num_facilities_rule )
```

Abstract p-Median (pmedian.dat)

```
param N := 3;

param M := 4;

param P := 2;

param d:  1     2     3     4 :=
     1   1.7   7.2   9.0   8.3
     2   2.9   6.3   9.8   0.7
     3   4.5   4.8   4.2   9.3 ;
```

In Class Exercise: Abstract Knapsack

$$\max \quad \sum_{i=1}^{N} v_i x_i$$

$$s.t. \quad \sum_{i=1}^{N} w_i x_i \le W_{\max}$$

$$x_i \in \{0,1\}$$

Item	Weight	Value
hammer	5	8
wrench	7	3
screwdriver	4	6
towel	3	11

Max weight: 14

Syntax reminders:

```
AbstractModel()
Set( [index, …], [initialize=list/function] )
Param( [index, …], [within=domain], [initialize=dict/function] )
Var( [index, …], [within=domain], [bounds=(lower,upper)] )
Constraint( [index, …], [expr=expression|rule=function] )
Objective( sense={maximize|minimize},
           expr=expression|rule=function )
```

Abstract Knapsack: *Solution*

```
from pyomo.environ import *

model        = AbstractModel()
model.ITEMS = Set()
model.v      = Param( model.ITEMS, within=PositiveReals )
model.w      = Param( model.ITEMS, within=PositiveReals )
model.W_max = Param( within=PositiveReals )
model.x      = Var( model.ITEMS, within=Binary )

def value_rule(model):
    return sum( model.v[i]*model.x[i] for i in model.ITEMS )
model.value = Objective( rule=value_rule, sense=maximize )

def weight_rule(model):
    return sum( model.w[i]*model.x[i] for i in model.ITEMS ) \
        <= model.W_max
model.weight = Constraint( rule=weight_rule )
```

Abstract Knapsack: *Solution Data*

```
set ITEMS := hammer wrench screwdriver towel ;

param:  v w :=
    hammer       8 5
    wrench       3 7
    screwdriver  6 4
    towel       11 3;

param W_max := 14;
```