ELECTRIC DRIVE OPTIMISATION

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LIST OF ABBREVIATIONS

AC	Alternating Current
BLDC	Brushless DC
DC	Direct Current
I/O	Input Output
PMBLDC	Permanent Magnet Brushless DC
PWM	Pulse Width Modulation
SOC	State of Charge

LIST OF SYMBOLS

lim	limit

ABSTRAK

Abstrak versi bahasa melayu

ELECTRIC DRIVE OPTIMISATION

ABSTRACT

English version of the abstract

CHAPTER 1

INTRODUCTION

1.1 Background

Electric motor can be classified into two major categories which are the DC electric motor and AC electric motor. An AC motor is an electric motor driven by alternating current whereas a DC motor is driven by direct current. There are various types of AC motor which includes induction motor, synchronous motor, eddy current motor and etc. The DC electric motor includes permanent magnet brushed motor, permanent magnet brushless motor, switched reluctance motor and etc.

Electric motor is used in many application which includes in machine for driving the pulley and belts, the conveyor belt, in drilling and lathe machine. Apart from the heavy industry, electric motor is used in home appliances for powering the washing machine, fan, blower of air-conditioner and blender machine. Moreover, electric motor is also used in automobile industry as the starter motor for firing up the internal combustion engine of cars and trucks and last but not least, as the drive train for electric vehicle.

The PMBLDC is a synchronous motor. In other words, the frequency of the magnetic field generated at the stator and the rotor is the same. PMBLDC comes in single-phase, 2-phase and 3-phase configuration which the 3-phase configuration is the most popular among the three. There are basically two major components inside a PMBLDC motor which is the stator and the rotor. The stator of a PMBLDC motor is made up of a series of laminated steel with wire windings around it. The rotor is build up of permanent magnet that has at least 2 poles.

Unlike brushed motor, PMBLDC does not have brushes for comutation, instead a controller is needed for controlling the rotation of the PMBLDC by sending out AC signal to the PMBLDC. There are two types of AC signal sent to the PMBLDC for controlling the motor which are the Trapezoidal type and the Sinusoidal type which depends on the winding of the stator.

In order for the controller to send out the correct signal, the position of the rotor must be sent to the controller so that a sequence of AC signal can be generated which energized the winding of the stator for rotating the motor. Hall effect sensors is used as the rotor position detection sensor which has an analog signal output. When the magnetic field is detected by the hall effect sensor, the voltage output will be changes from lowest to the highest or vice versa depending on the circuit configuration. Normally there are three hall effect sensor mounted on the stator of PMBLDC motor which are 60°or 120°apart depending on the number of poles and the comutation sequence required.

Figure 1.1 shows the hall effect sensors signal for a 2 pole PMBLDC motor, the back emf, the phase current and the output torque. As shown in the figure, there are 3 hall effect sensors with labels A, B and C respectively, sensor A is leading sensor B by 60°and sensor B is leading sensor C by 60°.

Ideally, the phase current should behave as a digital square wave signal where the current rises and drops immediately. But in real world, the current would take some time to rises from zero to maximum/minimum. Hence, the torque produced would behave as a series of ripple instead of a constant output torque.

In this project, a PMBLDC hub motor is used as the sole powertrain of a electric vehicle. The electric vehicle will be used for participating the Shell Eco-Marathon which is a competition which teams compete to build a higher mileage vehicle. The competition requires the participating teams to build their own car for the categories they are participating(for example, the urban concept category or the prototype category). For our case, a four-wheel electric vehicle is built for participation in urban concept category. The vehicle will be driven around a race track, which is the Sepang International Track, North Track for year 2012 for four laps with 10 seconds of stop between each lap. The energy consumption will be collected and the mileage will be calculated for each attempt and every team will have 3 attempts for mileage improvement.

1.2 Problem Statement

PMBLDC motor is better compare to brushed DC motor because brushless motor increases the efficiency by dropping the friction between the brush and the comutator which happens in brushed DC motor. However, the inefficient PWM in controlling the speed of the motor and the torque ripple cause by the phase current limits the efficiency of a PMBLDC motor.

There are three types of torque produced by a permanent magnet electric motor which are:

- cogging torque

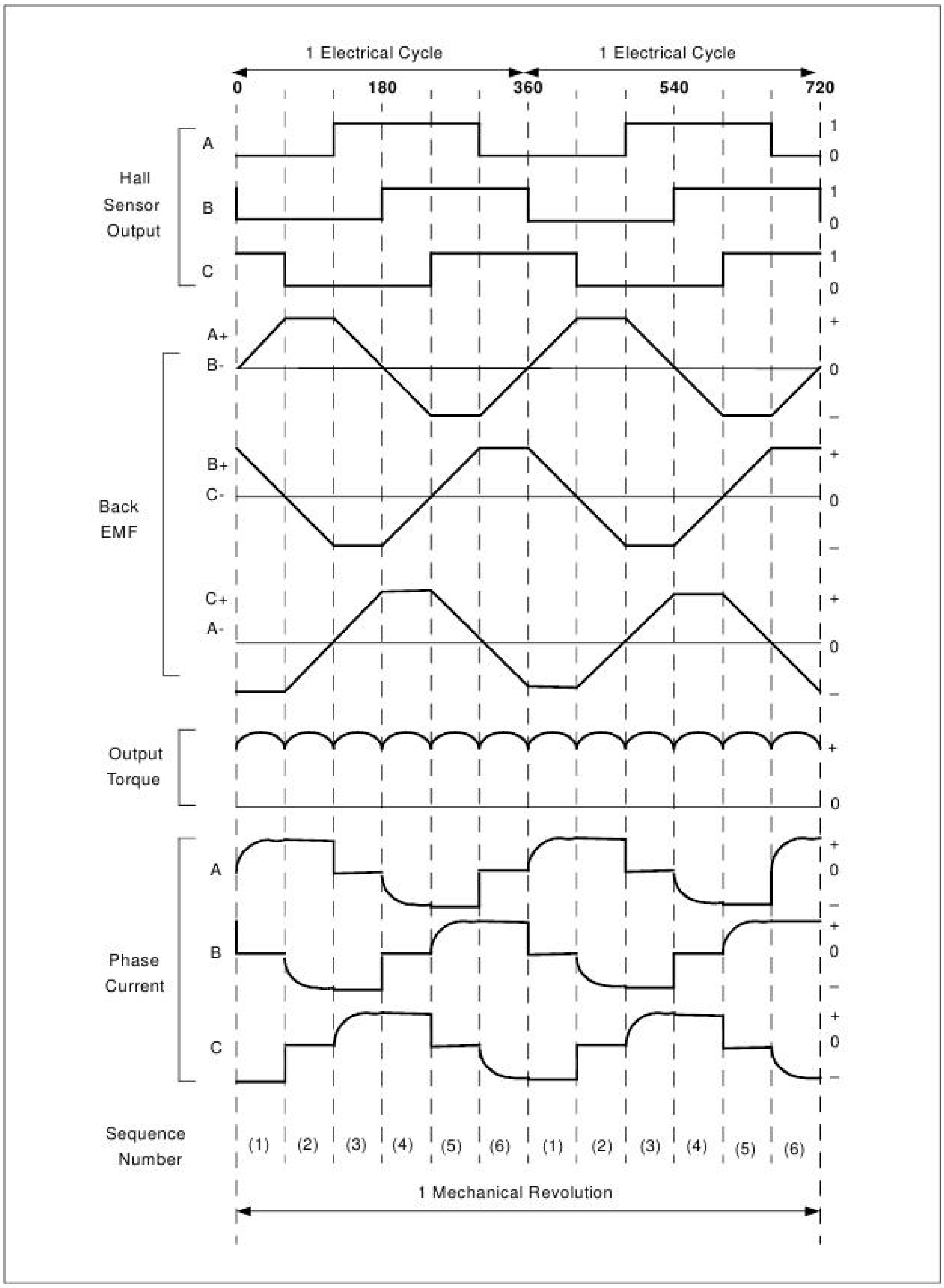

Figure 1.1: Hall effect sensors signal, back emf, output torque and phase current (Yedamale, 2003)

- reluctance torque
- mutual torque

The cogging torque is produced by the interaction between the permanent magnet at the rotor and the stator slots. Cogging torque is an undesireable torque generated by the electric motor which dominates at low speed and results in speed ripple. Cogging torque can only be minimize by means of hardware tuning which includes altering the number of poles, teeth at the stator or editing the controller setting which changes the drive current waveform.

The reluctance torque is generated by the difference in position of the rotor and the phase induction at the rotor. The ripple produced by the reluctance torque could be negligible with a good number of poles and slots of the windings at the stator.

The third type of torque produced by PMBLDC motor is the mutual torque which is caused by the non-sinusoidal signal reaching the stator windings or the magnet at the rotor. Since the torque is generated by the current, ripple reduction for mutual torque could be achieved by fine-tuning the current signal.

The torque ripple occurs in PMBLDC is mainly contributed by the different rise and decay time of the phase current as shown in figure 1.1. The cogging torque does contribute to torque ripple in low speed but it's negligible at high speed. Torque ripple contributed by reluctance torque could also be ignored since the high number of poles and stator slot minimize the ripple. Mutual torque would be the major contributor to torque ripple due to the direct dependency on the current.

Based on figure 1.2 (a), the rate of decay of i_a is faster than the rate of increase of i_b. Therefore, at certain point where i_a reaches 0 but i_b still rising, there will be a surge in current at phase C, i_c. As the result, there will be a sudden drop in torque output. The same case happens as shown in figure 1.2 (c) where i_a drops slower than i_b causing an sudden drop in i_c and sudden increase in torque output. The same circumstances apply for i_a-i_c and i_b-i_c hence creating a ripple torque output.

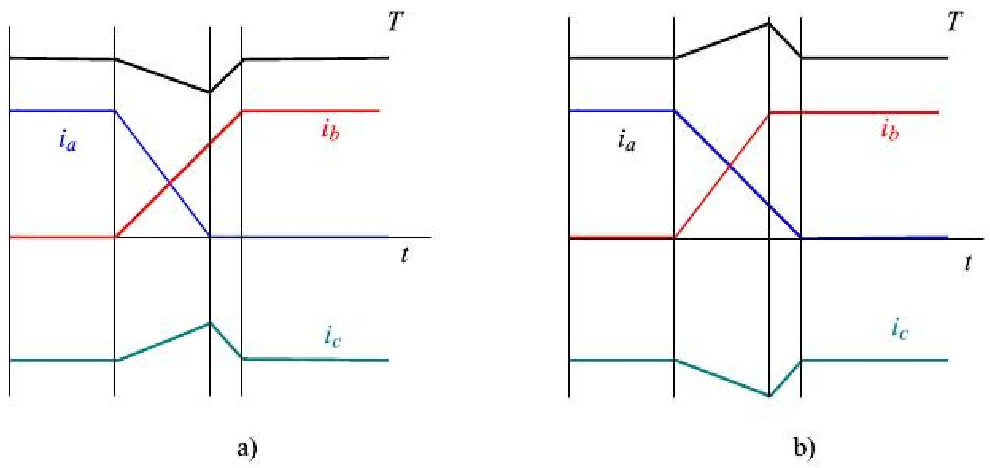

Figure 1.2: The phase current and torque during an alternating comutation event (Salah et al., 2011)

Apart from torque ripple in PMBLDC that reduce the efficiency of the electric motor and cut down the mileage of the electric vehicle, poor driving strategy will also contributes to poor mileage. The Sepang North Track is a racing track that contains 3 uphill section and 3 uphill section and an approximately 800m long straight for the starting and ending line. This track is especially challenging for electric vehicle because the factor of torque generated by the electric motor, cruise speed when uphill and downhill, rolling resistance and drag factor need to be taken into consideration for minimum energy consumption.

1.3 Objectives

The objectives for this project are:

1. To identify the output signal of the controller circuit and the hall effect sensor of the PMBLDC and develop a set of instrument for measuring the mileage of the electric vehicle.

2. To study the track profile of Sepang North Track and compose a set of strategy to increase the mileage of the electric vehicle.

3. To suggest methods for improving the efficiency of the electric vehicle through altering the drive current waveform and reducing coefficient of drag.

1.4 Scope of Research

In this project, the proprietary controller and PMBLDC motor signal output port will be identified. After that, by utilizing the signal output of the hall effect sensor and the controller speed output signal port, a set of instrument will be build for measuring the speed, input current and input voltage. The power input will then be calculated based on the voltage and current input.

After the instrumentation for measuring the performance of the electric vehicle is established, methods for improving the overall electric vehicle efficiency will be suggested which includes reducing the frontal area and improving the coefficient of drag of the electric vehicle and hence reducing the drag force. Modification on the phase current signal using indirect method also will be suggested for reducing the torque ripple and hence improving the overall efficiency of the electric vehicle.

Next up, the behaviour of the electric vehicle on the Sepang North Track will be simulated with taking the drag force, rolling resistance and track gradient into consideration. With the frontal area, coefficient of drag, coefficient of rolling resistance and the mass of the vehicle as manipulating variables, a set of strategies could be created for electric vehicle at different mass, tyre pressure at the same track with using the same electric motor as drive train.

1.5 Research Approach

For tapping the signal from the PMBLDC motor's hall effect sensors as well as the analog/digital output from the controller circuit board for the speed signal, multimeter will be used. For building the measurement tools for measuring and logging the input voltage, input current and the speed of the vehicle, Arduino boards will be used in conjunction with the self-made transducing circuit.

For simulating the vehicle's behaviour on track, self-made codes using C++ language will be used for iteration. Data's for each set of simulation and strategies will be saved and the graph will be plot for analysing the effectiveness of the strategy.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, the research paper of different researchers around the world will be reviewed. The research on tuning the PWM controller signal, modification of phase current signal for minimizing the torque ripple as well as driving strategy for cars and trains will be reviewed.

2.2 Torque Ripple

Park, Sung Jun *et al.* (Park et al., 2000) found that the back EMF generated by each of the three windings are slightly different in shape and magnitude from each other. Since the back EMF for each winding is different, therefore different phase current should be apply to each of the three windings. By assuming the three stator windings are in Y-shape connection, the cogging torque and the reluctance torque component is negligible and the mutual torque is directly proportional to the phase current, phase current for each of the three stator windings should be seperately excited in phase with the back EMF to minimize the losses and maximizing the torque-per-ampere generation.

Kim, Tae-Sung *et al.* (Kim et al., 2001) shows that by using the rectangular shape phase current that changes to high at the flat part of the back-emf wave can minimize the torque ripple, but this method is too ideal to be used in practical condition. In this paper, another method which is the unipolar PWM is introduced but it has a slow dynamic response, making this method not feasible at reducing the torque for current control commutation. The proposed current control algorithm is to seperate the phase current that contains all harmonic components to each harmonical components and then transformed into stationary frame. Then, each stationary frame is added together and the output is the current command. After this process, the rectangular wave will appear more rectangular and hence reducing the torque ripple.

Nam, Ki-Yong *et al.* (Nam et al., 2006) suggested reducing the torque ripple by varying the input voltage to the PMBLDC. The idea of this paper is that by maintaining the current at a constant value, the torque ripple could be minimized. The method for reducing the torque ripple used in this paper is to supply varied input voltage during the freewheeling region.

Wael A. Salah *et al.* (Salah et al., 2011) proposed a method which apply a modified PWM signal to the PMBLDC. The modified PWM signal used will delay the build up of current in the in-coming phase gradually at low speed region. At high speed region, it will speed up the build up of current which results in overcoming the tips and dips of current during phase current commutation which contributes to reduce in torque ripple.

Mohamed. A. Enany *et al.* (Enany et al., 2010) presented a method for improving the performance of BLDC with varying the switch-on and switch-off angle of the phase current. By advancing the switch-on and switch-off phase current, it enables current at each windings to reach to the maximum value earlier hence reducing the tips and dips of the current at the other winding which results in reducing torque ripple.

G. H. Jang and C. J. Lee (Jang and Lee, 2006) proposed a method to reduce the torque ripple through eliminating cogging torque by implementing a modified current wave form. The modified current wave form consists of main and auxiliary wave which the main wave is the conventional wave whereas the auxiliary wave generates a torque which has the same magnitude but opposite direction to the cogging torque.

Vanisri A. and Devarajan N. (A. and N., 2011) describe the design of a controller with minimize torque ripple which different than the conventional controller for PM-BLDC motor by filter components. The methodology of the method used in this research is passing through the signal through an inductor-capacitor filter which filters out the high frequency waveform. The capacitor is selected in a way that it can charge and discharge effectively and the inductor is responsible for reduce the current pulsation hence reducing the torque ripple.

Leila Parsa and Lei Hao (Parsa and Hao, 2008) studied the effect of magnetization, winding distribution, skew angle and current angle on torque pulsation minimization. The switching instance has been calculated in a way that the reluctance torque is utilized in reducing the torque pulsation. It is also shown in the paper that by using the proper switching interval and applying suitable current waveform, the torque pulsation is reduced.

2.3 Driving Strategy

Michiel Koot *et al.* (Koot et al., 2005) showed the work of analysing the engine, battery and alternator of an Hybrid Electric Vehicle (HEV). After the value of the parameters of an HEV is discretized, Dynamic Programming (DP) and Quadratic Programming (QP) techniques are used for the control strategy of the HEV.

Phil Howlett *et al.* (Howlett et al., 1997) discussed the making of an optimal strategy for a solar powered vehicle on a level road for participation in the World Solar Challenge. The draft strategy pointed out in the paper is:

- Accelerate using maximum available power.
- Hold speed at a lower critical speed, V, until mid-morning.
- Follow solar power up to an upper critical speed, W.
- Hold speed W until mid-afternoon.
- Follow solar power back down to the lower critical speed, V.
- Hold speed V until late in the afternoon.
- Follow solar power.
- Apply full regenerative braking, if available, or coast.
- Apply full regenerative braking and mechanical braking.

The mathematical model for the vehicle is built from the power flow from the solar panel to the traction system, from the battery to the traction system, from the solar panel to the battery and from the traction system to the battery. Vehicle dynamics, for instance, the rolling resistance and drag force were also modelled. Finally, the control strategy is made based on the mathematical model.

The paper written by Yasuo Shimizu *et al.* (Shimizu et al., 1998) introduced 2 system which is the Supervision Support System as shown in figure 2.1 and the Cruising Simulation as shown in figure 2.2. The Supervision Support System is a system where a control centre vehicle is followed behind the solar vehicle and collect all the data from the solar vehicle, the data is logged and calculated and the instruction is sent via transmitter back to the solar vehicle and displayed on the panel meters

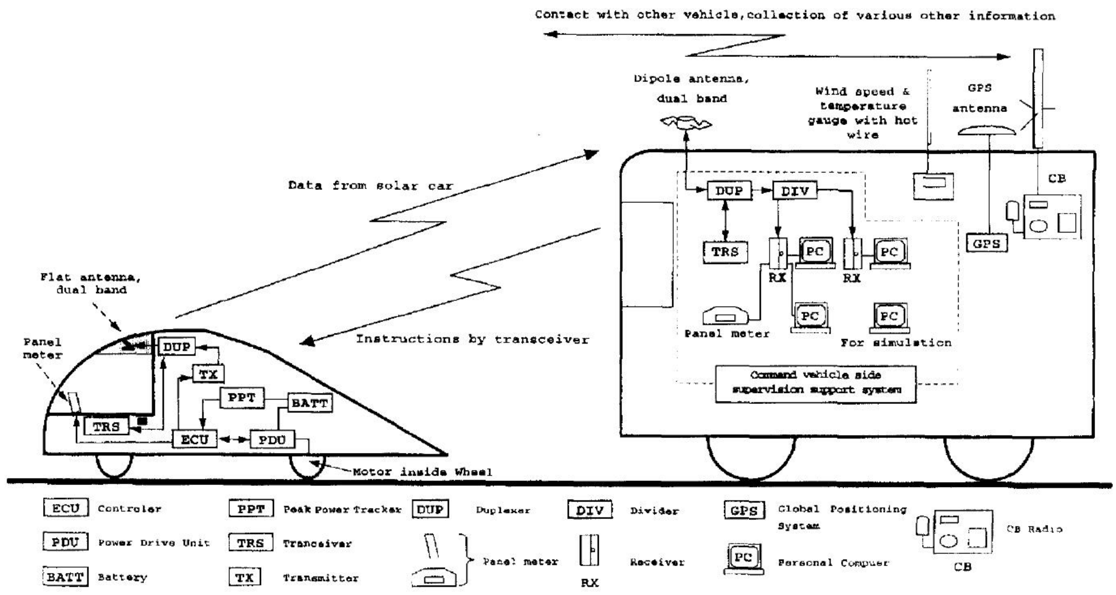

Figure 2.1: Supervision Support System hardware configuration schematic (Shimizu et al., 1998)

The Cruising Simulation on the other hand is a simulation proccess where the condition component is set, for example, the environment condition settings, the vehicle specification settings and etc. Next, the result is calculated through calculation of every single condition, for instance, the reading of geographical data, the calculation of motor power and etc. By utilising the Supervision Support System and the Cruising Simulation, the on-road calculation could be done so that the power or speed control strategy can be set and implemented when the solar vehicle is running.

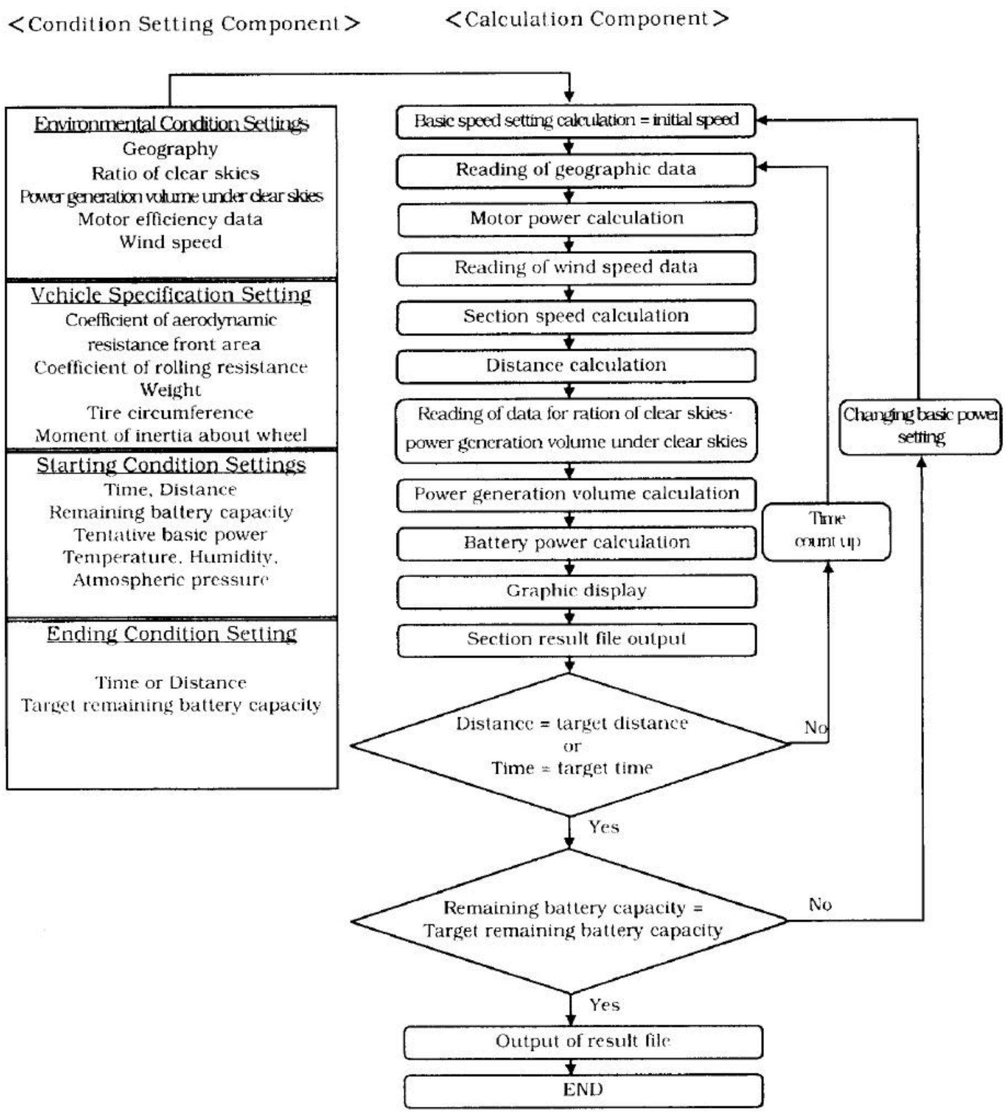

Figure 2.2: Flowchart of the Cruising Simulation (Shimizu et al., 1998)

Y.V. Bocharnikov *et al.* (Bocharnikov et al., 2007) researched on the optimal driving strategy for suburban railways. The strategy is constrained by two conditions which are energy saving and running fast so that it would not misses the schedule. There are 3 components for suburban railways which are the Motoring(M), Coasting(C) and Braking(B). By building mathematical model based on this 3 components and using fuzzy logic for optimization of energy function, a strategy with balance between the energy saving and running fast is achieved.

Peter Pudney and Phil Howlett Pudney and Howlett (1994) described building of strategies for a train journey which have speed limits at certain part of the journey. The method used in this paper is to build mathematical model for both the vehicle model and the journey model. Then, the speed profile was build and the optimal driving strategies was derived using mathematical models.

2.4 Summary

CHAPTER 3

METHODOLOGY

3.1 Introduction

In this chapter, the procedure for tapping the signal from the proprietary controller and PMBLDC motor and then building a performance measurement system based on the identified signal will be listed. Furthermore, the process of developing the mathematical model and the simulation of electric vehicle on Sepang North Track will be described so that the optimal mileage strategy could be identified.

The terminology of this project is to hack and get the signal from a proprietary contoller and a PMBLDC nub motor, then a performance measurement system is built upon the signal obtained and then simulation is used for building optimal mileage strategy.

3.2 Signal Identification

The drive train of the electric vehicle that will be used for participation in Shell Eco-Marathon consists of a single hub motor with no transmission. The hub motor is a PMBLDC motor, hence a controller is needed for controlling the speed as well as running the hub motor. The controller has two parts which is the controller circuit that is responsible for signal handling and a power unit which receives the signal from the signal handling unit and create the phase current for running the PMBLDC.

The signal unit of the controller takes input signal from the throttle, the key and also the hall effect sensors signal from the PMBLDC motor before it can do the calculation and output the signal to the power unit. Apart from that, the signal unit of the controller powers the speedometer and delivers the signal (for example: speed, battery voltage, vehicle mileage covered and the state of charge (SOC) of the battery) for displaying at the speedometer.

Since the speedometer acts solely as a display unit, it does not have signal outputs or data logging capability. Logging the data manually by reading and recording the speed, voltage and SOC when the vehicle is running is appropriate because the electric vehilce is a single seater vehicle and it's impossible for the driver to drive and record the data simultaneously.

Therefore, the signals need to be tapped from the signal unit of the controller in order to build a system that is able to measure, display and log the data needed. Tapping all the signals is redundant because some reading is primary (for example the voltage) whereas some readings are derived from the primary signal (for instance, the SOC of the battery is based on the voltage). Hence, identification of the primary signals should be adequate.

There are a few signal that need to be tapped from the controller and the PMBLDC motor such as:

- Speed from the controller
- Voltage from the controller

- Hall effect sensors signal from the PMBLDC motor

Before the signal can be tapped and identified, the nature of the signal should be known. The voltage signal is an analog signal which is delivered directly from the battery through the controller. On the other hand, the speed signal should be an analog signal as well since the controller takes the signal of hall effect sensors, calculate the interval between each state change and multiply with the number of poles and the wheel diameter and get the speed of the vehicle as shown in equation 3.1, where V is the wheel speed, D is the wheel diameter, N is the number of poles of the PMBLDC motor and t is the interval between changing state of the hall effect sensor.

$$V = \frac{\pi D}{2Nt} \tag{3.1}$$

Figure 3.1 shows the hall effect signals which is a square wave. The hall effect sensor is a sensor where the output voltage from the sensor will change from minimum to maximum or vise versa depending on the circuit configuration. Since the sensor circuit is set in a way that it ampliefies the signal and output high/low when the magnetic field of the rotor is detected as shown in figure 3.2, the signal output of the hall effect sensors is a square wave.

There are 16 ports for signal and power from the controller to the speedometer as shown in figure **....** The method for detecting the signal is trial and error and elimination. The ground port is found using the digital multimeter where the two probes of the digital multimeter is plug in to any 2 ports. The port where it is constantly lower voltage than

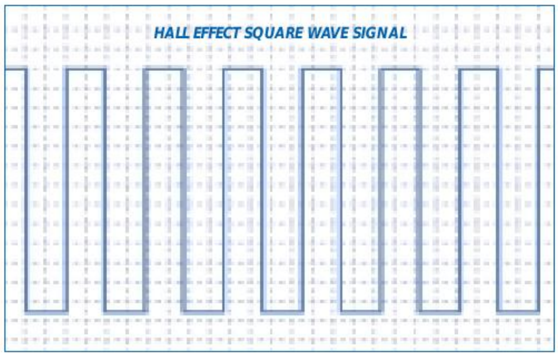

Figure 3.1: Hall effect sensor square wave signal (Point, 1999)

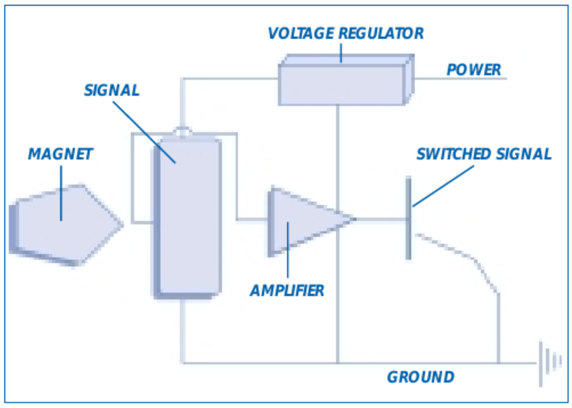

Figure 3.2: Circuit of hall effect sensor (Point, 1999)

the other 15 ports is the ground port.

After the ground port is found, the procedure above is repeated by testing the 15 ports by taking the ground port as reference. The voltage reading for those 15 ports is measured and recorded when the system is idle and when the motor is running. The ports for the following signal should be found:

- Speed where the reading changes when the motor is rotating and static
- Battery voltage which it is at 50+V when the motor is idle and drops 2V when the motor is running
- A constant voltage 12V for powering the speedometer

The hall effect sensor terminal is a 8 ports input/output as shown in figure ...... Again, the method for detecting the signal is trial and error with step by step elimination that is similar to the method used for identifying the speed signal at the controller circuit. The hall effect sensor circuit has the same circuit that is shown in figure 3.2 except that it has 3 sensors signal output instead of 1 output.

The hall effect sensor tapping is started with the identification of the voltage input to the hall effect sensors circuit. The voltage input to the circuit is 12V, hence, using the same method as speed signal detection, the digital multimeter's probe is plug into the any 2 ports of the 8 I/O ports within the terminal and detect the port with 12V reading.

After the voltage input ports is identified, the 3 hall effect sensors output signal is tapped by again plugging in the ground probe to the ground input port and the other positive probe to any 1 of the 6 remaining ports. The wheel is rotated manually over a revolution and the ports that produce ups and downs voltage signal would be the hall effect sensors output port.

After the hall effect sensors output port has been identified, the probes of the multimeter is plugged into the first hall effect sensor output port and the ground port. The wheel is rotated by hand over a precise 1 revolution and the number of square wave over a wheel revolution is measured and the signal output at each 10 °is measured and recorded. The same procedures apply to the other 2 hall effect sensors output ports. A graph of hall effect sensors signals versus the angle is then plot.

3.3 Measurement System

After the primary signal (speed and voltage) is identified, a measurement that has the ability to measure, display and log the parameters need to be built. The purpose of building the measurement system is to

- Measure and log the power consumption of the electric vehicle so that the mileage of the vehicle can be known.
- Measure and log the power consumtion at each vehicle on-road speed.
- Building a speed and power consumption display for the electric vehicle.

For measuring the power input, based on equation 3.2, the input voltage and input current need to be measured. Apart from measuring the power input to the electric vehicle, the data output from the measurement system can also be used to calculate the instantaneous deceleration of the vehicle when it's cruising, when it's braking with regenerative braking and when it's braking with regenerative and mechanical braking.

$$P_{input} = V_{input} I_{input} \tag{3.2}$$

The measurement system uses a circuit with microcontroller, crystal and I/O pins, which is the Arduino Mega 2560 board for processing the signal from the controller circuit, calculate the real value based on the signal, log the data into a SD card through the ITDB02 shield and display the speed and power consumption value through the ITDB02-3.2(WC) LCD screen.

Since the microcontroller board can only accept I/O less than 5V and more than 0V but the battery voltage is in the range of 48V - 60V and the speed signal from the controller is negative voltage, hence an intermediate circuit has to be built so that the signal can be read by the microcontroller.

For the voltage measurement circuit, a voltage divider is built for measuring the voltage of the battery. The voltage divider is a 2 resistor circuit that has same or different value of resistance for each of the resistor as shown in figure 3.3 The voltage output, V_{out} can be calculated using equation 3.3 where *R1* is the resistance for the first resistor, *R2* is the resistance of the second resistor and V_{in} is the voltage that need to be measured.

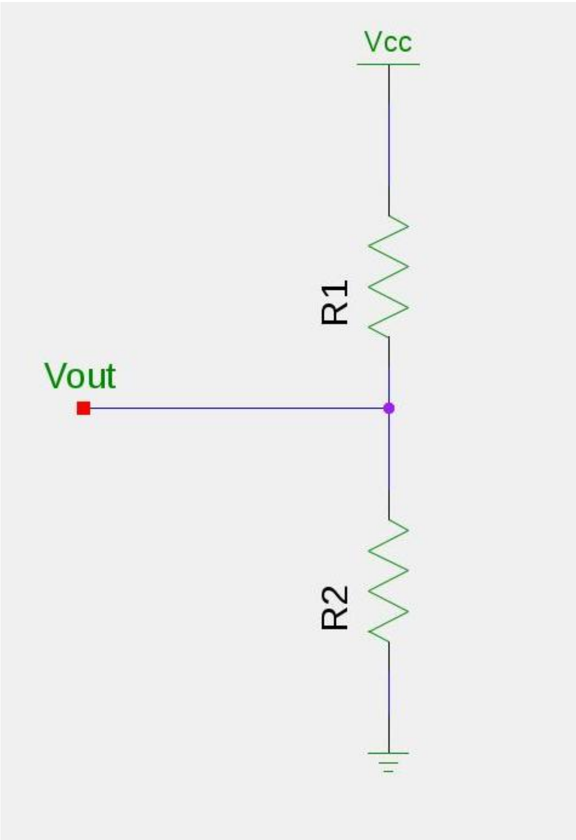

Figure 3.3: Voltage Divider

$$V_{out} = \frac{V_{in}R2}{R1+R2} \tag{3.3}$$

For measuring the speed, the same method which is the voltage divider is used with a little modification which replaces the V_{in} to +5V and the ground to V_{in} as shown in figure 3.4 The V_{out} for the modified voltage divider circuit could be calculated using equation 3.4

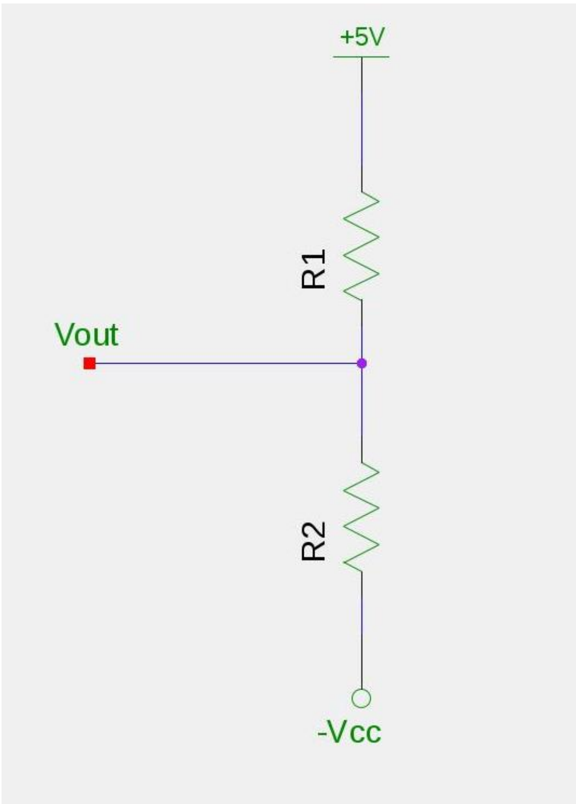

Figure 3.4: Voltage Divider for negative voltage

$$V_{out} = \frac{(5-V_{in})(R2)}{R1+R2} + V_{in} \tag{3.4}$$

The resistance is selected based on the value of the negative voltage. For example, if the negative voltage is -5V, 1kΩ resistor can be selected for both *R1* and *R2* which yields:

$$V_{out} = \frac{(5+5)(1000)}{1000+1000} - 5$$

$$V_{out} = 0V$$

If the V_{in} is 0V, V_{out} will be:

$$V_{out} = \frac{(5+0)(1000)}{1000+1000} - 0$$

$$V_{out} = 2.5V$$

Therefore, using a voltage divider, a negative voltage can be converted to positive voltage with minimum half of the original resolution.

The third parameter that need to be included in the measurement system is the input current. The input current is needed for calculating the input power to the electric vehicle and thus the overall vehicle mileage can be calculated. Since the rules of Shell Eco-Marathon restricted the nominal current input to the electric vehilce to be 60A and the peak current must be less than 150A, therefore a 200A current sensor is used.

The current sensor used in detecting the input current is the ALLEGRO MICROSYSTEMS ACS758ECB-200B-PFF-T which is a 200A current sensor with 5 terminals and analog voltage output signal as shown in figure **....** In order for the microcontroller to measure the current, a current measuring circuit with the current sensor is built. The schematic of the current sensor circuit is shown in figure **....**

After the prototype circuit of the current measuring circuit, the voltage measurement circuit and the speed voltage divider circuit is built, the 3 circuits is integrated into a single circuit and the microcontroller is program in a way that it can read, log and display the speed of the vehicle, the instantaneous current and voltage and the mileage covered every 100ms interval.

3.4 Vehicle Simulation

CHAPTER 4

RESULT AND DISCUSSION

CHAPTER 5

CONCLUSION

REFERENCES

A., V. and N., D. (2011). Torque Ripple Minimization in Indirect Position Detection of Permanent Magnet Brushless DC Motor. *European Journal of Scientific Research*, 65(4):481–489.

Bocharnikov, Y., Tobias, A., Roberts, C., Hillmansen, S., and Goodman, C. (2007). Optimal Driving Strategy for Traction Energy Saving on DC Suburban Railways. *IET Electr. Power Appl.*, 19(5):675–682.

Enany, M. A., Elshewy, H. M., and Abdel-kader, F. E. (2010). Brushless DC Motor Performance Improvement through Switch-on and Switch-off Angles Control. *Proceedings of the 14th International Middle East Power System Conference (MEPCON'10)*, pages 786–790.

Howlett, P., Pudney, P., Tarnopolskaya, T., and Gates, D. (1997). Optimal Driving Strategy for a Solar Car on a Level Road. *IMA Journal of Mathematics Applied in Business & Industry*, pages 59–81.

Jang, G. H. and Lee, C. J. (2006). Design and Control of the Phase Current of a Brushless DC Motor to Eliminate Cogging Torque. *Journal of Applied Physics*, page 3.

Kim, T.-S., Ahn, S.-C., and Hyun, D.-S. (2001). A New Current Control Algorithm for Torque Ripple Reduction of BLDC Motors. *IECON'01: The 27th Annual Conference of the IEEE Industrial Electronics Society*, pages 521–526.

Koot, M., Kessels, J. T. B. A., de Jager, B., Heemels, W. P. M. H., van den Bosch, P. P. J., and Steinbuch, M. (2005). Energy Management Strategies for Vehicular Electric Power Systems. *IEEE Transactions on Vehicular Technology*, 54(3):771–782.

Nam, K.-Y., Lee, W.-T., Lee, C.-M., and Hong, J.-P. (2006). Reducing Torque Ripple of Brushless DC Motor by Varying Input Voltage. *IEEE Transactions on Magnetics*, 42(4):1307–1310.

Park, S. J., Park, H. W., et al. (2000). A New Approach for Minimum-Torque-Ripple Maximum-Efficiency Control of BLDC Motor. *IEEE Transactions on Industrial Electronics*, 47(1):109–114.

Parsa, L. and Hao, L. (2008). Interior Permanent Magenet Motors With Reduced Torque Pulsation. *IEEE Transactions on Industrial Electronics*, 55(2):602–609.

Point, W. C. (1999). Understanding Hall Effect Sensors. `http://www.wellsve.com/sft503/Counterpoint3_1.pdf`.

Pudney, P. and Howlett, P. (1994). Optimal Driving Strategies for a Train Journey with Speed Limits. *J. Austral. Math. Soc. Ser. B*, 36:38–49.

Salah, W. A., Ishak, D., and Hammadi, K. J. (2011). PWM Switching Strategy For Torque Ripple Minimization in BLDC Motor. *Journal of Electrical Engineering*, 62(3):141–146.

Shimizu, Y., Komatsu, Y., Torii, M., and Takamuro, M. (1998). Solar Car Cruising Strategy and Its Supporting System. *JSAE Review*, 19:143–149.

Yedamale, P. (2003). *Brushless DC (BLDC) Motor Fundamentals*. Microchip Technology Inc.

APPENDICES

APPENDIX A

SCHEMATIC