CNC06A Course Project Review: Throughput Analysis of Wireless Relay Slotted ALOHA Systems with Network Coding (NC)

Phetxomphou Zalita
CCL Lab, The University of Aizu
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Outlines

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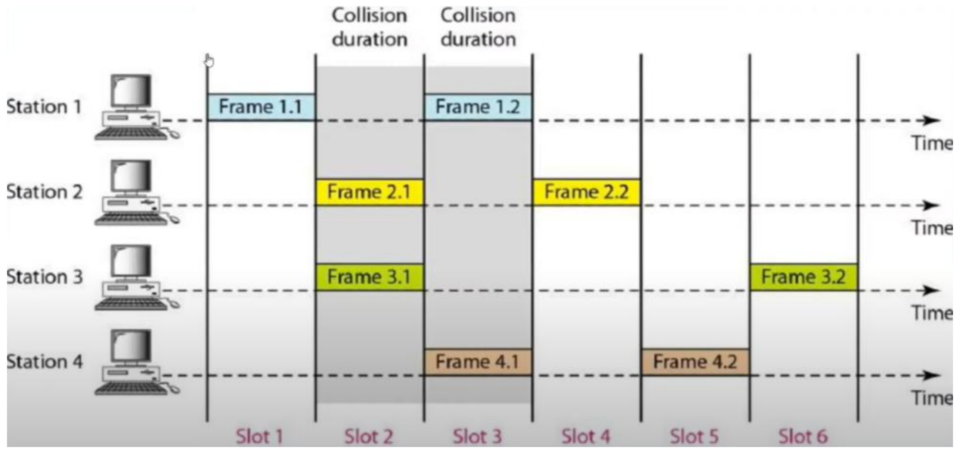
Introduction

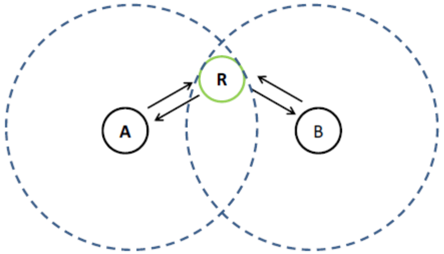
What is Slotted Aloha?

- A time-slotted random access protocol where nodes transmit at the beginning of any time slot.
- If two or more nodes transmit in the same slot → Collision occurs → Packets are lost.
- Application such as: Satellite Communication, RFID & Wireless Sensor Networks.



What is Relay Network?

- Intermediate node (relay) assists in forwarding data from the source to the destination.
- Two-hop relay network model used in this study, involving two end nodes (A and B) and one relay node (R).



Network Coding Improves Performance

- Non-NC Limitation: The relay forwards packets separately, leading to inefficiency.
- NC Advantage: Relay combines packets using XOR, reducing redundancy & improving throughput. The key idea of network coding is that (a XOR b) XOR b = a.

Network Model

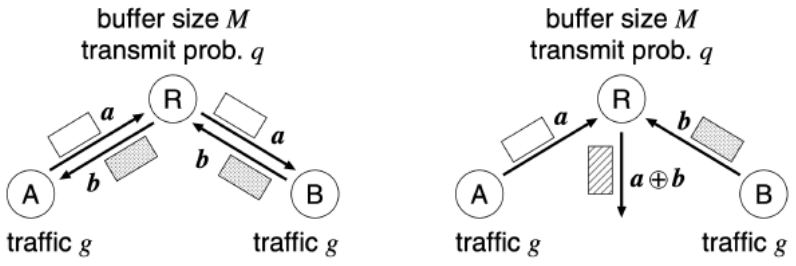
Node A & Node B:
- End nodes that need to communicate (Half duplex).
- Out of range of each other, so they must rely on R.

Relay Node R:
- Receives & forwards packets between A & B.
- Stores packets temporarily in a buffer before transmission.
- If R has packets in buffer, send packet in slot with prob. q

Buffer Size (M):
- Infinite buffer (if need) storage capacity at R
- Operates on FIFO (First-In-First-Out) basis.

Transmission Probability (g):
- End nodes transmit with a probability of 1 in normal mode
- $g = g_a + g_r$ where g_a is prob. of new transmissions; g_r for retransmissions.

Offered Traffic (G):
- Average number of packets transmitted per time slot (G=2g)



(a) non-NC system (b) NC system

The process without Network Coding:

1. A sends packet “a" → Relay receives “a".
2. B sends packet “b" → Relay receives “b".
3. Relay forwards “a" to B.
4. Relay forwards “b" to A.

Network Coding Process:

If both buffers are non-empty, instead of forwarding each packet separately, the relay combines them using XOR.

Step 1:
- A sends packet “a".
- B sends packet “b".
- Relay receives both “a" and “b".

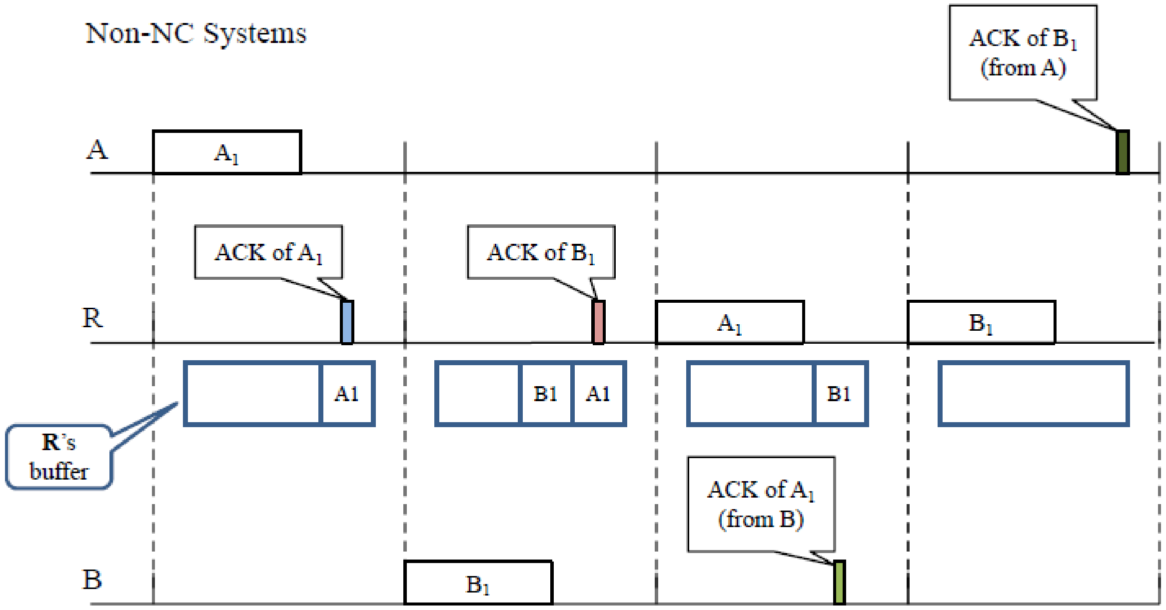
Step 2 (XOR Encoding at Relay):
- Relay XORs the packets: a ⊕ b
- The relay sends this single coded packet (a ⊕ b) to both A & B.

Step 3 (Decoding at End Nodes):
- A already knows its own packet (a).
- A receives (a ⊕ b) from the relay.
- A can recover B by computing: (a ⊕ b) ⊕ a = b
- B does the same.

Network Model (con.)

Non-NC System:

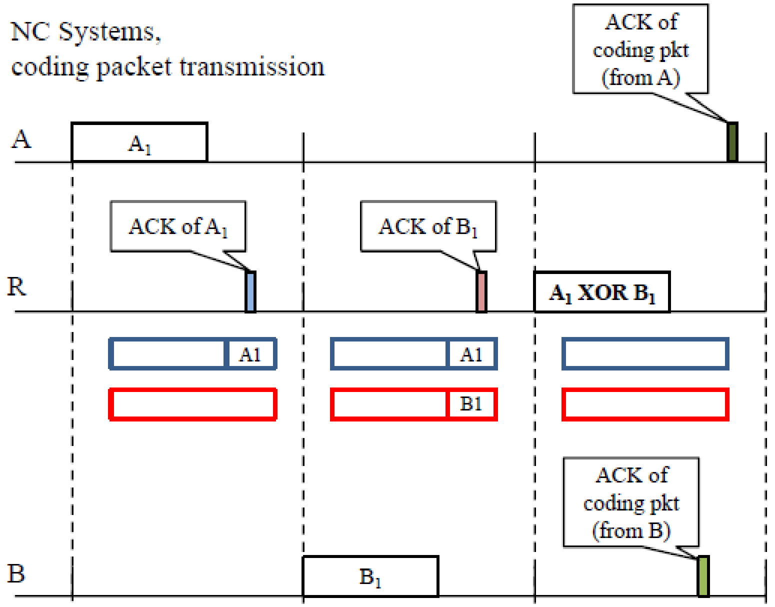
- Relay forwards packets without coding
- Use slotted ALOHA for medium access: receiver broadcasts ACK (very short length) for each correctly received packet
- If the buffer is non-empty, R sends the packet at its physical buffer's head



Network Model (con.)

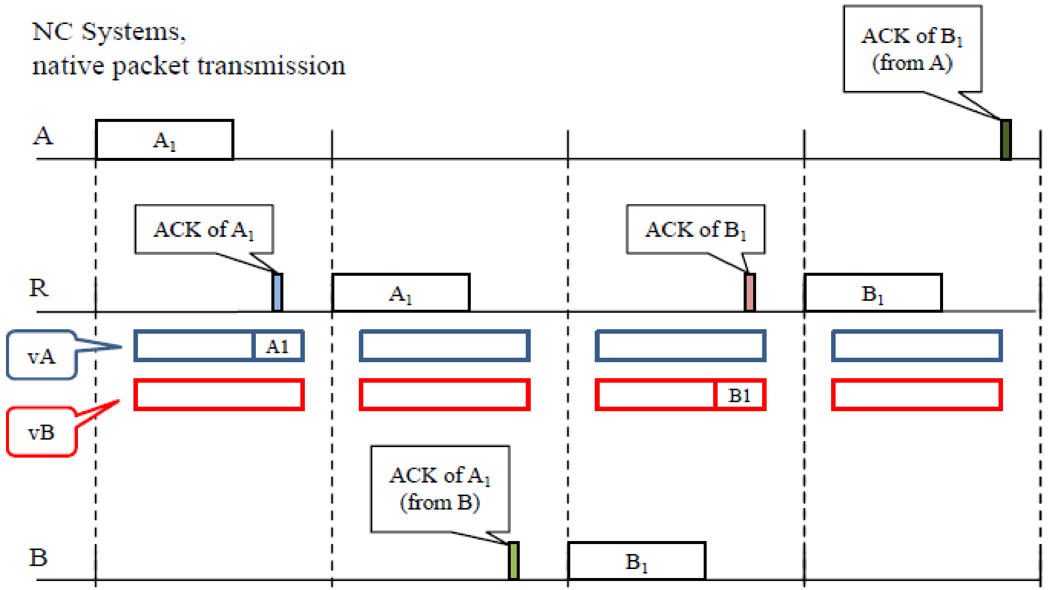
NC System:

- R maintains two virtual buffers: vA → Packets from A, and vB → Packets from B.
- If both buffers are non-empty → R XOR packets at the head of vA and vB
- If only one buffer is non-empty → R sends a native packet.



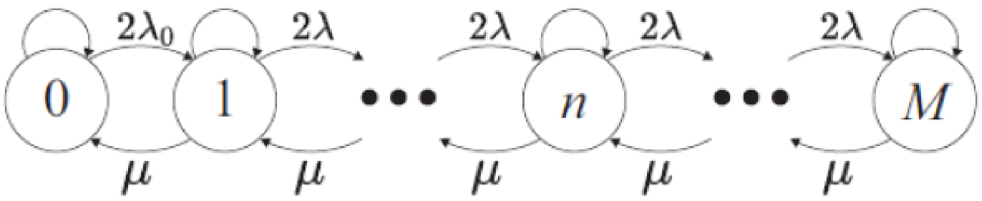
How does A and B decodes the coding packet "A_1 XOR B_1"?

- <u>A needs to get B_1</u>: use A_1 XOR "A_1 XOR B_1" = get B_1
- <u>B needs to get A_1</u>: use B_1 XOR "A_1 XOR B_1" = get A_1



Throughput Analysis: Slotted ALOHA non-NC

Approach: Use Discrete-Time Markov Chain (DTMC) Analysis



State (i)	Meaning
i = 0	No packets in relay buffer (empty)
i = 1	One packet waiting to be forwarded
i = n	n packets in the buffer
i = M	Maximum buffer size (if finite)

- The relay buffer stores packets before transmission.
- ***State*** $\boldsymbol{i}$ represents the number of packets in the relay's buffer.
- The buffer changes based on:

  New packet arrivals → buffer increases;

  Successful transmissions → buffer decreases

Action (Buffer Changes)	Transition	Probability Formula
New packet arrives (when buffer is empty)	$0 \rightarrow 1$	$P(0,1) = 2g(1-g)$
New packet arrives (when buffer is not empty)	$i \rightarrow i+1$	$P(i, i+1) = (1-q)\, 2g(1-g)$
Relay successfully transmits	$i \rightarrow i-1$	$P(i, i-1) = q(1-g)$

- Throughput S = average number of successfully delivered packets from **R** to **A & B** per slot time.
- From steady-state Markov analysis, the probability that the buffer is empty (no packets to send) is:

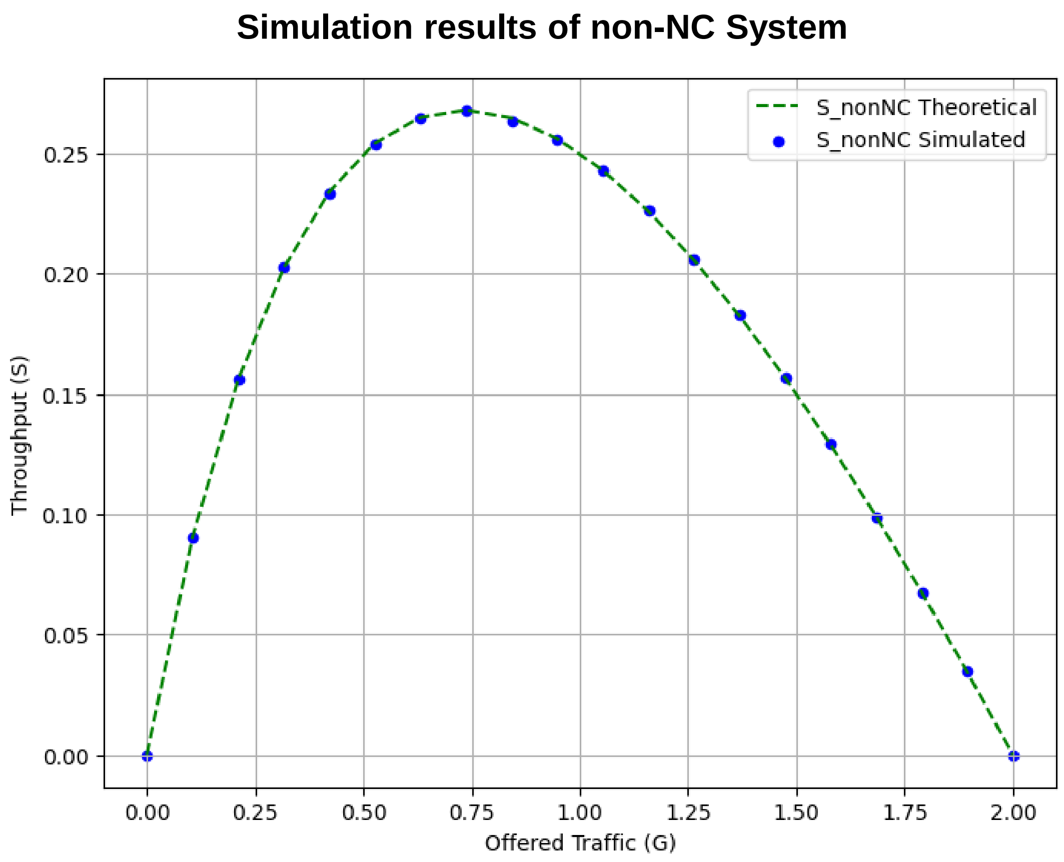
$$P(0) = \frac{1}{1+G}$$

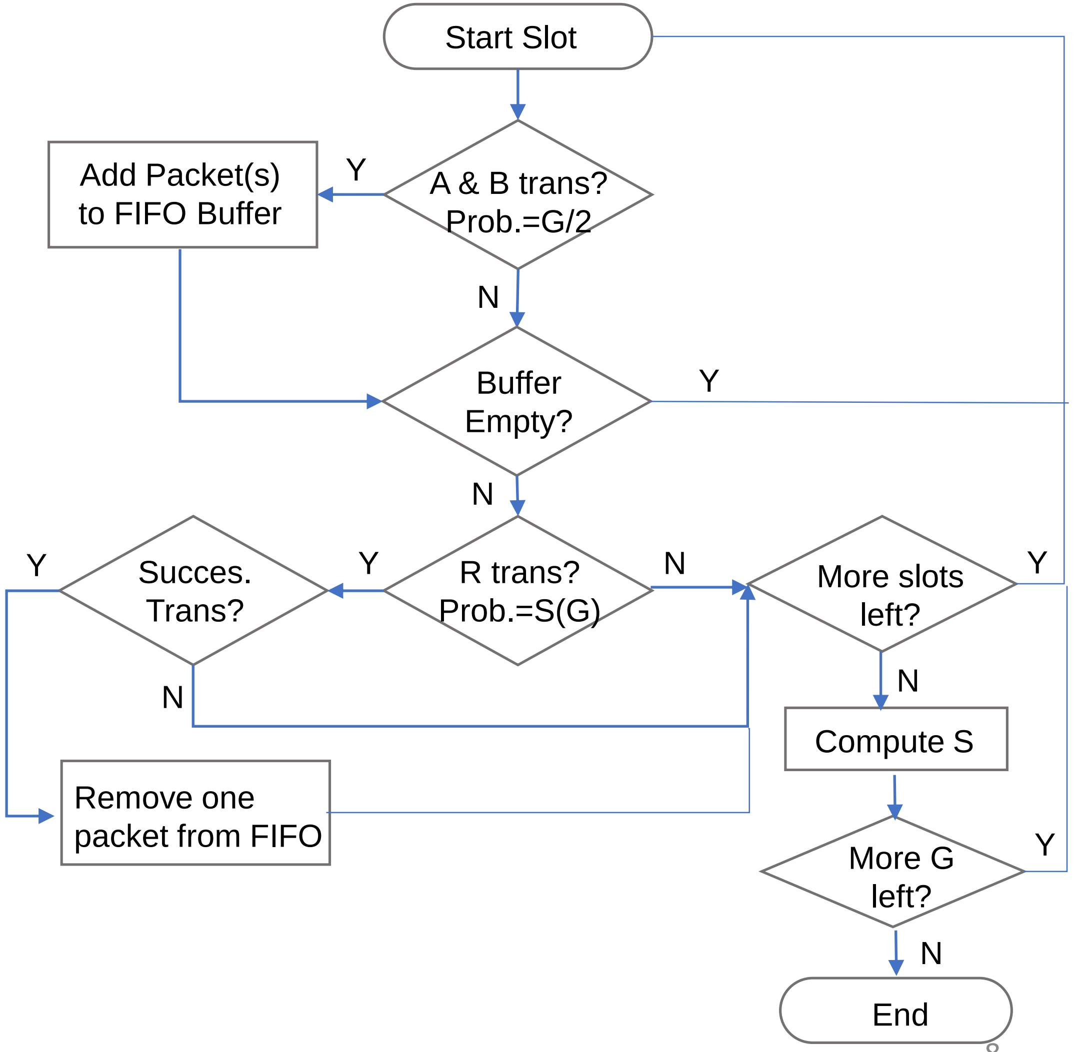
- Substituting into the throughput equation: $S = 1 * (\mu)[1 - P(0)] = (\mu) \times \left[1 - \frac{1}{1+G}\right] = \left(1 - \frac{G}{2}\right) \times \left[\frac{G}{1+G}\right] = \boxed{\frac{G(1-G/2)}{1+G}}$, where $G = 2g$

- Stabilization Condition: $2\lambda < \mu$ (to prevent buffer overflow)

Does not depends on q (the relay forwards packets one by one)

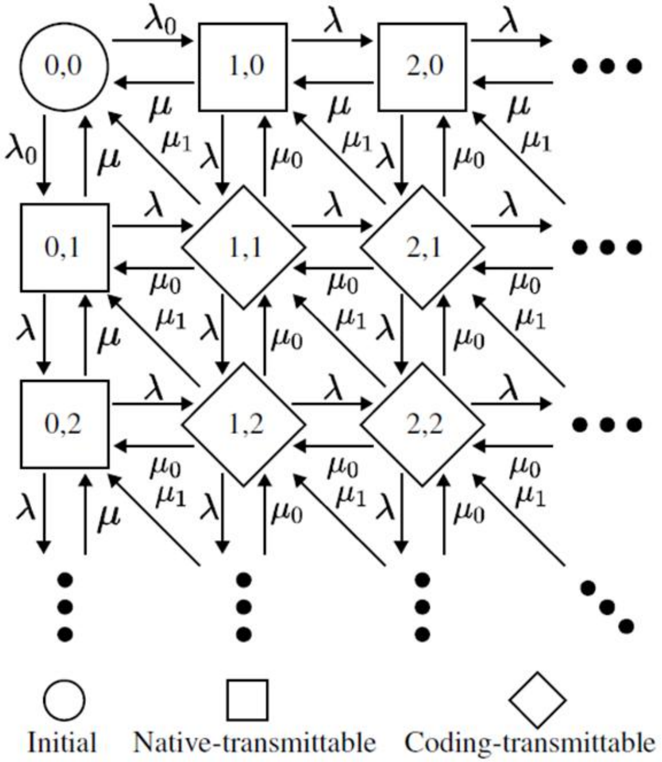
Throughput Analysis: Slotted ALOHA non-NC (con.)





Throughput Analysis: Slotted ALOHA NC

To analyze throughput, we model the number of packets in R's buffer using DTMC to find throughput *S*.



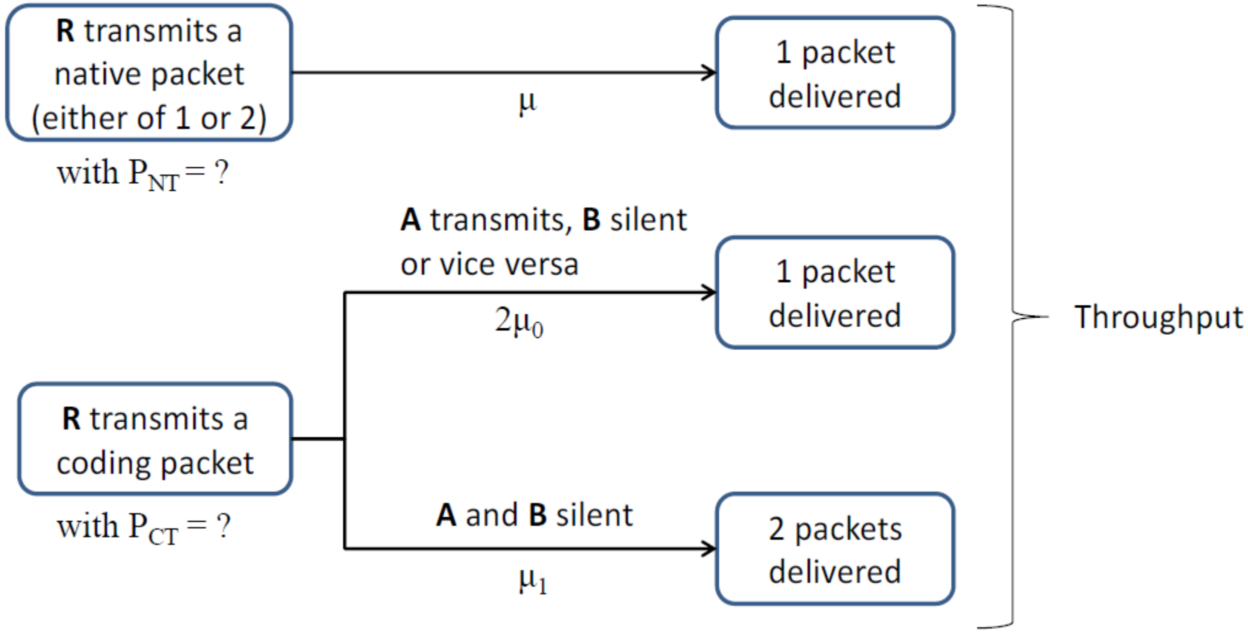
- In NC-Aloha, the relay maintains two **virtual buffers** for packets from A and B.
- The relay transmits packets using two mechanisms:
  - ➢ Native Transmission: When only one buffer is non-empty.
  - ➢ Coded Transmission: When both buffers are non-empty (XOR operation).
- (Ak, Bk) represents the number of packets in virtual buffers vA and vB in time slot k

Transition probabilities: λ_0, λ, μ same as non-NC

- $\mu_0 = qg(1-g)$: Relay transmits a coded packet and only one node is silent.
- $\mu_1 = q(1-g)^2$: Relay transmits a coded packet and both nodes are silent.
- Total transmission probability: $\mu = \mu_0 + \mu_1$

Throughput Analysis: Slotted ALOHA NC (con.)

How to calculate throughput?



$$S = 1 * \mu P_{NT} + (2\mu_0 + \mu_1 * 2)P_{CT} = \boxed{\mu P_{NT} + 2\mu P_{CT}}$$

$$\begin{cases} P_{NT} = P_A(0) + P_B(0) - 2P(0,0) \\ P_{CT} = 1 - P(0,0) - P_{NT} \end{cases}$$

To compute throughput, we need the prob. that the relay has packets to transmit $\boldsymbol{P(n,m)}$, which is complex to get exact formula for all states, involving $\boldsymbol{P(0,0)}$.

Instead of solving for all $P(n,m)$, we estimate:

$$P_A(0) = \sum_m P(0,m), \qquad P_B(0) = \sum_n P(n,0)$$

This approximation treats virtual buffers independently, but in reality, they influence each other. Errors accumulate at high G.

Using the approximation, the derived throughput expression is:

$$S = \frac{2qG(2-G)}{q(2+G)^2 - G}, \quad \text{where } G = 2g$$

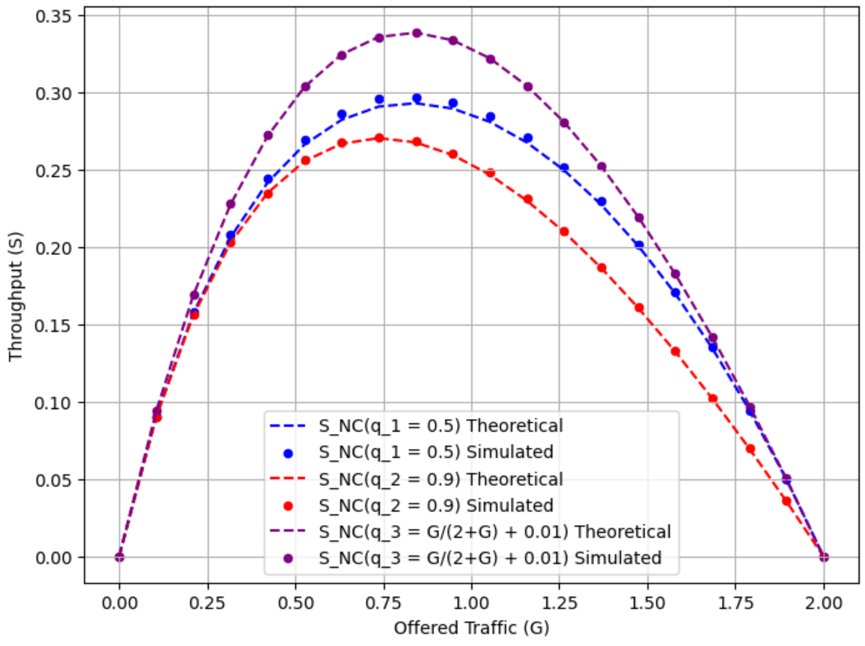
Stabilization Condition (Prevents Buffer Overflow):

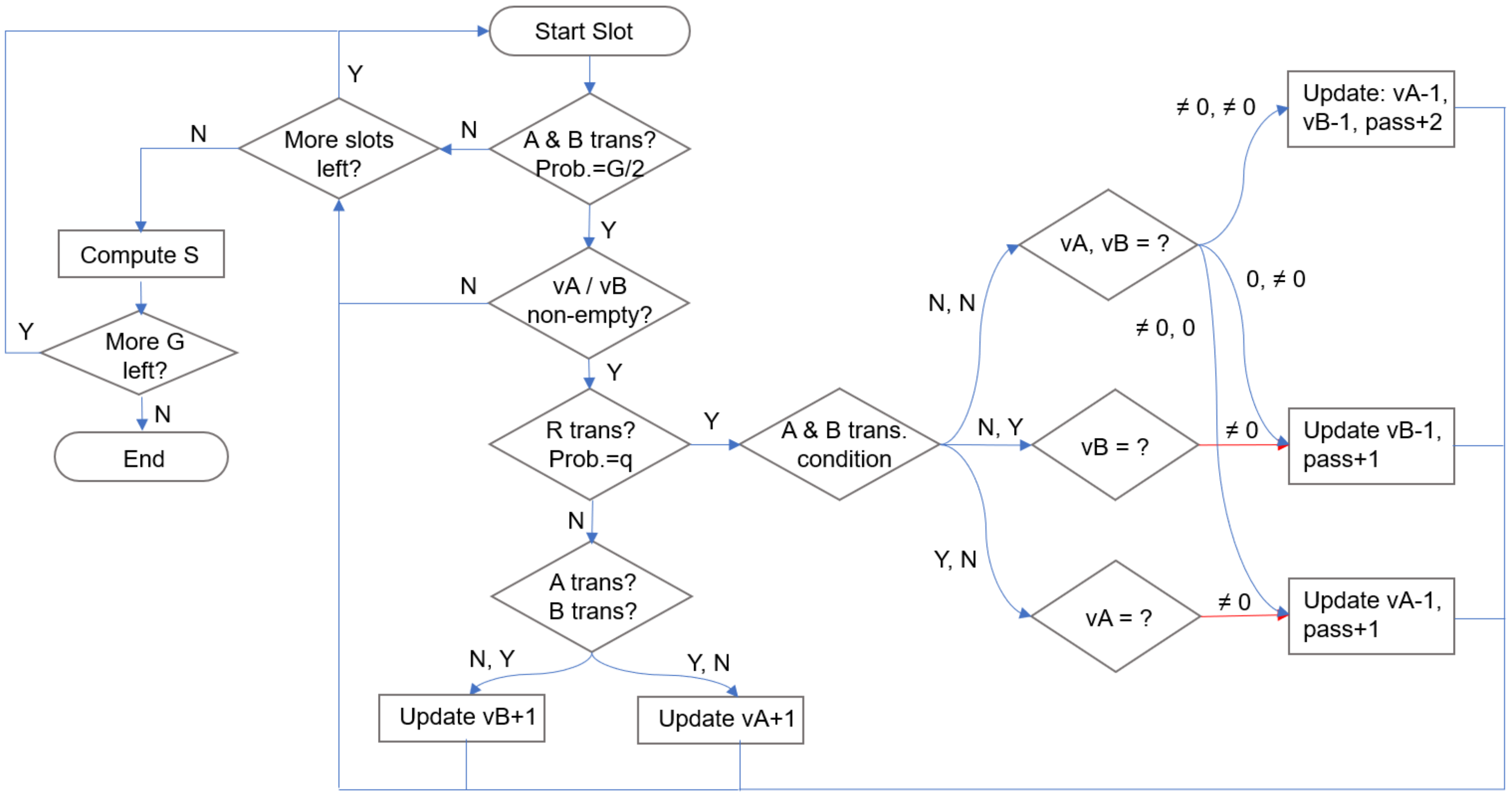
$$\lambda < \mu \quad \text{i.e.,} \quad q = \frac{G}{2+G}$$

Relay transmission probability for maximum throughput

Throughput Analysis: Slotted ALOHA NC (con.)

Simulation results of NC System





Note: (Y, Y) case in “A & B trans. Condition” is omitted as the relay does not transmit when both A & B send packets; it only updates buffers.

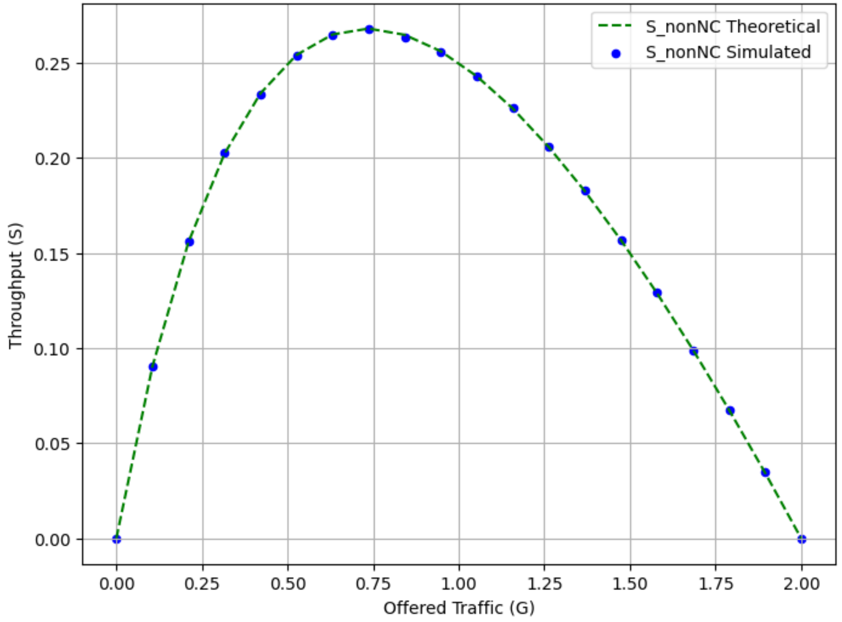
Conclusion

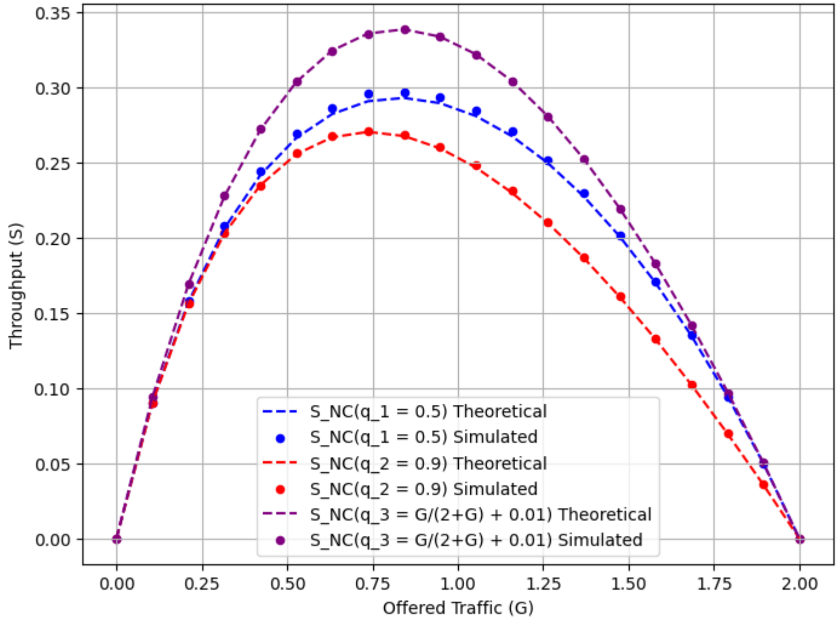
1. Non-NC: Simple but Less Efficient

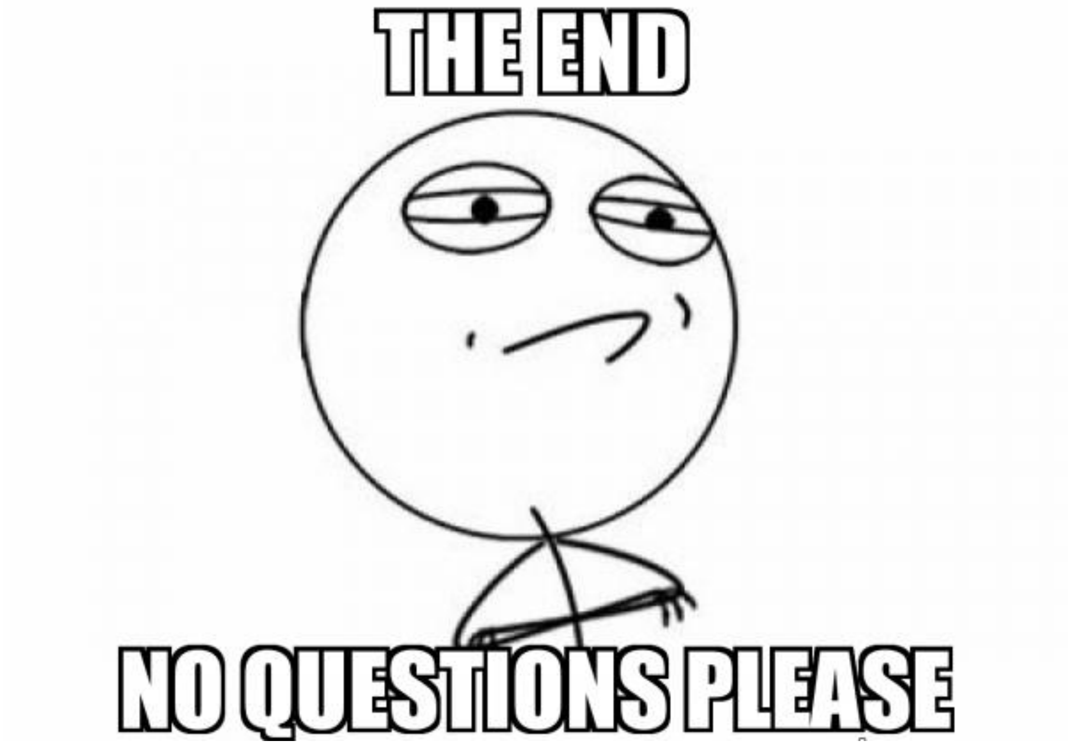
- The relay **forwards packets one by one**, leading to **limited throughput**.
- **Does not depend on relay probability q** since it forwards packets independently.

2. NC: Higher Throughput via Coded Transmission

- **Relay transmits XOR-coded packets**, reducing required transmissions.
- Two-dimensional DTMC needed to model **virtual buffers vA, vB**.
- Approximation of $P(0,0)$ **introduces errors at high G.**







References paper: D. Umehara, T. Hirano, S. Denno, and M. Morikura, “Throughput analysis of wireless relay slotted aloha systems with network coding,” in IEEE GLOBECOM 2008 - 2008 IEEE Global Telecommunications Conference, Nov 2008, pp. 1–5.