

# Magic Circles with Embedded Magic Squares

## 1 Overview

This note explores a fascinating mathematical pattern that combines magic squares and magic circles, which can be displayed on a Rubik's cube.

A magic square is a grid of numbers where every row, column, and diagonal adds up to the same number. Magic circles are arrangements where numbers placed in circular patterns add up to the same value.

We'll look at how to place the numbers 1 through 54 on a Rubik's cube so that:

- Each face forms a magic square
- The numbers also form magic circles when viewed from a different perspective
- All of these patterns maintain a special sum of 330 across specific combinations

This mathematical structure was originally created by Harry A. Sayles in 1910.

## 2 Key Properties

### 2.1 Face Progression

Each face of the Rubik's cube contains a  $3 \times 3$  magic square. The magic sum increases by 27 as we move from face to face:

- Up Face: Base magic square with sum 15
- Right Face: Up + 27 (sum = 42)
- Front Face: Right + 27 (sum = 69)
- Back Face: Front + 27 (sum = 96)
- Left Face: Back + 27 (sum = 123)
- Down Face: Left + 27 (sum = 150)

### 2.2 Magic Square Structure

Each face forms a  $3 \times 3$  magic square where:

- Every row adds up to the magic sum for that face
- Every column adds up to the magic sum for that face
- Both main diagonals add up to the magic sum for that face

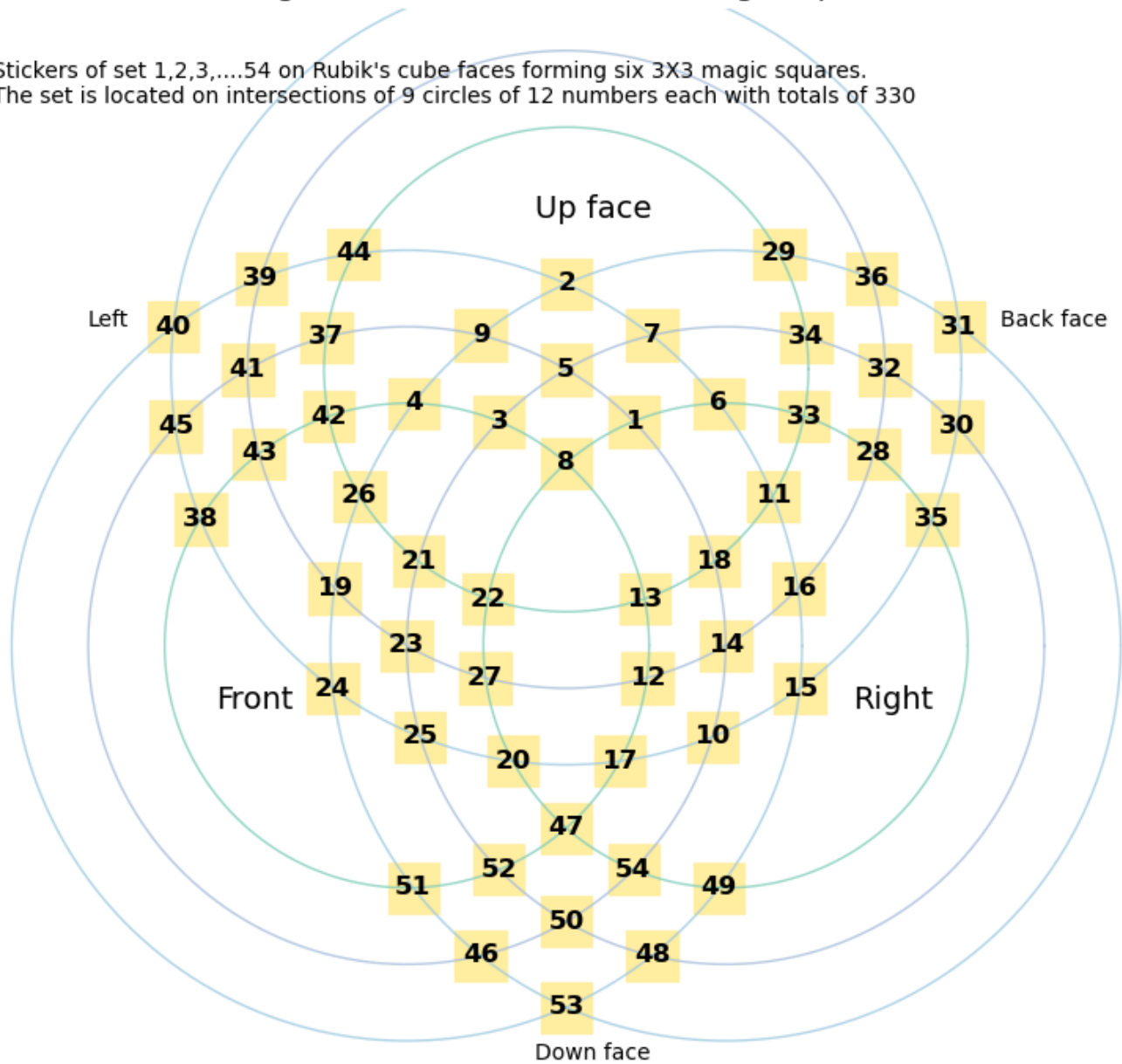
## 3 Key Properties

For example, the Up face has rows:

- Row 1:  $2 + 7 + 6 = 15$
- Row 2:  $9 + 5 + 1 = 15$
- Row 3:  $4 + 3 + 8 = 15$

## Magic circles with embedded magic squares

Stickers of set 1,2,3,...,54 on Rubik's cube faces forming six 3X3 magic squares.  
The set is located on intersections of 9 circles of 12 numbers each with totals of 330



# MAGIC SQUARES

Up  
Center: 5

2	7	6
9	5	1
4	3	8

Left  
Center: 41

44	37	42
39	41	43
40	45	38

Front  
Center: 23

26	21	22
19	23	27
24	25	20

Right  
Center: 14

13	18	11
12	14	16
17	10	15

Back  
Center: 32

33	34	29
28	32	36
35	30	31

Down  
Center: 50

51	52	47
46	50	54
53	48	49

magic circle sum = 330

### 3.1 Magic Circles

When looking at the flattened cube, we can see three concentric circles:

- The Outer Circle
- The Middle Circle: Contains the center element of each face's magic square
- The Inner Circle

Each circle connects specific positions across different faces of the cube.

### 3.2 The Magic Sum of 330

It will be noticed that all circle combinations create the same magic sum of 330:

- Left/Up/Right/Down: 330 (for each circle position)
- Left/Back/Right/Front: 330 (for each circle position)
- Up/Back/Down/Front: 330 (for each circle position)

This consistent sum of 330 creates a beautiful mathematical harmony across the entire structure.

## 4 Mathematical Properties

### 4.1 Sum Patterns

1. Total Sum: All 54 numbers (1-54) sum to 1485
2. Face Sum Progression: On each face, the magic sum equals three times the median number in that face's magic square.
  - Up: 15 ( $3 \times 5$ )
  - Right: 42 ( $3 \times 14$ )
  - Front: 69 ( $3 \times 23$ )
  - Back: 96 ( $3 \times 32$ )
  - Left: 123 ( $3 \times 41$ )
  - Down: 150 ( $3 \times 50$ )
3. Special Properties:
  - The sum of the four corner numbers on any face equals four times the median
  - The sum of two numbers on opposite sides of the center equals twice the median

## 5 How This Works on a Rubik's Cube

To apply this pattern to a Rubik's cube:

1. Place the 6 magic squares on the 6 faces of the cube
2. Put the numbers as stickers according to the pattern
3. When the cube is solved, each face shows a perfect  $3 \times 3$  magic square
4. The magic circles maintain their sum of 330 even during rotations

This creates a mathematically elegant arrangement where both the magic square properties and the magic circle properties work together perfectly.

## 6 Why This is Interesting

This structure is fascinating because:

- It combines two different mathematical patterns (magic squares and magic circles)
- It works perfectly with the geometry of a Rubik's cube
- It maintains the same magic sum of 330 across all circle combinations
- It was discovered over 100 years ago, showing the timelessness of mathematical patterns

## 7 Acknowledgement

The original creator of this mathematical structure was Harry A. Sayles, who published it in "The Monist" journal in 1910.

## 8 References

Sayles, Harry A. "MAGIC CIRCLES AND SPHERES." The Monist 20, no. 3 (1910): 454–72.  
<http://www.jstor.org/stable/27900270>.

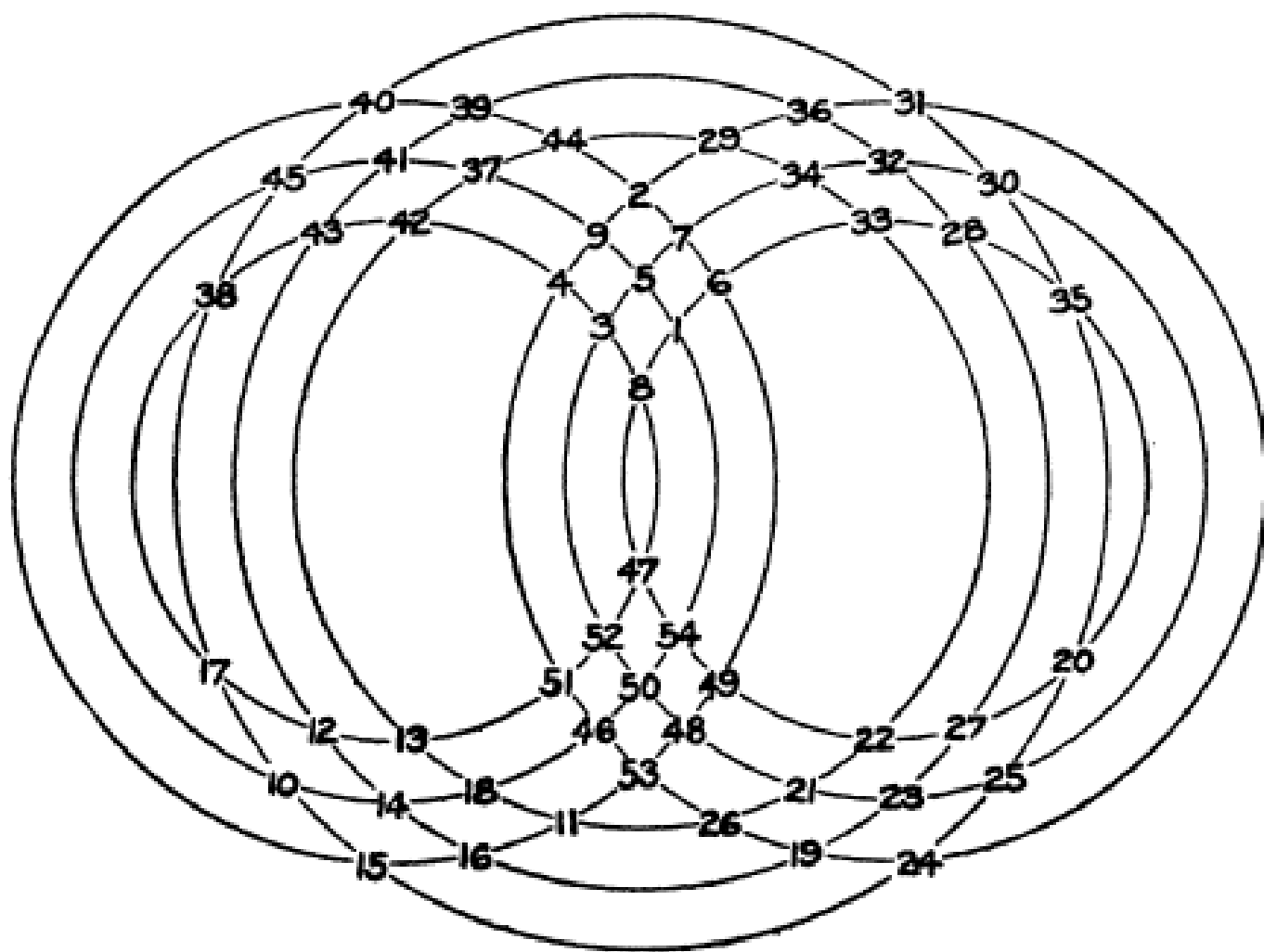


Fig. 20,

H.A. Sayles