Homework: Auctions

Pablo Maldonado

Instructions   Solve the following homework problems. You can ask me any questions by email.

Problem 1: First price, sealed bid auctions.   Two bidders have valuations a, b respectively. a, b are uniformly distributed real numbers in $[0, 1]$. They participate in a first-price sealed bid auction: they simultaneously write down their bids and submit them to the seller, who announces the winner (in this case, the highest bidder). Ties are broken down by flipping a coin. The bidders understand the rules of the game, but have no information on each others' valuations.

- If the winner of the auction pays its bid price, then truth-telling is *not* a dominant strategy. Why is this the case?

- If the winner of the auction pays half of its bid price, then truth-telling *is* a dominant strategy. This is the truth telling mechanism that should exist, thanks to the revelation principle. Justify this.

Problem 2: Modified English auction   Assume there is an auction mechanism such that works like an English auction, except that the winner of the auction, that is, the bidder who bid b_{max}, pays $b_{max}/2$. How does this compare to an English auction in terms of revenue for the seller?

Problem 3: Team collusion in hockey   Historically, the teams in the National Hockey League received two points for winning a game and 0 for losing. If the game is tied, an overtime period is played and if nobody wins, each team would get 1 point. At some point, league officials felt that teams played too conservative in overtime to avoid a loss. They decided to

experiment in 1999 with mechanism design, giving 1 point to a team that loses in overtime, instead of 0. The winning time would still get 2 points.

Suppose team A has probability r of winning in regular time, and probability p of winning in overtime, whereas team B has probability s of winning in regular time and probability q of winning in overtime.

a) Calculate the expected payoff for each team once overtime is reached.

b) What is the expected payoff for winning in normal play? This is the expected payoff of winning in regular time, plus the expected payoff of winning in overtime.

c) Under which conditions would there be an incentive for teams to agree to skate on regular time and try to win in overtime?