The Wave Function and its Collapse

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1 The Wave Function

On a microscopic scale all matter such as electrons, atoms and even molecules exhibit wave-like behaviour. This is what underlies all of quantum physics. All of these systems are described by some wave function, denoted by ψ. The wave function is a mathematical object that contains all the information about a quantum system such as the probability of finding the system in a certain position, momentum, and energy. Yet it does not give precise values, like classical mechanics does, but instead provides a probability amplitude which assigns probabilities to certain outcomes. This means that if we have 100 identical systems, say electrons, and measure the position of each, the wave function, more explicitly $|\psi|^2$, will correctly predict the distribution of measurement outcomes. But it does not predict the exact position for a single system.

One central property of the wave function is that it satisfies the Schrödinger equation that describes how the wave function of a system evolves over time. It is written as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial}{\partial t}\psi,$$

where the left side is the sum of the kinetic and potential energies and the right side is the time derivative of the wave function multiplied by $i\hbar$.

There is another form of the Schrödinger equation which is often referred to as the time-independent Schrödinger equation where the right side of the equation is simplified

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi.$$

This version allows us to describe systems that are not changing over time.

The Schödinger equation is analogous to other waves and their respective wave equations. For example, the wave equation for electromagnetic waves,

$$\nabla^2 E(\mathbf{r}, t) = \frac{1}{c^2}\frac{\partial^2 E(\mathbf{r}, t)}{\partial t^2},$$

or the equation for waves on a string,

$$\nabla^2 y(\mathbf{r}, t) = \frac{\rho}{T}\frac{\partial^2 y(\mathbf{r}, t)}{\partial t^2}.$$

All of them expressing the change in the wave functions in space and time. The difference being that a classical wave consists of multiple particles, whereas the quantum wave function can, for example, describe a single atom.

To illustrate the usefulness of the time-independent Schrödinger equation, consider a particle in a one-dimensional box where the potential on the inside is zero, $V = 0$, and infinite on the outside. This means that the particle is confined to the box. Now we can use the time-independent Schrödinger equation to derive the wave function of the system which then allows us to predict the probabilities of finding a particle at a certain position or energy level.

According to the Copenhagen interpretation of quantum mechanics, the system is in no definite state before any measurements are made. This means that, before a measurement is made, the system is in a superposition of multiple states, represented by its wave function. Therefore a particle can be in multiple positions, have multiple energies, or have multiple momenta. This led Erwin Schrödinger to famously propose the following thought experiment: A cat and radioactive isotope are placed inside a box, when the isotope decays an explosive is triggered and the cat disintegrates into a million pieces (Adaption of original experiment proposed by Schrödinger (1935)). The idea being that even though the isotope is in a superposition of having decayed and not having decayed, the cat cannot simultaneously be dead and alive. Posing the question of how far a superposition, or an uncollapsed wave function, can extend and what causes the collapse of the wave function.

2 Wigner's Friend - What Causes the Wave Function to Collapse?

Imagine the following: you and your friend want to measure the position of quantum particle. Due to limited space in the lab, you decide to wait outside and let your friend do the measurements and inform you of the results.

At what point does the wave function of the particle collapse? Your friend will tell you that it collapsed as soon as he made the measurement, however, as you have not been in the lab and have not made a measurement the superposition propagates from the quantum particle through the measurement device and even to your friend's brain. So, what did he actually observe when he made the measurement? Did he measure all possible outcomes at once? This paradoxical observation led Eugene Wigner and Paul von Neumann to conclude that the collapse of the wave function occurs at the point of observation by a conscious observer (which could perhaps be a cat) (Wigner 1995). The thought experiment is commonly known as Wigner's friend, it is similar to Schrödinger's cat but highlights the importance of consciousness and it solves some of the problems associated with the collapse.

In later years Wigner revised his view and distanced himself from the standpoint that consciousness plays a role in the collapse (Ballentine 2019). The idea remains controversial and is an ongoing topic of debate. It also raises the question of whether the universe could even exist independently from an observer or what even constitutes consciousness. Physicists like Sir Roger Penrose have argued that consciousness cannot be involved in the collapse of the wave function, because the theory fails to explain how consciousness arose in the first place: Supposing that gene mutations are caused by unlikely collapses of their respective wave functions, they would require a conscious observer. But how can gene mutations have caused consciousness if they themself need consciousness to occur (Penrose 1998).

Consciousness is one of many ways to approach the so-called measurement problem. One of the most popular approaches is sometimes called "Shut up and calculate", which highlights that even though there are some open questions in the foundation of quantum mechanics it is still so good that you can just apply it without worrying about some of the issues regarding the collapse.

Other theories trying to solve the issue suggest that the wave function never actually collapses but rather all possible outcomes occur in different universes, this is know as the many-worlds interpretation (Everett 1957).

Another approach is to modify Schrödinger's equation to include non-linear terms, which do not significantly change the behaviour at a quantum level but can induce the collapse of the wave function. These theories are collectively known as objective-collapse models. And they are attempting to find a physical and objective reasons for the collapse of the wave function. They are also, unlike other interpretations, testable. And some efforts are currently being made to design experiments capable of testing their predictions. The exact nature of the non-linear terms are still under debate, some theories suggest that it could be due to spontaneous collapses due to some noise field, as proposed by the Ghirardi-Rimini-Weber (GRW) theory (Ghirardi et al. 1986). Others propose that it is due to the gravitational interactions, as in the Diósi-Penrose (DP) model (Diósi 1987, Diósi 1989, Penrose 1996).

In conclusion, the Wigner's friend thought experiment highlights the paradoxes and complexities involved in the collapse of the wave function. Many possible interpretations have been proposed, including the idea of consciousness-induced collapse, many-worlds interpretation, and objective-collapse models. Each of them has their respective advantages and disadvantages and no definite answer to the measurement problem has been found.

3 Gravity and Collapse

Schrödinger realised early that something was not quite right with quantum mechanics or at least with the way the wave function collapses, famously illustrating this with his cat in a box thought experiment (Schrödinger 1935). Ever since then, physicists have been searching for and proposing possible answers, ranging from the involvement of consciousness (Wigner 1995) to the existence of multiple universes (Everett 1957).

It was not until 1986, more than 50 years after Schrödinger voiced his concerns, that Giancarlo Ghirardi, Alberto Rimini and Tullio Weber proposed a model, now commonly known as GRW, which allowed for objective physical processes to play a role in the collapse (Ghirardi et al. 1986). What they have done was to add non-linear terms to the Schrödinger equation which have a negligible, but possibly measurable, effect on a quantum scale but can induce the collapse of the wave function. According to GRW the collapse of a system is spontaneous according to some probability distribution. And for larger systems, every constituent can collapse the whole wave function, so increasing the probability of collapse. Hence, single atoms can spend millions of years in a superposition, whereas composite systems, like a measurement device, will very quickly collapse all superpositions.

Initially, GRW proposed a collapse rate of $\lambda = 10^{-16}$ s^{-1}, corresponding to a collapse every 320 million years. More recently, Stephen L. Adler proposed that the value would be closer to $\lambda = 10^{-8}$ s^{-1} with an uncertainty of two orders of magnitude, this corresponds to a collapse every 3.2 years (Adler 2007).

Various extensions of the initial GRW model exist, most notably the continuous spontaneous localization (CSL) model which treats the collapse as being continuous in time (Pearle 1989, Ghirardi et al. 1990), and the Diósi–Penrose (DP) model which postulates that the collapse happens due to gravity (Diósi 1987, Diósi 1989, Penrose 1996).

Unifying gravity and quantum mechanics is undoubtedly one of the biggest problems in modern physics. The usual approach is to try to quantise gravity, arguing that the more "fundamental" quantum physics has to be a building block of gravity.

However, Nobel laureate Roger Penrose believes that there is equally good reason to try to gravitize quantum mechanics (Penrose 1996, 2014). Even though the DP model is not a complete theory of quantum gravity, it is attempting to show the implications of gravity on quantum systems.

The idea behind the DP model is this: a massive system which is in a superposition of two positions will also lead to an unstable superposition of two space-times. The more massive the system is the bigger the instability. Both Diósi and Penrose, independently and using two different approaches, were able to make predictions about the decay time. While Diósi provided clear dynamical equations for the collapse, Penrose decided to not assume much about the mathematics of the collapse as he expects that a satisfactory answer can only be found with a complete theory of quantum gravity. Nevertheless both theories predict that for a small sphere of water with radius of about 10^{-5} cm, its lifetime would be about an hour and for a bigger sphere with radius of 10^{-3} cm, its lifetime would only be a millionth of a second.

It should be noted that although objective-collapse models are testable, they have not yet been verified. So far, it has only been possible to put certain bounds on some of the parameters: Adler (2007) giving bounds for the decay rate in the GRW model, and Tilloy & Stace (2019) for the mass density distribution, R_0, in the DP model. Yet various experiments are currently being designed to test the predictions objective-collapse models, including the DP model (Marshall et al. 2003, Carlesso et al. 2022).

Objective-collapse models also suffer from other criticisms. One of them is that in their initial formulation they did not conserve energy. But newer revised versions of the models have been developed which include dissipative effects to conserve the energy (Smirne et al. 2014, Smirne & Bassi 2015, Bahrami et al. 2014). Another issue is that they are very difficult to generalise to agree with relativistic effects, some work is being done in that directions but it is still open to research (Ghirardi et al. 1990, Tumulka 2006).

In conclusion, objective collapse models and the role of gravity in the collapse of the wave function are fascinating and provide reasonable predictions about lifetimes. They are still very much open for debate, neither having been proven or disproven but only parameter bounds have been found.

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