



# Output summary of the CUED program

The CUED program is developed and maintained by:

Chair of Computational Condensed Matter Theory  
Institute of Theoretical Physics  
University of Regensburg  
Universitätsstraße 31  
D - 93053 Regensburg  
Germany

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Responsible scientists:

Jan Wilhelm

Ferdinand Evers

Contributors (in alphabetic order):

Jack Crewse

Patrick Grössing

Adrian Seith

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## 1 Electric-field pulse

The following electric driving field is employed in the simulation:

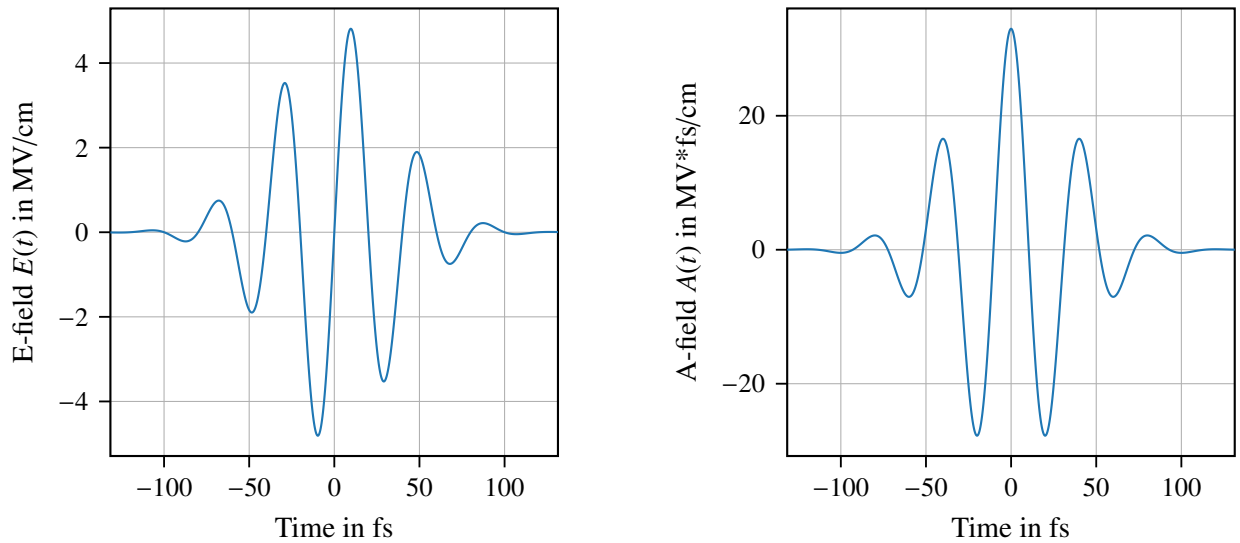
$$\mathbf{E}(t) = E(t) \hat{e}_\phi, \quad E(t) = E_0 \sin\left(2\pi f_0 (1 + f_{\text{chirp}} t) t + \varphi\right) e^{-t^2/(2\alpha)^2}. \quad (1)$$

The pulse is sketched in Fig. 1. The following parameters are used in the simulation:

- $\hat{e}_\phi = \hat{e}_x$
- Pulse frequency:  $f_0 = 25.0$  THz
- Chirp:  $f_{\text{chirp}} = 0.0$  THz
- Carrier-envelope phase:  $\phi = 0.0$
- $\alpha = 25.0$  fs, full width at half maximum (FWHM) of the Gaussian envelope = 83.255 fs

The gauge field  $\mathbf{A}(t) = A(t) \hat{e}_\phi$  follows from  $\dot{\mathbf{A}}(t) = -\mathbf{E}(t)$ . We compute  $A(t)$  for the sketch in Fig. 1 as

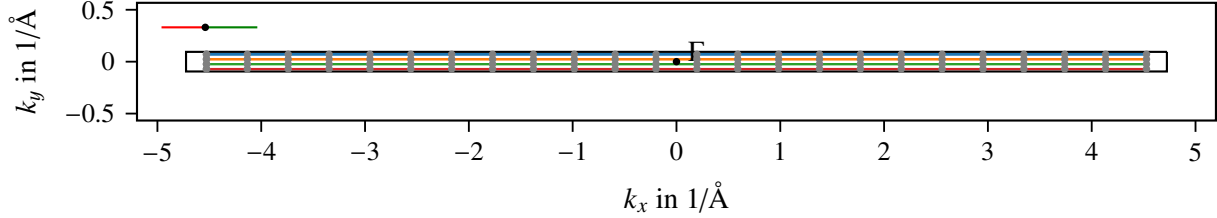
$$\dot{A}(t) = -E(t) \quad \Rightarrow \quad A(t) = - \int_{-\infty}^t E(t') dt'. \quad (2)$$



**Figure 1:** Left: Electric driving field  $E(t)$  from Eq. (1), right: Gauge field  $A(t)$  from Eq. (2).

## 2 Brillouin zone and $k$ -point grid

You are using a rectangle as Brillouin zone (BZ) with a mesh size of  $50 \times 4$ . The BZ and a  $24 \times 4$   $k$ -point mesh is sketched in Fig. 2. Please note that in the `params.py` file, the BZ size (in case of a rectangular BZ) and the lattice parameter  $a$  (in case of a hexagonal BZ) are both given in atomic units, while the output here is in  $1/\text{\AA}$  (note: 1 atomic length unit =  $1a_0 = 0.529 \text{\AA}$ ,  $a_0$ : Bohr radius, 1 atomic inverse length unit =  $1/(0.529 \text{\AA}) = 1.890 \text{\AA}^{-1}$ ).



**Figure 2:** Brillouin zone (BZ) and  $24 \times 4$   $k$ -point mesh. The BZ is indicated by the black rectangle. Gray points indicate  $k$ -points, colored lines indicate  $k$ -points that are coupled via the term  $\mathbf{E}(t) \cdot \nabla_{\mathbf{k}}$ . The green and red line at the top left corner sketch  $\mathbf{A}_{\max} := \hat{e}_\phi \max(-qA(t)/\hbar)$  and  $\mathbf{A}_{\min} := \hat{e}_\phi \min(-qA(t)/\hbar)$  indicating the extremal excursion of electrons in the BZ.

## 3 Hamiltonian, bandstructure and dipoles

This still needs to be filled; Gauge; how to print the Hamiltonian properly? (e.g. Dirac or  $\text{Bi}_2\text{Te}_3$ ?) Printing of band structure for rectangle along  $k_x, k_y = 0, k_y, k_x = 0$ , for hexagon along K- $\Gamma$ -M. Plotting of dipoles as in Patricks plot with 4 diagrams (dvv, dcc, Re(dvc), Im(dvc)), additionally along K- $\Gamma$ -M (also absolute value of dvc).

## 4 Time evolution of the density matrix

In case you choose the length gauge (gauge = 'length', that is also the default), we solve semiconductor Bloch equations in the length gauge, Eq. (50) in Ref. [1]:

$$\left[ \frac{\partial}{\partial t} + q\mathbf{E}(t) \cdot \frac{\partial}{\partial \mathbf{k}} \right] \rho_{nn'}(\mathbf{k}, t) = [i(\epsilon_{n'}(\mathbf{k}) - \epsilon_n(\mathbf{k})) - 1/T_2] \rho_{nn'}(\mathbf{k}, t) - i\mathbf{E}(t) \cdot \sum_{\underline{n}} (\rho_{n\underline{n}}(\mathbf{k}; t) \mathbf{d}_{\underline{n}n'}(\mathbf{k}) - \mathbf{d}_{nn}(\mathbf{k}) \rho_{\underline{n}n'}(\mathbf{k}; t)) \quad (3)$$

with a dephasing time  $T_2 = 1 \text{ fs}$ . **TODO:** Plots of initial vv and cc density matrix elements, plot of time evolution at a  $k$ -point, snapshot of density matrix at 3 time points for whole BZ.

## 5 Time-dependent current

In case you choose the length gauge (gauge = 'length', that is also the default), the current is computed from Eq. (67) in Ref. [1] as

$$\mathbf{j}(t) = q \sum_{nn'} \int_{\text{BZ}} \frac{d\mathbf{k}}{(2\pi)^2} \langle u_{n\mathbf{k}} | \frac{\partial h(\mathbf{k})}{\partial \mathbf{k}} | u_{n'\mathbf{k}} \rangle \rho_{n'n}(\mathbf{k}, t) . \quad (4)$$

The matrix element  $\langle u_{n\mathbf{k}} | (\partial_{\mathbf{k}} h(\mathbf{k})) | u_{n'\mathbf{k}} \rangle$  can be computed from Eq. (68) in Ref. [1] as

$$\langle u_{n\mathbf{k}} | \frac{\partial h(\mathbf{k})}{\partial \mathbf{k}} | u_{n'\mathbf{k}} \rangle = \delta_{nn'} \partial_{\mathbf{k}} \epsilon_n(\mathbf{k}) + \frac{i}{q} \mathbf{d}_{nn'}(\mathbf{k}) (\epsilon_n(\mathbf{k}) - \epsilon_{n'}(\mathbf{k})) . \quad (5)$$

In our case, the current is a two-dimensional vector. For generating meaningful plots, we project the current onto the axis  $\hat{e}_\phi$  of the incoming E-field and its orthogonal direction  $\hat{e}_{\phi+\pi/2}$ :

$$j_{\parallel}(t) = \hat{e}_\phi \cdot \mathbf{j}(t), \quad j_{\perp}(t) = \hat{e}_{\phi+\pi/2} \cdot \mathbf{j}(t), \quad (6)$$

and we recover

$$\mathbf{j}(t) = \hat{e}_\phi j_{\parallel}(t) + \hat{e}_{\phi+\pi/2} j_{\perp}(t). \quad (7)$$

## 6 Frequency-resolved emission spectrum

Experiments measure the frequency resolved emission intensity  $I$ , which is computed by Eq. (53) in Ref. [1]

$$I(\omega) = \frac{\omega^2}{3c^3} |\mathbf{j}(\omega)|^2, \quad (8)$$

where  $\mathbf{j}(\omega)$  is the Fourier transform of  $\mathbf{j}(t)$  and  $c$  is the speed of light. The emission is sketched in Fig. 4. When inserting Eq. (7) in the frequency domain, we have

$$I(\omega) = \frac{\omega^2}{3c^3} (|\mathbf{j}_{\parallel}(\omega)|^2 + |\mathbf{j}_{\perp}(\omega)|^2), \quad (9)$$

that motivates the definitions

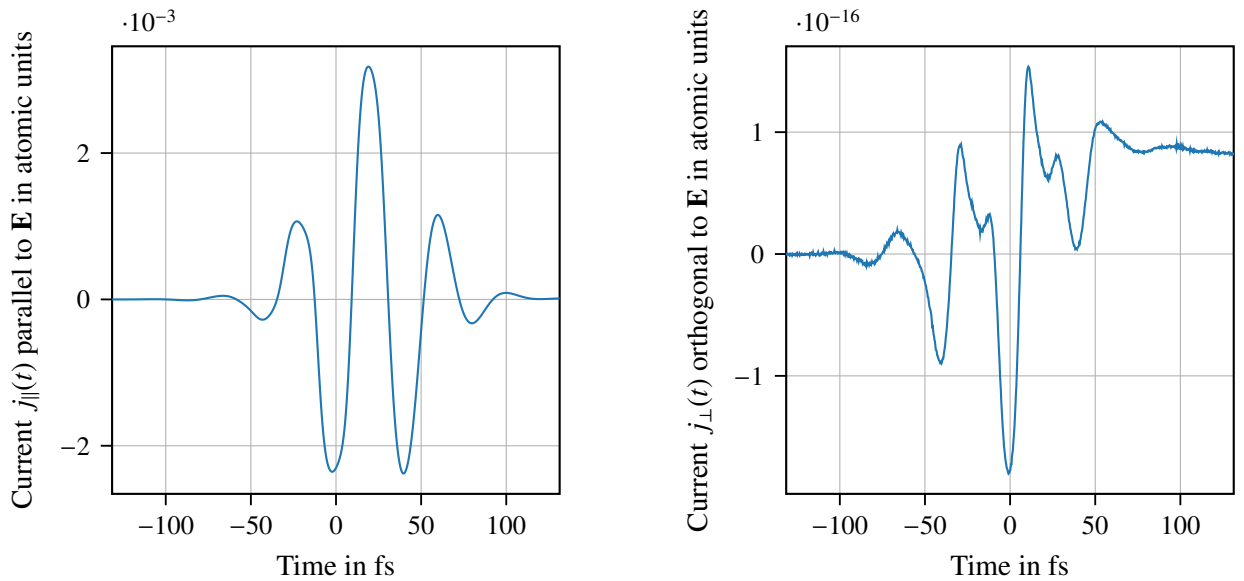
$$I_{\parallel}(\omega) = \frac{\omega^2}{3c^3} |\mathbf{j}_{\parallel}(\omega)|^2, \quad I_{\perp}(\omega) = \frac{\omega^2}{3c^3} |\mathbf{j}_{\perp}(\omega)|^2. \quad (10)$$

We recover  $I(\omega) = I_{\parallel}(\omega) + I_{\perp}(\omega)$ .

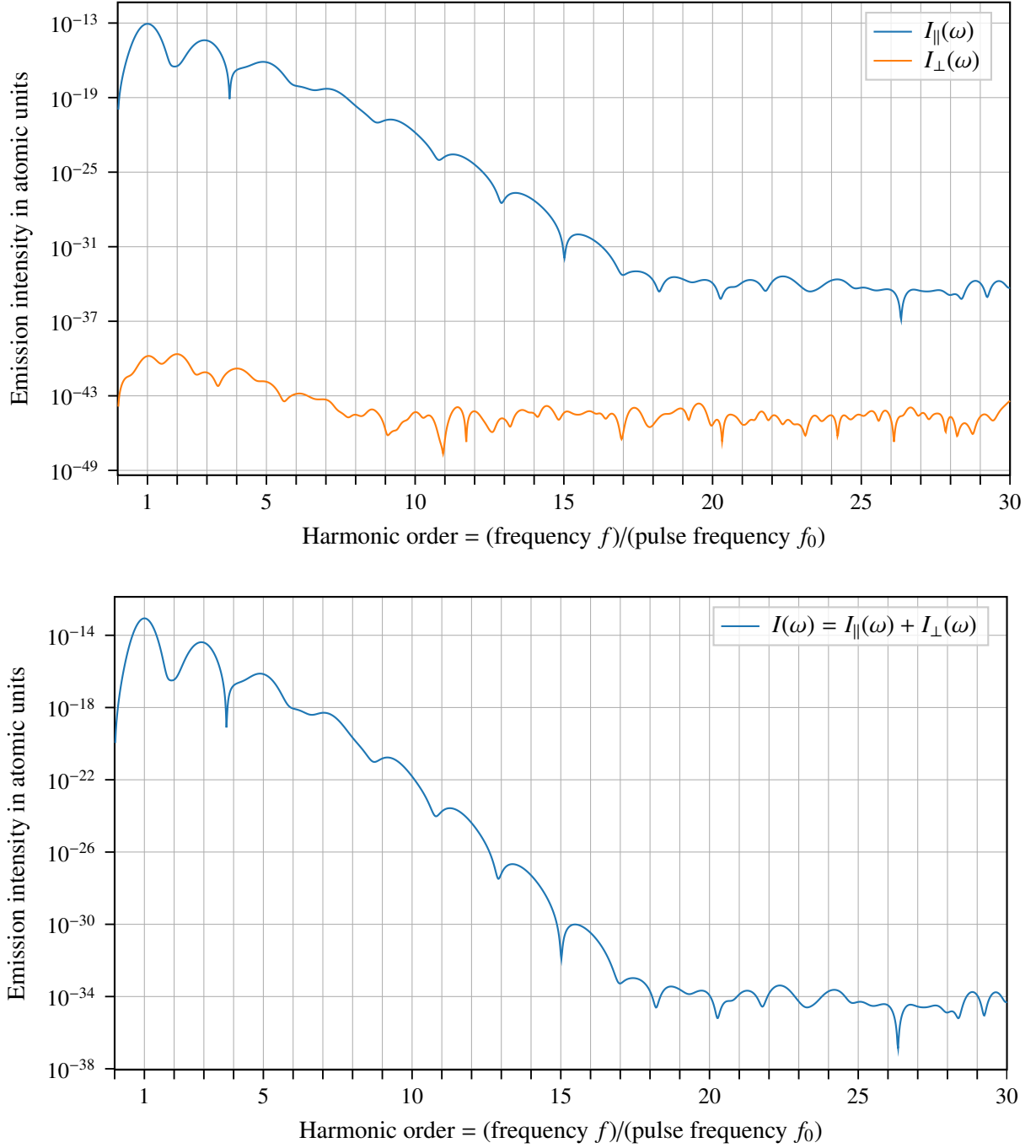
## 7 References

When using the CUED software package, please reference to CUED by citing the following publication:

- [1] J. Wilhelm, P. Grössing, A. Seith, J. Crewse, M. Nitsch, L. Weigl, C. Schmid, and F. Evers, *Semiconductor-Bloch Formalism: Derivation and Application to High-Harmonic Generation from Dirac Fermions*, *Phys. Rev. B* **x**, y (2021).



**Figure 3:** Components of the time-dependent current  $\mathbf{j}(t)$ : left: parallel to the driving field, right: orthogonal to the driving field, see Eq. (6).



**Figure 4:** Emission intensity from the irradiated material computed from Eqs. (8) and (10). The frequency is given by  $f = \omega/(2\pi)$ .