



Example 52 : As shown in the figure, in $\triangle ABC$, I is the center, AD , BE , CF are the angle bisectors, DF intersects BI at P , and DE intersects CI at Q . Prove: $\angle CAQ = \angle PAB$.

Proof: Suppose $I = \frac{aA + bB + cC}{a + b + c}$, then $P = \frac{aA + 2bB + cC}{a + 2b + c}$, $Q = \frac{aA + bB + 2cC}{a + b + 2c}$,

$$\frac{\frac{A-P}{A-B} + \frac{2(a+2b+c)}{a+b+2c} \frac{B-P}{B-A} + \frac{2(a+b+2c)}{a+2b+c} \frac{C-Q}{C-A}}{\frac{A-C}{A-Q}} = \frac{2a^2 + 4ab + 2b^2 + 4ac + 5bc + 2c^2}{(a+2b+c)(a+b+2c)}$$