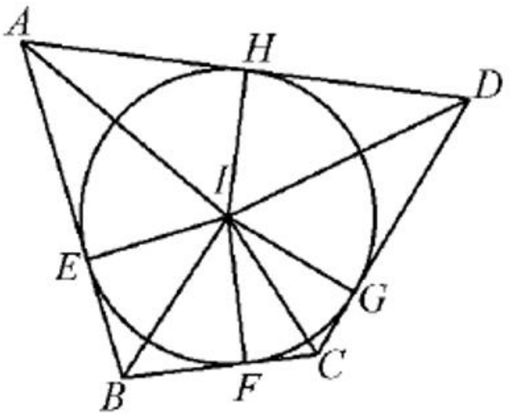
Example 32 : After exploring and obtaining the related properties of the inner triangle, try to generalize the triangle to the quadrilateral.

As shown in the figure , it is known that quadrilateral $ABCD$ is a circumscribed quadrilateral of $\frac{AI^2}{DA\cdot BA}+\frac{BI^2}{AB\cdot CB}+\frac{CI^2}{BC\cdot DC}+\frac{DI^2}{CD\cdot AD}=2$ circle I, then .

We want $\frac{AI^2}{DA\cdot BA}+\frac{BI^2}{AB\cdot CB}+\frac{CI^2}{BC\cdot DC}+\frac{DI^2}{CD\cdot AD}$ to be constant value. Assuming a fixed value, when the quadrilateral $ABCD$ is a square, it is easy to guess that the fixed value is 2 .



Suppose there is an inscribed circle in the quadrilateral $\frac{1}{z_1}$ *ABCD , and let the circle be the unit circle, the complex forms of the four tangent points are* , $\frac{1}{z_2}$, $\frac{1}{z_3}$, $\frac{1}{z_4}$, then

$a=\frac{2}{z_4+z_1}$, $b=\frac{2}{z_1+z_2}$, $c=\frac{2}{z_2+z_3}$, $d=\frac{2}{z_3+z_4}$, $x=0$, and it can be verified that $\frac{(x-a)^2}{(d-a)(b-a)}+\frac{(x-b)^2}{(a-b)(c-b)}+\frac{(x-c)^2}{(b-c)(d-c)}+\frac{(x-d)^2}{(c-d)(a-d)}=2$ the constant holds true.

Easy to verify $\frac{(x-a)^2}{(d-a)(b-a)}=\frac{(x-c)^2}{(b-c)(d-c)}$, ie $\frac{AI^2}{DA\cdot BA}=\frac{CI^2}{BC\cdot DC}$. the same way $\frac{BI^2}{AB\cdot CB}=\frac{DI^2}{CD\cdot AD}$.