

Example 179: As shown in Figure 3, if \triangle the center of ABC is I and the circumcenter of $\triangle BCI$ is D, then the three points A, I, and D are collinear, and the four points A, B, C, and D are in the same circle.

$$\left(\frac{A-I}{I-D}\right)^2 = \frac{\frac{B-I}{B-D}}{\frac{I-D}{I-B}} \frac{\frac{C-D}{C-B}}{\frac{B-C}{B-D}} \frac{\frac{I-C}{I-D}}{\frac{C-D}{C-I}} \frac{\frac{A-I}{A-B}}{\frac{A-B}{B-I}} \frac{\frac{B-A}{C-B}}{\frac{B-I}{C-I}} \frac{\frac{C-B}{C-I}}{\frac{C-I}{C-A}} \, .$$

$$\frac{\frac{C-D}{C-B}}{\frac{A-D}{A-B}} = \frac{\frac{B-I}{B-D}}{\frac{I-D}{I-B}} \frac{\frac{B-A}{B-I}}{\frac{B-I}{B-C}} \frac{\frac{C-D}{C-B}}{\frac{B-C}{B-D}} \frac{I-D}{A-D},$$