

Example 2 15 : As shown in the figure, circle A and circle B intersect at points E and F , and there is a point M on circle B , extend ME and MF to intersect circle A at E_1 and F_1 , and prove $E_1F_1 \perp MB$.

$$\frac{M-B}{E_1-F_1} = \frac{\frac{F_1-F}{F_1-E_1} \left(\frac{E-F}{E-E_1} \frac{M-B}{F-F_1} \right)}{E-E_1}$$

Example 23 From the midpoint C of arc AB of a circle, two secants are drawn meeting line AB at F , G , and the circle at D and E . Show that F , D , E , and G are on the same circle.

Point order: a, c, d, e, o, m, f, g .

Hypotheses: $\text{cong}(o, a, o, c)$, $\text{cong}(o, a, o, d)$, $\text{cong}(o, a, o, e)$,
 $\text{coll}(m, c, o)$, $\text{perp}(m, a, c, o)$, $\text{coll}(f, a, m)$, $\text{coll}(f, c, d)$,
 $\text{coll}(g, a, m)$, $\text{coll}(g, c, e)$.

Conclusion: $[ce, fg] + [cd, de]$.

The Machine Proof

$$-[gf, ec] - [ed, dc]$$

(Since a, f, g, m are collinear; $[gf, ec] = [ma, ec]$.)

$$= -[ma, ec] - [ed, dc]$$

(Since $ma \perp co$; $[ma, ec] = [oc, ec] + 1$.)

$$= -[oc, ec] - [ed, dc] - 1$$

(Since $\text{circumcenter}(o, c, e, d)$; $[oc, ec] = [oc, ce] = -[ed, dc] + 1$.)

$$= 0$$

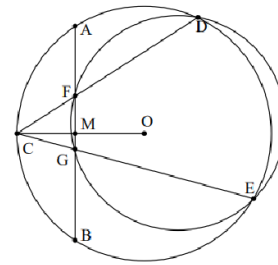


Figure 23