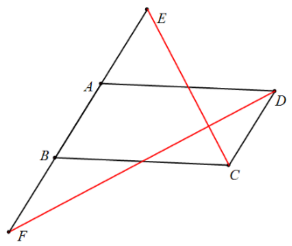
Example 1 0 : As shown in Figure 1, in the parallelogram $ABCD$, extend AB to E and F *in two directions*, so that $AE = AB = BF$, connect CE and DF, prove: $AD = 2AB \Leftrightarrow EC \perp FD$.



picture

Proof Suppose $A=0$, $4\frac{\frac{F-0}{F-D}}{\frac{D-F}{D-0}}+\left[\frac{-\frac{F}{2}-\left(\frac{F}{2}+D\right)}{D-F}\right]^2=1$,

$\frac{-4DF}{(D-F)^2}+\frac{(D+F)^2}{(D-F)^2}=1$, $(D-F)^2+4DF=(D+F)^2$.

The proof assumes $A=0$ that . $(-B-(B+D))(2B-D)+(4B^2-D^2)=0$

The way to convert it into a general vector is :

$(-\overrightarrow{AB}-(\overrightarrow{AB}+\overrightarrow{AD}))\cdot(2\overrightarrow{AB}-\overrightarrow{AD})=-(4\overrightarrow{AB}^2-\overrightarrow{AD}^2)$.

$4\frac{\frac{2B-0}{2B-D}}{\frac{D-2B}{D-0}}+\left(\frac{B+D+B}{2B-D}\right)^2=1$, $\frac{-8BD}{(2B-D)^2}+\frac{(2B+D)^2}{(2B-D)^2}=1$,