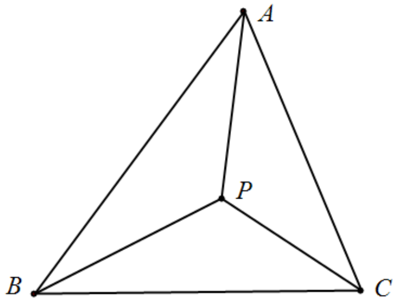
Example 30 : As shown in Figure 4 , in $\triangle ABC$, the angle bisectors of $\angle B$ and $\angle C$ intersect *at* point *P*. *Prove* that AP bisects $\angle A$. (inner theorem)



Proof: $\dfrac{\frac{P-A}{B-A}}{\frac{C-A}{P-A}}+\dfrac{\frac{P-B}{C-B}}{\frac{A-B}{P-B}}+\dfrac{\frac{P-C}{A-C}}{\frac{B-C}{P-C}}=1$.

Explanation: The above formula is equivalent to $\dfrac{(P-A)^2}{(B-A)(C-A)}+\dfrac{(P-B)^2}{(C-B)(A-B)}+\dfrac{(P-C)^2}{(A-C)(B-C)}=1$. If rewritten as $\dfrac{(x-a)^2}{(b-a)(c-a)}+\dfrac{(x-b)^2}{(c-b)(a-b)}+\dfrac{(x-c)^2}{(a-c)(b-c)}=1$, it is a very classic algebraic identity. The classic proof is that the left side of the formula is regarded as a quadratic function about x, *and it is easy to verify that when* $x=a$, b, c, the left side is equal to 1, so the formula is always true. It is hard to imagine that the complex geometric meaning of such a familiar algebraic identity is an inner theorem. What needs to be emphasized is that while the identity proves the angle relationship, it also proves the side length relationship. According to $\left|\dfrac{\frac{P-A}{B-A}}{\frac{C-A}{P-A}}\right|+\left|\dfrac{\frac{P-B}{C-B}}{\frac{A-B}{P-B}}\right|+\left|\dfrac{\frac{P-C}{A-C}}{\frac{B-C}{P-C}}\right|\geq\left|\dfrac{\frac{P-A}{B-A}}{\frac{C-A}{P-A}}+\dfrac{\frac{P-B}{C-B}}{\frac{A-B}{P-B}}+\dfrac{\frac{P-C}{A-C}}{\frac{B-C}{P-C}}\right|=1$, it can be obtained that the equality sign is established $\dfrac{PA^2}{BA\cdot CA}+\dfrac{PB^2}{CB\cdot AB}+\dfrac{PC^2}{AC\cdot BC}\geq 1$ if and only when P is $\triangle$the incenter.