

Example 28: As shown in Figure 9, in \triangle *ABC*, *G* is the center of gravity, and point *P* satisfies $\angle PAB = \angle PBC = \angle PCA$. If *A*, *B*, *P*, and *G* are four points in a circle, prove that: *C*, *E*, *G*, and *P* are in a circle, and *A*, *D*, and The four points *G* and *E* share a circle.

Proof:
$$\frac{\frac{A+C}{2}-P}{\frac{A+C}{2}-\frac{A+B+C}{3}} = \frac{\frac{A+B+C}{3}-P}{\frac{A+B+C}{2}-B} = \frac{\frac{C-P}{3}}{\frac{C-P}{C-A}} + 1,$$

description $\angle PEG = \angle PCG \Leftrightarrow \angle PGB = \angle PAB$,

$$\frac{\frac{A+B+C}{3} - \frac{A+B}{2}}{\frac{A+B+C}{3} - A} + \frac{\frac{1}{2}}{\frac{A+C}{2} - \frac{A+B}{2}} = 1,$$

$$\frac{\frac{A+C}{2} - \frac{A+B}{2}}{\frac{A+C}{2} - A} + \frac{\frac{A-P}{3}}{\frac{A-B}{3} - P} = \frac{\frac{A+B+C}{3} - P}{\frac{A-B}{B-C}} = \frac{\frac{B-P}{B-C}}{\frac{B-C}{B-C}}$$

description $\angle AED = \angle AGD \Leftrightarrow \angle PGB = \angle PBC$.