Example 87 : As shown in Figure 3, $\triangle$in ABC, O is the circumcenter, AP is the height, $PM \perp AB$ intersects AB at M, $PN \perp AC$ intersects AC at N, to prove: $AO \perp MN$.

Proof: $$\frac{M-N}{A-O}=\frac{\dfrac{A-C}{A-P}}{\dfrac{A-O}{A-M}}\frac{\dfrac{C-B}{C-A}}{\dfrac{M-A}{M-N}}\frac{P-A}{B-C}.$$

Explanation: According to $AN\cdot AC=AP^2=AM\cdot AB$ the four points M, B, C and N *are in a circle,* $\angle BCA=\angle AMN$. Or $\angle BCA=\angle NPA=\angle AMN$.