Example 2 13 : As shown in the figure, in $\triangle ABC$, H is the orthocenter, CI is the height, M and N *are the midpoints of* BC and AH respectively , to prove: $IM \perp IN$.

$$\frac{I-M}{I-N}=\frac{\left(\frac{I-M}{A-B}\right)^2}{\frac{M-I}{B-C}}\frac{\left(\frac{B-A}{I-N}\right)^2}{\frac{A-H}{N-I}}\frac{A-H}{B-C}$$

Example 58 *Let A, B, C, D be four points on circle (O). $E = CD \cap AB$. CB meets the line passing through E and parallel to AD at F. GF is tangent to circle (O) at G. Show that $FG = FE$.*

Point order: a, b, c, d, e, f.

Hypotheses: cyclic(a, b, c, d), coll(e, a, b), coll(e, c, d), coll(f, b, c), para(f, e, a, d).

Conclusion: eqangle(f, e, b, e, c, b).

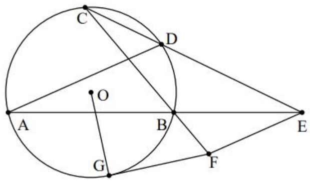

Figure 58

The Machine Proof

$[fe, eb] - [ec, cb]$

(Since $fe \parallel ad$; $[fe, eb] = -[eb, da]$.)

$= -[ec, cb] - [eb, da]$

(Since c, d, e are collinear; $[ec, cb] = [dc, cb]$.)

$= -[eb, da] - [dc, cb]$

(Since a, b, e are collinear; $[eb, da] = -[da, ba]$.)

$= -[dc, cb] + [da, ba]$

(Since c, d, b, a are cyclic; $[dc, cb] = [da, ba]$.)

$= 0$