



Example 85 : As shown in Figure 3, in $\triangle ABC$, BE and CF are heights, B_1 and C_1 are the midpoints of AC and AB respectively, which intersect EC_1 at X .

Prove: $\angle B_1XC_1 = 3\angle A$.

Analysis: 1) $\angle FAC = \angle CFB_1 \Leftrightarrow \frac{A-C}{F-C} / \frac{F-C}{F-B_1} \in R$; 2) $\angle ABE = \angle BEC_1 \Leftrightarrow$

$\frac{E-B}{E-C_1} / \frac{B-A}{B-E} \in R$; 3) $CF \perp AB \Leftrightarrow \left(\frac{C-F}{A-B} \right)^2 \in R$; 4) $EB \perp AC \Leftrightarrow$

$\left(\frac{A-C}{E-B} \right)^2 \in R$;

5) $\angle B_1XC_1 = 3\angle A \Leftrightarrow \left(\frac{A-C}{A-B} \right)^3 / \frac{C_1-E}{B_1-F} \in R$.

Proof: $\left(\frac{A-C}{A-B} \right)^3 / \frac{C_1-E}{B_1-F} = \left(\frac{C-F}{A-B} \right)^2 \left(\frac{A-C}{E-B} \right)^2 \left(\frac{A-C}{F-C} / \frac{F-C}{F-B_1} \right) \left(\frac{E-B}{E-C_1} / \frac{B-A}{B-E} \right)$.

Since the parallel (including collinear) and perpendicular conclusions involve only two minor terms, it is relatively easy to express them with conditions. However, the angle relationship (including the proof that the triangle is an isosceles triangle) and the four points cocircle involve four small terms, so it is relatively difficult to express it with conditions. Of course this is not absolute.