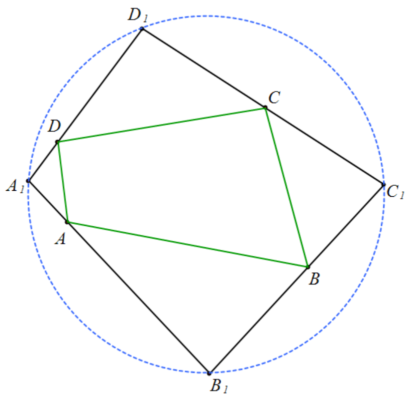
Example 81 : As shown in Figure 3, the quadrilateral $ABCD$, the bisectors of the four angles form a quadrilateral $A_1B_1C_1D_1$, verify that the quadrilateral $A_1B_1C_1D_1$ is a quadrilateral inscribed in a circle.

Proof: $$\frac{\frac{A-D}{A_1-D_1}}{\frac{A_1-D_1}{A-B}}\frac{\frac{C_1-D_1}{B-C}}{\frac{B-A}{C_1-D_1}}\frac{\frac{C-B}{C_1-B_1}}{\frac{C_1-B_1}{C-D}}\frac{\frac{A_1-B_1}{D-A}}{\frac{D-C}{A_1-B_1}}=\left(\frac{\frac{B_1-A_1}{A_1-D_1}}{\frac{C_1-B_1}{C_1-D_1}}\right)^2.$$

Another proof: $\angle A_1D_1C_1+\angle A_1B_1C_1=180^\circ-\frac{\angle A}{2}-\frac{\angle B}{2}+180^\circ-\frac{\angle C}{2}-\frac{\angle D}{2}=180^\circ$.



$\angle A_1+\angle C_1+\angle A_1AD+\angle A_1DA+\angle C_1CB+\angle C_1BC$

$=\angle B_1+\angle D_1+\angle B_1AB+\angle B_1BA+\angle D_1DC+\angle D_1CD=360^\circ$, easy to get

$\angle A_1+\angle C_1=\angle B_1+\angle D_1=180^\circ$.