



Example 209 : As shown in Figure 1 , *if the intersections of the* bisectors of the diagonals B and D of the inscribed quadrilateral $AECD$ and the circumscribed circle are E and F respectively , then EF is the diameter of the circle.

$$\text{prove: } -\left(\frac{C-E}{C-F}\right)^2 = \frac{\frac{B-E}{B-C} \frac{D-F}{D-A} \frac{B-A}{D-C} \left(\frac{E-C}{D-F}\right)^2 \left(\frac{B-C}{F-E}\right)^2}{\frac{B-A}{B-E} \frac{D-F}{D-C} \frac{A-D}{D-F} \left(\frac{E-F}{D-C}\right) \left(\frac{B-E}{F-C}\right)}$$

Another proof: $\angle CEF = \angle CDF = \frac{1}{2} \angle ADC$, $\angle CFE = \angle CBE = \frac{1}{2} \angle ABC$,

$$\angle CEF + \angle CFE = \frac{1}{2} (\angle ADC + \angle ABC) = 90^\circ, \text{ thus } \angle ECF = 90^\circ .$$