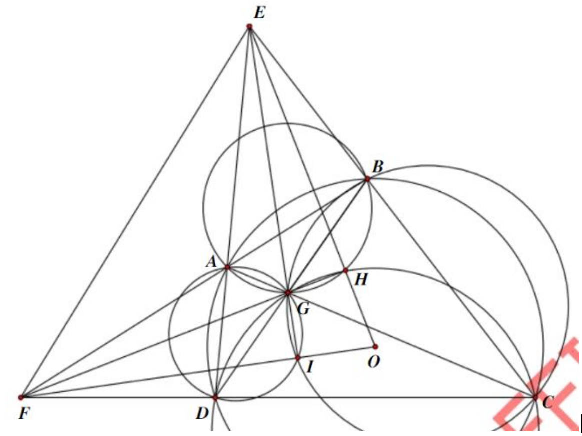

Inscribed quadrilateral

Example 2 33 : As shown in the figure, the circle O inscribes the quadrilateral $ABCD$, AD intersects BC at E , AB intersects CD at F , AC intersects BD at G , the circumcircle of △ ABG and the circumcircle of △ CDG intersect at G , H , △ The circumscribed circle of ADG and the circumscribed circle of △ BCG intersect at G , I , and prove that: F , I , O are collinear; O *is* the orthocenter of △ EFG .

Proof: First prove that A , B , I , O are four points in a circle.

$$\frac{\dfrac{I-A}{I-B}}{\dfrac{O-A}{O-B}}\frac{\dfrac{D-A}{D-G}}{\dfrac{I-A}{I-G}}\frac{\dfrac{I-B}{I-G}}{\dfrac{C-B}{C-G}}\frac{C-A}{C-G}\frac{D-G}{D-B}\frac{\dfrac{O-A}{O-B}}{\dfrac{C-A}{C-B}\dfrac{D-A}{D-B}}=1,$$

Similarly, four points D , C , I , O *share a circle.* According to the circular power theorem, F *is on* the root axis of the circumcircle of △ ABI and △ DCI *, so the three points* F , I , O are collinear. Similarly, the three points E , G , and I are collinear. The three points E , H and O are collinear.

$$\frac{I-O}{I-G}=-\frac{\dfrac{I-O}{I-C}}{\dfrac{D-O}{D-C}}\frac{\dfrac{I-C}{I-G}}{\dfrac{B-C}{G-B}}\frac{B-D}{B-G}\left(\frac{B-C}{B-D}\frac{D-O}{D-C}\right),$$ so $IO \perp IG$, similarly $HO \perp HG$,

so O is the orthocenter of △ EFG .