Example 34 : Convex quadrilateral $ABCD$ with side lengths a, b, c and d is circumscribed on circle O. To prove: $OA\cdot OC+OB\cdot OD=\sqrt{abcd}$.

Proof: Suppose $O=0$, $A=\frac{2}{z_4+z_1}$, $B=\frac{2}{z_1+z_2}$, $C=\frac{2}{z_2+z_3}$, $D=\frac{2}{z_3+z_4}$, then there is an identity

$$\frac{\frac{A-O}{A-B}\frac{C-O}{C-D}}{\frac{A-D}{A-O}\frac{C-B}{C-O}}+\frac{\frac{B-O}{B-C}\frac{D-O}{D-C}}{\frac{B-A}{B-O}\frac{D-A}{D-O}}-2\frac{A-O}{A-B}\frac{B-O}{B-C}\frac{C-O}{C-D}\frac{D-O}{D-A}=1,$$

its geometric meaning $\frac{AO^2}{AB\cdot AD}\frac{CO^2}{CB\cdot CD}+\frac{BO^2}{BC\cdot BA}\frac{DO^2}{DC\cdot DA}+2\frac{AO}{AB}\frac{BO}{BC}\frac{CO}{CD}\frac{DO}{DA}=1$,

ie $OA\cdot OC+OB\cdot OD=\sqrt{abcd}$.