

Example 94: As shown in Figure 3, in \triangle *ABC*, *O* is the circumcenter, A_1 is the midpoint of *BC*, *S* is on *BC*, and \angle BAA $I = \angle$ SAC, AA $I_{\text{intersects}}$ the circle *O* at *P*, *AS* intersects the circle *O* at *Q*, the feet of *Q* on *BA* and *BC* are *G* and *D* respectively. To prove: $AP \perp DG$.

$$\frac{A-P}{D-G} = \frac{A-P}{A-A_1} \frac{A-B}{B-G} \frac{A-Q}{A-S} \left(\frac{A-C}{A-Q} / \frac{B-C}{B-Q} \right) \left(\frac{A-S}{A-B} / \frac{A-C}{A-A_1} \right) \left(\frac{B-G}{B-Q} \frac{D-Q}{D-G} \right) \frac{B-C}{D-Q}$$