Example 2 14 : As shown in the figure, four points A , B , C and D *are on the circle* O , AB intersects CD at E , make EF // AD intersect CB at F , and draw the tangent line FG *of* circle O *through* F , to prove: $FG = FE$.

$$\frac{\frac{C-E}{C-F}}{\frac{E-F}{E-B}}=\frac{C-B}{C-F}\frac{A-D}{F-E}\frac{C-E}{C-D}\frac{B-E}{A-B}\frac{\frac{C-D}{C-B}}{\frac{A-D}{A-B}}$$

Explanation: To change the certificate $FG^2=FE^2=FB\cdot FC$, it means to prove that $\triangle\ CFE \backsim \triangle\ EFB$, that is, to prove that $\angle\ FEB = \angle\ ECB$.

Example 64 *Given two circles* (A), (B) *intersecting in* E, F, *show that the chord* E_1F_1 *determined in* (A) *by the lines* MEE_1, MFF_1 *joining* E, F *to any point* M *of* (B) *is perpendicular to* MB.

Point order: $e, f, m, b, d, a, e1, f1$.

Hypotheses: circumcenter(b, e, f, m), midpoint(d, e, f), coll(a, d, b), coll$(e1, m, e)$, cong$(e1, a, e, a)$, coll$(f1, m, f)$, cong$(f1, a, e, a)$.

Conclusion: perp$(e1, f1, m, b)$.

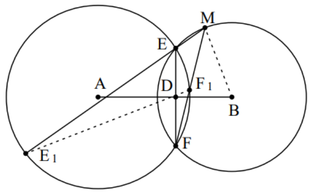

Figure 64

The Machine Proof

$[f1e1, bm] + 1$

(Since f, $f1$, m are collinear; $f1$, $e1$, f, e are cyclic;
$[f1e1, bm] = [f1e1, f1f] + [fm, bm] = [e1e, fe] - [bm, mf]$.)

$= [e1e, fe] - [bm, mf] + 1$

(Since e, $e1$, m are collinear; $[e1e, fe] = [me, fe]$.)

$= -[bm, mf] + [me, fe] + 1$ (Since circumcenter(b,m, f, e); $[bm, mf] = [bm, mf] = [me, fe] + 1$.)

$= 0$