

Example 34 : Suppose I the center of the inscribed circle of $\triangle ABC$, c_1, c_2, c_3 the three circumscribed circles of I_a, I_b, I_c is, $\triangle ABC$ the centers of the three circumscribed c_1, c_2, c_3 circles of respectively $\triangle ABC$, then

$$\frac{AB \cdot AC}{I_a A^2} + \frac{BA \cdot BC}{I_b B^2} + \frac{CA \cdot CB}{I_c C^2} = 1.$$

$$\frac{I_a A^2}{AB \cdot AC} - \frac{I_b B^2}{BA \cdot BC} - \frac{I_c C^2}{CA \cdot CB} = 1.$$

Suppose $A = a^2$, $B = b^2$, $C = c^2$, $I_a = ab - bc + ca$, $I_b = ab + bc - ca$,

$$I_c = -ab + bc + ca,$$

$$\text{verifiable } \frac{(c^2 - a^2)(b^2 - a^2)}{(a^2 - I_a)^2} + \frac{(a^2 - b^2)(c^2 - b^2)}{(b^2 - I_b)^2} + \frac{(b^2 - c^2)(a^2 - c^2)}{(c^2 - I_c)^2} = 1,$$

$$\text{ie } \frac{AB \cdot AC}{I_a A^2} + \frac{BA \cdot BC}{I_b B^2} + \frac{CA \cdot CB}{I_c C^2} = 1.$$

$$\text{Verifiable } \frac{(a^2 - I)(a^2 - I_a)}{(b^2 - a^2)(c^2 - a^2)} = 1, \text{ ie } \frac{IA \cdot I_a A}{BA \cdot CA} = 1.$$