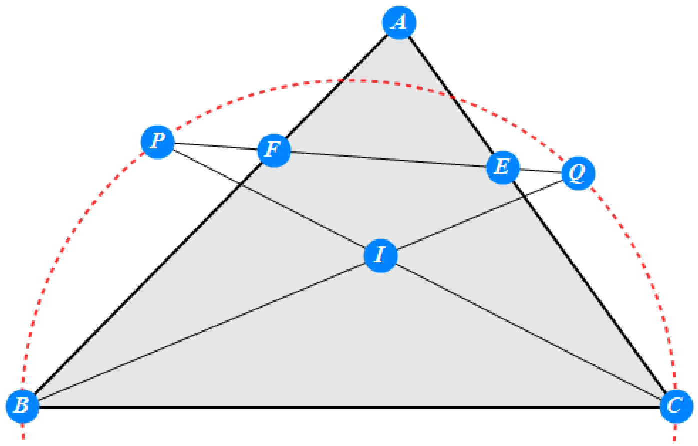

Example 133 : As shown in Figure 9 , in $\triangle ABC$, the inscribed circle I intersects AC and AF at E and F *respectively*, and the straight line EF intersects BI and CI at Q and P *respectively*. Prove that: B, C, P and Q share a circle.

Proof: $$\left(\frac{\frac{E-F}{I-B}}{\frac{C-I}{C-B}}\right)^2=\frac{\frac{E-F}{C-A}\frac{B-A}{B-I}\frac{C-B}{C-I}}{\frac{B-A}{F-E}\frac{B-I}{B-C}\frac{C-I}{C-A}}.$$

Explanation: B, C, P, Q four points cocircle are equivalent to $\angle PQB=\angle PCB$. In fact, we only need to slightly change the above formula to get a new identity $$\left(\frac{\frac{F-E}{A-C}}{\frac{B-I}{I-C}}\right)^2=\frac{\frac{E-F}{C-A}\frac{B-A}{B-I}\frac{C-I}{C-A}}{\frac{B-A}{F-E}\frac{B-I}{B-C}\frac{C-B}{C-I}},$$ which shows that $\angle QEC=\angle QIC$, and the four points I, C, Q, and E share a circle. It's a way of discovering new propositions. Transform some conditions in the title, including but not limited to: taking the reciprocal, adding and subtracting a real number, etc. After the recombination, if the obtained expression has obvious geometric meaning in the form, it is considered that a new proposition has been obtained.

The identities correspond to $\angle PQB=\angle PCB$, $\angle QEC=\angle QIC$.

Automatically discover new questions