Example 133 : As shown in Figure 8 , two circles intersect at points A and B, draw a straight line passing through A and B respectively and intersect the two circles at C, D, F, F *respectively*. Prove: CE // DF.

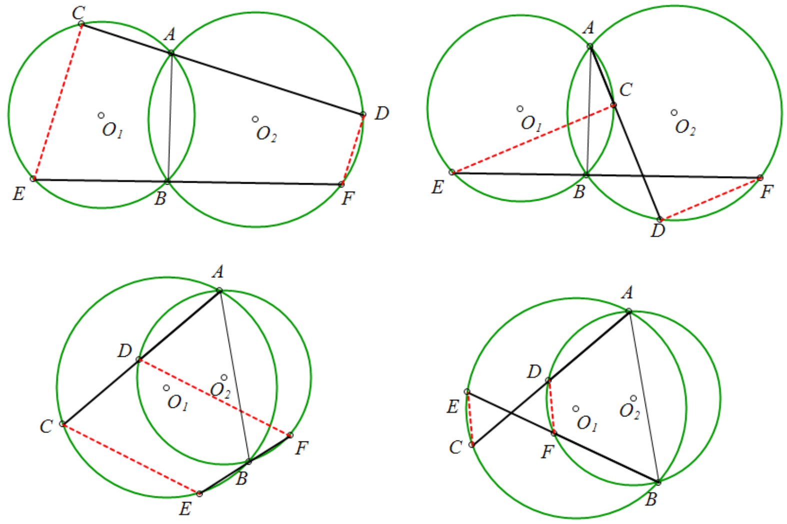

Figure 8 Multiple situations where two intersecting circles produce parallel lines

Proof: $\dfrac{C-E}{D-F}=\dfrac{C-A}{A-D}\dfrac{E-B}{B-F}\dfrac{(B-A)(C-E)}{(B-E)(C-A)}\dfrac{(A-D)(F-B)}{(A-B)(F-D)}$.

Explanation: $\dfrac{C-E}{D-F}\in R \Leftrightarrow CE$ // DF, $\dfrac{C-A}{A-D}\in R \Leftrightarrow C$, A, D are collinear,

$\dfrac{E-B}{B-F}\in R \Leftrightarrow E$, B, F are three points collinear, $\dfrac{(B-A)(C-E)}{(B-E)(C-A)}\in R \Leftrightarrow A$, C, E, B are four points collinear,

$\dfrac{(A-D)(F-B)}{(A-B)(F-D)}\in R \Leftrightarrow A$, B, F, D share a circle,

The establishment of the above identity is clear at a glance. Based on the identity, it can be known that any four of the five conditions are true, and the remaining one can be deduced to be true. This means that the identity method not only proves the original proposition, but also obtains a new proposition. Paper [24] also established the identity proof on this problem, but it needs to use more complex mathematical knowledge such as conformal geometric algebra, and the amount of calculation is large.

The situation of this question is diverse (Figure 8), and the default solution of traditional geometry is only for the first situation , so the proof is considered simple: $\angle ECA=\angle ABF=180^{\circ}-\angle ADF$, so $CE//DF$. In fact, $\angle ECA=\angle ABF$ it does not hold in the second case , at this time