



**Example 6 :** As shown in Figure 10, in  $\triangle ABC$ ,  $O$  and  $H$  are the circumcenter and orthocenter respectively, to prove:  $\angle BAO = \angle CAH$ . If  $\angle B < \angle C$ , then  $\angle ACB = \angle ABC + \angle HAO$ .

$$\left( \frac{\frac{B-A}{C-B} \frac{A-H}{C-A}}{\frac{B-C}{C-A}} \right)^2 = - \left( \frac{A-H}{B-C} \right)^2 \frac{\frac{B-A}{A-O} \frac{A-C}{C-O} \frac{B-O}{C-B}}{\frac{A-B}{C-A} \frac{C-O}{C-B}},$$

$$\left( \frac{\frac{A-C}{A-H} \frac{A-H}{A-O}}{\frac{A-B}{A-O}} \right)^2 = - \left( \frac{B-C}{A-H} \right)^2 \frac{\frac{B-A}{A-O} \frac{A-C}{C-O} \frac{B-O}{C-B}}{\frac{A-B}{C-A} \frac{C-O}{C-B}},$$

Make a question, use the combination of known conditions

Another proof: Suppose  $O = 0$ ,  $H = A + B + C$ ,  $\frac{\frac{A-0}{A-B}}{\frac{A-C}{A-(A+B+C)}} = T$ ,

$$\frac{\frac{A-0}{B-0}}{\frac{A-B}{B-0}} = T_1, \quad \frac{\frac{C-0}{B-0}}{\frac{B-C}{B-0}} = T_2, \quad \frac{\frac{C-0}{A-C}}{\frac{C-A}{A-0}} = T_3, \quad \text{then } T = T_1 + T_3 - T_1 T_2 T_3, \quad \text{the relationship}$$

between line segments  $AB^2 + AC^2 = BC^2 + AB \cdot AC \cdot \frac{AH}{AO}$ .