

Example 31: As shown in Figure 5, in \triangle *ABC*, the bisectors of the exterior angles of \angle B and \angle C intersect *at* point *P. Prove* that *AP* bisects \angle A. (sidecenter theorem)

Proof:
$$\frac{\frac{P-A}{B-A}}{\frac{C-A}{P-A}} - \frac{\frac{P-B}{B-A}}{\frac{C-B}{P-B}} - \frac{\frac{P-C}{B-C}}{\frac{C-A}{P-C}} = 1.$$

Explanation: It is not difficult to find that the proof of the inner theorem and the peripheral theorem are the same identity, but a little deformation has been made for the convenience of understanding. This means that the two theorems are somewhat equivalent. However, it is different when it is reflected in the relationship of side lengths, and the above-mentioned identity means that

$$\frac{PA^2}{BA \cdot CA} - \frac{PB^2}{CB \cdot AB} - \frac{PC^2}{AC \cdot BC} = 1_{\text{it is established.}}$$