Example 78 *ABC is triangle inscribed in a circle; DE is the diameter bisecting BC at G; from E a perpendicular EK is drawn to one of the sides, and the perpendicular from the vertex A on DE meets DE in H. Show that EK touches the circle GHK.*

Point order: $a, b, c, o, g, e, k, h, n$.

Hypotheses: circumcenter(o, b, a, c), midpoint(g, b, c), coll(g, o, e), pbisector(o, b, e), perp(k, e, a, b), coll(k, a, b), perp(a, h, o, g), coll(h, o, g), circumcenter(n, g, h, k).

Conclusion: perp(e, k, k, n).

The Machine Proof

$-[nk, ke] + 1$

(Since $ke \perp ab$; $[nk, ke] = [nk, ba] + 1$.)

$= -[nk, ba]$

(Since circumcenter(n,k, g, h); $[nk, ba] = [nk, kg] + [kg, ba] = [hk, hg] + [kg, ba] + 1$.)

$= -[hk, hg] - [kg, ba] - 1$

(Since $hg \perp bc$; $[hk, hg] = [hk, cb] + 1$.)

$= -[hk, cb] - [kg, ba]$

(Since e, g, h are collinear; h, k, e, a are cyclic; $[hk, cb] = [hk, he] + [eg, cb] = [ka, ea] + [eg, cb]$.)

$= -[kg, ba] - [ka, ea] - [eg, cb]$

(Since a, b, k are collinear; k, g, b, e are cyclic; $[kg, ba] = [kg, kb] = [eg, eb]$.)

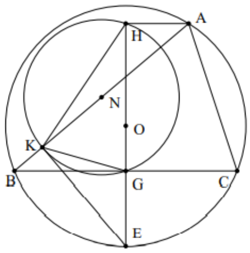

Figure 78

$= -[ka, ea] - [eg, eb] - [eg, cb]$

(Since a, b, k are collinear; $[ka, ea] = -[ea, ba]$.)

$= -[eg, eb] - [eg, cb] + [ea, ba]$

(Since $eg \perp bc$; $[eg, eb] = -[eb, cb] + 1$.)

$= -[eg, cb] + [eb, cb] + [ea, ba] - 1$

(Since $eg \perp cb$; $[eg, cb] = 1$.)

$= [eb, cb] + [ea, ba]$

(Since b, e, c, a are cyclic; $[eb, cb] = [ea, ca]$.)

$= [ea, ca] + [ea, ba]$

(Since circumcenter(o,a, e, c, b); $oc \perp eb$; $[ea, ca] = -[ea, ba]$.)

$= 0$