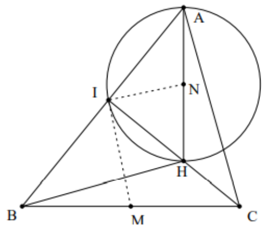
Example 2 12 : As shown in the figure, in $\triangle ABC$, O is the circumcenter, E *is* the midpoint of the inferior arc BC , G is the midpoint of BC , $EK \perp AB$ is at K , $AH \perp GE$ is at H , and N is the circumcenter of $\triangle GHK$, to prove: E is on AB .

$$\frac{N-K}{B-K}=\left(\frac{H-G}{H-K}\frac{K-N}{K-G}\right)\frac{\left(\frac{E-G}{G-H}\frac{A-K}{A-B}\right)}{\left(\frac{A-E}{A-B}\frac{E-B}{E-H}\right)}\frac{\frac{A-E}{A-K}\frac{K-G}{K-B}}{\frac{H-E}{H-K}\frac{E-G}{E-B}}$$

Example 66 *Show that in a triangle ABC the circles on AH and BC as diameters are orthogonal.*



Point order: a, b, c, i, h, n, m.

Hypotheses: foot(i, c, a, b), coll(h, c, i), perp(h, a, b, c), midpoint(m, c, b), midpoint(n, a, h).

Conclusion: perp(m, i, n, i).

The Machine Proof

$[mi, ni] + 1$

(Since circumcenter(m, i, b, c); $[mi, ni] = [mi, ib] + [ib, ni] = -[ni, ib] + [ic, cb] + 1$.)

$= -[ni, ib] + [ic, cb]$

(Since circumcenter(n, i, a, h); $[ni, ib] = [ni, ia] = [hi, ha] + 1$.)

$= -[hi, ha] + [ic, cb] - 1$

(Since $hi \parallel ic$; $-[hi, ha] + [ic, cb] = [ha, cb]$.)

$= [ha, cb] - 1$ (Since $ha \perp cb$; $[ha, cb] = 1$.)

$= 0$