

Example 26: As shown in Figure 1, in \triangle ABC, G is the center of gravity, point P satisfies $\angle PAB = \angle PBC = \angle PCA$, if the four points A, B, P and G share a circle, prove that: C, E, G and P share a circle, A, D, The four points G and E share a circle.

Proof:

$$\frac{\frac{A+C}{2}-P}{\frac{A+C}{2}-\frac{A+B+C}{3}} = \frac{\frac{A+B+C}{3}-P}{\frac{A+B+C}{2}-B} = \frac{\frac{C-P}{3}}{\frac{C-P}{C-A}} + 1 \qquad , \qquad \text{stating}$$

 $\angle PEG = \angle PCG \Leftrightarrow \angle PGB = \angle PAB$,

$$\frac{\frac{A+B+C}{3} - \frac{A+B}{2}}{\frac{A+B+C}{2} - A} + \frac{\frac{1}{2}}{\frac{A+C}{2} - \frac{A+B}{2}} + \frac{\frac{1}{2}}{\frac{A+B+C}{3} - P} = 1$$
, explain
$$\frac{A+C}{\frac{A+C}{2} - A} = \frac{\frac{A-P}{A-B}}{\frac{A-B}{B-P}} - \frac{\frac{A+B+C}{3} - B}{\frac{B-P}{B-C}}$$

 $\angle AED = \angle AGD \Leftrightarrow \angle PGB = \angle PBC$.