Problem 1. 設 $\triangle ABC$ 是一個以 A 為頂點的等腰三角形。兩點 P, Q 滿足

$$\measuredangle ABP = \measuredangle BCQ \quad 及 \quad \measuredangle PCA = \measuredangle QBC.$$

證明：A, P, Q 共線。

Example 39 : In $\triangle ABC$, $AB = AC$, two points P and Q satisfy $\angle ABP = \angle BCQ$, $\angle PCA = \angle QBC$, prove: A, P, Q are collinear.

Proof: Suppose $\dfrac{A-P}{P-Q} = t_1, \dfrac{\dfrac{B-A}{B-C}}{\dfrac{C-B}{C-A}} = t_2, \dfrac{\dfrac{B-A}{B-P}}{\dfrac{C-B}{C-Q}} = t_3, \dfrac{\dfrac{C-P}{C-A}}{\dfrac{B-Q}{B-C}} = t_4,$

$-t_2 - t_1t_3 + t_2t_4 + t_1t_2t_4 - t_2t_3t_4 = 0,$