

Example 180: As shown in Figure 3, there are four circles on the plane C_1 , C_2 , C_3 , C_4 . C_1 Intersecting C_2 at Z_1 sum W_1 , C_2 intersecting C_3 at Z_2 sum W_2 , C_3 intersecting C_4 at Z_3 sum W_3 , C_4 intersecting C_1 at Z_4 sum W_4 , to prove Z_1 that, Z_2 , Z_3 , Z_4 the necessary and sufficient condition for the four points to be in the same circle is that W_1 , W_2 , W_3 , W_4 the four points are in the same circle.

$$\frac{Z_1 - Z_2}{W_2 - Z_2} \frac{Z_2 - Z_3}{W_3 - Z_3} \frac{Z_3 - Z_4}{W_4 - Z_4} \frac{Z_4 - Z_1}{W_1 - Z_1} = \frac{Z_1 - Z_2}{Z_3 - Z_2} \frac{W_1 - W_2}{W_3 - W_2} \frac{Z_2 - W_2}{W_3 - W_2} \frac{Z_3 - W_3}{W_1 - W_4} = \frac{Z_1 - Z_2}{Z_3 - Z_2} \frac{W_1 - W_2}{W_3 - W_2} \frac{Z_3 - Z_2}{W_3 - W_2} \frac{W_1 - W_2}{W_3 - W_4}.$$