Full－ angle method

Example 81 *If the two bisectors of the angle A of the triangle ABC are equal, and the circle having BC for diameter cuts the sides AB, AC in the points P, Q, show that $CP \equiv CQ$.*

Point order: u, v, a, b, c, o, p, q.

Hypotheses: perp(a, u, a, v), cong(a, u, a, v), coll(u, v, b), eqangle(c, a, u, u, a, b), coll(u, v, c), midpoint(o, b, c), coll(a, b, p), coll(a, c, q), pbisector(o, b, q), pbisector(o, b, p).

Conclusion: pbisector(c, p, q).

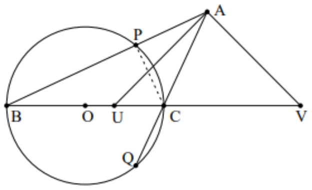


Figure 81

The Machine Proof

$-[qp, qc] - [qp, pc]$

(Since q, p, c, b are cyclic; $[qp, qc] = [pb, cb]$.)

$= -[qp, pc] - [pb, cb]$

(Since $pc \perp ab$; $[qp, pc] = [qp, ba] + 1$.)

$= -[qp, ba] - [pb, cb] - 1$

(Since a, c, q are collinear; q, p, c, b are cyclic; $[qp, ba] = [qp, qc] + [ca, ba] = [pb, cb] + [ca, ba]$.)

$= -2[pb, cb] - [ca, ba] - 1$

(Since a, b, p are collinear; $[pb, cb] = -[cb, ba]$.)

$= 2[cb, ba] - [ca, ba] - 1$

(Since b, c, u, v are collinear; $[cb, ba] = -[ba, vu]$.)

$= -[ca, ba] - 2[ba, vu] - 1$

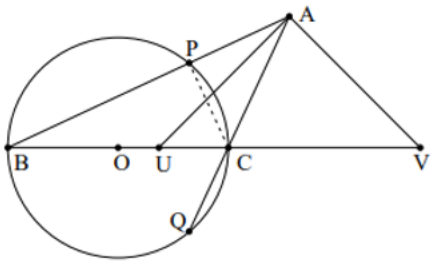
(Since $\angle[ac, ab] = \angle[ua, ab]$; $[ca, ba] = [ca, ua] + [ua, ba] = -2[ba, au]$.)

$= 2[ba, au] - 2[ba, vu] - 1$

(Since $ba \parallel ba$; $2[ba, au] - 2[ba, vu] = -2[au, vu]$.)

$= -2[au, vu] - 1$

(Since $au = av$ $ua \perp av$; $[au, vu] =_1 423772$.)



Example 2 11 : As shown in the figure, in $\triangle ABC$, AU and AV are the bisectors of the inner and outer angles of $\angle A$ respectively. A circle with BC as *the* diameter intersects AB and AC at P and Q *respectively* . If $AU = AV$, then $CP = CQ$.

$$\frac{\dfrac{P-C}{P-Q}}{\dfrac{Q-P}{C-A}} = \frac{\dfrac{B-A}{B-C}}{\dfrac{Q-P}{C-A}} \frac{\dfrac{P-C}{P-Q}}{\dfrac{B-C}{B-Q}} \frac{\dfrac{C-B}{V-A}}{\dfrac{U-A}{B-C}} \frac{\dfrac{A-U}{A-B}}{\dfrac{A-C}{A-U}} \frac{A-V}{A-U} \frac{A-C}{B-Q}$$