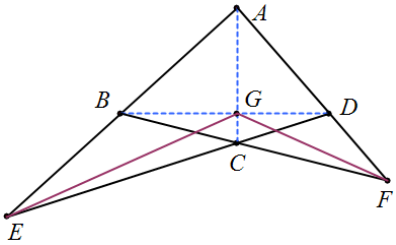

Example 51 : As shown in the figure, in the quadrilateral $ABCD$, AC intersects BD at G, AB intersects CD at E, and AD intersects BC at F. Prove that the necessary and sufficient condition for $AC \perp BD$ is $\angle EGC = \angle FGC$.

Proof: Suppose $D=\dfrac{xA+yB+zC}{x+y+z}$, $G=\dfrac{xA+zC}{x+z}$, $F=\dfrac{yB+zC}{y+z}$, $E=\dfrac{xA+yB}{x+y}$,

solve the equation

$$k_1\left(\frac{B-D}{A-C}\right)^2+k_2\frac{\dfrac{G-F}{A-C}}{\dfrac{A-C}{G-E}}=k_3$$, available

$$y^2(x+y+z)^2\left(\frac{B-D}{A-C}\right)^2-(x+y)(x+z)^2(y+z)\frac{\dfrac{G-F}{A-C}}{\dfrac{A-C}{G-E}}=x^2z^2.$$