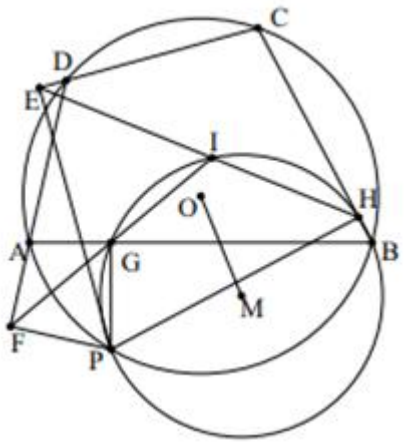
Example 178 : As shown in Figure 3, P is a point on the circumscribed circle O of the quadrilateral $ABCD$, and the feet of P on CD, DA, AB, and BC are E, F, G, and H respectively. EH intersects FG and I. Prove: G, P, H, I are four points in a circle.



$$\frac{I-H}{I-G}\frac{P-G}{P-H}=\left(\frac{H-E}{H-C}/\frac{P-E}{P-C}\right)\left(\frac{P-F}{P-A}/\frac{G-F}{G-A}\right)\left(\frac{C-D}{C-P}\frac{A-P}{A-D}\right)\left(\frac{C-H}{H-P}\frac{G-P}{A-G}\right)\frac{I-H}{H-E}\frac{F-G}{G-I}\frac{A-D}{F-P}\frac{E-P}{C-D}$$