Example 79 : As shown in Figure 3, there is a *point* P inside $\triangle ABC$, which satisfies $\angle PBA + \angle PCA = \angle PBC + \angle PCB$. Prove that: B, C, P, *and* I share a circle, and I is the center of $\triangle ABC$.

Proof: $\left(\dfrac{\dfrac{P-C}{P-B}}{\dfrac{I-C}{I-B}}\right)^2 = \dfrac{\dfrac{B-I}{B-C}\dfrac{C-B}{C-I}}{\dfrac{B-A}{B-I}\dfrac{C-I}{C-A}}\left(\dfrac{\dfrac{B-A}{B-P}\dfrac{C-P}{C-A}}{\dfrac{B-P}{B-C}\dfrac{C-B}{C-P}}\right)$.