



**Example 60 :** As shown in Figure 3,  $I$  is the heart of  $\triangle ABC$ , to prove:

$$\angle BIC = 90^\circ + \frac{1}{2} \angle A.$$

Proof: 
$$\frac{\left(\frac{I-C}{I-B}\right)^2 \frac{B-I}{A-C} \frac{C-B}{B-A} \frac{C-I}{C-I}}{\frac{A-B}{B-I} \frac{C-A}{C-I}} = -1.$$

Explanation: 
$$\frac{\left(\frac{I-C}{I-B}\right)^2}{\frac{A-C}{A-B}} \in R$$
 Can only describe  $2\angle BIC = \angle A$  or

$$2\angle BIC - \angle A = 180^\circ.$$

Method 1: With the aid of graphs, it is impossible to hold  $\angle BIC > \angle A$  since  $2\angle BIC = \angle A$ .

Method 2: Further clarify the positive and negative of each item, not just judge whether it is a real number. Among them, known conditions:  $\angle IBC = \angle$

$$ABI; \angle BCI \Leftrightarrow \frac{\frac{B-I}{B-C} \frac{B-C}{B-A}}{\frac{B-I}{B-I}} \in R^+ = \angle ICA; \text{ so } \Leftrightarrow \frac{\frac{C-B}{C-I} \frac{C-I}{C-A}}{\frac{C-B}{C-A}}, \frac{\left(\frac{I-C}{I-B}\right)^2}{\frac{A-C}{A-B}} \in R^- \text{ explain}$$

$$2\angle BIC - \angle A = 180^\circ.$$