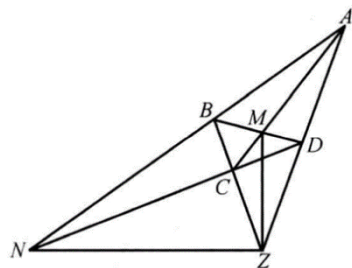


**Example 50 :** As shown in the figure, in the quadrilateral  $ABCD$ ,  $AC$  intersects  $BD$  at  $M$ ,  $AB$  intersects  $CD$  at  $N$ , and  $AD$  intersects  $BC$  at  $Z$ . Prove that the necessary and sufficient condition for  $ZM \perp ZN$  is  $\angle AZM = \angle BZM$ .



Proof: Suppose  $D = \frac{xA + yB + zC}{x + y + z}$ ,  $M = \frac{xA + zC}{x + z}$ ,  $Z = \frac{yB + zC}{y + z}$ ,  $N = \frac{xA + yB}{x + y}$ ,

solve the equation

$$k_1 \left( \frac{Z - N}{Z - M} \right)^2 + k_2 \frac{\frac{Z - B}{Z - M}}{\frac{Z - A}{Z - M}} = k_3, \quad \text{available} \quad (x + y)^2 \left( \frac{Z - N}{Z - M} \right)^2 - 4xy \frac{\frac{Z - B}{Z - M}}{\frac{Z - A}{Z - M}} = (x + z)^2.$$