USAJMO 2012
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by BOGTRO, tc1729, rusczyk

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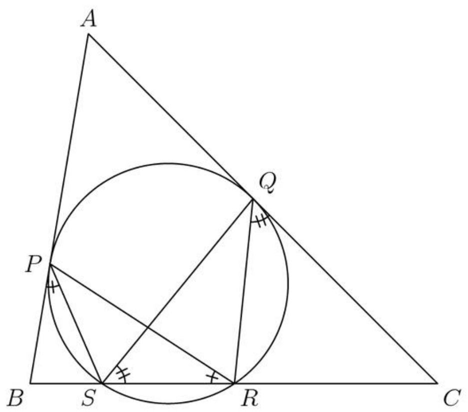
1 Given a triangle ABC, let P and Q be points on segments $\overline{AB}$ and $\overline{AC}$, respectively, such that $AP = AQ$. Let S and R be distinct points on segment $\overline{BC}$ such that S lies between B and R, $\angle BPS = \angle PRS$, and $\angle CQR = \angle QSR$. Prove that P, Q, R, S are concyclic (in other words, these four points lie on a circle).

Example 210 : As shown in Figure 1 , given $\triangle ABC$, P and Q *are points* on sides AB and AC *respectively* , and $AP = AQ$; S and R are two different points on side BC , and point S is located between B and R , ∠BPS = ∠PRS , ∠CQR = ∠QSR . _ _ Proof: The four points P , Q , R , S share a circle. (2012 USA Mathematical Olympiad *USAJMO* test questions)



$$\frac{\frac{P-S}{P-B}}{\frac{C-B}{R-P}}\frac{\frac{S-Q}{B-C}}{\frac{Q-C}{Q-R}}\frac{\frac{Q-C}{Q-P}}{\frac{P-Q}{P-B}}\left(\frac{P-Q}{P-S}/\frac{R-Q}{B-C}\right)\left(\frac{C-B}{R-P}/\frac{Q-S}{Q-P}\right)=1$$

This question is a bit confusing to understand. It is easy to understand if it is said another way: As shown in the figure, given $\triangle ABC$, P and Q *are* points on sides AB and AC respectively , S and R are two different points on side BC , *and point* S *is* located between B and R , ∠BPS = $\angle PRS$, ∠CQR = ∠QSR . _ _ If P , Q , R , S share a circle , prove that $AP = AQ$.