



Example 58 : As shown in Figure 3, O is the circumcenter of $\triangle ABC$, to prove: (Circle center angle theorem).

Proof:
$$\frac{\left(\frac{A-C}{A-B}\right)^2 \frac{C-O}{O-C} \frac{B-A}{O-B}}{\frac{C-A}{A-C} \frac{B-O}{A-O} \frac{B-A}{A-B}} = 1.$$

If the three points O , B and C are collinear, it is Thales' theorem.

The known conditions: $\angle OCA = \angle OAC \Leftrightarrow \frac{\frac{C-O}{A-C}}{\frac{A-O}{A-B}} \in R$; $\angle OBA = \angle OAB \Leftrightarrow$

$\frac{\frac{B-A}{A-B}}{\frac{B-O}{A-O}} \in R$; Prove the conclusion $2\angle BAC = \angle BOC \Leftrightarrow \frac{\left(\frac{A-C}{A-B}\right)^2 \frac{C-O}{O-C}}{\frac{B-O}{A-O} \frac{B-A}{A-B}} \in R$. In

Euclidean geometry, it is often necessary to give corresponding proofs according to different positions of points. However, using the method of complex identities, the proof can be unified. Moreover, identities are good for understanding inverse propositions. O is the circumcenter of $\triangle ABC$, available $2\angle BAC = \angle BOC$. But

not the other way around, because $\frac{\left(\frac{A-C}{A-B}\right)^2 \frac{C-O}{O-C}}{\frac{B-O}{A-O} \frac{B-A}{A-B}} \in R$ they cannot be rolled out

separately $\frac{\frac{C-O}{A-C}}{\frac{A-O}{A-B}} \in R, \frac{\frac{B-O}{A-O}}{\frac{B-A}{A-B}} \in R.$