MSDS 6371 HW 2

1. The world's smallest mammal is the bumblebee bat, also known as the Kitti's hog nosed bat. Such bats are roughly the size of a large bumblebee! Listed below are weights (in grams) from a sample of these bats. Test the claim that these bats come from the same population having a mean weight equal to 1.8 g. *(Beware: This data is **NOT** the same as in the lecture slides!)*
Sample: 1.7 1.6 1.5 2.0 2.3 1.6 1.6 1.8 1.5 1.7 1.2 1.4 1.6 1.6 1.6

a. Perform a complete analysis using SAS. Use the six step hypothesis test with a conclusion that includes a statistical conclusion, a confidence interval and a scope of inference (as best as can be done with the information above … there are many correct answers given the vagueness of the description of the sampling mechanism.)

Step1: H_0 : u = 1.8 H_a: u ≠ 1.8

Step2: Critical values: (α = .05 = significance level) & df = 15 – 1 = 14

$t_{.975,14}$ = -2.145 & $t_{.975,14}$= 2.145

Step3:

Sample mean = (1.7+1.6+1.5+2.0+2.3+1.6+1.6+1.8+1.5+1.7+1.2+1.4+1.6+1.6+1.6)/15 = 1.6467

Standard deviation = 0.2532

- $t = \frac{1.6467-1.8}{0.2532/\sqrt{15}} = -2.345$

Step4: p-value = 0.0342 < 0.05

Step5: The sample mean we found is very unusual under the assumption that the true mean weight is 1.8g. So we reject the assumption that the true mean weight is 1.8g. We reject the H_0.

Step6: At the .05 significance level, we reject the null hypothesis (p=0.0342). There is sufficient evidence to conclude that the mean weight of bumblebees is different than 1.8g. A 95% confidence interval for the mean weight is (1.5065 1.7869)g. The problem was ambiguous on the randomness of the sample thus we will assume that it was not a random sample which makes inference to all bats strictly speculative.

b. Inspect and run this R Code and compare the results (t statistic, p-value and confidence interval) to those you found in SAS. To run the code, simply copy and paste the below code into R.

sample = c(1.7, 1.6, 1.5, 2.0, 2.3, 1.6, 1.6, 1.8, 1.5, 1.7, 1.2, 1.4, 1.6, 1.6, 1.6)

t.test(x=sample, mu = 1.8, conf.int = "TRUE", alternative = "two.sided")

One Sample t-test

data: sample

t = -2.3457, df = 14, p-value = 0.03424

alternative hypothesis: true mean is not equal to 1.8

95 percent confidence interval:

1.506466 1.786868

sample estimates:

mean of x

1.646667

2. In the United States, it is illegal to discriminate against people based on various attributes. One example is age. An active lawsuit, filed August 30, 2011, in the Los Angeles District Office is a case against the American Samoa Government for systematic age discrimination by preferentially firing older workers. Though the data and details are currently sealed, suppose that a random sample of the ages of fired and not fired people in the American Samoa Government are listed below:

Fired

34 37 37 38 41 42 43 44 44 45 45 45 46 48 49 53 53 54 54 55 56

Not fired

27 33 36 37 38 38 39 42 42 43 43 44 44 44 45 45 45 45 46 46 47 47 48 48 49 49 51 51 52 54

a. Perform a permutation test to test the claim that there is age discrimination. Provide the Ho and Ha, the p-value, and full statistical conclusion, including the scope (inference on population and causal inference). Note: this was an example in Live Session 1. You may start from scratch or use the sample code and PowerPoints from Live Session 1.

```
##
## Two Sample t-test
##
## data:  Fired and Not_fired
## t = 1.0991, df = 49, p-value = 0.2771
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -1.593635  5.441254
##sample estimates:
## mean of x mean of y
## 45.85714  43.93333

proc iml;
Fired = {34, 37, 37, 38, 41, 42, 43, 44, 44, 45, 45, 45, 46, 48, 49, 53, 53, 54, 54, 55, 56};
NotFired = {27, 33, 36, 37, 38, 38, 39, 42, 42, 43, 43, 44, 44, 44, 45, 45, 45, 45, 46, 46, 47, 47, 48, 
48, 49, 49, 51, 51, 52, 54};
obsdiff = mean(Fired) - mean(NotFired);
```

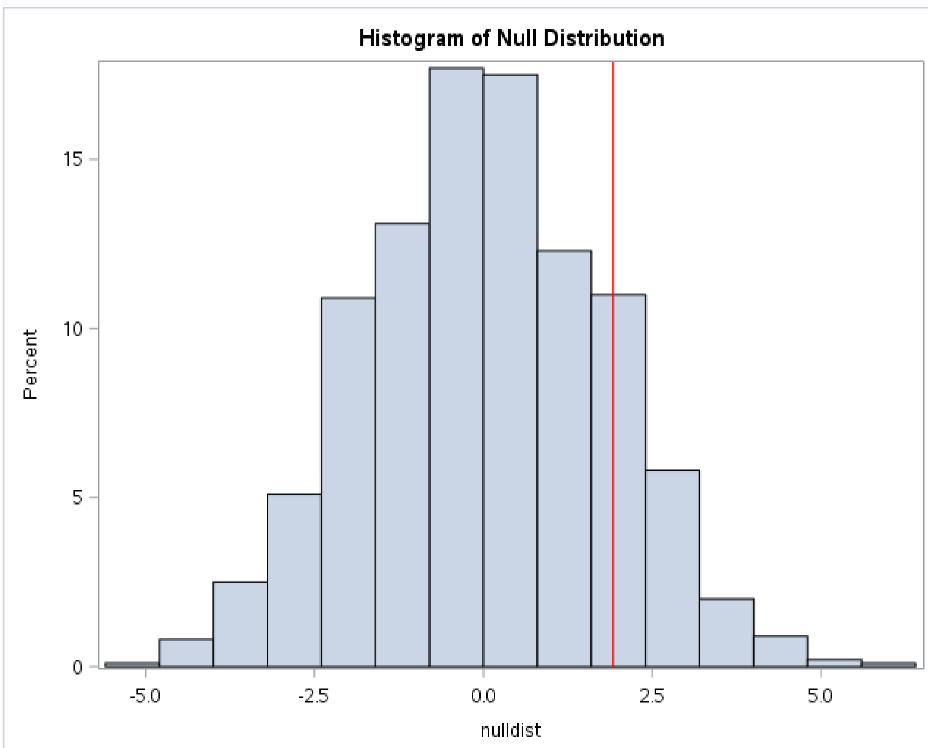
```
print obsdiff;

call randseed(12345);                    /* set random number seed */
alldata = Fired // NotFired;             /* stack data in a single vector */
N1 = nrow(Fired);  N = N1 + nrow(NotFired);
NRepl = 1000;                            /* number of permutations */
nulldist = j(NRepl,1);               /* allocate vector to hold results */
do k = 1 to NRepl;
  x = sample(alldata, N, "WOR");                  /* permute the data */
  nulldist[k] = mean(x[1:N1]) - mean(x[(N1+1):N]);  /* difference of means */
end;

title "Histogram of Null Distribution";
refline = "refline " + char(obsdiff) + " / axis=x lineattrs=(color=red);";
call Histogram(nulldist) other=refline;

pval = (1 + sum(abs(nulldist) >= abs(obsdiff))) / (NRepl+1);
print pval;
```

obsdiff
1.9238095



Histogram of Null Distribution

pval
0.2997003

To test for a difference of population means between the Fired and NotFired groups, a permutation test was conducted on 1,000 random permutations of the data. A histogram of the 1,000 differences of sample means from the 1,000 permutations can be seen above. The observed difference was 1.9238095 and 299 of the 1000 permutations yielded a difference in sample means that was as extreme or more extreme than this observed difference (p-value = 135/1000 = 0.299).

The null hypothesis is Ho: µFired- µNotFired=0

The alternate hypothesis is Ha: µFired-µNotFired≠0

The mean for Fired was 45.8571 and NotFired 43.9333

Statistical conclusion: (From a two-sided t-test) P-value = 0.299> 0.05. We failed to reject that the true mean age of those who got fired is equal to that of those who were not fired. A 95% confidence interval for this difference is (-1.5936 to 5.4413).

b. Now run a two sample t-test appropriate for this scientific problem. (Use SAS.) *(Note: we may not have talked much about a two-sided versus a one-sided test. If you would like to read the discussion on pg. 44 (Statistical Sleuth), you can run a one-sided test if it seems appropriate. Otherwise, just run a two-sided test as in class. There are also examples in the Statistics Bridge Course.)* Be sure to include all six steps, a statistical conclusion, and scope of inference.

The TTEST Procedure

Variable: age

status	Method	N	Mean	Std Dev	Std Err	Minimum	Maximum
Fired		21	45.8571	6.5214	1.4231	34.0000	56.0000
NotFired		30	43.9333	5.8835	1.0742	27.0000	54.0000
Diff (1-2)	Pooled		1.9238	6.1519	1.7503		
Diff (1-2)	Satterthwaite		1.9238		1.7830		

status	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
Fired		45.8571	42.8886	48.8256	6.5214	4.9893	9.4173
NotFired		43.9333	41.7364	46.1303	5.8835	4.6857	7.9093
Diff (1-2)	Pooled	1.9238	-1.5936	5.4413	6.1519	5.1389	7.6661
Diff (1-2)	Satterthwaite	1.9238	-1.6790	5.5266			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	49	1.10	0.2771
Satterthwaite	Unequal	40.268	1.08	0.2870

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	20	29	1.23	0.6005

c. Compare this p-value to the randomized p-value found in the previous sub-question.

This p-value = 0.2771

Randomized p-value in previous HW: p-value = 0.2997

They are very close

d. The jury wants to see a range of plausible values for the difference in means between the fired and not fired groups. Provide them with a confidence interval for the difference of means and an interpretation.

A 95% confidence interval for this difference is (-1.5936 to 5.4413,). We are 95% confident that the true difference in means between the fired and not fired groups is contained within the interval (-1.5936,5.4413).

e. Given the sample standard deviations from SAS, calculate by hand

i. Pooled standard deviation (s_p)

Sp = √(20*6.52142a^2 +29*5.88352^2)/(51-2) = 6.15186

ii. The standard error of $(\bar{X}_{FIRED} - \bar{X}_{Not\ Fired})$
$(\bar{X}_{FIRED} - \bar{X}_{Not\ Fired})$= 45.8571 - 43.9333 = -.0762

f. Inspect and run this R Code and compare the results (t statistic, p-value, and confidence interval) to those you found in SAS. To run the code, simply copy and paste the code below into R.

Fired = c(34, 37, 37, 38, 41, 42, 43, 44, 44, 45, 45, 45, 46, 48, 49, 53, 53, 54, 54, 55, 56)

Not_fired = c(27, 33, 36, 37, 38, 38, 39, 42, 42, 43, 43, 44, 44, 44, 45, 45, 45, 45, 46, 46, 47, 47, 48, 48, 49, 49, 51, 51, 52, 54)

t.test(x = Fired, y = Not_fired, conf.int = .95, var.equal = TRUE, alternative = "two.sided")

Two Sample t-test

data: Fired and Not_fired

t = 1.0991, df = 49, p-value = 0.2771

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-1.593635 5.441254

sample estimates:

mean of x mean of y

45.85714 43.93333

3. In the last homework, it was mentioned that a Business Stats professor here at SMU polled his class and asked students them how much money (cash) they had in their pockets at that very moment. The idea was that we wanted to see if there was evidence that those in charge of the vending machines should include the expensive bill / coin acceptor or if it should just have the credit card reader. However, another professor from Seattle University was asked to poll her class with the same question. Below are the results of our polls.
SMU
34, 1200, 23, 50, 60, 50, 0, 0, 30, 89, 0, 300, 400, 20, 10, 0
Seattle U
20, 10, 5, 0, 30, 50, 0, 100, 110, 0, 40, 10, 3, 0

a. Run a two sample t-test to test if the mean amount of pocket cash from students at SMU is different than that of students from Seattle University. Write up a complete analysis: all 6 steps including a statistical conclusion and scope of inference (similar to the one from the PowerPoint). (This should include identifying the Ho and Ha as well as the p-value.) Also include the appropriate confidence interval. **FUTURE DATA SCIENTIST'S CHOICE!: YOU MAY USE SAS *OR* R TO DO THIS PROBLEM!**

The TTEST Procedure

Variable: amount

school	Method	N	Mean	Std Dev	Std Err	Minimum	Maximum
SEATTLE		14	27.0000	36.7193	9.8136	0	110.0
SMU		16	141.6	304.3	76.0670	0	1200.0
Diff (1-2)	Pooled		-114.6	224.1	82.0131		
Diff (1-2)	Satterthwaite		-114.6		76.6974		

school	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
SEATTLE		27.0000	5.7989	48.2011	36.7193	26.6198	59.1564
SMU		141.6	-20.5079	303.8	304.3	224.8	470.9
Diff (1-2)	Pooled	-114.6	-282.6	53.3711	224.1	177.8	303.1
Diff (1-2)	Satterthwaite	-114.6	-277.6	48.3948			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	28	-1.40	0.1732
Satterthwaite	Unequal	15.499	-1.49	0.1551

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	15	13	68.66	<.0001

Step1: Ho: µSMU- µSEATTLE=0 Ha: µSMU-µSEATTLE≠0

Step2: Critical values: (α = .05 = significance level) & df = 28 – 2 = 26

$t_{.975,26}$ = -2.056 & $t_{.975,26}$= 2.056

Step3: The mean for SMU was 141.6 and SEATTLE 27 & Standard deviation = 0.2532

$$sp=\frac{(14-1)36.7193^2+(16-1)304.3^2}{14+16-2}=224.1$$

$$t=\frac{27-141.6}{224.1\sqrt{\frac{1}{14}+\frac{1}{16}}}=1.397$$

Step4: p-value = 0.1732> 0.05

Step5: Failed to reject

Step6: (From a two-sided t-test) P-value = 0.173 > 0.05. This does not provide sufficient evidence against the null hypothesis that the mean pocket cash of SMU students is equal to that of Seattle University students at a significance level of 0.5 or even 0.10. A 95% confidence interval for this difference is (-282.6, 53.3711).Since this is not a random sample of the students we can't generalize this inference to all the students in the university. Since we failed to reject Ho, there is no need to write up whether casual inference can be drawn.

b. Compare the p-value from this test with the one you found from the permutation test from last week. Provide a short 2 to 3 sentence discussion on your thoughts as to why they are the same or different.

A p-value of 0.1538 last week with permutation test compared to .1732 with SAS t-test.

4. A. Calculate the estimate of the pooled standard deviation from the Samoan discrimination problem. Use this estimate to build a power curve. Assume we would like to be able to detect effect sizes between 0.5 and 2 and we would like to calculate the sample size required to have a test that has a power of .8. Simply cut and paste your power curve and SAS code. HINT: USE THE CODE FROM DR. McGEE's lecture. Instead of using **groupstddevs**, use **stddev** since we are using the pooled estimate.

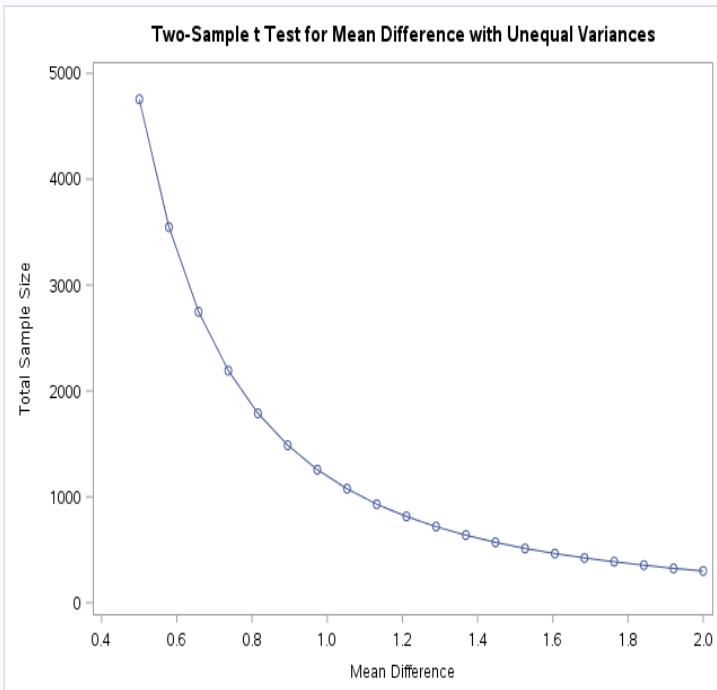
```
proc power;
    twosamplemeans test=diff_satt
    power = 0.8
    ntotal = .
    stddev = 6.1519
    meandiff = 1.9238
    alpha = 0.05
    sides = 2;
    plot x=effect min=0.5 max=2;
    run;
```

The POWER Procedure
Two-Sample t Test for Mean Difference with Unequal Variances

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Number of Sides	2
Nominal Alpha	0.05
Mean Difference	1.9238
Standard Deviation	6.1519
Nominal Power	0.8
Null Difference	0
Group 1 Weight	1
Group 2 Weight	1

Computed N Total		
Actual Alpha	Actual Power	N Total
0.05	0.801	324

The POWER Procedure
Two-Sample t Test for Mean Difference with Unequal Variances



B. Now suppose we decided that we may be able to live with slightly less power if it means savings in sample size. Provide the same plot as above but this time calculate curves of sample size (y-axis) vs. effect size (.5 to 2) (x axis) for power = 0.8, 0.7, and 0.6. There should be three plots on your final plot. Simply cut and paste your power curve and SAS code. HINT: USE THE CODE FROM DR. McGEE's lecture. Instead of using **groupstddevs**, use **stddev** since we are using the pooled estimate. The effect size here refers to a difference in means, though there are many effect size metrics, such a Cohen's D.

```
proc power;
    twosamplemeans test=diff_satt
    power = 0.8, 0.7, 0.6
    ntotal = .
    stddev = 6.1519
```

```
meandiff = 1.9238
alpha = 0.05
sides = 2;
plot x=effect min=0.5 max=2;
run;
```

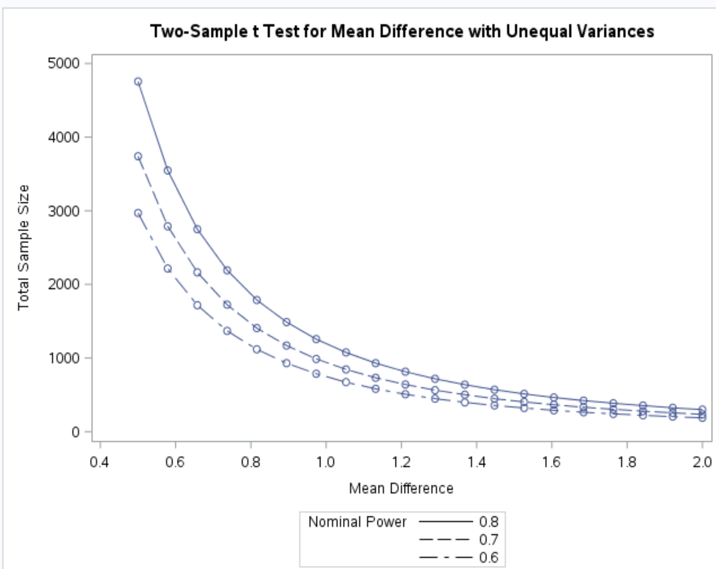
The POWER Procedure
Two-Sample t Test for Mean Difference with Unequal Variances

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Number of Sides	2
Nominal Alpha	0.05
Mean Difference	1.9238
Standard Deviation	6.1519
Null Difference	0
Group 1 Weight	1
Group 2 Weight	1

Computed N Total

Index	Nominal Power	Actual Alpha	Actual Power	N Total
1	0.8	0.05	0.801	324
2	0.7	0.05	0.703	256
3	0.6	0.05	0.604	204

The POWER Procedure
Two-Sample t Test for Mean Difference with Unequal Variances



C. Using similar code, estimate the savings in sample size from a test aimed at detecting an effect size of 0.8 with a power of 80% versus a power of 60%.
The difference between 80% power and 60% is 120.

Note: You will learn how to do this in R in a future HW!