UNIT 6 HW

1. **Handicap Study.** Use the Bonferroni method to construct simultaneous confidence intervals for $\mu_2 - \mu_3, \mu_2 - \mu_5$, and $\mu_3 - \mu_5$ (to see whether there are differences in attitude toward the mobility type of handicaps).

$\mu_1, \mu_2, \mu_3, \mu_4$, and μ_5, are the mean scores in the none, amputee, crutches, hearing, and wheelchair groups respectively. Be careful when identifying 'k' here. This study is mentioned throughout Chapter 6 of <u>Statistical Sleuth</u>.

T= 2.90602 SP=1.633 SE=$(1.633)\sqrt{\frac{1}{14}+\frac{1}{14}}$=0.6172

$\mu_2 - \mu_3 \in 4.4286 - 5.9214 \pm 2.90602(0.6172)$

-1.6354 ± 1.7936

(-3.429 ,0.1582)

$\mu_2 - \mu_5 \in 4.4286 - 5.3429 \pm 2.90602(0.6172)$

-0.9143 ± 1.7936

(-2.7079,0.8793)

$\mu_3 - \mu_5 \in 5.9214 - 5.3429 \pm 2.90602(0.6172)$

0.5785 ± 1.7936

(-1.2151,2.3721)

2. **Handicap Study.**

See what multiple comparison procedures are available within the one-way analysis of variance procedure. Verify the 95% confidence interval half-widths in Display 6.6.

DISPLAY 6.6 Summary of 95% confidence interval procedures for differences between treatment means in the handicap study

		Difference with . . .			
Group	Average	Hearing	Amputee	Control	Wheelchair
Crutches	5.921	1.871	1.492	1.021	0.578
Wheelchair	5.343	1.293	0.914	0.443	
Control	4.900	0.850	0.471		
Amputee	4.429	0.379			
Hearing	4.050				

A confidence interval is centered at a difference with half-width given by one of the procedures.

Procedure	95% interval half-width
LSD	1.233
Dunnett	1.545 (for comparisons with control only)
Tukey–Kramer	1.735
Bonferroni	1.794
Scheffé	1.957

```
proc print data = handi;
run;


proc glm data = handi;
class Handicap;
model Score = Handicap;

/* Construct CI for Treatment Means*/
means Handicap /alpha=.05 lsd clm;
means Handicap / alpha=.05 bon clm;
/* Pairwise Comparison*/
means Handicap /alpha=.05 lines lsd;
means Handicap /alpha=.05 lines bon;
means Handicap /alpha=.05 lines scheffe;
means Handicap /alpha=.05 lines tukey;
means Handicap /dunnett;
run;
```

LSD: $HW = t_{\left(1-\frac{\alpha}{2}\right),\, df} \cdot SE$

= 1.9714 *.6172

= 1.2326 rounded to 1.233

The GLM Procedure

t Tests (LSD) for Score

Note: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of t	1.99714
Least Significant Difference	1.2326

Bonferroni: $HW = t_{\left(1-\frac{\alpha}{2k}\right),\, df} \cdot SE$

= 2.90602*.6172

= 1.79359 rounded to 1.7936

The GLM Procedure

Bonferroni (Dunn) t Tests for Score

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of t	2.90602
Minimum Significant Difference	1.7936

Dunnett: $HW = t_{v,1-\alpha/2}(\mu, \Sigma) \cdot SE$

= 2.50316*.6172

= 1.5449

The GLM Procedure

Dunnett's t Tests for Score

Note: This test controls the Type I experimentwise error for comparisons of all treatments against a control.

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of Dunnett's t	2.50316
Minimum Significant Difference	1.5449

Tukey: $HW = \frac{q_{I,n-I}(1-\alpha)}{\sqrt{2}} \cdot SE$

== in the sas the 95th percentile is 3.96804 and in the book it is 3.975

= in SAS 2.8057 * .6172 which equals 1.7317

= in the book it is 2.8107 * .6172 which equals 1.735

= so difference of .0018

The GLM Procedure

Tukey's Studentized Range (HSD) Test for Score

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of Studentized Range	3.96804
Minimum Significant Difference	1.7317

Scheffe: $HW = \sqrt{(I-1) \cdot F_{I-1,n-I}(1-\alpha)} \cdot SE$

= F distribution is 2.51304 so multiplier is 3.1705

= 3.1705*.6172

= 1.9568

The GLM Procedure

Scheffe's Test for Score

Note: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of F	2.51304
Minimum Significant Difference	1.9568

Show your work for this problem by simply copying the code and relevant output for each comparison. (Cut and paste your code and relevant output.) The half-width might be found directly from your output. If so, note where it is found. Do this for both R and SAS.

3. **Education and Future Income.** Reconsider the data problem of Exercise 5.25 concerning the distributions of annual incomes in 2005 for Americans in each of five education categories. (a) Use the Tukey–Kramer procedure to compare every group to every other group. Which pairs of means differ and by how many dollars (or by what percent)? (Use p-values and confidence intervals in your answer.) (b) Use the Dunnett procedure to compare every other group to the group with 12 years of education. Which group means apparently differ from the mean for those with 12 years of education and by how many dollars (or by what percent)? (Use p-values and confidence intervals in your answer.)

This question is obviously from the book, but assume you are starting this problem from scratch. Show all parts:

(1) Discussion of Assumptions (This could result in the inferences no longer being about the means. IF that happens, you should still compare the groups, just use the appropriate parameters when making inferences. Remember that you already did the work for addressing assumptions in prior homeworks.)

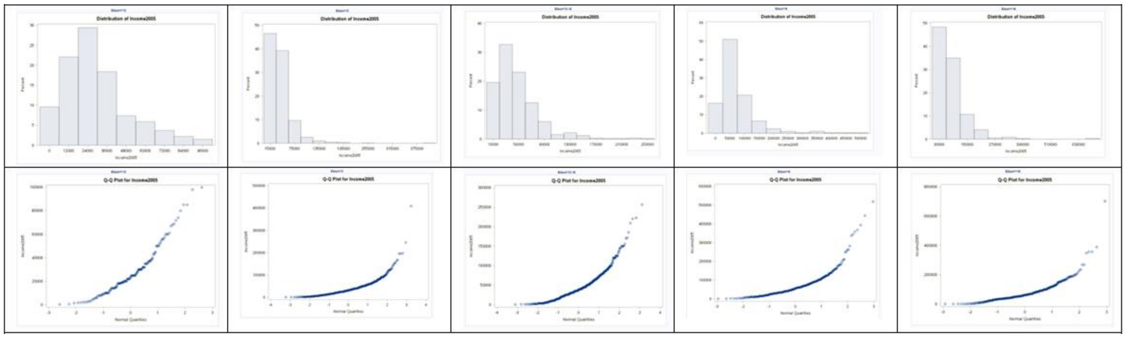
(2) Selection and Execution of Tests

(3) Interpretation and Conclusion.

In short, perform a complete analysis like you usually do. Provide and interpret all the confidence intervals that suggest a significant difference in incomes; provide your SAS and R code as well. (Generate your statistics using both softwares.)

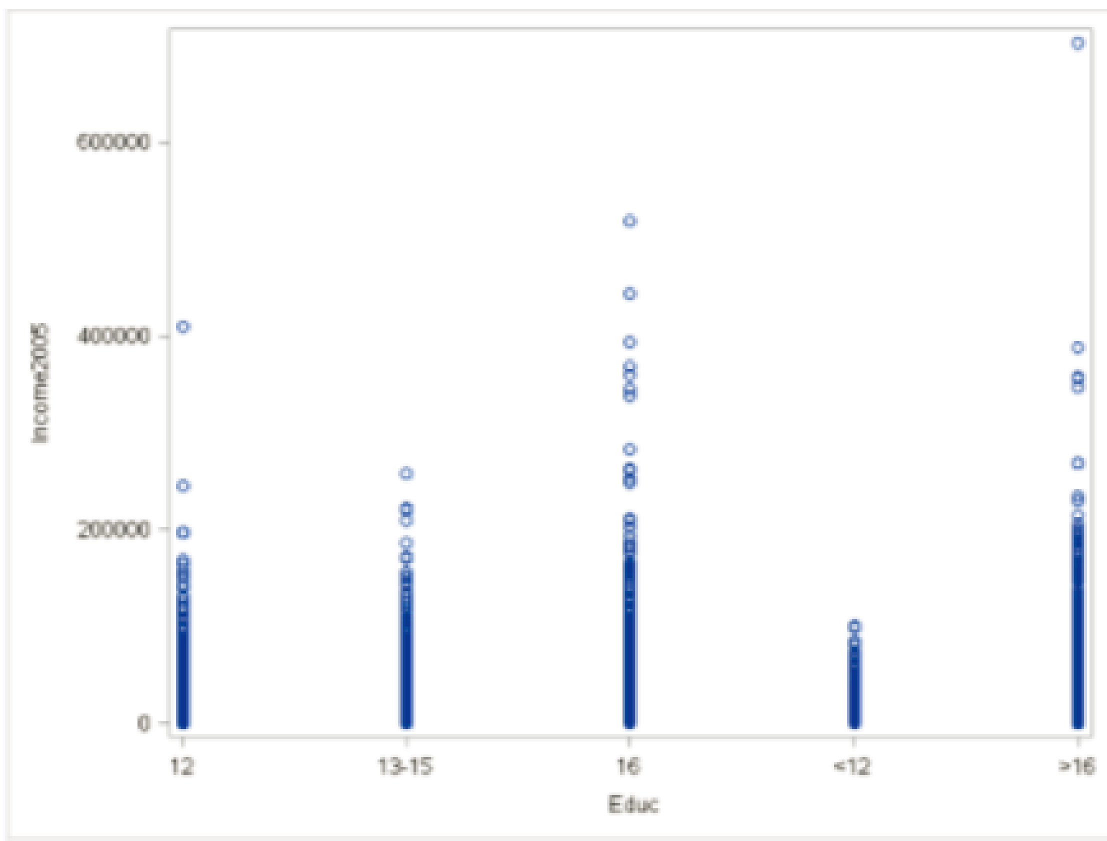
Finally, you should first test to see if any of the groups are different before you consider pairwise comparisons.

```
*To address ANOVA assumptions on original data with histograms and QQ plots;
proc univariate data = incomedata;
by educ;
histogram income2005;
qqplot income2005;
run;
```



Equal Standard Deviations (3 points): It appears that the original data shows evidence against equal standard deviations in the scatter plot. We were given in the problem that we are able to assume that the standard deviations between the groups are equal (homoscedasticity) for log transformed data. This is a safe assumption visually. Small deviations in sd will have less effect on the test than larger deviations. Remember, all models are wrong, but some are useful. [George Box]

```
*To address ANOVA assumptions on original data with scatter plots;
proc sgplot data = incomedata;
scatter x= educ y = income2005;
run;
```



Independence : We will assume the data are independent, both between and within groups, and proceed with the Tukey-Kramer test for differences in mean income between the five levels of education.

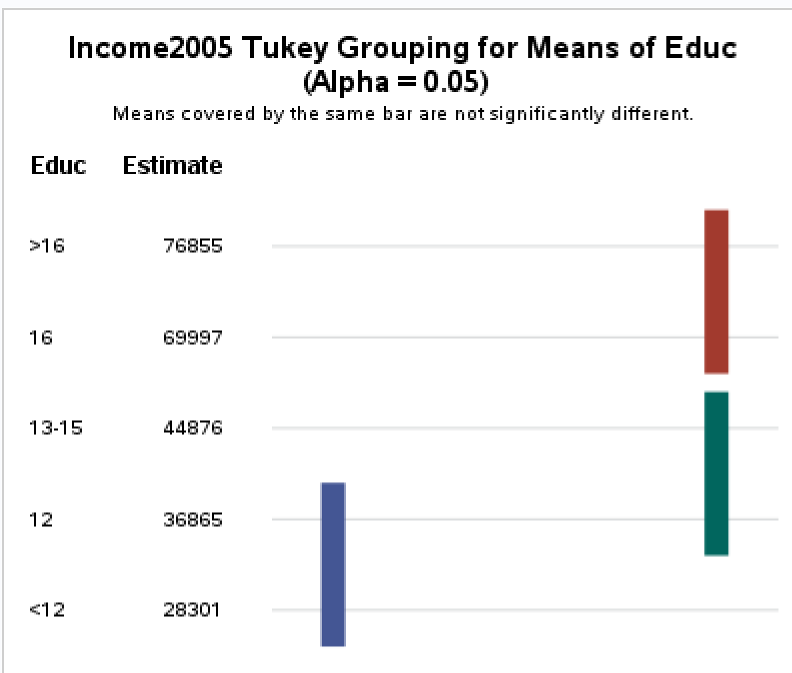
Part A

```
proc import datafile = '/home/chec0/New Folder/ex0525.csv'
 out = annual
 dbms = CSV
 ;
proc print data=annual;
run;

*Tukey-kramer test;
proc glm data = annual;
class Educ;
model Income2005 = Educ;
means Educ /alpha=.05 lines tukey;
run;
```

Alpha	0.05
Error Degrees of Freedom	2579
Error Mean Square	1.92E9
Critical Value of Studentized Range	3.86039
Minimum Significant Difference	9269.1
Harmonic Mean of Cell Sizes	333.036

Note: Cell sizes are not equal.



Tukey's Studentized Range (HSD) Test for Income2005

Note: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	2579
Error Mean Square	1.92E9
Critical Value of Studentized Range	3.86039

Comparisons significant at the 0.05 level are indicated by ***.

Educ Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
>16 - 16	6858	-1714	15431	
>16 - 13-15	31980	24212	39747	***
>16 - 12	39991	32760	47221	***
>16 - <12	48554	36577	60531	***
16 - >16	-6858	-15431	1714	
16 - 13-15	25121	17550	32692	***
16 - 12	33132	26113	40151	***
16 - <12	41696	29845	53546	***
13-15 - >16	-31980	-39747	-24212	***
13-15 - 16	-25121	-32692	-17550	***
13-15 - 12	8011	2002	14020	***
13-15 - <12	16575	5293	27856	***
12 - >16	-39991	-47221	-32760	***
12 - 16	-33132	-40151	-26113	***
12 - 13-15	-8011	-14020	-2002	***
12 - <12	8563	-2355	19482	
<12 - >16	-48554	-60531	-36577	***
<12 - 16	-41696	-53546	-29845	***
<12 - 13-15	-16575	-27856	-5293	***
<12 - 12	-8563	-19482	2355	

```
annual <-read.csv('/ex0525.csv')
annual2 <- aov(Income2005 ~ Educ, data = annual)
TukeyHSD(annual2)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Income2005 ~ Educ, data = annual)

$`Educ`
                diff         lwr        upr     p adj
>16-<12    48554.014  36576.984  60531.044 0.0000000
12-<12      8563.448  -2355.447  19482.342 0.2031235
13-15-<12  16574.508   5292.917  27856.100 0.0005961
16-<12     41695.524  29845.030  53546.019 0.0000000
12->16    -39990.566 -47221.008 -32760.125 0.0000000
13-15->16 -31979.506 -39746.830 -24212.182 0.0000000
16->16     -6858.490 -15431.193   1714.213 0.1860668
13-15-12    8011.061   2002.372  14019.750 0.0025813
16-12      33132.077  26113.227  40150.927 0.0000000
16-13-15   25121.016  17550.263  32691.769 0.0000000

>
```

Step 1 - Hypotheses:

$\mathbf{H_o}$:

$\mathbf{H_a}$:

Step 2 – critical value 3.86039

Step 3 –HSD 9269.1

Step 4 -

Step 5 - Decision: Reject $\mathbf{H_o}$

Step 6 - Conclusion: The above 95% familywise confidence intervals are constructed using Tukey-Kramer procedure.

Part B

***dunnett;**

proc glm data = annual;

class Educ;

model Income2005 = Educ;

means Educ / HOVTEST= BF dunnett ('12');

run;

The GLM Procedure

Dunnett's t Tests for Income2005

Note: This test controls the Type I experimentwise error for comparisons of all treatments against a control.

Alpha	0.05
Error Degrees of Freedom	2579
Error Mean Square	1.92E9
Critical Value of Dunnett's t	2.48068

Comparisons significant at the 0.05 level are indicated by ***.				
Educ Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
>16 - 12	39991	33420	46561	***
16 - 12	33132	26754	39511	***
13-15 - 12	8011	2551	13472	***
<12 - 12	-8563	-18486	1359	

gout <- glht(annual2, mcp(Educ = 'Dunnett'))

confint(gout)

annual.count <- table(annual$Educ)

new <- contrMat(annual.count, base=3, type="Dunnett")

dunnett.handi <- glht(annual2, linfct=mcp(Educ=new))dunn

confint(dunnett.handi)

```
         Simultaneous Confidence Intervals

Multiple Comparisons of Means: User-defined Contrasts


Fit: aov(formula = Income2005 ~ Educ, data = annual)

Quantile = 2.4805
95% family-wise confidence level


Linear Hypotheses:
                Estimate    lwr         upr
<12 - 12 == 0    -8563.4475 -18485.3450   1358.4499
>16 - 12 == 0    39990.5665  33420.3322  46560.8008
13-15 - 12 == 0   8011.0607   2551.0211  13471.1003
16 - 12 == 0     33132.0768  26754.1135  39510.0401
```

Problem : How strong is the evidence that

Assumptions: The Assumptions of the Anova are: the incomes in each educational group come from a normal distribution, the variances of these normal distributions are equal, the data are independent within each group, and the data are independent between each group.

Normality : The histograms and QQ plots below (of original data) appear to each show strong evidence of right skew, and thus provide evidence against coming from a normal distribution. This is not unexpected, as income data is often right skewed.

Independence : We will assume the data are independent, both between and within groups, and proceed with the Dunnett's test for differences in the groups(>16,16,13-15,<12) means to controls mean group(12) .

Step 1 - Hypotheses:

H_o: All mean incomes are the same across education levels.
H_a: The mean of control group(12) is different than the means of the other groups

Step 2 – critical value 2.48068

Step 3 –

Step 4 -

Step 5 - Decision: Reject H_o

Step 6 - Conclusion : There is sufficient evidence to suggest that there are differences between the means of the groups >16 , 13-15 and 16 and to the mean of the of the control group 12. The above 95% familywise confidence intervals are constructed using Dunnett's procedure.

The mean of group >16 differs from the mean of group 12 by$39,991
The mean of group 16 differs from the mean of group 12 by $33132
The mean of group 13-15 differs from the mean of group 12 by $8011

This is an observational study, and thus, we cannot assign causal inference to this relationship. The NLSY is a random sample of households and, thus, is a random sample but not a simple random sample of subjects in the desired population. Inference can be generalized to the population of areas sampled in the United States.

Bonus: Max 5 pts

Equity in Group Learning. [Continuation of Exercise 5.22.] (a) To see if the performance of low-ability students increases steadily with the ability of the best student in the group, form a linear contrast with increasing weights: -3 = Low, -1 = Low–Medium, $+1$ = Medium–High, and $+3$ = High. Estimate the contrast and construct a 95% confidence interval. (b) For the High-ability students, use multiple comparisons to determine which group composition differences are associated with different levels of test performance.

DISPLAY 5.24 Achievement test scores of Low ability students who worked in different study groups

	Highest ability level in the study group			
	Low	Low-medium	Medium-high	High
Average:	0.26	0.37	0.36	0.47
St. Dev.:	0.14	0.21	0.17	0.21
n:	17	24	25	14

(c) Give the levels of ability a quantitative representation (Low = 1, Low-Medium = 2, etc.) for the low ability students. After completing the questions above, conduct a linear regression (we haven't studied this yet!) of the **AVERAGE** performance against the level variable you just created. Be sure and address the assumptions. Defend the ones you can and assume the others are met. Include a scatterplot and residual plot. Is there evidence of linear trend? Is this inferred from the contrast? Assume the levels are equidistant in ability from each other.

Note: the data for Part b above is in Display 5.25 in your textbook.