

Natural number  $n ::= \mathbb{Z} \cup s(n)$   
 list  $l ::= \text{nil} \cup \text{cons}(n; l)$   
 tree  $t ::= \text{empty} \cup \text{node}(n; t_1; t_2)$

(even1) even  $\mathbb{Z}$  ✓  
 (even2) even  $s(n)$  : even  $n$  ✓

(Odd1) odd  $\mathbb{Z}$   
 (Odd2) odd  $s(n)$  : odd  $n$

(Sum1) sum  $\mathbb{Z} n n$   
 (Sum2) sum  $s(n) m s(x) ::= \text{sum } n mx$

(mul1) Times  $\mathbb{Z} n \mathbb{Z}$   
 (mul2) Times  $s(n) m s(x) ::= \text{Times } n m y, \text{ plus}(y, m y)$

(l1)  $|n| = \mathbb{Z}$   
 (l2)  $|\text{cons}(n; l)| = s(|l|)$

(c1)  $\text{nil} \cdot l = l$   
 (c2)  $\text{cons}(n; l_1) \cdot l_2 = \text{cons}(n; l_1 \cdot l_2)$

(Rev1)  $\text{rev}(\text{nil}) = \text{nil}$   
 (Rev2)  $\text{rev}(\text{cons}(n; l)) = \text{rev}(l) \cdot \text{cons}(n; \text{nil})$

(T1) size(empty) =  $\mathbb{Z}$   
 (T2) size(node(n;t1;t2)) =  $s(\text{size}(t1) + \text{size}(t2))$   
 (T3) height(empty) =  $\mathbb{Z}$   
 (T4) height(node(n;t1;t2)) =  $s(\max(\text{height}(t1), \text{height}(t2)))$

(Pre1) pre(empty) =  $\text{nil}$   
 (Pre2) pre(node(n;t1;t2)) =  $\text{cons}(n; \text{nil}) \cdot (\text{pre}(t1) \cdot \text{pre}(t2))$

(In1) In(empty) =  $\text{nil}$   
 (In2) In(node(n;t1;t2)) =  $\text{In}(t1) \cdot (\text{cons}(n; \text{nil}) \cdot \text{In}(t2))$

(Post1) post(empty) =  $\text{nil}$   
 (Post2) post(node(n;t1;t2)) =  $\text{post}(t1) \cdot (\text{post}(t2) \cdot \text{cons}(n; \text{nil}))$

Odd  $s(\mathbb{Z})$

odd  $s(s(n)) = \text{odd } n$

$l_1 \cdot l_2$

C1:  $\text{nil} \cdot l_2 = l_2$

C2:  $\text{cons}(n; l_1) \cdot l_2 = \text{cons}(n; l_1 \cdot l_2)$

rev(l)  
 rev(nil) = nil

rev2:  $\text{rev}(\text{cons}(n; l)) = \text{rev}(l) \cdot \underline{\text{cons}(n; \text{nil})}$