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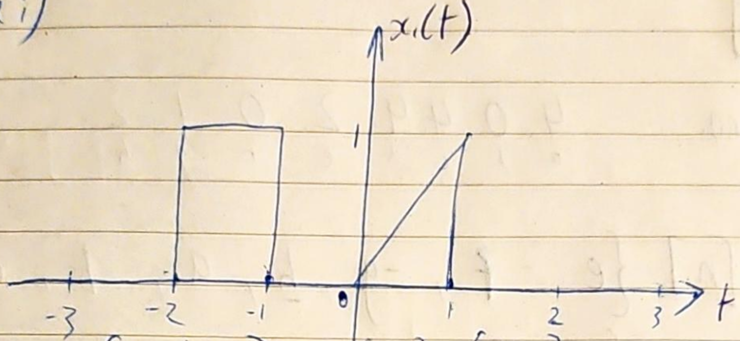
Coursework

Q1) a i)



$$u(t) = \begin{cases} 0, & t \le 0 \\ 1, & t \ge 0 \end{cases}$$

$$u(t-1) = \begin{cases} 0, & t < 1 \\ 1, & t \ge 1 \end{cases}$$

$$u(t+2) = \begin{cases} 0, & t < -2 \\ 1, & t \ge -2 \end{cases}$$

$$u(t+1) = \begin{cases} 0, & t < -1 \\ 1, & t \ge -1 \end{cases}$$

$$x_{1a}(t) = \begin{cases} 0, & t<0 \\ t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases} \quad x_{1b}(t) = \begin{cases} 0, & t<-2 \\ 1, & -2 \le t < -1 \\ 0, & t \ge -1 \end{cases} \quad x_1 = \begin{cases} 0, & t \le -2 \\ 1, & -2 \le t \le -1 \\ 0, & -1 \le t \le 0 \\ t & 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$

ii) $x_1(2 - 2t)$

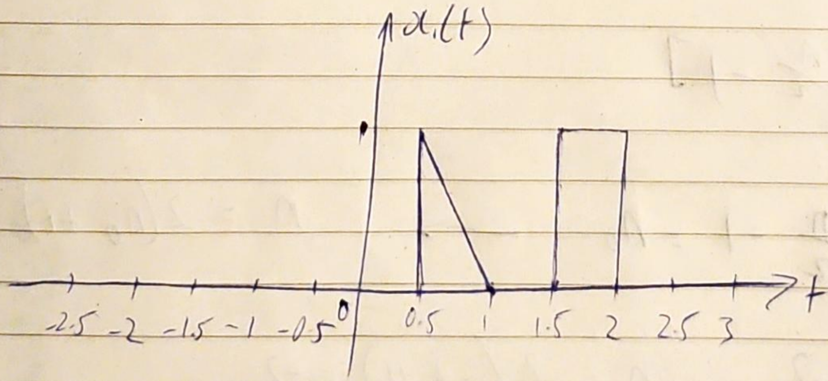
$2 - 2t = t_0 \longrightarrow t = \dfrac{2 - t_0}{2}$

$\therefore\ t_0 = -2, \quad t = (2 - (-2))/2 = 2$

$t_0 = -1 \quad t = (2 - (-1))/2 = 1.5$

$t_0 = 0 \quad t = (2 - (0))/2 = 1$

$t_0 = 1 \quad t = (2 - (1))/2 = 0.5$



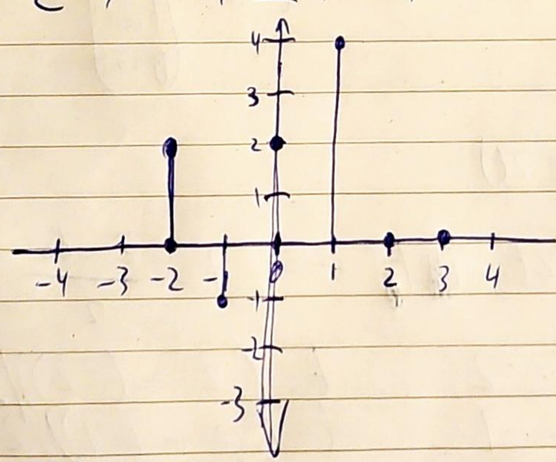
a ii) $x_2[n]$

Student no. $= \underset{a}{4}\ \underset{b}{0}\ \underset{c}{4}\ \underset{d}{4}\ \underset{e}{2}\ \underset{f}{0}\ \underset{g}{1}\ \underset{h}{2}$

$$x_2[n] = \{e - f,\ -g,\ \underline{h},\ a,\ -b,\ c-d\}$$

$$x_2[n] = \{2 - 0,\ -1,\ \underline{2},\ 4,\ -(0),\ 4-4\}$$

$$= \{2,\ -1,\ \underline{2},\ 4,\ 0,\ 0\}$$



$x_2[\frac{n}{2} - 1]$

$$\frac{n}{2} - 1 = n_0 \longrightarrow n = 2(n_0 + 1)$$

$n_0 = -2$, $n = 2(-2+1) = -2$

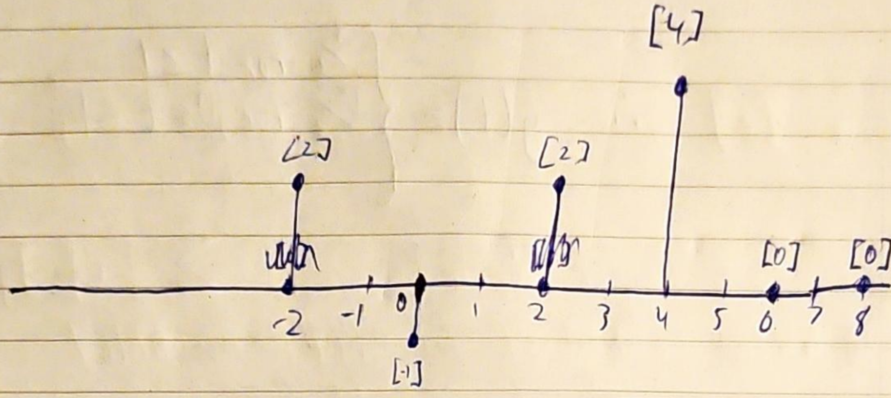
$n_0 = -1$, $n = 2(-1+1) = 0$

$n_0 = 0$, $n = 2(0+1) = 2$

$n_0 = 1$, $n = 2(1+1) = 4$

$n_0 = 2$, $n = 2(2+1) = 6$

$n_0 = 3$, $n = 2(3+1) = 8$



aiii) $x_1(t) * u(t)$

$$y(t) = x_1(t) * u(t) = \int_{-\infty}^{\infty} x_1(\tau)\, u(t-\tau)\, d\tau$$

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$$

$$\therefore \quad y(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$$

$$x_1(t) = \begin{cases} 0, & t < -2 \\ 1, & -2 \leq t < -1 \\ 0, & -1 \leq t < 0 \\ t, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

$t < -2$, $y(t) = 0$

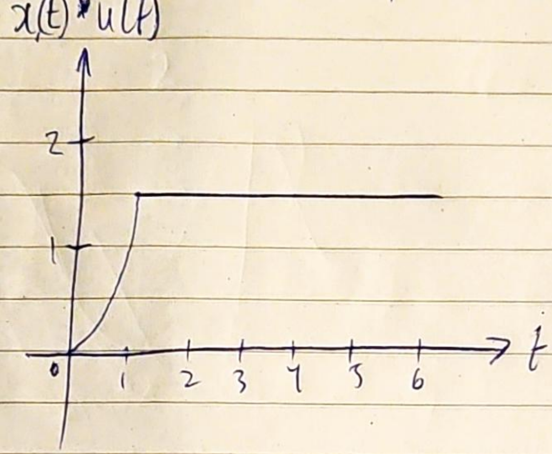
$-2 \leq t < -1$, $y(t) = \int_{-2}^{t} 1\, d\tau = t+2$

$-1 \leq t < 0$, $y(t) = \int_{-2}^{-1} 1\, d\tau = 1$

$0 \leq t < 1$, $y(t) = 1 + \int_{0}^{t} \tau\, d\tau = 1 + \frac{t^2}{2}$

$t \geq 1$, $y(t) = 1 + \int_{0}^{1} \tau\, d\tau = 1 + \frac{1}{2} = \frac{3}{2}$

$$\therefore\ y(t) = \begin{cases} 3/2, & t \geq 1 \\ t^2/2, & 0 \leq t < 1 \\ 0, & t < 0 \end{cases}$$



~~aiii) [x_1(t) * u(t)]~~

1b) $x_2[n] = 2 \cdot \delta[n+2] - \delta[n+1] + 2 \cdot \delta[n] + 4 \cdot \delta[n-1]$

1c) $x_1(t) = [u(t) - u(t-1)] \times t + [u(t+2) - u(t+1)]$

$$x_1(t) = \begin{cases} t, & \text{for } 0 \leq t < 1 \\ 1, & \text{for } -2 \leq t < -1 \\ 0, & \text{elsewhere} \end{cases}$$

Continuous time energy: $\int_{t_1}^{t_2} |x(t)|^2 \, dt$

$$\therefore \quad Ex_1 = \int_0^1 t^2 \, dt + \int_{-2}^{-1} 1^2 \, dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3} \qquad [t]_{-2}^{-1} = 1$$

$$\underline{\underline{Ex_1 = \frac{1}{3} + 1 = \frac{4}{3}}}$$

Since this is an energy signal (finite energy), the power is 0.

$x_2[n] = \{2, -1, \underline{2}, 4, 0, 0\}$

Energy of Discrete time signal: $\sum_{n=n_1}^{n_2} |x[n]|^2$

$\therefore \quad E_{x_2} = \sum_{n=-\infty}^{\infty} |x_2[n]|^2$

$E_{x_2} = |2|^2 + |-1|^2 + |2|^2 + |4|^2$

$= 4 + 1 + 4 + 16$

$= 25$

$\cancel{P_\infty = \lim_{T \to \infty} \frac{1}{2T} \sum_{n=-T}^{T} |x[n]|^2}$

x_2 has finite energy, so 0 power.

1d) $x_1(t)$ $\longrightarrow$ repeats every 3 units (see 1a i), period $T = 3$

$$P_\infty = \frac{1}{T}\int_0^T |x_1(t)|^2 \, dt$$

$$x_1(t) = \begin{cases} t, & \text{for } 0 \le t < 1 \\ 1, & \text{for } -2 \le t < -1 \\ 0, & \text{elsewhere} \end{cases}$$

$\therefore$ $$P_{x_1} = \frac{1}{3}\left(\int_0^1 t^2 \, dt + \int_{-2}^{-1} 1 \, dt\right)$$

$$\left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3} \qquad [t]_{-2}^{-1} = 1$$

$$P_{x_1} = \frac{1}{3}\left(\frac{1}{3} + 1\right) = \frac{4}{9}$$

Energy for a periodic signal $= \infty$

$x_2[n]$ $\longrightarrow$ repeats every 6 samples, so period $T = 6$

$$P = \frac{1}{T}\sum_{n=0}^{T-1} |x_2[n]|^2$$

$$\sum_{n=-2}^{3} |x_2[n]|^2 = 2^2 + (-1)^2 + 2^2 + 4^2 = 25$$

$$P = \frac{1}{6} \times 25 = \frac{25}{6}$$

Energy $= \infty$

Q2a) ① A memoryless system depends only on the input at the time of input

Since the output, $y(t)$ of a linear time invariant system is the result of convolution: $y(t) = (x*h)(t)$, this involves integrating over a range of time.

Non zero interval depends on values at previous / future times.
Not memoryless.

② System is causal if output at any time t depends only on present & past values of the input
Causal if $h(t) = 0$ for $t<0$

non zero for $-1 \leq t \leq 1$

non causal

③ Stable, BIBO, stable if absolutely integrable, i.e.

$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty$$

Since area under graph $h(t)$ is bounded + finite, stable.

2b)

2b) $$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) = e^{-2t} u(t) \longrightarrow x(t) = 0 \quad t<0$

$h(t)$ = impulse response , non zero between $-1 \leq t \leq 1$

$$y(t) = \int_0^{\infty} e^{-2\tau} h(t-\tau)\, d\tau$$

for $t \geq 1$, $y(t) = \int_{t-1}^{t+1} e^{-2\tau}\, d\tau$

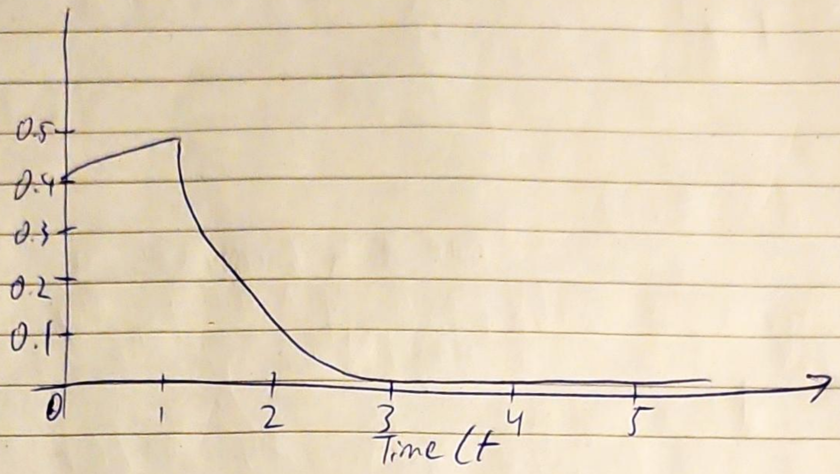
$$y(t) = \left[\frac{e^{-2\tau}}{-2}\right]_{t-1}^{t+1} = \frac{e^{-2(t+1)} - e^{-2(t-1)}}{-2} \text{ for } t \geq 1$$

for $0 \leq t \leq 1$, $y(t) = \int_0^{t+1} e^{-2\tau}\, d\tau$

$$y(t) = \left[\frac{e^{-2\tau}}{-2}\right]_0^{t+1} = \frac{e^{-2(t+1)} - e^0}{-2}$$

$$= \frac{1 - e^{-2t-2}}{2} \text{ for } 0 \leq t < 1$$

$$\therefore \quad y(t) = \begin{cases} \dfrac{1-e^{-2t-2}}{2}, & 0 \leq t < 1 \\ \dfrac{e^{-2t-2} - e^{-2t+2}}{-2}, & t \geq 1 \end{cases}$$



2.c)

$$y(t) = \int_{-\infty}^{\infty} e^{-(2\tau+\alpha)} u(\tau-\alpha)\, h(t-\tau)\, d\tau$$

since the step function ensures $=0$ for $\tau < \alpha$

$$y(t) = \int_{\alpha}^{\infty} e^{-(2\tau+\alpha)}\, h(t-\tau)\, d\tau$$

$\alpha = 1$, since this delays the input until after the impulse response has finished so that there is no overlap.

3a) $\frac{d^2}{dt^2} y(t) + (e+f)\frac{d}{dt} y(t) + (ef)\, y(t) = a\frac{d}{dt} x(t) + x(t)$

$a = 4$
$e = 2 \rightarrow \frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) = 4\frac{d}{dt} x(t) + x(t)$
$f = 0$

Laplace transform: $\mathcal{L}\left\{\frac{d}{dt} y(t)\right\} = sY(s) - y(0)$

$\mathcal{L}\left\{\frac{d^2}{dt^2} y(t)\right\} = s^2 Y(s) - sy(0) - y'(0)$

$$s^2 Y(s) + 2sY(s) = 4sX(s) + X(s)$$

$$(s^2 + 2s)\,Y(s) = (4s+1)\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4s+1}{s^2+2s}$$

$s = j\omega \longrightarrow H(j\omega) = \frac{4(j\omega)+1}{(j\omega)^2 + 2(j\omega)}$

$$= \frac{4(j\omega)+1}{-\omega^2 + 2j\omega}$$

3b) $H(j\omega) = \dfrac{4j\omega + 1}{-\omega^2 + 2j\omega}$

$x(t) = \delta(t)$ dirac delta function

Laplace transform : $X(s) = 1$

$\therefore \quad Y(s) = H(s)X(s)$

$Y(s) = H(s)$

$\therefore \; Y(s) = \dfrac{4s+1}{s^2+2s}$

Find inverse Laplace Transform

$$\frac{4s+1}{s^2+2s} = \frac{4s+1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$4s+1 = A(s+2) + Bs$

$s=-2$, $8+1 = -2B$

$-7 = -2B$

$B = \frac{-7}{-2} = \frac{7}{2}$

$s=1 \quad 5 = 3A + \frac{7}{2}$

$3A = \frac{3}{2}$

$A = \frac{1}{2}$

$$\frac{4s+1}{s^2+2s} = \frac{1/2}{s} + \frac{7/2}{s+2}$$

$$\mathcal{L}^{-1}\left\{\frac{1/2}{s}\right\} = \frac{1}{2}$$

$$\mathcal{L}^{-1}\left\{\frac{7/2}{s+2}\right\} = \frac{7}{2}e^{-2t}$$

$$y(t) = \frac{1}{2} + \frac{7}{2}e^{-2t}$$

This is impulse response (Q2 input was Dirac delta, typically used to determine impulse response of an LTI system)

3c) $x(t) = 2\cos(2t)$

Fourier transform of $\cos(\omega_0 t)$:

$$\mathcal{F}[\cos(\omega_0 t)] = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$\mathcal{F}\{2\cos(2t)\} = 2\pi[\delta(\omega - 2) + \delta(\omega + 2)]$$

$$\therefore X(j\omega) = 2\pi[\delta(\omega - 2) + \delta(\omega + 2)]$$

$$H(j\omega) = \frac{4j\omega + 1}{-\omega^2 + 2j\omega}, \quad Y(j\omega) = H(j\omega) \times X(j\omega)$$

$$Y(j\omega) = \left(\frac{4j\omega + 1}{-\omega^2 + 2j\omega}\right) \cdot 2\pi[\delta(\omega - 2) + \delta(\omega + 2)]$$

At $\omega = 2$: $H(j2) = \dfrac{4j(2) + 1}{-(2)^2 + 2j(2)} = \dfrac{8j + 1}{-4 + 4j}$

At $\omega = -2$: $H(j(-2)) = \dfrac{4j(-2) + 1}{-(-2)^2 + 2j(-2)} = \dfrac{-8j + 1}{-4j - 4}$

PTO

$$Y(j\omega) = 2\pi\left[H(j2)\,\delta(\omega-2) + H(j(-2))\,\delta(\omega+2)\right]$$

$$\therefore Y(j\omega) = 2\pi\left[\frac{8j+1}{-4+4j}\,\delta(\omega-2) + \frac{-8j+1}{-4-4j}\,\delta(\omega+2)\right]$$

$$= 2\pi\left[\left(\frac{7}{8} - \frac{9}{8}j\right)\delta(\omega-2) + \left(\frac{7}{8} + \frac{9}{8}j\right)\delta(\omega+2)\right]$$

3d) Input: $x(t) = 2\cos(2t)$

Angular freq: $\omega_0 = 2$ rad/s
Nyquist ": $2\omega_0 = 4$ rad/s
sample freq: $\omega_s = 4/2 = 2$ rad/s
sample period: $T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{2} = \pi s$

$$X(j\omega) = 2\pi\left[\delta(\omega-2) + \delta(\omega+2)\right]$$

$$X_s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$\therefore X_s(j\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$= \frac{1}{\pi}\sum_{k=-\infty}^{\infty} 2\pi\left[\delta(\omega - k\omega_s - 2) + \delta(\omega - k\omega_s + 2)\right]$$

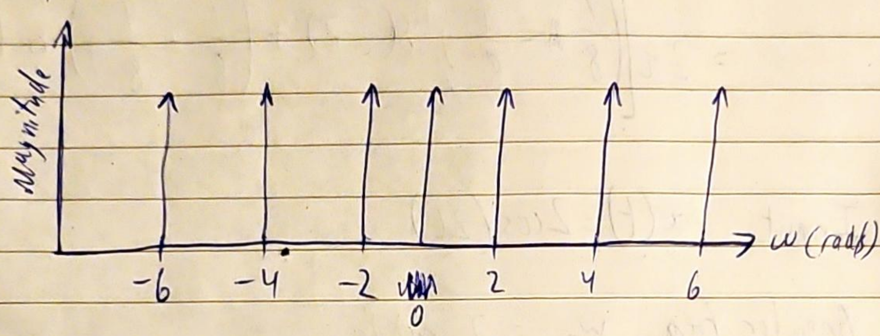
$$= \frac{2\pi}{\pi}\sum_{k=-\infty}^{\infty}\left[\delta(\omega-(2+2k)) + \delta(\omega-(-2+2k))\right]$$

$$= 2\sum_{k=-\infty}^{\infty}\left[\delta(\underbrace{\omega-(2+2k)}_{\omega = 2+2k}) + \delta(\underbrace{\omega-(-2+2k)}_{\omega = -2+2k})\right]$$

PTO

$k=0, \quad w = 2, -2$
$k=1, \quad w = 4, 0$
$k=-1, \quad w = 0, -4$
$k=2, \quad w = 6, 2$
$k=-2, \quad w = -2, -6$

$$X_s(jw) = 2\sum_{k=-\infty}^{\infty} \delta(w-2k)$$



3e) $X_s(jw) = 2\sum_{k=-\infty}^{\infty} \delta(w-2k)$

$T_s = \pi, \quad w_s = 2$

$$w_c \leq w_s/2$$

if $w_c = 1$ rad/s, and $H_r(jw) = \begin{cases} 1, & |w| \leq 1 \\ 0, & |w| > 1 \end{cases}$

$\therefore \quad X_{rec}(jw) = H_r(jw)\, X_s(jw)$

But all impulses are at $w = 2m$

For $|w| \leq 1$, the only pulse is at $w = 0$.

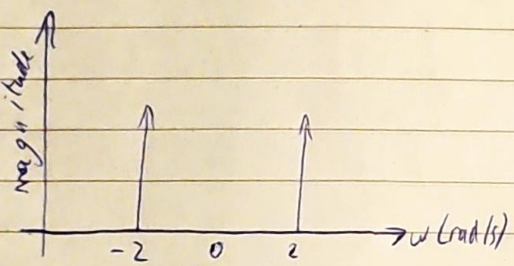
$\therefore$ After filtering, $X_{rec}(jw) = C\delta(w)$

$$X_{rec}(jw) \rightarrow x_{rec}(t)$$

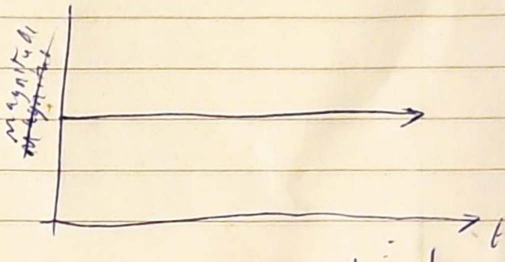
$$x_{rec}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{rec}(jw) e^{jwt} \, dw$$

$$x_{rec}(t) = \frac{1}{2\pi} \cdot C e^{j0t} = \frac{C}{2\pi}$$

The reconstructed value is constant in time, DC signal.



original signal



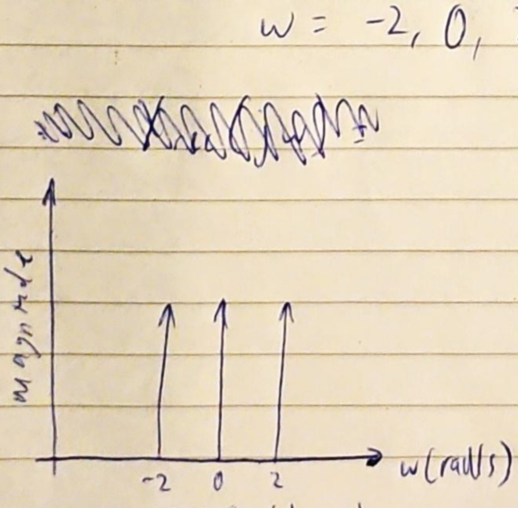
reconstructed signal

Because the signal was sampled at $\frac{1}{2}$ Nyquist rate, severe aliasing that low pass filter can't fix. With 1rad/s bandwidth only DC component remains.
However by increasing w_c:

cutoff

$$w_c = 3 \text{rad/s} \quad , \quad |w| \leq 3$$

$$w = -2, 0, 2$$



reconstructed signal

This is still not an accurate reconstruction of the original signal, however it is more accurate than using a lower cut off frequency.

4a) $y[n-2] - (2+a)\,y[n-1] + (1+a)\,y[n] = x[n]$

$a=4 \rightarrow y[n-2] - 6y[n-1] + 5y[n] = x[n]$

input: $x[n] = \delta[n], \quad -2 \leq n \leq 2$
$y[n] = 0, \quad n < 0$

Since input is unit impulse, output is <u>impulse response</u> $h[n]$

$h[n-2] - 6h[n-1] + 5h[n] = \delta[n]$

$n=-2$, since $y[n]=0 \quad n<0, \quad h[-2]=0$

$n=-1$, " $, \quad h[-1]=0$

$n=0$, $h[-2] - 6h[-1] + 5h[0] = \delta[0]$
$0 - 0 + 5h[0] = 1$
$h[0] = 1/5$

$n=1$, $h[-1] - 6h[0] + 5h[1] = \delta[1]$
$0 - 6(1/5) + 5h[1] = 0$
$5h[1] = 6/5$
$h[1] = 6/25$

$n=2$, $h[0] - 6h[1] + 5h[2] = \delta[2]$
$1/5 - 6(6/25) + 5h[2] = 0$
$5h[2] = 31/25$
$h[2] = 31/125$

$\therefore y[n] = h[n] = \begin{cases} 0 & n=-2,-1 \\ 1/5 & n=0 \\ 6/25 & n=1 \\ 31/125 & n=2 \end{cases}$

since nonzero only for $n \geq 0$, <u>right sided</u>.

4b) $y[n-2] - 6y[n-1] + 5y[n] = x[n]$

LTI system function: $H(z) = \frac{Y(z)}{X(z)}$

Time shift property:

$$Z\{y[n-k]\} = z^{-k}Y(z), \quad Z\{x[n]\} = X(z)$$

$$Z\{y[n-2]\} = z^{-2}Y(z)$$

$$Z\{y[n-1]\} = z^{-1}Y(z)$$

$$Z\{y[n]\} = Y(z)$$

$$z^{-2}Y(z) - 6z^{-1}Y(z) + 5Y(z) = X(z)$$

$$Y(z)(z^{-2} - 6z^{-1} + 5) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-2} - 6z^{-1} + 5} \quad [\times z^2]$$

$$H(z) = \frac{z^2}{1 - 6z + 5z^2} = \frac{z^2}{(1-z)(1-5z)}$$

ROC $\longrightarrow$ $1 - z = 0$ $\quad$ $1 - 5z = 0$

$z = 1$ $\quad$ $z = \frac{1}{5}$

$\therefore \quad |z| > 1$

4c) Outputs at any given time depend only on past + present inputs $\longrightarrow$ Causal.

$$y[n-2]-6y[n-1]+5y[n]=x[n]$$

$$5y[n]=x[n]-y[n-2]+6y[n-1]$$

$$y[n]=\frac{1}{5}x[n]+\frac{6}{5}y[n-1]-\frac{1}{5}y[n-2]$$

$n,\ n-1,\ n-2,\ \longrightarrow$ past + present

Therefore, causal.

4d)

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{z^{-2}-6z^{-1}+5}\quad,\ ROC=|z|>1$$

$$w=z^{-1}\implies w^2-6w+5=(w-1)(w-5)$$

$$\therefore\ H(z)=\frac{1}{(z^{-1}-1)(z^{-1}-5)}\quad \to\ z^{-1}-1=-(1-z^{-1}),\quad z^{-1}-5=-5(1-\tfrac{1}{5}z^{-1})$$

$$\therefore H(z)=\frac{1}{5(1-z^{-1})(1-\frac{1}{5}z^{-1})}=\frac{C}{1-z^{-1}}+\frac{D}{1-\frac{1}{5}z^{-1}}\quad[\times(1-z^{-1})(1-\tfrac{1}{5}$$

$$C(1-\tfrac{1}{5}z^{-1})+D(1-z^{-1})=\frac{1}{5}$$

$z^{-1}=1,\ \frac{4}{5}C=\frac{1}{5},\quad C=\frac{1}{4}$

$z^{-1}=0,\ \frac{1}{4}+D=\frac{1}{5},\quad D=-\frac{1}{20}$

$$\therefore \quad H(z) = \frac{1}{4}\,\frac{1}{1-z^{-1}} - \frac{1}{2}\,\frac{1}{1-\frac{1}{5}z^{-1}}$$

$$\hookrightarrow z^{-1}\left\{\frac{1}{1-az^{-1}}\right\} = a^n u[n]$$

$$h[n] = z^{-1}\left\{H(z)\right\} = \frac{1}{4}u[n] - \frac{1}{20}\left(\frac{1}{5}\right)^n u[n]$$

$n = -2$, $u[n] = 0 \quad \therefore h[n] = 0$

$n = -1$, $u[n] = 0 \quad \therefore h[n] = 0$

$n = 0$, $h[0] = \frac{1}{4} - \frac{1}{20}\left(\frac{1}{5}\right)^0 = \frac{1}{4} - \frac{1}{20} = \frac{5}{20} - \frac{1}{20} = \frac{1}{5}$

$n = 1$, $h[1] = \frac{1}{4} - \frac{1}{20}\left(\frac{1}{5}\right) = \frac{1}{4} - \frac{1}{100} = \frac{25}{100} - \frac{1}{100} = \frac{6}{25}$

$n = 2$, $h[2] = \frac{1}{4} - \frac{1}{20}\left(\frac{1}{5}\right)^2 = \frac{1}{4} - \frac{1}{20}\cdot\frac{1}{25}$

$$= \frac{1}{4} - \frac{1}{500} = \frac{125}{500} - \frac{1}{500}$$

$$= \frac{31}{125}$$

5a) Input : $x(t) = u(t)$

Output: $y(t) = [e^{-5t} \cos(2t)] u(t)$

$$X(s) = \mathcal{L}\{u(t)\} = \frac{1}{s}, \quad \mathbb{R}(s) > 0$$

$$\mathcal{L}\{e^{-at} \cos(w_0 t)\, u(t)\} = \frac{s+a}{(s+a)^2 + w_0^2} = \frac{s+5}{(s+5)^2+4}$$

$$H(s) = \frac{Y(s)}{X(s)} = Y(s)\, s$$

$$H(s) = \frac{s+5}{(s+5)^2+4} \cdot s$$

$$(s+5)^2 + 4 = s^2 + 10s + 25 + 4 = s^2 + 10s + 29$$

$$s = jw \longrightarrow H(s) = \frac{s(s+5)}{s^2+10s+29} = \frac{s^2+5s}{s^2+10s+29}$$

$$H(jw) = \frac{(jw)^2 + 5(jw)}{(jw)^2 + 10(jw) + 29} = \frac{-w^2 + j5w}{-w^2 + j10w + 29}$$

5b) $$\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0 y = b_2\frac{d^2x}{dt^2} + b_1\frac{dx}{dt} + b_0 x$$

$L\{y''\} = s^2 Y(s)$

$L\{y'\} = sY(s)$

$L\{x''\} = s^2 X(s)$

$L\{x'\} = sX(s)$

$$s^2 Y(s) + a_1 s Y(s) + a_0 y = b_2 s^2 X(s) + b_1 sX(s) + b_0 X$$

$$(s^2 + a_1 s + a_0)\,Y(s) = (b_2 s^2 + b_1 s + b_0)\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{s^2 + 5s}{s^2 + 10s + 29}$$

$\therefore$ $a_1 = 10$ $\qquad$ $b_0 = 0$

$a_2 = 29$ $\qquad$ $b_1 = 5$

$b_2 = 1$

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 29y = \frac{d^2x}{dt^2} + 5\frac{dx}{dt}$$

5c) $X(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega)$