1 Control systems CW

0. Given Parameters

Mechanical/Geometry:

- $m = 0.462\,kg$
- $g = 9.81\,ms^{-2}$
- $d = 0.42$
- $\delta = 0.65m$
- $r = 0.123m$
- $\phi = 41°$
- $k = 1885\,Nm^{-1}$
- $b = 10.4\,Nsm^{-1}$

Electrical:

- $R = 2.2k\Omega = 2200\Omega$
- $L_0 = 0.125H$
- $L_1 = 0.0241H$
- $\alpha = 1.2m^{-1}$
- $C = 6.811\,m^3g/(A^2s^2)$ (given constant in force law)

Sensor:

- $\tau_m = 0.03s$

Unknown / Assumed

- K_m (sensor gain) unknown → Assume 1.

↓ Assumption **A2**

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Control Systems CW - My part 1.

Goal: Derive circuit dynamics and measurement equation.

Electrical and sensor modelling.

1. Variables:

- $x(t)$: position of the ball (m)
- $y(t)$: distance from the electromagnet (m)
- $i(t)$: Current in the circuit (A)
- $V(t)$: input Voltage (V)
- R : Resistance (Ω)
- L : inductance (H)
- $y_m(t)$: measured position (sensor output).

Key relationship:

$$y(t) = \delta - x(t)$$

The electromagnet force depends on the distance y, not position x.

2. Inductance Model.

$$L = L_0 + L_1 e^{-\alpha y}$$

Sub in $y = \delta - x$

$$L(x) = L_0 + L_1 e^{-\alpha(\delta - x)}$$

The inductance changes as the ball moves, making the system non-linear.

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3. Electrical Model

3.1 Applying KVL:

$$V(t) = Ri(t) + \frac{d}{dt}\left(L(x)i(t)\right)$$ *

where $Ri(t)$ is the voltage across the resistor and the inductor voltage is given by $V_L = \frac{d}{dt}(Li)$. The inductance depends on the position of the ball. Since the inductance varies with position the product rule must be applied when differentiating $L(x)i(t)$

3.2 Expand derivative (product rule).

$$\frac{d}{dt}(Li) = L(x)\frac{di}{dt} + i(t)\frac{dL}{dt}$$

So:

$$V(t) = Ri(t) + L(x)\frac{di}{dt} + i(t)\frac{dL}{dt}$$

3.3 Simplify - Engineering simplification.

Assumption A1 (model simplification): *

The term $i(t)\frac{dL}{dt}$ is neglected to simplify the model and avoid coupling with mechanical dynamics.

$$V(t) \overset{approx}{\approx} Ri(t) + L(x)\frac{di}{dt}$$

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3.4 Rearranging to state equation i:

$$L(x)\frac{di}{dt} = V(t) - Ri(t) \xrightarrow{\div L(x)}$$

$$\boxed{\frac{di}{dt} = \frac{V(t) - Ri(t)}{L(x)}} \rightarrow E1 \rightarrow \text{Electrical ODE}$$

4 Sensor Model Dynamics

4.1 First order sensor model - already given.

Sensor modelled with time constant τ_m
Let sensor output be $y_m(t)$

Model:

$$\tau_m \frac{dy_m}{dt} + y_m = K_m x(t)$$

4.2 Rearrange:

$$\frac{dy_m}{dt} = \frac{K_m x(t) - y_m(t)}{\tau_m}$$

4.3 Gain assumption:
Assumption A2: $K_m = 1$

So:

$$\boxed{\frac{dy_m}{dt} = \frac{x(t) - y_m(t)}{\tau_m}} \rightarrow S1 \rightarrow \text{Sensor ODE}$$

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5 Mini Checklist

Phase 1 outputs:

- $L(x) = L_0 + L_1 e^{-\alpha(g-x)}$
- $\dot{i} = \dfrac{V - Ri}{L(x)}$
- $\dot{y}_m = \dfrac{x - y_m}{\tau_m}$

Assumptions:

A1 → Neglect $\dot{L}$

A2 → $K_m = 1$