Week Commencing 04/02/2026 Dara McCallum

FOR

down slope $\oplus$
up slope $\ominus$

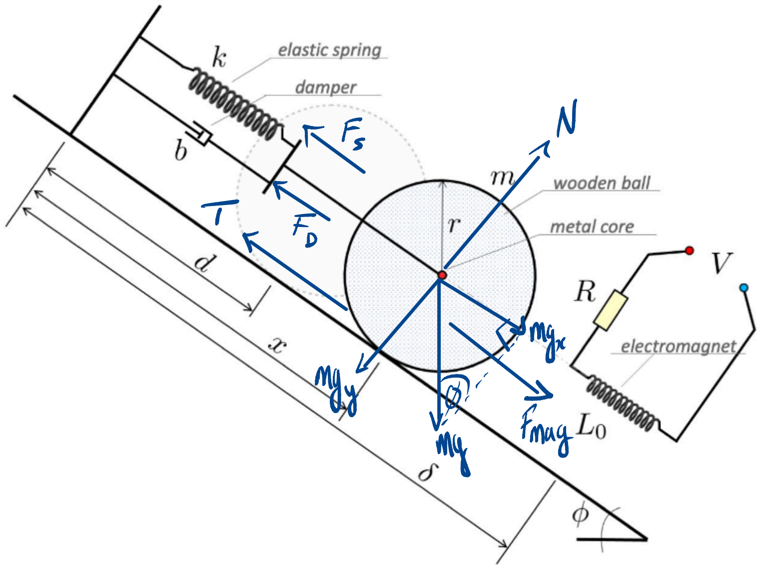
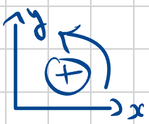


Figure 1. System of a wooden ball on an inclined plane. The ball can be attracted downwards by an electromagnet, which is controlled by the voltage V.



Summary:
- → the ball rolls without slipping on an incline at angle ϕ
- → spring stiffness k
- → viscous damper b
- → spring is unstretched at $x = d$
- → the electromagnet centre is at $x = \delta$
- → current is i
- → $F_{mag} = c\dfrac{i^2}{y^2}$, where y = distance from centre of ball to centre of magnet.

Force components on x-axis

a) Gravity → $mg\sin\phi$ {+ve as down the slope}

b) Spring (unstretch @ $x = d$) → $F_s = -k(x-d)$ {acts up the slope so −ve} → pulls back to $x = d$

c) Viscous damper → $F_d = b\dot{x}$

d) Magnetic (acts towards magnet at $x = \delta$) → $F_{mag} = c\dfrac{i^2}{(\delta - x)^2}$ {$y = \delta - x$ assuming $x < \delta$}

e) Friction → $-T$

From Newton's 2^{nd} Law

$$\Rightarrow \quad \sum F_x = m\ddot{x}$$

$$m\ddot{x} = mg\sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2} - T$$

Rotational Dynamics

from Newton's 2^{nd} Law

$$\sum \tau = I\ddot{\theta}$$

only component to create torque is friction, T. This is due to it acting at a point at radius r from the centre point.

$$\tau = -Tr$$

$$I\ddot{\theta} = -Tr$$

$$x = -r\theta$$ sig due to friction producing a clockwise torque.

$$\dot{x} = -r\dot{\theta}$$

$$\ddot{x} = -r\ddot{\theta} \quad \Rightarrow \quad \ddot{\theta} = -\frac{\ddot{x}}{r}$$

$$\Rightarrow I\ddot{\theta} = -Tr$$

$$I\left(-\frac{\ddot{x}}{r}\right) = -Tr$$

$$-\frac{I\ddot{x}}{r} = -Tr$$

$$-T = \frac{-I\ddot{x}}{r^2}$$

$$T = \frac{I}{r^2}\ddot{x}$$

$$\therefore \quad m\ddot{x} = mg\sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2} - \frac{I}{r^2}\ddot{x}$$

$$\Rightarrow m\ddot{x} + \frac{I}{r^2}\ddot{x} = mg\sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2}$$

$$\Rightarrow \left(m + \frac{I}{r^2}\right)\ddot{x} = mg\sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2}$$

System is a solid sphere (ball)

$$\therefore \quad I = \frac{2}{5} m r^2$$

$$\Rightarrow \quad m + \frac{I}{r^2} = m + \frac{\left(\frac{2}{5} m \cancel{r^2}\right)}{\cancel{r^2}}$$

$$\Rightarrow \quad m + \frac{2}{5} m = \frac{7}{5} m$$

$$\therefore \Rightarrow \frac{7}{5} m \ddot{x} = mg \sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2}$$

$$\ddot{x} = \frac{5}{7m}\left(mg \sin\phi - k(x-d) - b\dot{x} + c\frac{i^2}{(\delta - x)^2}\right)$$

Convert to state form.

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x}_1 \end{aligned}\right\} \quad \Rightarrow \quad \dot{x}_2 = \frac{5}{7m}\left(mg \sin\phi - k(x_1 - d) - b x_2 + c\frac{i^2}{(\delta - x_1)^2}\right)$$