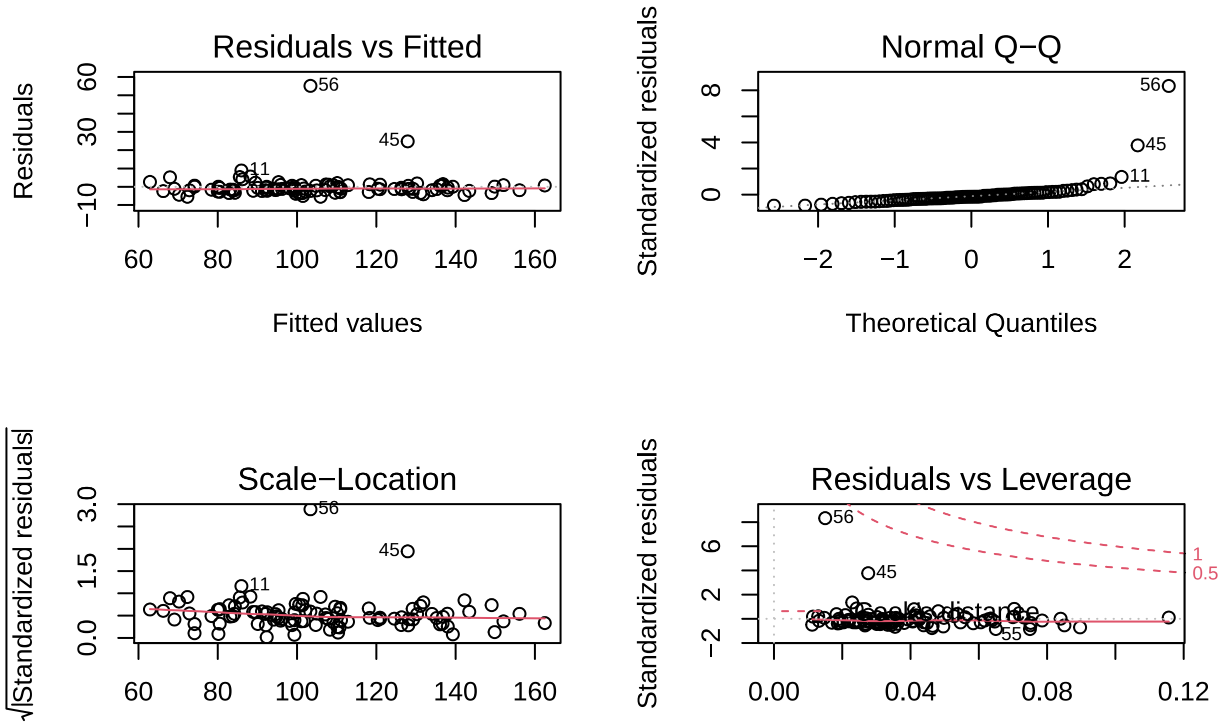
Bootstrap Project: Dataset 3

Arturo Prieto Tirado, Cesar Conejo Villalobos

Introduction

In this project, we study the dataset `data_3.csv` which contains four columns referring to a response variable `y` and 3 covariates `x1`, `x2`, and `x3` for 100 observations. The goal of the task is to build a linear regression model to `y` in terms of some of the other three variables. However, if we apply a linear model, due to the data contains several outliers, it can be checked in the following picture how the error residuals are not normally distributed..

```r
model.lm <- lm(y ~  x1 + x2 + x3, data = data3)
par(mfrow=c(2,2))
plot(model.lm)
```



The coefficients for the linear model are given by $\beta_1 = 6.29$, (corresponding to the intercept) $\beta_2 = 0.91$, $\beta_3 = 2.02$, and $\beta_4 = 3.99$. As a result, we build a robust linear regression model `rlm()` from `MASS` package and use the bootstrap to study the significance of the regressors.

Robust regression

In this section, we calibrate a robust linear model with the function `rlm()`. From the output, we can see a significant change in the coefficients of the linear `linear model` with respect to the `robust linear model`. First, the values related to the intercept and the variable `x3` decrease in the robust model. On the other hand, the weight assigned to the variables `x1` and `x2` increase considerably. As a result, the first insight that we can get is related to the covariate `x3`, in which the increase of one unit of the variable `x3` decreases from 4 in the linear model to 1.3.

```r
model.rlm <- rlm(y ~  x1 + x2 + x3, data = data3, maxit = 200)
knitr::kable(coef(summary(model.rlm)), digits = 3)
```

	Value	Std. Error	t value
(Intercept)	4.735	0.905	5.230
x1	3.707	0.959	3.864
x2	4.653	0.959	4.851
x3	1.317	0.958	1.374

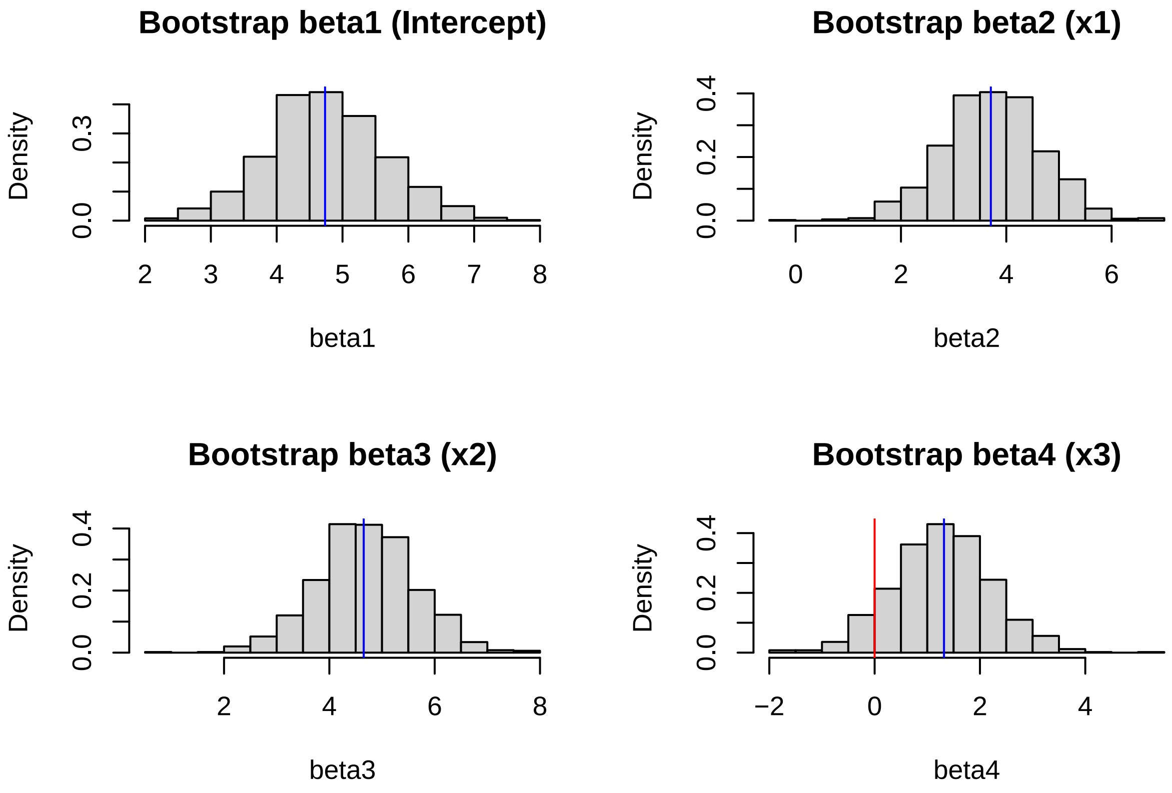
Additionally, we must consider that the standard error reported in `rml()` for the coefficients is based on asymptotic results. Moreover, the sample size of 100 is relatively small, resulting in not trustworthy estimations of the variability in the coefficient's regression. Therefore, we can explore a 95% bootstrap confidence interval on the regressors' coefficients in order to study their significance. In this case, we have two alternatives: bootstrap in pairs and bootstrap in residuals. However, given the presence of outliers and influential observations the bootstrap in pairs can lead to low-quality estimators. For example, in some samples, it is possible to exclude some influential observations, given a high variability in the results. In conclusion, we tackle this estimation problem using the bootstrap in residuals technique. The following code shows how to realize the bootstrap configuration:

```r
res_rob_rg <- function(x, beta, xdata){
  y_fit  <- beta[1] + beta[2]*xdata$x1 + beta[3]*xdata$x2 + beta[4]*xdata$x3 + x
  bmodel <- rlm(y_fit ~ x1 + x2 + x3, data = xdata)
  return(coef(bmodel))
}

rres    <- model.rlm$residuals
rbeta.h <- coef(model.rlm)

B <- 1000
set.seed(1)
coeff.res <- bootstrap(rres, B, res_rob_rg, beta = rbeta.h, xdata = data3)$thetastar
```

Then, the following figure shows the distribution of the estimate parameters. we can see how the distributions of β_1 and β_2 are symmetric, unimodal, and takes all values greater than zero. In the case of the distribution of β_3, we can see notice a slightly right tail distribution, however, all the values are greater than zero. Finally, if we look at the distribution of β_4 corresponding to the variable `x3` we can see a left-skewed distribution taking positive and negative values. As a result, in some of the resampling scenarios, the coefficient β_4 can be not significant.

```r
par(mfrow=c(2,2))
hist(coeff.res[1,], main = "Bootstrap beta1 (Intercept)", probability = T, xlab = "beta1")
abline(v = rbeta.h[1], col ="blue")
hist(coeff.res[2,], main = "Bootstrap beta2 (x1)", probability = T, xlab = "beta2")
abline(v = rbeta.h[2], col ="blue")
```

```r
hist(coeff.res[3,], main = "Bootstrap beta3 (x2)", probability = T, xlab = "beta3")
abline(v = rbeta.h[3], col ="blue")
hist(coeff.res[4,], main = "Bootstrap beta4 (x3)", probability = T, xlab = "beta4")
abline(v = 0, col ="red")
abline(v = rbeta.h[4], col ="blue")
```



Moreover, we can compute the basic bootstrap confidence interval for each coefficient. We can confirm that even in the space corresponding to 95% probability we have chances of taking negative values for β_4.

```r
basic_beta1 <- 2*rbeta.h[1] - quantile(coeff.res[1,], c(0.975,0.025))
basic_beta2 <- 2*rbeta.h[2] - quantile(coeff.res[2,], c(0.975,0.025))
basic_beta3 <- 2*rbeta.h[3] - quantile(coeff.res[3,], c(0.975,0.025))
basic_beta4 <- 2*rbeta.h[4] - quantile(coeff.res[4,], c(0.975,0.025))
knitr::kable(rbind(basic_beta1, basic_beta2, basic_beta3, basic_beta4), digits = 3)
```

	97.5%	2.5%
basic_beta1	3.217	6.613
basic_beta2	2.195	5.169
basic_beta3	3.107	6.128
basic_beta4	-0.213	2.872

Backward elimination

From the previous section, we are suspicious about the significance of the coefficient β_4. For this reason, we will explore the significance of this covariate deeply with the bias corrected accelerated BC_a bootstrap. In this case, we use again the residual bootstrap, but we fix the values corresponding to $\beta 1$, $\beta 2$ and $\beta 3$. In other words, we compute only the effect of the residuals only over the covariate `x3`.

```
# Fitted values for theoretical betas
y_predict <-  predict(model.rlm)

res_rob_rg_x3 <- function(x, beta, xdata){

  y_bt <-  y_predict + x
  y_bt <-  y_bt - (beta[1] + beta[2]*xdata$x1 + beta[3]*xdata$x2)
  data1_BT <-  data.frame(x3 = data3$x3,
                          y = y_bt)
  fit_BT <- rlm(y ~ x3 -1, data = data1_BT)$coefficients
  return(fit_BT)
}

B = 1000
bcanon(rres, B, res_rob_rg_x3, beta = rbeta.h, xdata = data3,alpha = c(0.025, 0.975))$confpoints
```

```
##      alpha bca point
## [1,] 0.025  1.296253
## [2,] 0.975  1.339254
```

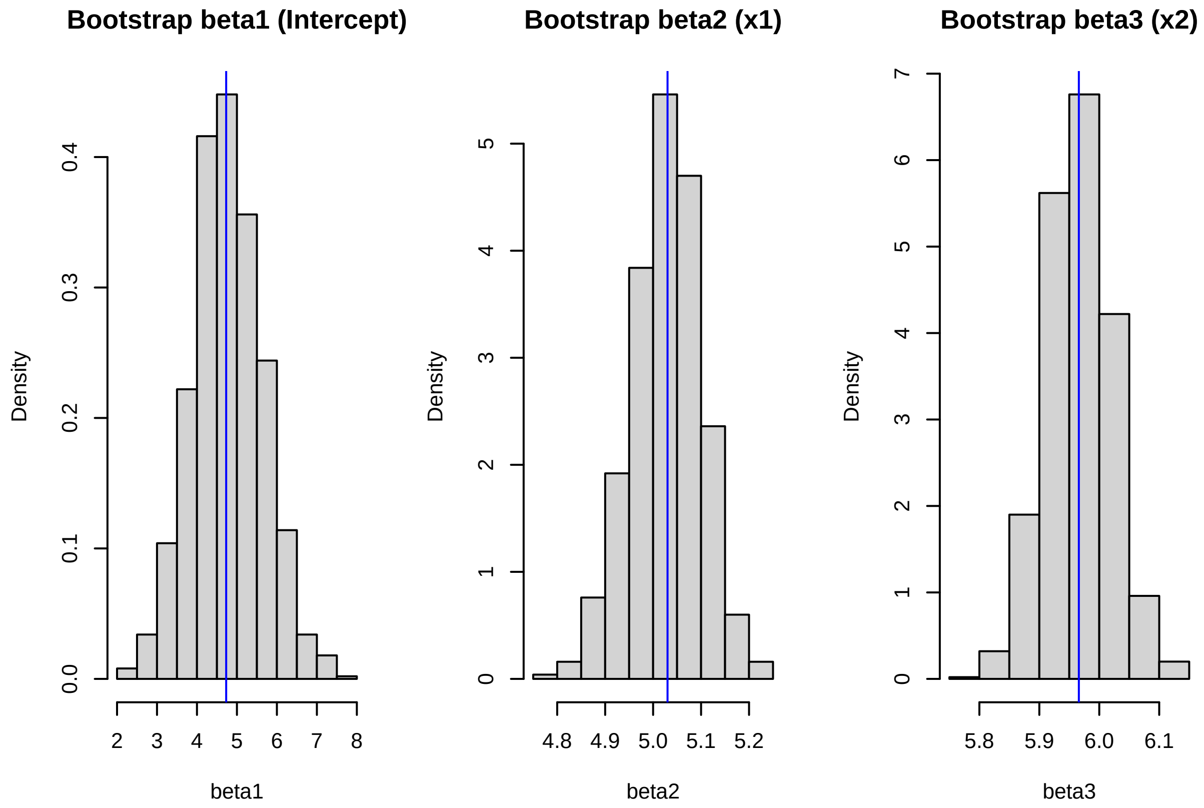
Confidence Intervals

```
model.rlm <- rlm(y ~  x1 + x2, data = data3, maxit = 200)
knitr::kable(coef(summary(model.rlm)), digits = 3)
```

	Value	Std. Error	t value
(Intercept)	4.731	0.898	5.266
x1	5.030	0.076	66.555
x2	5.966	0.059	101.300

```
res_rob_rg <- function(x, beta, xdata){
  y_fit  <- beta[1] + beta[2]*xdata$x1 + beta[3]*xdata$x2 + x
  bmodel <- rlm(y_fit ~ x1 + x2, data = xdata)
  return(coef(bmodel))
}

rres    <- model.rlm$residuals
rbeta.h <- coef(model.rlm)

B <- 1000
set.seed(1)
coeff.res <- bootstrap(rres, B, res_rob_rg, beta = rbeta.h, xdata = data3)$thetastar
```

```r
par(mfrow=c(1,3))
hist(coeff.res[1,], main = "Bootstrap beta1 (Intercept)", probability = T, xlab = "beta1")
abline(v = rbeta.h[1], col ="blue")
hist(coeff.res[2,], main = "Bootstrap beta2 (x1)", probability = T, xlab = "beta2")
abline(v = rbeta.h[2], col ="blue")
hist(coeff.res[3,], main = "Bootstrap beta3 (x2)", probability = T, xlab = "beta3")
abline(v = rbeta.h[3], col ="blue")
```



```r
basic_beta1 <- 2*rbeta.h[1] - quantile(coeff.res[1,], c(0.975,0.025))
basic_beta2 <- 2*rbeta.h[2] - quantile(coeff.res[2,], c(0.975,0.025))
basic_beta3 <- 2*rbeta.h[3] - quantile(coeff.res[3,], c(0.975,0.025))
knitr::kable(rbind(basic_beta1, basic_beta2, basic_beta3), digits = 3)
```

	97.5%	2.5%
basic_beta1	2.927	6.395
basic_beta2	4.891	5.178
basic_beta3	5.857	6.074

Mean Response

```r
# Model with only x1 and x2
```

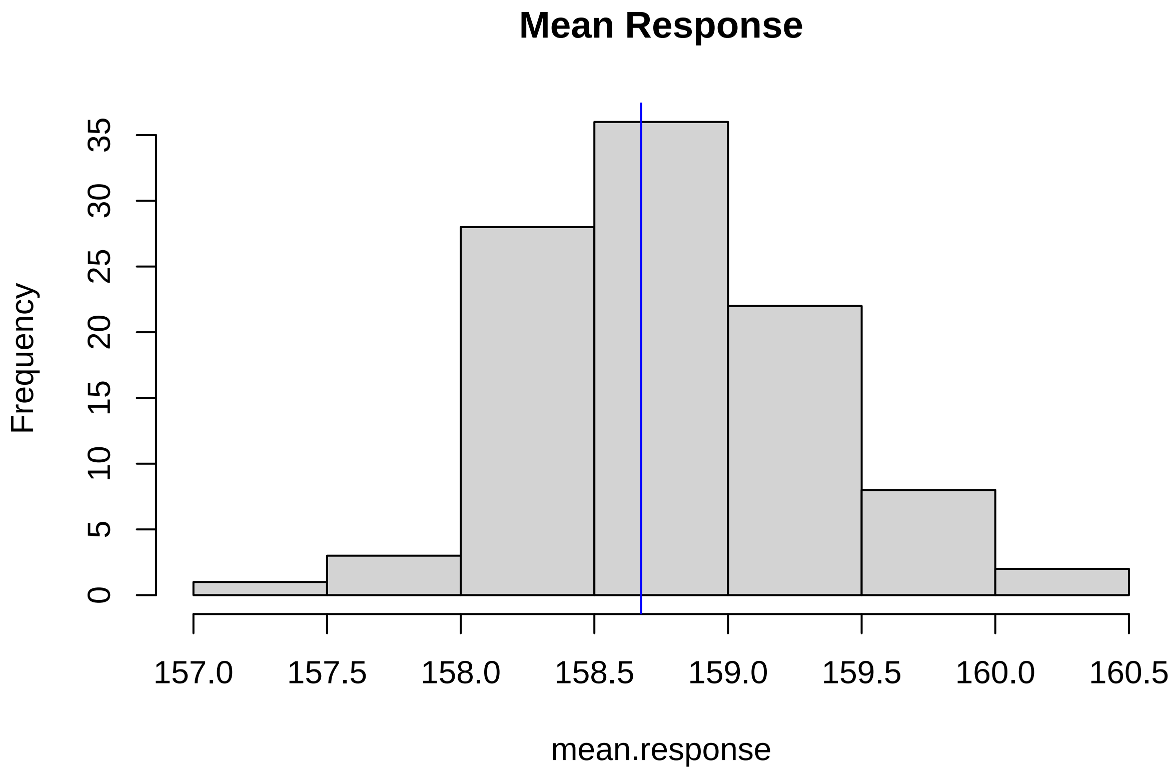
```
model.rlm <- rlm(y ~  x1 + x2, data = data3)
coef(summary(model.rlm))
```

```
##                Value Std. Error   t value
## (Intercept) 4.730836 0.89835999   5.26608
## x1          5.030091 0.07557784  66.55511
## x2          5.965955 0.05889420 101.29954
```

```
fit_value <- predict(model.rlm, newdata = data.frame(cbind(x1 = 14, x2 = 14, x3 = 14)))
fit_value
```

```
##        1
## 158.6755
```

```
res_rob_mean <- function(x,beta,xdata){
  y_fit  <- beta[1] + beta[2]*xdata$x1 + beta[3]*xdata$x2 + x
  bmodel <- rlm(y_fit ~ x1 + x2, data = xdata)
  return(predict(bmodel, newdata = data.frame(cbind(x1 = 14, x2 = 14, x3 = 14))))
}

#take the best model in 3 (change this)
rres    <- model.rlm$residuals
rbeta.h <- coef(model.rlm)

B <- 100
set.seed(1)
mean.response <- bootstrap(rres, B, res_rob_mean, beta = rbeta.h, xdata = data3)$thetastar


hist(mean.response, main = "Mean Response")
abline(v = fit_value, col = "blue")
```



```r
# Basic Bootstrap
2*fit_value - quantile(mean.response, c(0.975,0.025))
```

```
##   97.5%    2.5%
## 157.437 159.462
```