$\sum_{n\geq 1}^{\infty} a_n$

$a_n \geq 0$

- comp.
- comp limite
- integral
- serie p
- cociente
- raiz

$\sum_{n=1}^{\infty} (-1)^n a_n$

- $a_n$ dec
- $\lim_{n\to\infty} a_n = 0$

MA-2115: Matemáticas 4

# Semana 4

## 4.1 Convergencia absoluta

1. Definición

   $\sum |a_n|$ conv $\Rightarrow$ $\sum a_n$ conv. absolutamente

   conv. abs $\Rightarrow$ conv.

2. Criterios del cociente y de la raiz (revisado)

   $$\underbrace{\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = c \;\begin{matrix} > \\ = \\ < \end{matrix}\; 1 \;\begin{matrix} \text{diverge} \\ \text{inconcluso} \\ \text{converge abs.} \end{matrix}}_{\text{cociente}} \quad \underbrace{1 \;\begin{matrix} < \\ = \\ > \end{matrix}\; R = \lim_{n\to\infty} \sqrt[n]{|a_n|}}_{\text{raiz}}$$

3. *Ejercicio 4.1*

   Determine la convergencia absoluta de $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$. $= \sum_{n=1}^{\infty} (-1)^n a_n$ $\quad b_n = (-1)^n a_n$

   $a_n = \frac{1}{n}$ , $n > n+1 \Rightarrow a_n = \frac{1}{n} < \frac{1}{n+1} = a_{n+1}$ dec.

   $\cdot \lim_{n\to\infty} \frac{1}{n} = 0$

   $\Big\}$ serie alter. converge

   $|b_n| = \left|(-1)^n \frac{1}{n}\right| = \frac{1}{n}$ div. $\Rightarrow \sum \frac{(-1)^n}{n}$ no conv. abs

4. *Ejercicio 4.2*

   Determine la convergencia absoluta de $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{n\ln^2(n)}$.

$\boxed{\cos n\pi = (-1)^n}$

$\left|\frac{(-1)^n}{n\ln^2 n}\right| = a_n = \frac{1}{n\ln^2(n)}$ $\quad f(x) = \frac{1}{x\ln^2(x)}$

- continua en $[2,\infty)$
- $f'(x) = -\frac{\ln^2(x) - 2\ln(x)}{(x\ln^2(x))^2} < 0$ $\quad x \geq 2 > 1$ decreciente
- $f(x) > 0, \; x \geq 2 > 0$

Usemos el crit. de la integral

$$\int_2^\infty \frac{dx}{x\ln^2(x)} \overset{u=\ln x}{=} \quad du = \frac{1}{x}dx \quad = \int_{\ln(2)}^\infty \frac{du}{u^2} = -\frac{1}{u}\Big|_{\ln 2}^\infty = \frac{1}{\ln 2} < \infty$$

$\Longrightarrow \sum_{n=2}^\infty \frac{\cos(n\pi)}{n\ln^2 n}$ conv. absolutamente.

5. Convergencia condicional

$\sum a_n$ converge pero no converge absolutamente.

| | | conv. | conv. abs | |
|---|---|---|---|---|
| $\frac{(-1)^n}{n^p}$ | $p>1$ | ✓ | ✓ | $1+\frac{1}{2^2}+\frac{1}{3^2}+\dots$ $\quad p=2$ |
| | $0<p\leq 1$ | ✓ | ✗ | $1+\frac{1}{2}+\frac{1}{3}+\dots$ $\quad p=1$ |
| | $p<0$ | ✗ | ✗ | $1+2+3+\dots$ $\quad p=-1$ |

6. *Ejercicio 4.3*

   Determine si $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ converge absolutamente, condicionalmente o diverge.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n+1}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{n}{n+1} = 1 = c \qquad 0 < c < \infty$$

Como $\sum \frac{1}{n^{1/2}}$ diverge por criterio P,

entonces por crit. de comparación usando

límite $\sum \frac{\sqrt{n}}{n+1}$ diverge.

## 4.2 Series de potencias

1. Definición

centro

term. lineal

term cuadratico

$$\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots + \underbrace{a_n(x-c)^n}_{\text{n-simo termino}} + \cdots$$

variable.

termino ind. principal

coeficiente

2. Convergencia

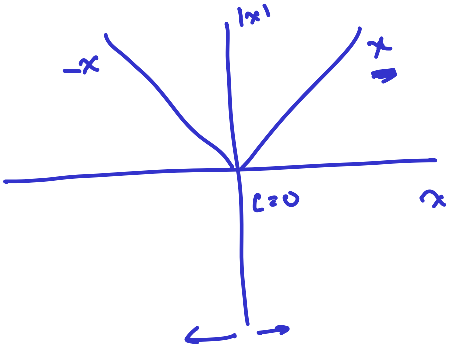
$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L = \frac{1}{R}$

R radio de conv.

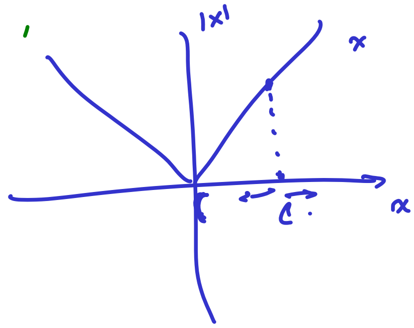
a. conv. solo $x = c$ $\qquad R = 0$

b. conv. absoluta $x \in (c-R, c+R)$ $\qquad 0 < R < \infty$

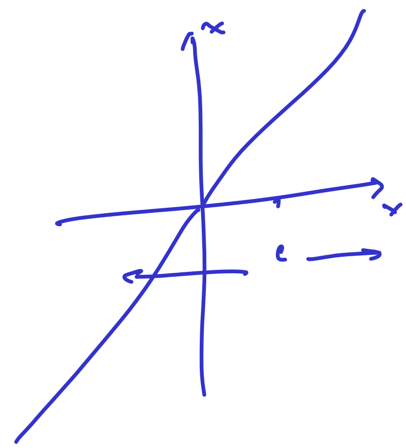
c conv. $x \in (-\infty, \infty)$ $\qquad R = \infty$




a

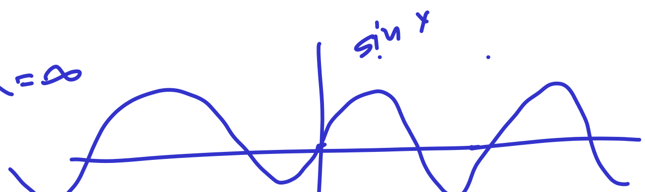


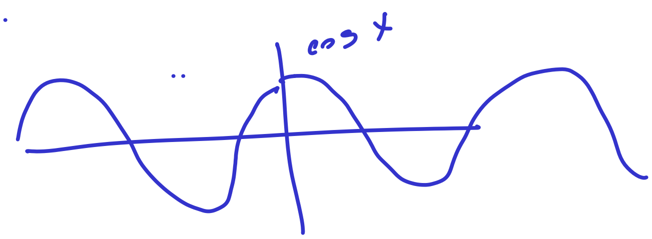

b

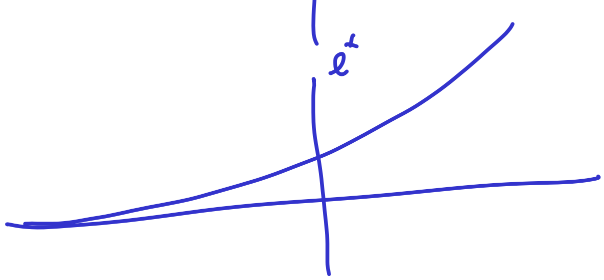



c

$R=\infty$




$R = \frac{\pi}{2}$

3. Propiedades. Sean $f(x) = \sum_{n=0}^{\infty} a_n x^n$ con radio $R$, y $g(x) = \sum_{n=0}^{\infty} b_n x^n$

$(Kx)^n$

(a) $f'(x) = \sum_{n=1}^{\infty} a_n n x^{n-1}$ radio. $R$

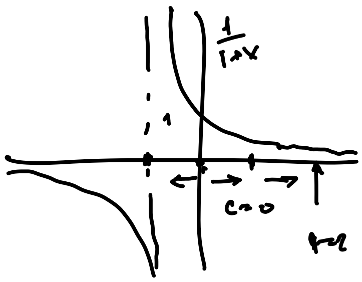
(b) $\int f(x)dx = c + \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$

(c) $f(x) + g(x) = \sum_{n=0}^{\infty} (a_n + b_n) x^n$ lineal.

(d) $f(x)g(x) = \sum_{k=0}^{\infty} c_k x^n$ donde $c_k = \sum_{n=0}^{k} a_n b_{k-n}$ producto de Cauchy

$f(Kx) = \sum a_n K^n x^n$. $f(x^m) = \sum a_n x^{m\cdot n}$

4. Series de potencias notables

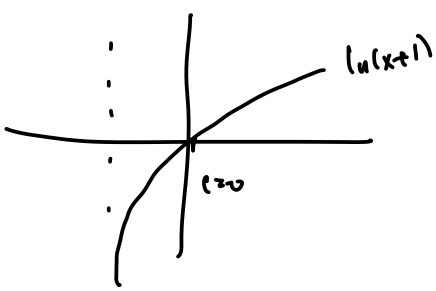



$(-1, 1)$

$$\frac{1}{x+1} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots \quad R=1$$

$$\ln(x+1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots \quad R=1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots \quad R=\infty$$




$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \quad R=\infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \quad R=\infty$$

Dos tipos de problemas

- $R$? (crit. del cociente)
- $f(x) \doteq \sum a_n x^n$ (prop. + potencias notables) $a_n$ ?

en Semana 5
- Problemas de series de potencias
- Ecuaciones diferenciales