

Divisible by Five

```
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
```

YES, the product is divisible by 5
No, the product is not divisible by 5

Run

```
361 636 277 129 434 577 796 596 727 586
156 109 714 716 548 979 386 766 137 243
331 999 922 304 657 314 634 303 877 597
363 174 431 193 361 877 403 926 279 892
749 401 346 202 783 314 333 244 798 897
674 651 517 349 337 887 617 484 379 793
542 464 962 148 946 199 302 899 606 126
519 203 137 517 146 724 898 699 747 883
126 247 469 953 398 502 582 847 384 214
348 648 331 426 783 291 557 764 939 856
753 581 797 224 537 381 283 493 196 162
382 102 629 936 883 279 966 241 907 677
945 416 122 583 667 394 654 592 977 177
866 199 483 581 954 924 991 383 754 754
199 451 798 586 829 651 517 167 704 749
622 299 486 559 973 243 839 276 803 753
```

Clear IDE Clear Output Pick a template

Chain Network

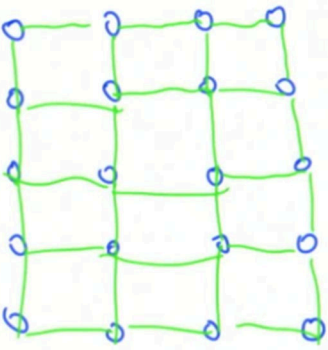
Chain Network

$n = 5$ nodes
 $m = 4$ edges

- $m = n - 1$
- $m = n$
- $m = n + 1$

Grid Network

Grid Network



$n = 20$
 $m = 15 + 16 = 31$

If we have 256 nodes arranged as a square grid, how many edges?

480

Big-Theta Reflexive

Big Θ Reflexive?

If $f(n) \in \Theta(g(n))$, is $g(n) \in \Theta(f(n))$?

- No, it doesn't follow from the definition.
- Yes, big Θ is like "=", and equality is reflexive
- Yes, because $\frac{1}{c_1}$ & $\frac{1}{c_2}$ sandwich g by f .

Big-Theta Practice

$$\underline{2n^2 + 6n + 20\log n} \in \Theta(?)$$

☒ $2n^2 + 6n + 20\log n$

☐ $20\log n$

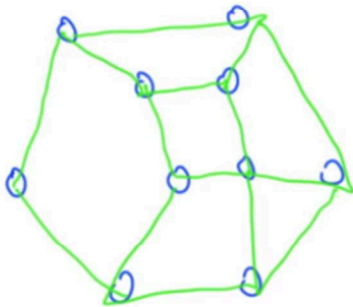
☒ $20n^2$

☒ n^2

☐ $n^2 \log n$

Regions on a Planar Graph

Regions In a Planar Graph



How many regions
in this graph?

7

Complete Graph

```
1 def make_link(G,node1,node2):
2     if node1 not in G:
3         G[node1]={}
4         (G[node1])[node2]=1
5     if node2 not in G:
6         G[node2]={}
7         (G[node2])[node1]=1
8     return G
9 #
10 # How many edges in a complete graph on n nodes?
11 #
12
13 def clique(n):
14     # Return the number of edges
15     # Try to use a mathematical formula...
16     #Graph
17     G={}
18     #edges
19     for i in range(n):
20         for j in range(n):
21             if i<j: make_link(G,i,j)
22
23     return sum([len(G[node]) for node in G.keys()])/2
24
25 for n in range (1,10):
26     print n, clique(n),n*(n-1)/2
```

Hypercube Edges

How many edges in an
n-node hypercube?

☐ $\Theta(n)$

☐ $\Theta(2^n)$

☐ $\Theta(\log n)$

☐ $\Theta(\sqrt{n})$

☐ $\Theta(n^2)$

☒ $\Theta(n \log n)$

☐ $\Theta(1)$

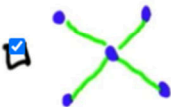
☐ $\Theta(\log^2 n)$

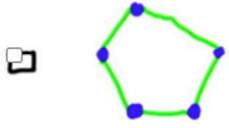
Tree Graphs

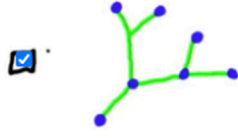
Tree Graphs

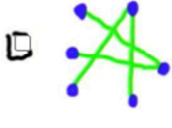
- connected
- no cycles

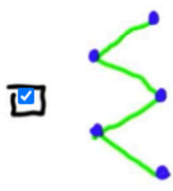
Which graphs are trees?


☒ 

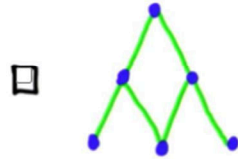
☐ 


☒ 

☐ 

☒ 

☒ 

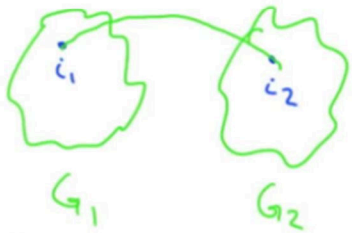
☐ 

☒ 

Recursive Graphs

What Graph Is This?

```
def makeG(n):  
    if n==1: return a single node  
    G1 = makeG(n/2)  
    G2 = makeG(n/2)  
    i1 = random node from G1  
    i2 = random node from G2  
    Make-link(G1, i1, i2)
```



G_1 G_2

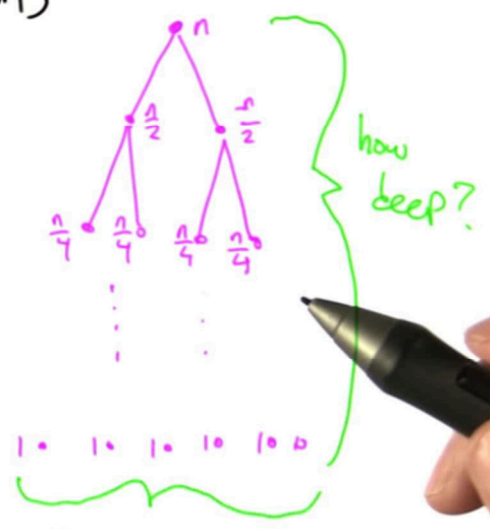
- ☒ tree
- ☐ chain
- ☐ ring

Recurrence Relation

Recurrence Relations

T : # of edges
 $T(1) = 0$
 $T(n) = 2T(n/2) + 1$

depth	leaves
<input checked="" type="checkbox"/> $\log n$	<input type="checkbox"/> $\log n$
<input type="checkbox"/> n	<input checked="" type="checkbox"/> n
<input type="checkbox"/> 2^n	<input type="checkbox"/> 2^n



Tangled Hypercube

```

G1 = makeG(n/2)
G2 = makeG(n/2)
i1 = list of nodes of G1 in random order
i2 = list of nodes of G2 in random order
for i in range(n/2): makeLink(G, i1[i], i2[i])
return G
    
```

G_1
 G_2

What Graph Structure Did we Make?

☐ ring
☐ tree
☐ hypercube
☒ none of the above