equations and variables $\qquad\qquad$ desired units

$\rho = 1.94 \left(\frac{slug}{ft^3}\right)$ density $\times \frac{32.2\ lbm}{1 slug}$ - - - $\frac{lbm}{ft^3}$

$\mu = 2.05E-5 \left(\frac{lbf \cdot s}{ft^2}\right)$ viscosity

L: length (ft)

D: diameter (in)

VA = Q: volumetric flow rate $\left(\frac{gal}{min}\right) \times \frac{231\ in^3}{1\ gal} \times \frac{1\ ft^3}{12^3\ in^3} \times \frac{1\ min}{60 s}$ = - - $\frac{ft^3}{s}$

$A = \frac{\pi}{4}D^2$: Area (ft^2)

Re: Reynold's number (dimensionless)

$$Re = \frac{\rho V D}{\mu} = \frac{\rho Q D}{A\mu}$$

f: friction factor (dimensionless)

$$f = \frac{64}{Re} = \frac{64\mu}{\rho V D} = \frac{64 A\mu}{\rho Q D} \qquad \text{for } Re \leq 2300$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \qquad \text{for } Re > 2300$$

$$\text{sol'n: } f = 0.25\left(\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right)^{-2}$$

- Darcy-Weisbach

$$f\left(\frac{L}{D}\right)\frac{V^2}{2} = \frac{\Delta P}{\rho} \quad \text{- - - - (i)}$$

where V: affective fluid velocity (ft/s)

ΔP: pressure drop $\left(\frac{lbf}{ft^2}\right) \times \left(\frac{1ft^2}{144 in^2}\right)$ - - - $\frac{lbf}{in^2}$

EQ(i) can be written in terms of f, V, or Q and ΔP

to yield a system of four equations and four unknowns

$$f_i\left(\frac{L_i}{D_i}\right)\frac{V_i^2}{2} = \frac{\Delta P}{\rho} \quad \text{- - - - (i)} \quad \text{for } i = 1,2,3$$

$$Q_1 + Q_2 + Q_3 = Q \quad \text{- - - - (4)}$$

- EQ ⋯ (i) in terms of V_1, V_2, V_3, and ΔP

$$f\left(\frac{L}{D}\right)\frac{V^2}{2} = \frac{\Delta P}{\rho} \quad \text{where } f = \frac{64\mu}{\rho V D}$$

assuming laminar (smooth) flow for $Re \leq 2300$

$$\left(\frac{64\mu}{\cancel{\rho}\cancel{V}D}\right)\left(\frac{V^2}{2}\right)\left(\frac{L}{D}\right) = \frac{\Delta P}{\cancel{\rho}}$$

$$\frac{64\mu V L}{2D^2} = \boxed{\Delta P = \frac{32\mu V L}{D^2}}$$

check the units

$$\frac{\frac{lbf\cdot \cancel{s}}{ft^2}\left(\frac{ft}{\cancel{s}}\right)(\cancel{ft})}{ft^2} = \frac{lbf}{ft^2} \checkmark$$

correct pressure units

EQ ⋯ (4) in terms of V_1, V_2, V_3

$$Q_1 + Q_2 + Q_3 = Q$$

$$V_1A_1 + V_2A_2 + V_3A_3 = Q$$

units $\frac{ft}{s}ft^2 = ft^3/s$ ✓ good as long as the initial value for Q is given as ft^3/s -NOT- gal/min

- EQ ⋯ (i) in terms of $f_1, f_2, f_3, \Delta P$

$$f\left(\frac{L}{D}\right)\frac{V^2}{2} = \frac{\Delta P}{\rho} \quad \text{where } f = \frac{64\mu}{\rho V D}, \text{ so } V = \frac{64\mu}{\rho f D}$$

$$f\left(\frac{L}{D}\right)\left(\frac{1}{2}\right)\left(\frac{64^2\mu^2}{\rho^2 f^2 D^2}\right) = \frac{\Delta P}{\rho}$$

$$\frac{(2^{11})L\mu^2}{f\rho D^3} = \Delta P \quad \text{check units}$$

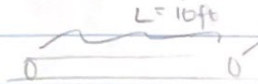
$$\frac{ft\ \frac{lbf^2\cdot \cancel{s^2}}{ft^4}}{\frac{lbm}{\cancel{ft^3}}\ \cancel{ft^3}} \times \frac{(1\,\cancel{lbm} \times 32.2\ ft/\cancel{s^2})}{(1\ \cancel{lbf})} = \frac{lbf}{ft^2}$$

but we need the 32.2 factor

So//

$$\boxed{\frac{(32.2)(2^{11})L\mu^2}{f\rho D^3} = \Delta P} \leftarrow \frac{lbf}{ft^2}$$

- solving for f and V on a single pipe



take the flow rate to be

$$Q_1 = \left(250\ \frac{gal}{min}\right)\left(\frac{231\ in^3}{1\ gal}\right)\left(\frac{1\ ft^3}{12^3\ in^3}\right)\left(\frac{1\ min}{60 s}\right)$$

fluid properties : μ, ρ $\quad Q_1 = .557\ ft^3/s$

so// $Q_1 = V_1 A_1$, $V_1 = \frac{Q_1}{A_1}$ $\qquad V_1 = \frac{.557\ ft^3/s}{.02182\ ft^2}$

$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(2in \times \frac{1ft}{12in}\right)^2$

$A_1 = .02182\ ft^2$ $\qquad \underline{V_1 = 25.53\ ft/s}$

- find reynolds number

$$Re = \frac{\rho V D}{\mu} = \frac{(1.94\ slug/ft^3)(25.53\ ft/s)(1/6\ ft)}{(2.05\ E\text{-}5)\ \frac{lbf \cdot s}{ft^2}}\left[\frac{1\ lbf}{1\ slug \cdot 1\ ft/s^2}\right]$$

$Re = 402668$

- so we use the second equation to find the friction factor

$$f = 0.25\left(\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right)^{-2} \qquad \text{with } e = .00085\ ft$$

$$f = 0.25\left[\log_{10}\left(\frac{.00085(6)}{3.7} + \frac{5.74}{(402668)^{0.9}}\right)\right]^{-2}$$

$\underline{f = .0309}$

- solve for pressure drop

$$\rho f \left(\frac{L}{D}\right)\frac{v^2}{2} = \Delta P$$

$$\Delta P = \left(1.94\ \frac{slug}{ft^3}\right)(.0309)\left(\frac{10\ ft(6)}{1\ ft}\right)\left(\frac{25.53^2\ ft^2/s^2}{2}\right)\left[\frac{1\ lbf}{1\ slug \cdot 1\ ft/s^2}\right]$$

$\Delta P = 8.14\ psi$

$$f = 0.25\left(\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right)^{-2}$$ solve for Re

where $Re = \frac{\rho V D}{\mu}$

$$0.25\left[\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{\left(\frac{\rho V D}{\mu}\right)^{0.9}}\right)\right]^{-2}$$

$$\frac{\rho V D}{\mu} = \frac{\left[\frac{slug}{ft^3}\right]\left(\frac{ft}{s}\right)(ft)}{\frac{lbf \cdot s}{ft^2}} \cdot \left(\frac{1 lbf}{1 slug \cdot ft/s^2}\right)$$ ✓ keep

ρ in slug/ft^3

so EQ... (i) for nasty friction equation

$$f\left(\frac{L}{D}\right)\frac{V^2}{2} = \frac{\Delta P}{\rho}$$

$$\underbrace{\left[\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{\left(\frac{\rho V D}{\mu}\right)^{0.9}}\right)\right]^{-2}}_{\text{unitless}}\underbrace{\left(\frac{L}{D}\right)}_{ft/ft}\left(\frac{V^2}{8}\right)\rho - \Delta P = 0$$

$$\left(ft^2/s^2\right)\left(\frac{slug}{ft^3}\right) \cdot \frac{1 lbf}{1 slug \cdot 1 ft/s^2} = \frac{lbf}{ft^2}$$ ✓

leave density in slugs/ft^3

$eq_1 <$ f_A, f_B

$eq_2 <$ f_A, f_B

$eq_3 <$ f_A, f_B

$f_A\ f_A\ f_B$

$f_A\ f_B\ f_A$

$f_A\ f_A\ f_B$

$f_B\ f_A\ f_A$

$f_B\ f_A\ f_B$

$f_B\ f_B\ f_B$