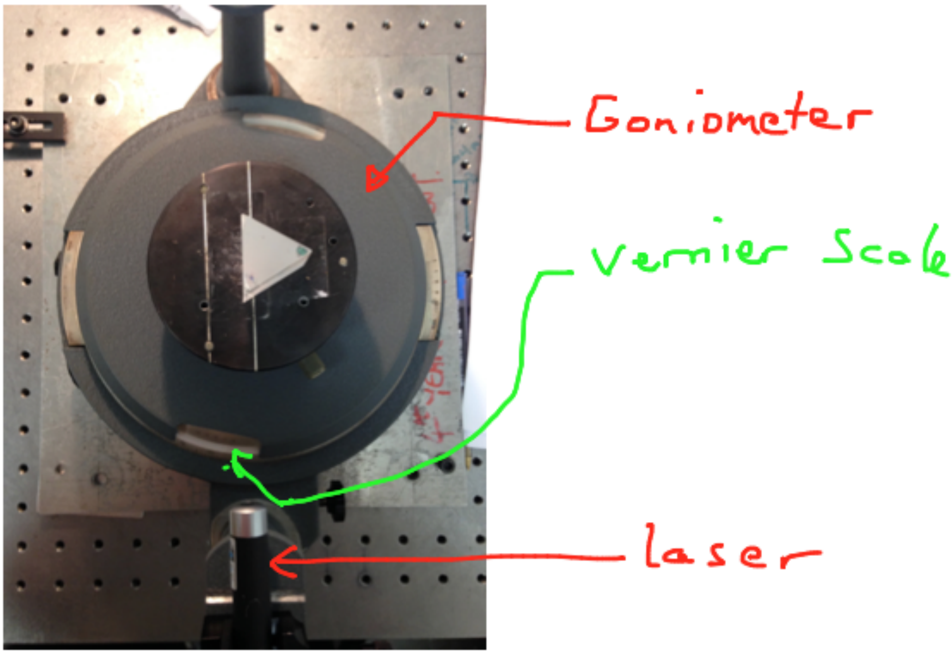
Prism Lab

Out[38]=

Out[47]=

Experiment Setup



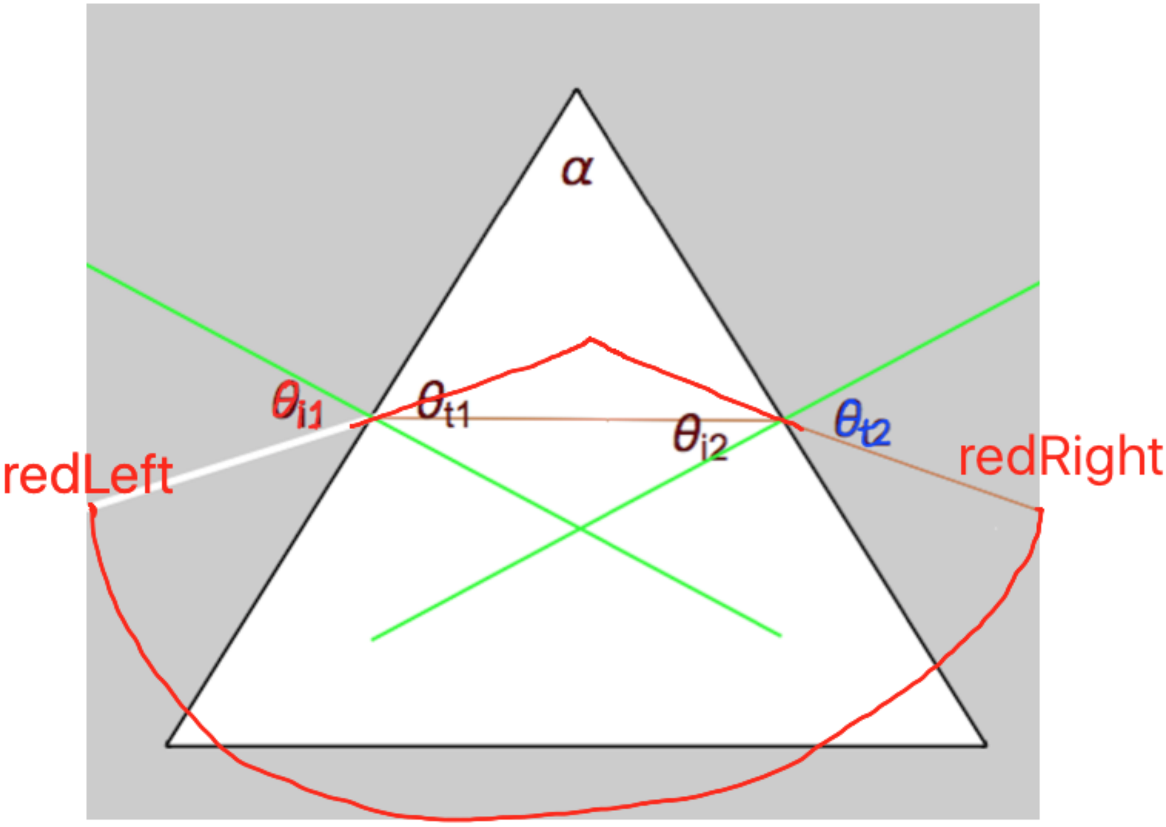
Lab Procedure

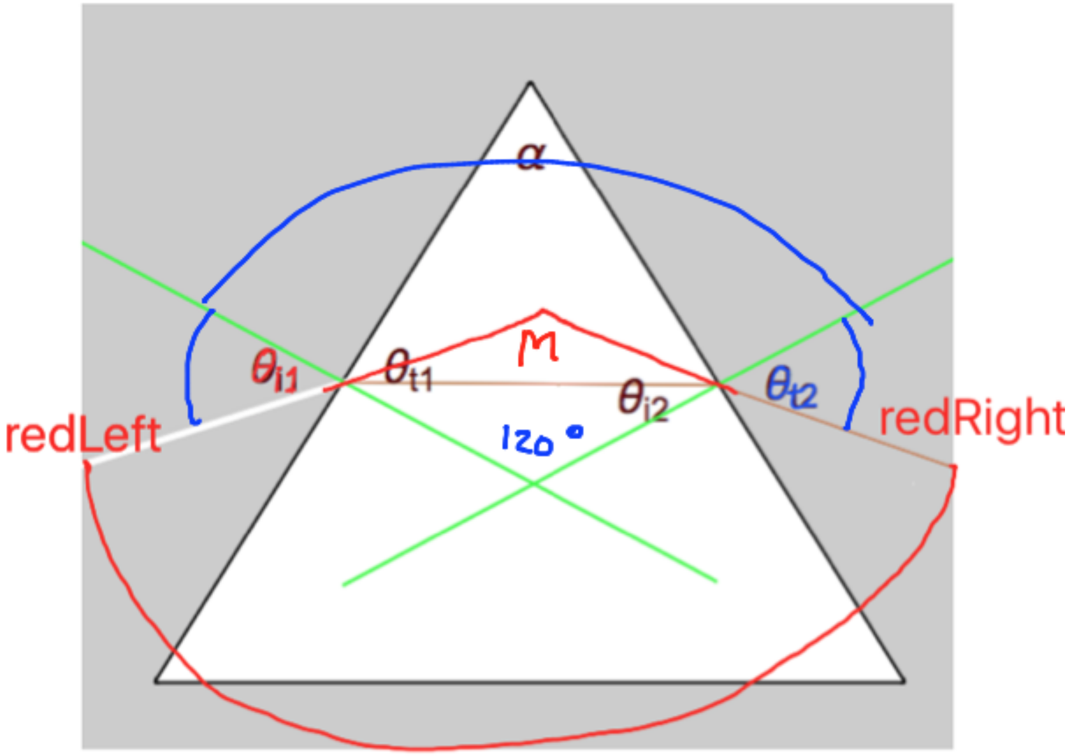
- We took four measurements 16 times.
- The corner angle of the prism $\boldsymbol{\alpha}$
- The angle of incidence $\boldsymbol{\theta_i}$ corresponding to the minimum deviation angle of the $\boldsymbol{\delta_m}$ for 3 different laser wavelengths ().

Method

We first measured the minimum deviation using the “Glance method”. Taking the difference in angle when the laser is parallel to the base of the prism and the angle when the ray is in the minimum deviation state.

- We eliminated human error in judging when the laser was “glanced” of the base of the prism by just measuring the angle at which minimum deviation occurs at two sides (joined to the corner from which we measured $\boldsymbol{\alpha}$)





- we measured angle **M** see diagram above 16 times down and recorded the reading on the vernier scale (including arc minutes as we used the SDOM to get the error).

- $$360° - (120° + \theta_{i2} + \theta_{t2}) = M$$

- We made an assumption that $\boldsymbol{\theta_{i1}} = \boldsymbol{\theta_{t2}}$ during the state of minimum deviation.

$$360° - (120° + 2\theta_i) = m$$

$$\theta_i = \frac{M - 360^\circ + 120^\circ}{2}$$

$$\theta_i = \frac{M - 240^\circ}{2}$$

we didn’t propagate error through these computations as we applied the Standard Deviation of the Mean to the data afterwards (we reasoned that the error would propagate itself through the vector computations through to the SDOM).

- I go into this in laborious detail deeper in this report.

Summary of corner angle measurement

- We reflected a laser of both sides of the corner angle α.
- We measured the difference in the angle between these position.
- We had to move the goniometer table approximately 120° in order to get the reflection point to co-allign with the reflection point on the other side. We utilized a crack on the wall far away from the experiment.

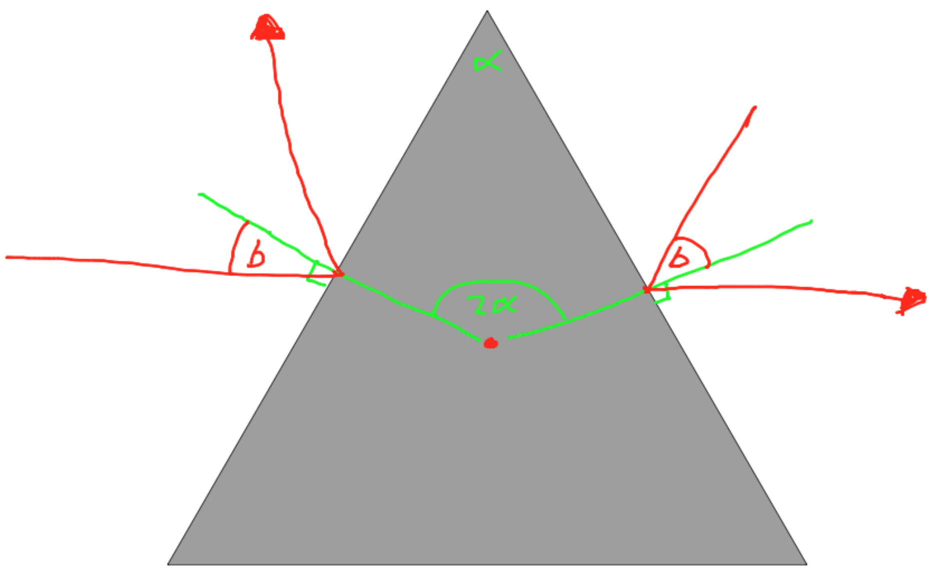
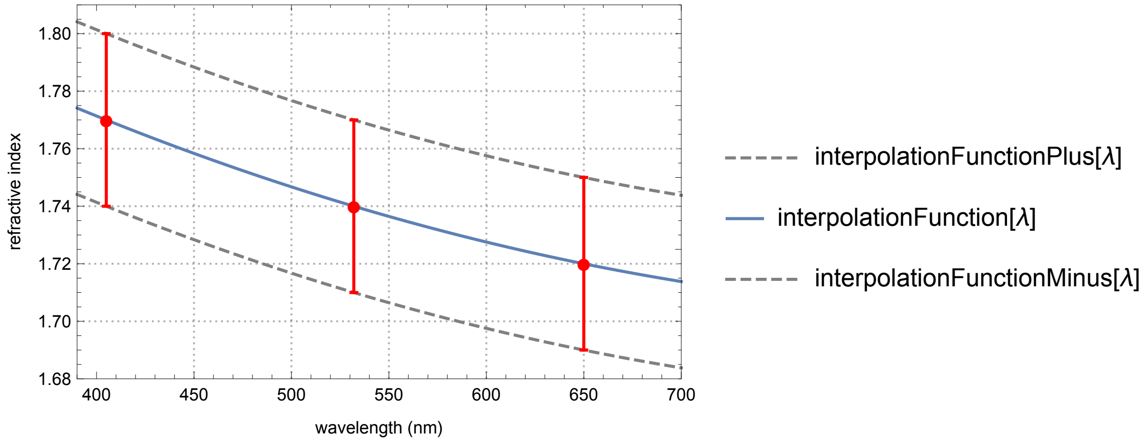
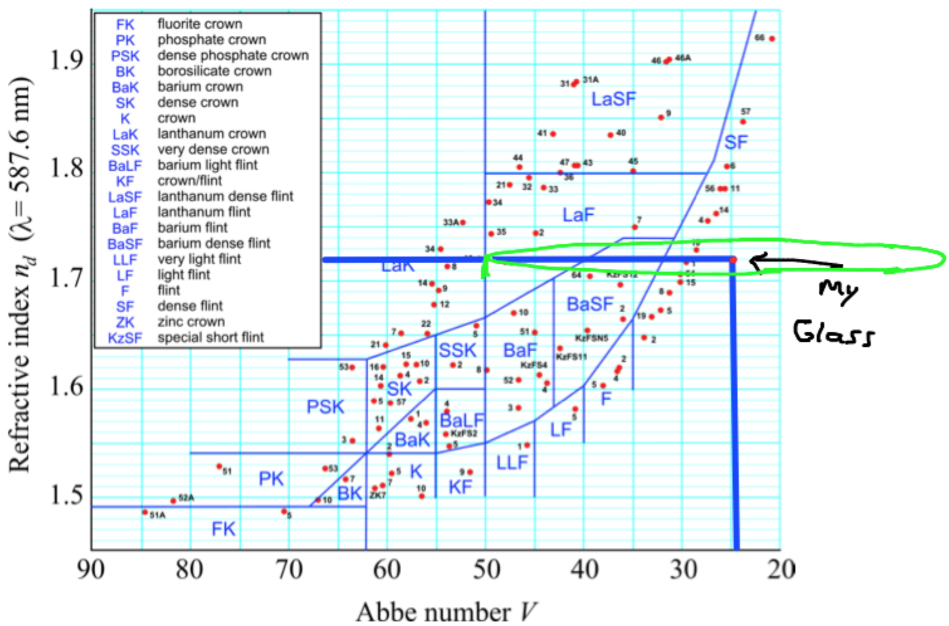


Table of results

"symbol"	"measurement"	"uncertainty"
α	60.05 °	0.01 °
δRed	58.73 °	0.04 °
δGreen	60.49 °	0.04 °
δBlue	65.21 °	0.04 °
nRed	1.72	0.03
nGreen	1.74	0.03
nBlue	1.77	0.03



- From this we calculated an Abbe number Vd of 24 with a uncertainty of 35.
- Even with this monstrous uncertainty, interpolation of refractive index gave us **dense flint glass**

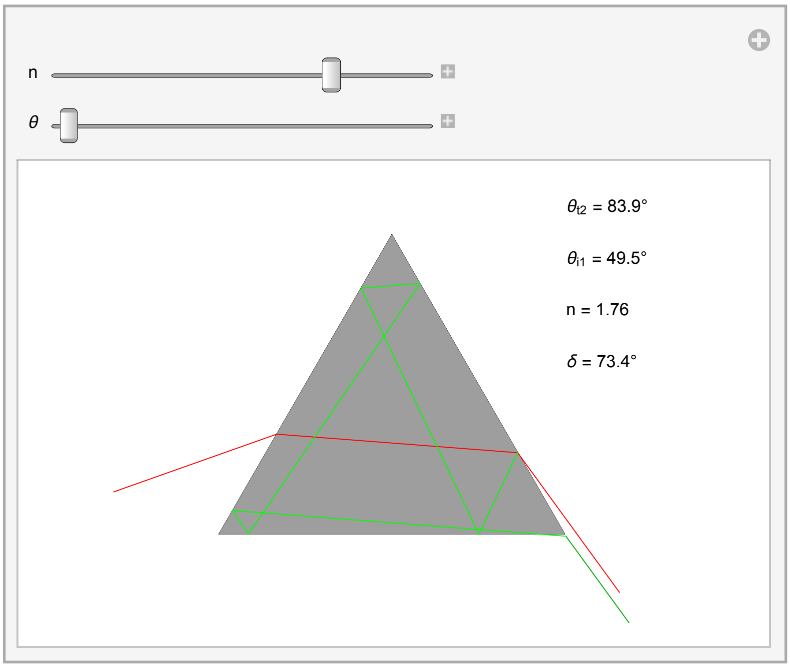


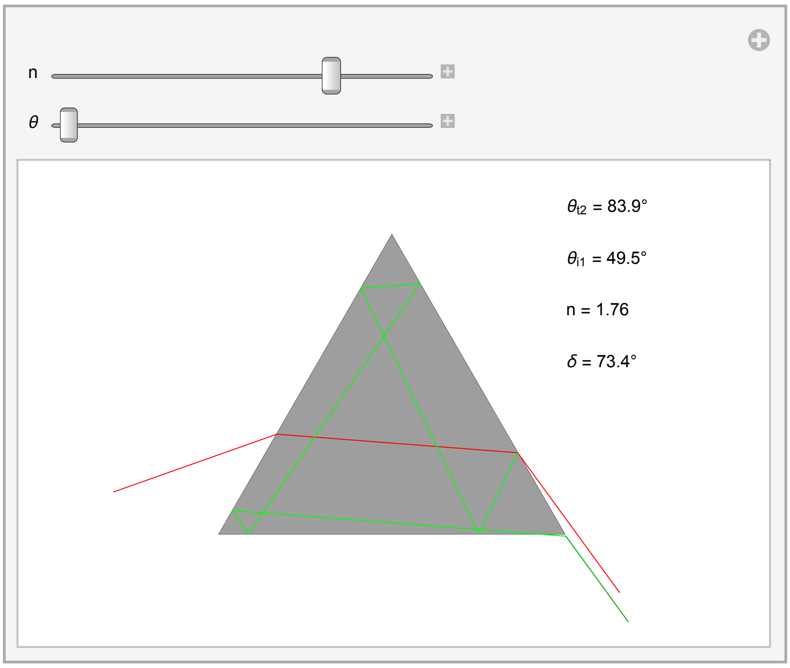
note that the green ellipse is actually much narrower (the one above is for illustrative purposes.)

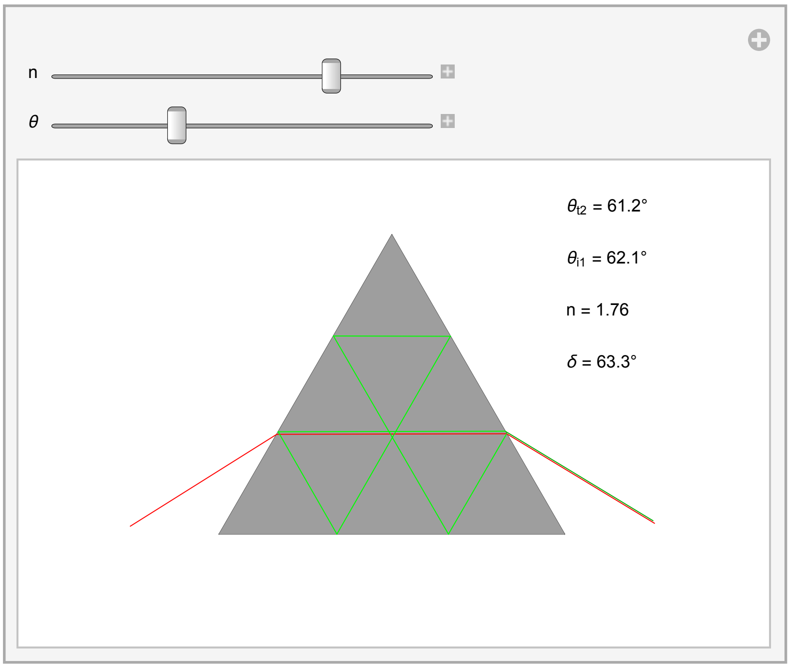
Explanation of ghost

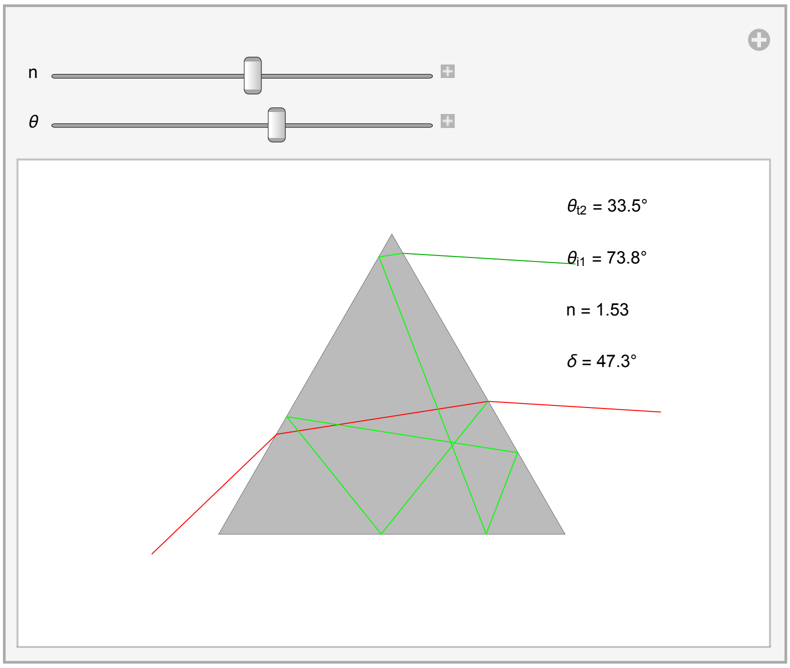
We show in the following simulation (available on GitHub (link for instructions { https://youtu.be/-2_L2nJJICE })) that the ghost could be caused by stray internal reflections. When the inner ray is parallel with the base of the prism the system undergoes a unique state of symmetry and the internal reflections merge with the transmitted ray.

```
<< prism`
```



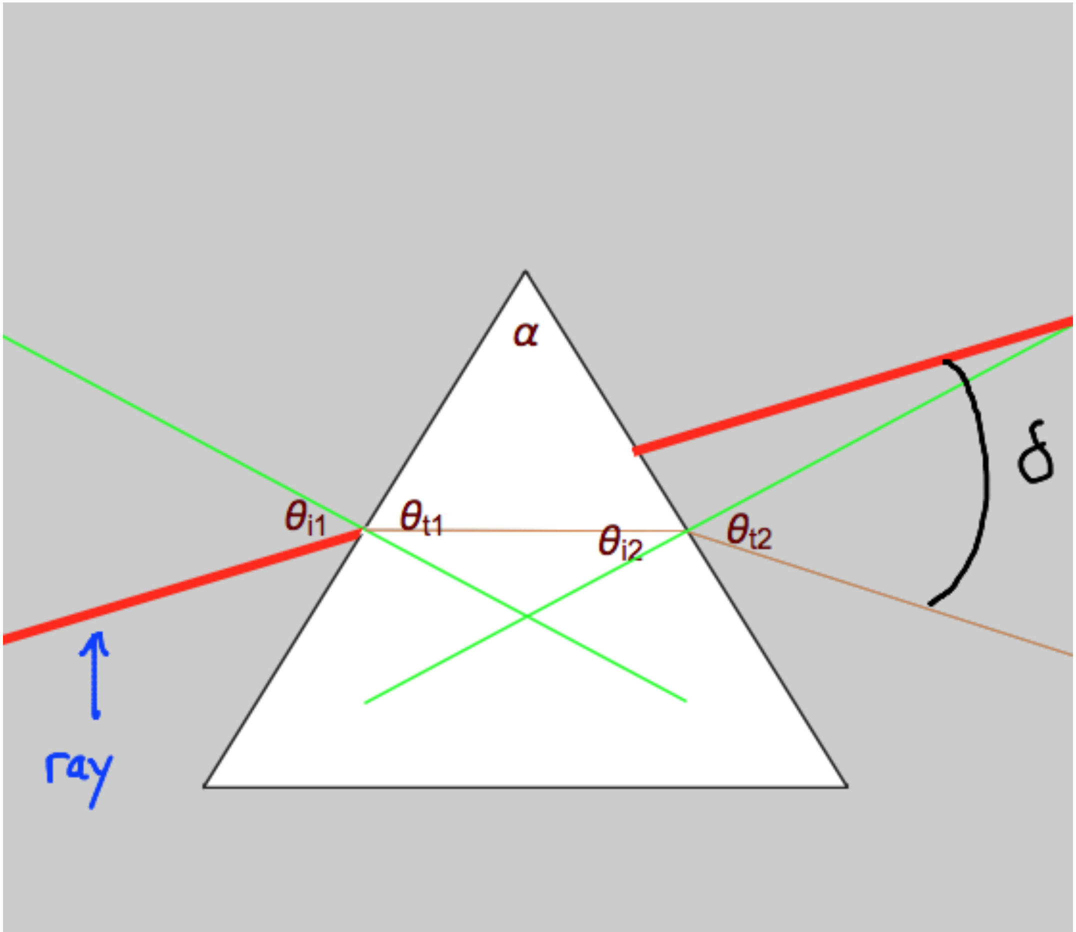



n
θ
θt2 = 61.2°
θi1 = 62.1°
n = 1.76
δ = 63.3°

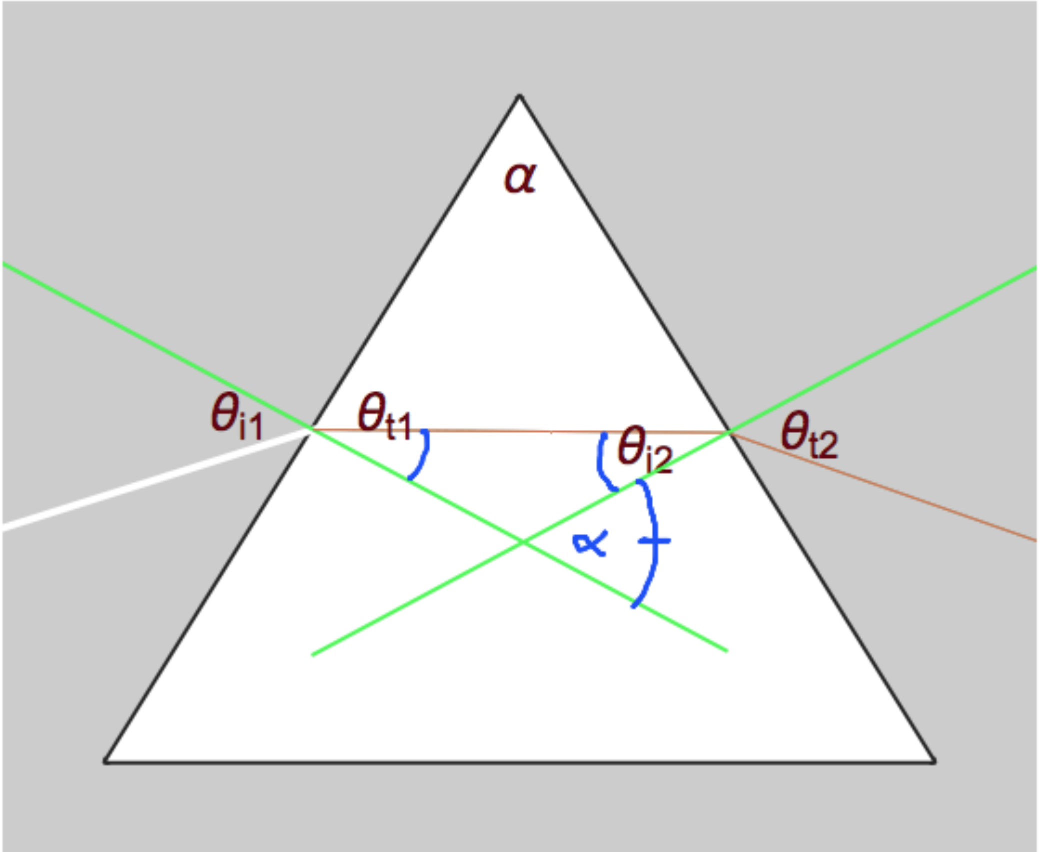

n
θ
θt2 = 33.5°
θi1 = 73.8°
n = 1.53
δ = 47.3°

Proof

consider a ray traversing a prism



- It deviates from its path by an angle δ (the angular deviation)
- apex angle α is trivially related to the internal incidence and transmission angles



$$\alpha = \theta_{t1} + \theta_{i2}$$

- By repeated application of Snell's law (see appendix B) we can express δ in terms of the incident angle θ_{i1}, the corner angle α and the refractive index of the prism **n**

$$\delta(\theta_{i1}, \alpha, n)$$

$$\delta(\theta_{i1}, \alpha, n) = \theta_{i1} + \sin^{-1}\left[(\sin\alpha)\sqrt{n^2 - \sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

Lets plot δ as a function of incidence angle θ_{i1} and we shall prove the proof :)

- let **n = 1.48** refractive index for red light for **fluorite crown glass**

- let $\boldsymbol{\alpha} = \frac{\pi}{3}$

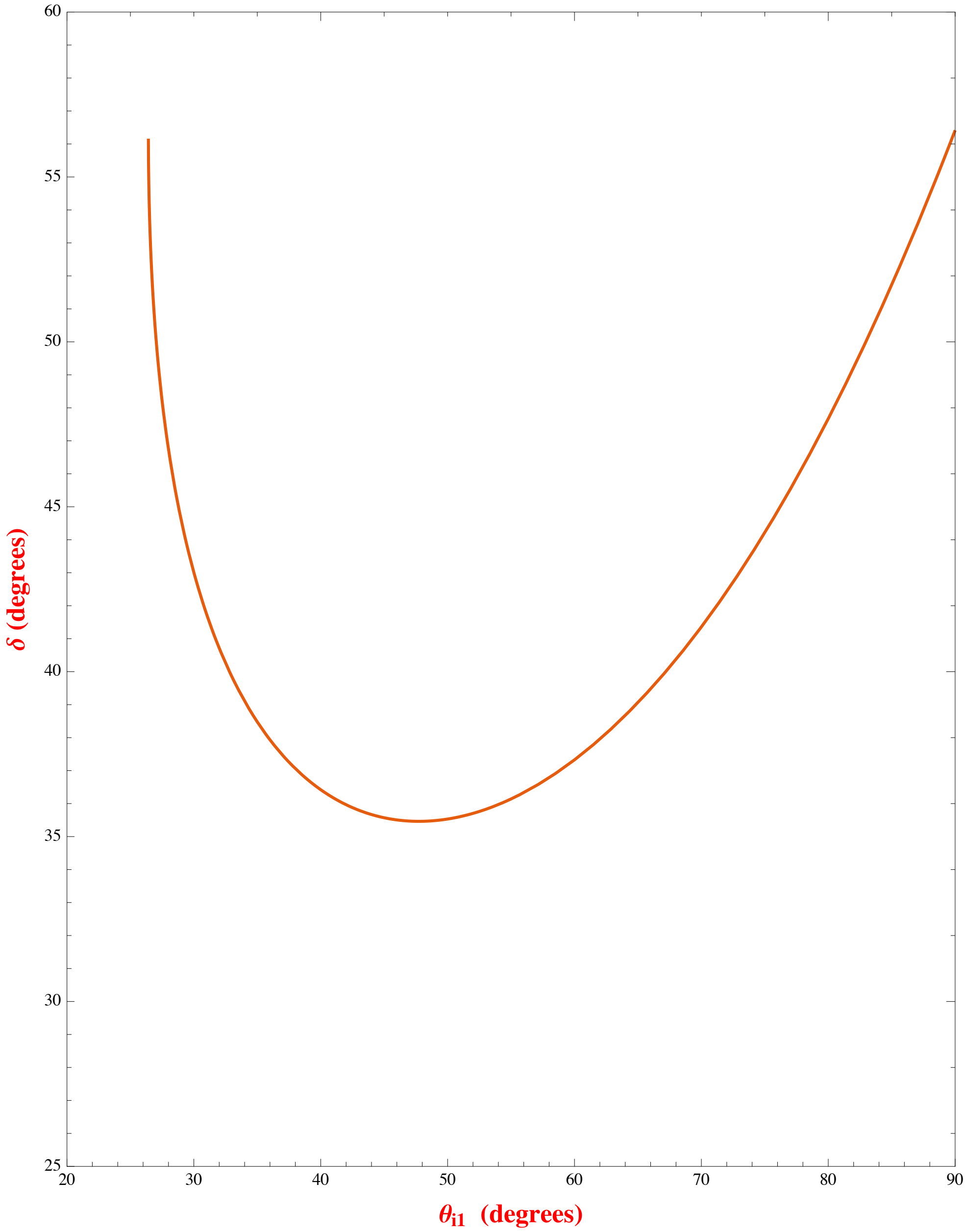
$$\delta(\theta_{i1}, \alpha, n) = \theta_{i1} + \sin^{-1}\left[(\sin\alpha)\sqrt{n^2 - \sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

```
n = Rationalize[1.48];
α = π/3;

δ[θi1_] := θi1 + ArcSin[(Sin[α]) Sqrt[n^2 - Sin[θi1]^2] - Sin[θi1] Cos[α]] - α;

radians2Degrees[rads_] := (180.0/π) rads;
degrees2Radians[degs_] := (π/180.0) degs;
r2D = radians2Degrees;
d2R = degrees2Radians;
```

```
Plot[r2D[δ[d2R[deg]]], {deg, 20, 90}, PlotRange → {{20, 90}, {25, 60}},
 AspectRatio → 1.3, PlotTheme → "Scientific",
 FrameLabel → (Style[#, Red, FontWeight → Bold, FontSize → 14] & /@
    {"θi1  (degrees)", "δ (degrees)"})]
```



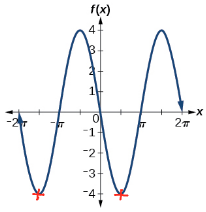
From this graph we can see that there is only 1 unique δ or angle of minimum deviation with a corresponding unique θ_{i1}

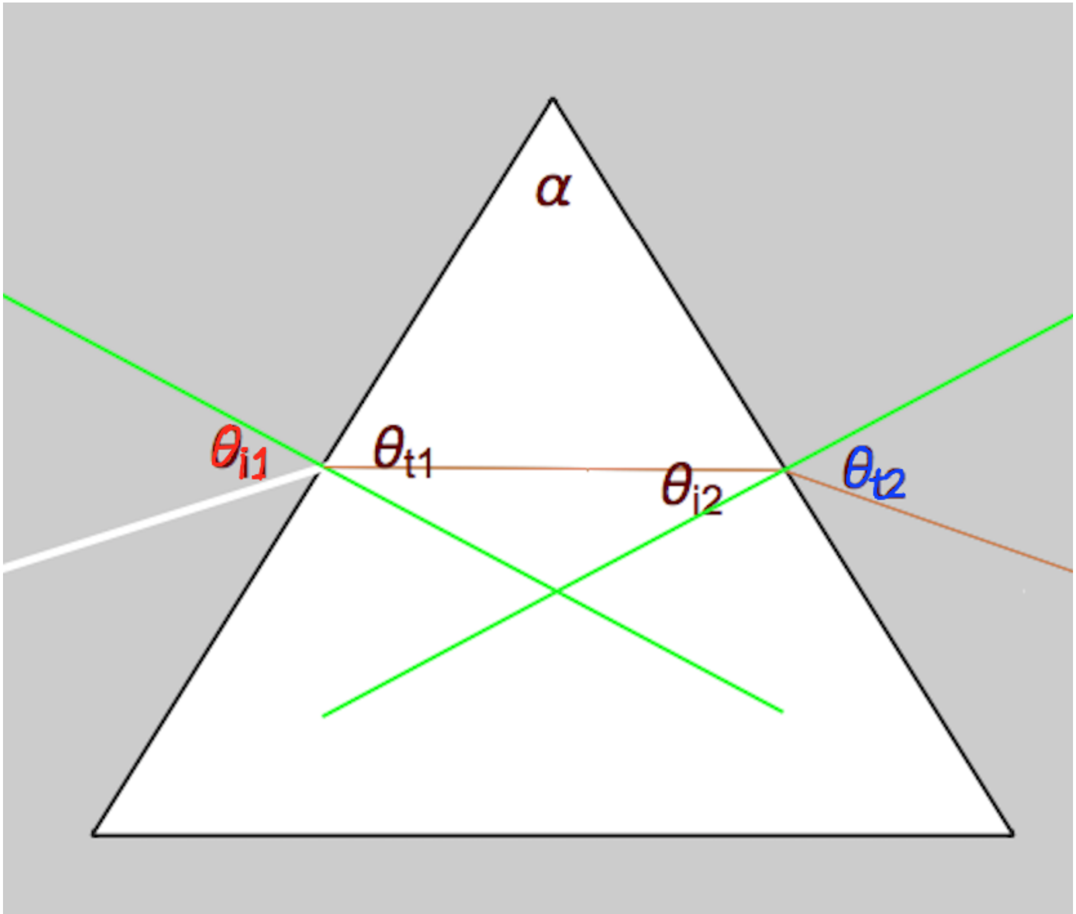
(One can prove that there is only 1 minima (more rigorously) by taking the derivative of the function, setting it equal to zero and solving for the unique θ_{i1} , this will be done below)

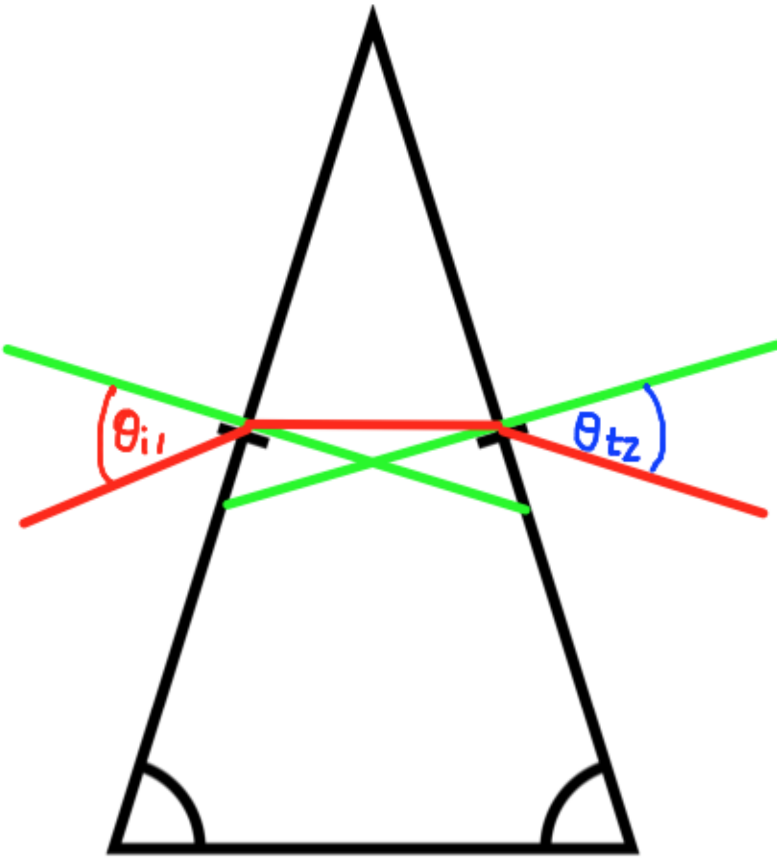
From this observation we can prove that $\theta_{i1} = \theta_{t2}$

Pretend we reverse the ray. And strike the prism from the right (see diagram below). In order to achieve the δ for this prism we have to put the ray in at the unique angle θ_{i1} relative to the normal.

If this is false then the graph would have two minima.







For a prism that is an isosceles triangle

if $\theta_{i1} = \theta_{t2}$

$\Rightarrow \theta_{t1} = \theta_{i2}$

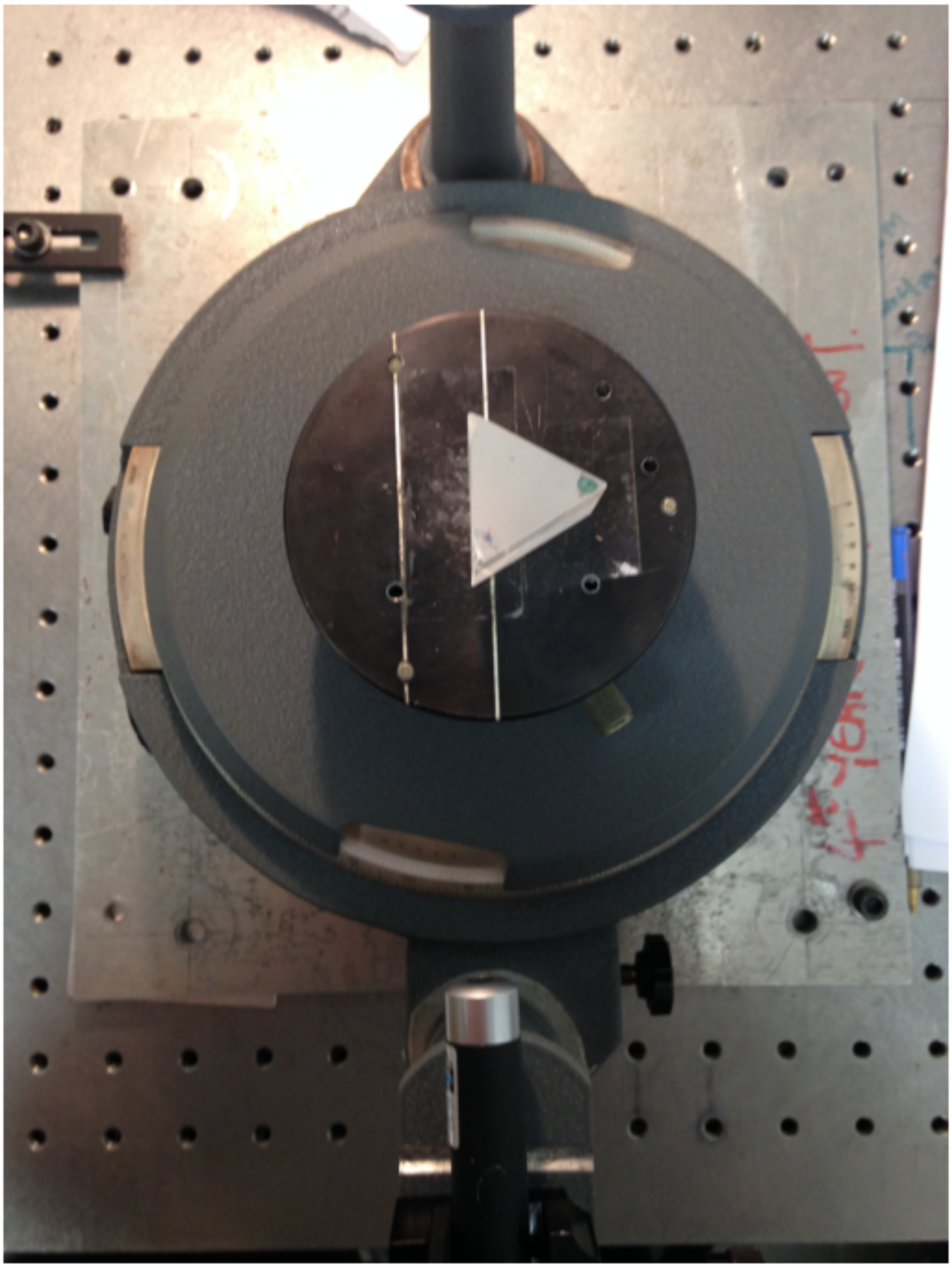
$\Rightarrow$ the internal path is parallel with the base

And these conditions only hold when the angular deviation δ is at its unique extrema for a unique θ_{i1}

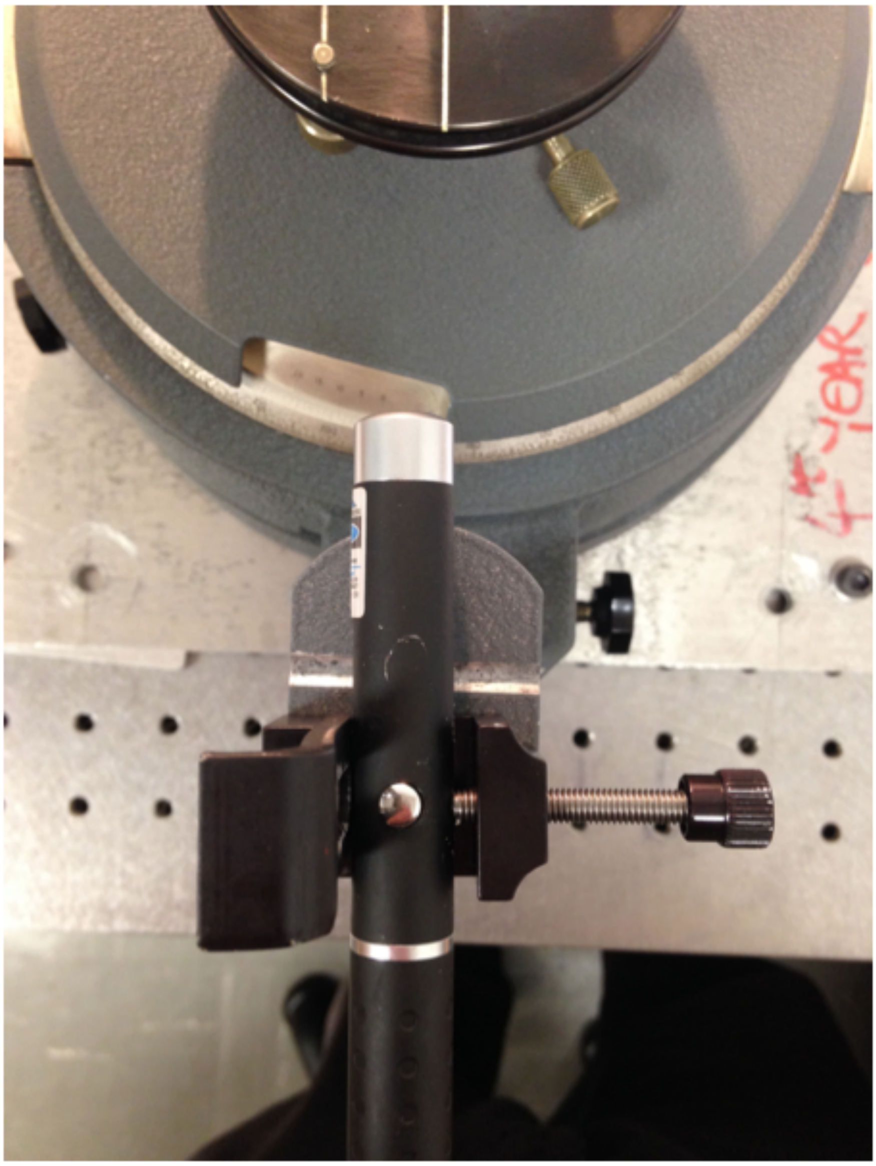
There is only one path a light ray can go to “traverse the prism symmetrically” and this path has to be parallel to the base in order to enable the light ray to be reversed.

Error of the prisms corner angle α

- Here is the initial configuration we found the experiment in. The prism is off center. Fortunately The circular table has a white line that we hope goes through the centre of rotation. We tried to orientate the laser along this white line.



- We attempted to center our prism on the goniometer table as best we could.
- We tried to move our laser beam through the center of the circular table. We had to find bits of metal from around the lab to try to do this.

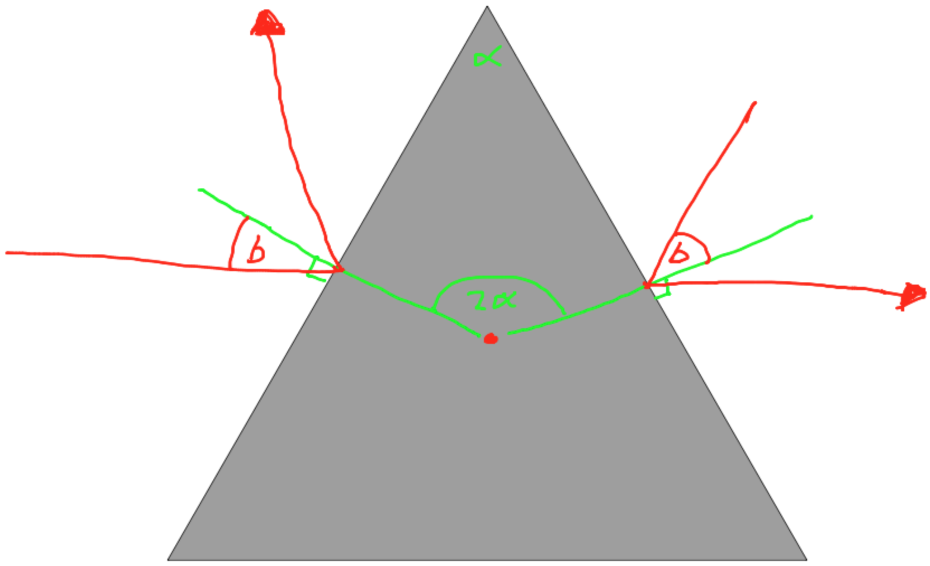


- We reflected the laser off a face of the prism (see diagram below)

```
data = {{139, 55, 19, 52},
        {139, 54, 19, 50},
        {139, 56, 19, 52},
        {140, 6, 19, 54},
        {140, 0, 19, 54},
        {140, 0, 19, 51},
        {139, 55, 19, 52},
        {140, 0, 19, 55}};

convertToArcMinutes2[{a_, b_, c_, d_}] :=
  (Quantity[a, "Degrees"] + Quantity[b, "ArcMinutes"]) -
   (Quantity[c, "Degrees"] + Quantity[d, "ArcMinutes"]);
dataInMinutes = convertToArcMinutes2 /@ data;
dataInDegrees = UnitConvert[#, "Degrees"] & /@ dataInMinutes // N;
```

- We had to move the goniometer table approximately 120° in order to get the reflection point to co-allign with the reflection point on the other side. We utilized a crack on the wall far away from the experiment.




```
αVector = dataInDegrees / 2
```

{60.025°, 60.0333°, 60.0333°, 60.1°, 60.05°, 60.075°, 60.025°, 60.0417°}

```
meanα = Mean[αVector]
```

60.0479°

```
SDOM = StandardDeviation[αVector] / Sqrt[8]
```

0.00941625°

$\alpha = 60.05°$

$\Delta\alpha = 0.01°$

Measuring δm for the Red Laser

- we collected data from the Vernier scale at the positions of minimum deviation at both sides of angle α

In fact at the positions of **incident angles** corresponding to minimum deviation

Readings of the vernier scale of the instrument.

$$\text{redLeft} = \begin{pmatrix} 175 & 40 & 18 \\ 176 & 0 & 2 \\ 176 & 0 & 0 \\ 175 & 40 & 10 \\ 176 & 0 & 0 \\ 176 & 0 & 0 \\ 175 & 40 & 12 \\ 176 & 0 & 0 \\ 177 & 0 & 2 \\ 177 & 0 & 17 \\ 177 & 0 & 6 \\ 177 & 0 & 8 \\ 177 & 0 & 18 \\ 176 & 40 & 19 \\ 177 & 0 & 11 \\ 177 & 0 & 12 \end{pmatrix}; \text{redRight} = \begin{pmatrix} 54 & 40 & 5 \\ 54 & 20 & 16 \\ 54 & 40 & 7 \\ 54 & 20 & 15 \\ 54 & 40 & 0 \\ 54 & 40 & 4 \\ 54 & 40 & 2 \\ 54 & 40 & 7 \\ 55 & 40 & 7 \\ 56 & 20 & 3 \\ 56 & 0 & 0 \\ 55 & 40 & 18 \\ 56 & 0 & 5 \\ 56 & 0 & 6 \\ 55 & 40 & 4 \\ 55 & 40 & 13 \end{pmatrix};$$

```
{{54, 40, 5}, {54, 20, 16}, {54, 40, 7}, {54, 20, 15}, {54, 40, 0},
 {54, 40, 4}, {54, 40, 2}, {54, 40, 7}, {55, 40, 7}, {56, 20, 3}, {56, 0, 0},
 {55, 40, 18}, {56, 0, 5}, {56, 0, 6}, {55, 40, 4}, {55, 40, 13}}
```

We calculate an incident angle corresponding to the instruments readings

```
convertToArcMinutes[{a_, b_, c_}] :=
 UnitConvert[Quantity[a, "Degrees"] + Quantity[b, "ArcMinutes"] +
    Quantity[c, "ArcMinutes"], "ArcMinutes"] // N
```

```
convertToArcMinutes /@ redLeft
```

```
{10 558.', 10 562.', 10 560.', 10 550.', 10 560.', 10 560.', 10 552.', 10 560.',
 10 622.', 10 637.', 10 626.', 10 628.', 10 638.', 10 619.', 10 631.', 10 632.'}
```

```
convertToArcMinutes /@ redRight
```

```
{3285.', 3276.', 3287.', 3275.', 3280.', 3284.', 3282.', 3287.',
 3347.', 3383.', 3360.', 3358.', 3365.', 3366.', 3344.', 3353.'}
```

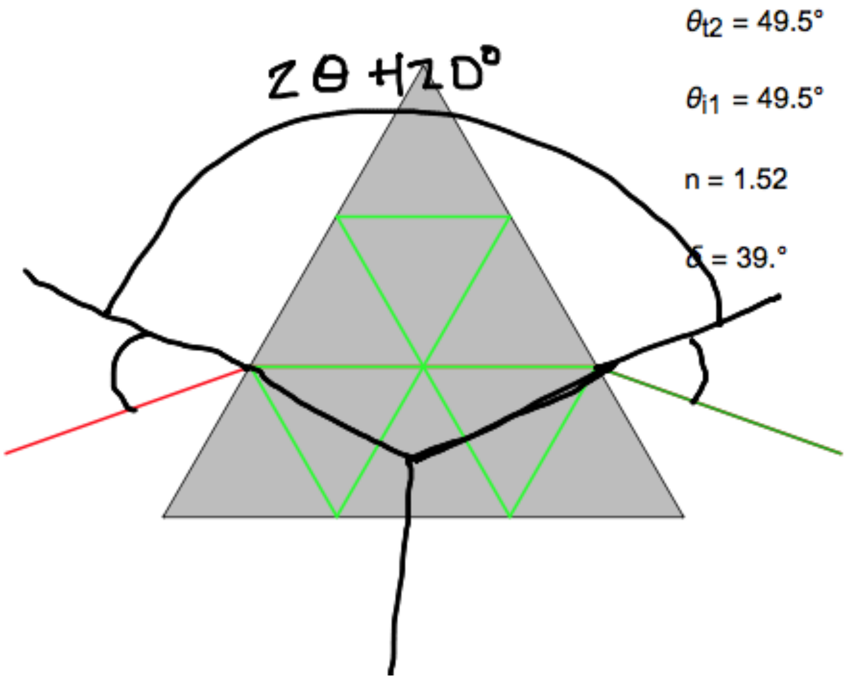
```
LRmatrix =
 Transpose[{convertToArcMinutes /@ redLeft, convertToArcMinutes /@ redRight}]
```

{{10 558.', 3285.'}, {10 562.', 3276.'},
{10 560.', 3287.'}, {10 550.', 3275.'},
{10 560.', 3280.'}, {10 560.', 3284.'}, {10 552.', 3282.'},
{10 560.', 3287.'}, {10 622.', 3347.'}, {10 637.', 3383.'},
{10 626.', 3360.'}, {10 628.', 3358.'}, {10 638.', 3365.'},
{10 619.', 3366.'}, {10 631.', 3344.'}, {10 632.', 3353.'}}

```
subTractor[{a_, b_}] := Quantity[360, "Degrees"] - (a - b)
```

```
DD = subTractor /@ LRmatrix
```

{14 327.', 14 314.', 14 327.', 14 325.', 14 320.', 14 324.', 14 330.', 14 327.',
14 325.', 14 346.', 14 334.', 14 330.', 14 327.', 14 347.', 14 313.', 14 321.'}



$$DD = 2\theta + 120^\circ$$

$$\frac{DD - 120^\circ}{2} = \theta$$

```
THETA = (DD - Quantity[120, "Degrees"]) / 2
```

{3563.5', 3557.', 3563.5', 3562.5', 3560.', 3562.', 3565.', 3563.5', 3562.5', 3573.', 3567.', 3565.', 3563.5', 3573.5', 3556.5', 3560.5'}

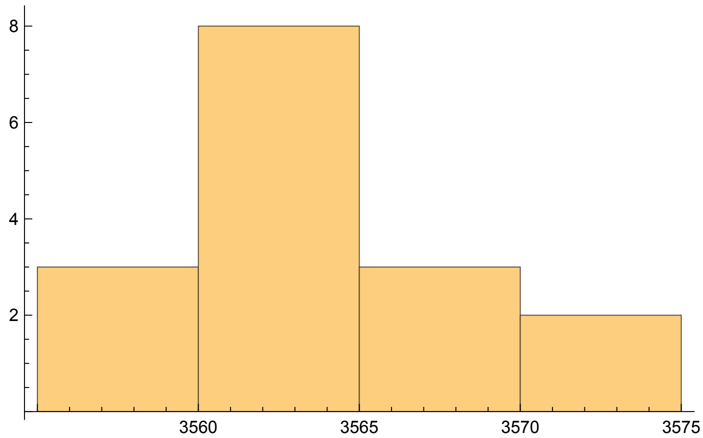
```
Grid[{#} & /@ THETA]
```

3563.5'
3557.'
3563.5'
3562.5'
3560.'
3562.'
3565.'
3563.5'
3562.5'
3573.'
3567.'
3565.'
3563.5'
3573.5'
3556.5'
3560.5'

```
Insert[ReplacePart[Grid[{#} & /@ THETA],
  1 → Prepend[First[Grid[{#} & /@ THETA]], {"incident angle"}]],
 {Background → {None, {GrayLevel[0.7], {White}}}, Dividers → {Black, {2 → Black}},
  Frame → True, Spacings → {2, {2, {0.7}, 2}}}, 2]
```

incident angle
3563.5'
3557.'
3563.5'
3562.5'
3560.'
3562.'
3565.'
3563.5'
3562.5'
3573.'
3567.'
3565.'
3563.5'
3573.5'
3556.5'
3560.5'

```
Histogram[THETA]
```



This looks normally distributed, so lets use the Standard

Deviation of the Mean formula.

Standard Deviation of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$\sigma_x =$ standard deviation of all our measurements

$N =$ number of measurements

```
SDOM = StandardDeviation[THETA] / Sqrt[16]
```

```
1.16075'
```

```
Δθ = UnitConvert[SDOM, "ArcMinutes"]
```

```
1.16075'
```

this is 1 Arc Minute

```
UnitConvert[Δθ, "Degrees"]
```

```
0.0193458°
```

- The error in θ is 0.02°

```
meanIncidentAngle = Mean[THETA] ~ UnitConvert ~ "Degrees"
```

```
59.3943°
```

δ

$$\delta = \theta_{i1} + \theta_{t2} - \alpha \qquad (5.52)$$

```
Clear[δm, α];
α = Quantity[60.05, "Degrees"];
δm = 2 (meanIncidentAngle) - α
```

```
58.7385°
```

- Error propagation rules

Addition or subtraction: If

$$Q = a + b$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2}$$

Measured quantity times exact number:

$$Q = Ax$$

then

$$\delta Q = |A|\,\delta x$$

```
UnitConvert[2 (meanIncidentAngle) - α, "Degrees"]
```

58.7385°

```
Clear[α, Δα, θ, Δθ, δm, Δδm]
Δα = 0.01;
α = 60.05;
θ = 59.39;
Δθ = 0.02;
```

$$\delta_m = 2\theta - \alpha$$

$$\Delta\delta_m = \sqrt{(2\,\Delta\theta)^2 + (\Delta\alpha)^2}$$

```
δm = 2 θ - α
```

58.73

```
Δδm = √((2 Δθ)² + (Δα)²)
```

0.0412311

- so our angle of minimum deviation for the red laser is:

$\delta m = 58.73°$

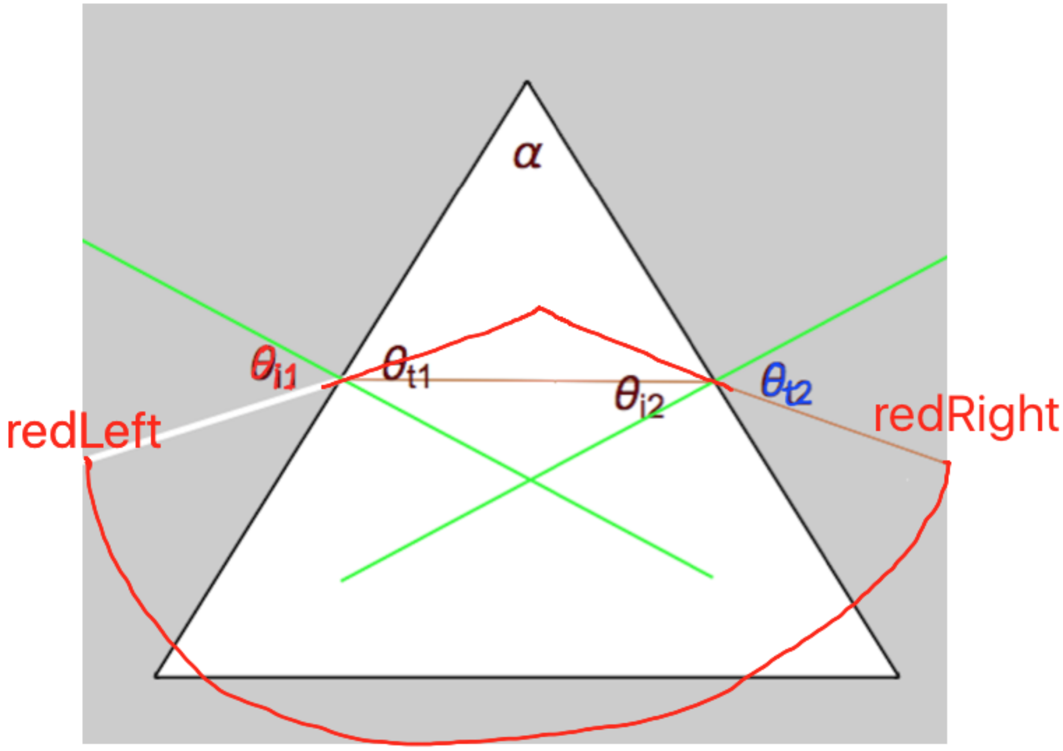
$\Delta\delta m = 0.04°$

Measuring δm for the Green laser and finding:

Error in δ_m for green laser (λ ~ 532nm)

Here I collect data from the Vernier scale at the positions of minimum deviation at both sides of angle α

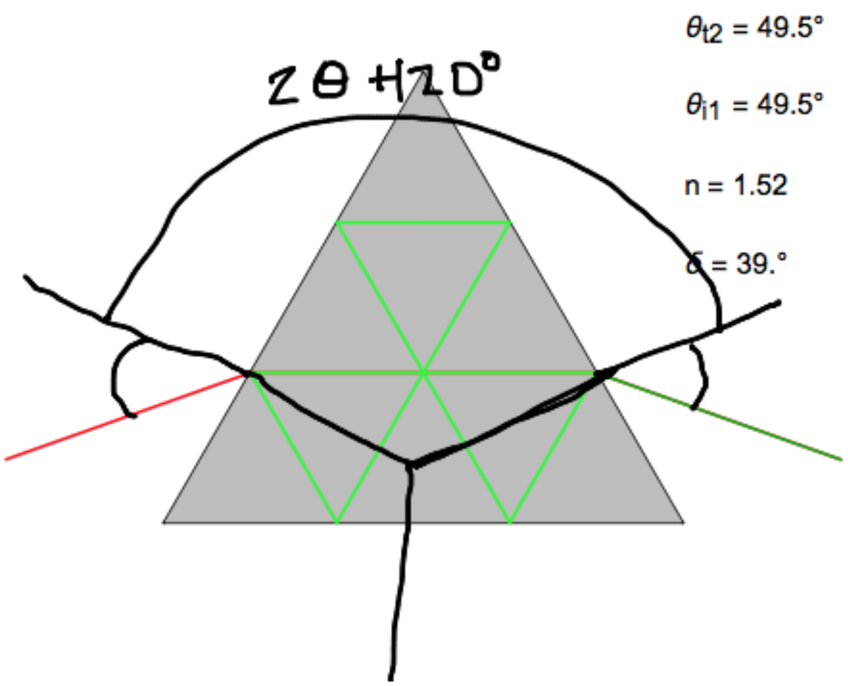
In fact at the positions of **incident angles** corresponding to minimum deviation




```
(*Readings of the vernier scale of the instrument.*)
```

$$\text{greenLeft} = \begin{pmatrix} 173 & 0 & 11 \\ 173 & 0 & 10 \\ 173 & 0 & 16 \\ 173 & 0 & 1 \\ 173 & 0 & 5 \\ 173 & 0 & 2 \\ 173 & 0 & 5 \\ 173 & 0 & 2 \\ 173 & 0 & 2 \\ 172 & 40 & 18 \\ 172 & 40 & 11 \\ 172 & 40 & 18 \\ 172 & 40 & 16 \\ 172 & 40 & 5 \\ 172 & 40 & 19 \\ 172 & 40 & 11 \end{pmatrix}; \text{greenRight} = \begin{pmatrix} 53 & 20 & 15 \\ 53 & 20 & 15 \\ 53 & 20 & 5 \\ 53 & 40 & 5 \\ 53 & 20 & 15 \\ 53 & 20 & 10 \\ 53 & 40 & 3 \\ 53 & 40 & 2 \\ 53 & 20 & 10 \\ 53 & 20 & 5 \\ 53 & 20 & 14 \\ 53 & 20 & 15 \\ 53 & 20 & 10 \\ 53 & 20 & 7 \\ 53 & 20 & 8 \\ 53 & 20 & 10 \end{pmatrix};$$

```
convertToArcMinutes[{a_, b_, c_}] :=
  UnitConvert[Quantity[a, "Degrees"] + Quantity[b, "ArcMinutes"] +
     Quantity[c, "ArcMinutes"], "ArcMinutes"] // N;
convertToArcMinutes /@ greenLeft;
convertToArcMinutes /@ greenRight;
LRmatrix = Transpose[
   {convertToArcMinutes /@ greenLeft, convertToArcMinutes /@ greenRight}];
subTractor[{a_, b_}] := Quantity[360, "Degrees"] - (a - b);
DD = subTractor /@ LRmatrix;
```



$$DD = 2\theta + 120^\circ$$

$$\frac{DD - 120^\circ}{2} = \theta$$

```
THETA = (DD - Quantity[120, "Degrees"]) / 2
```

{3612.' , 3612.5' , 3604.5' , 3622.' , 3615.' , 3614.' , 3619.' , 3620.' , 3614.' , 3613.5' , 3621.5' , 3618.5' , 3617.' , 3621.' , 3614.5' , 3619.5'}

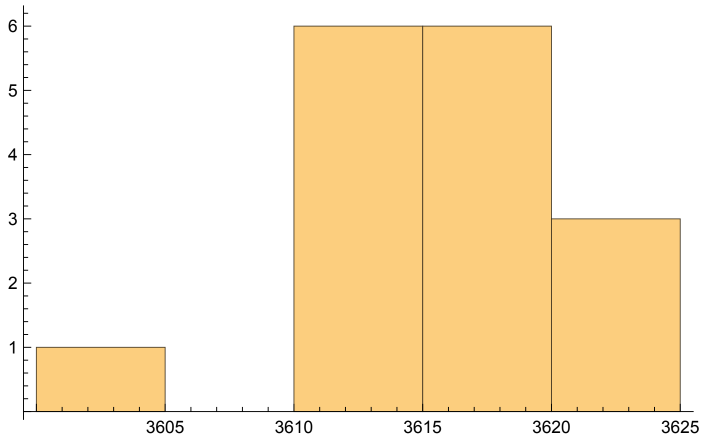
```
Grid[{#} & /@ THETA]
```

3612.'
3612.5'
3604.5'
3622.'
3615.'
3614.'
3619.'
3620.'
3614.'
3613.5'
3621.5'
3618.5'
3617.'
3621.'
3614.5'
3619.5'

```
Insert[ReplacePart[Grid[{#} & /@ THETA],
  1 → Prepend[First[Grid[{#} & /@ THETA]], {"incident angle"}]],
 {Background → {None, {GrayLevel[0.7], {White}}}, Dividers → {Black, {2 → Black}},
  Frame → True, Spacings → {2, {2, {0.7}, 2}}}, 2]
```

incident angle
3612.'
3612.5'
3604.5'
3622.'
3615.'
3614.'
3619.'
3620.'
3614.'
3613.5'
3621.5'
3618.5'
3617.'
3621.'
3614.5'
3619.5'

```
Histogram[THETA]
```



This looks normally distributed, so lets use the Standard

Deviation of the Mean formula.

Standard Deviation of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$\sigma_x =$ standard deviation of all our measurements
$N =$ number of measurements

```
SDOM = StandardDeviation[THETA] / Sqrt[16]
```

1.14266'

- 1 Arc minute can be rounded up to 0.02 Degrees

```
meanIncidentAngle = Mean[THETA] ~UnitConvert~ "Degrees"
```

60.2693°

δ

$$\delta = \theta_{i1} + \theta_{i2} - \alpha \qquad (5.52)$$

```
Clear[δm, α];
α = Quantity[60.05, "Degrees"];
δm = 2 (meanIncidentAngle) - α
```

60.4885°

- Error propagation rules

Addition or subtraction: If

$$Q = a + b$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2}$$

Measured quantity times exact number:

$$Q = Ax$$

then

$$\delta Q = |A|\,\delta x$$

```
UnitConvert[2 (meanIncidentAngle) - α, "Degrees"]
```

60.4885°

```
Clear[α, Δα, θ, Δθ, δm, Δδm]
Δα = 0.01;
α = 60.05;
θ = 60.27;
Δθ = 0.02;
```

$$\delta_m = 2\theta - \alpha$$

$$\Delta\delta_m = \sqrt{(2\Delta\theta)^2 + (\Delta\alpha)^2}$$

```
δm = 2 θ - α
```

60.49

$$\Delta\delta m = \sqrt{(2\,\Delta\theta)^2 + (\Delta\alpha)^2}$$

0.0412311

- so our angle of minimum deviation for the green laser is:

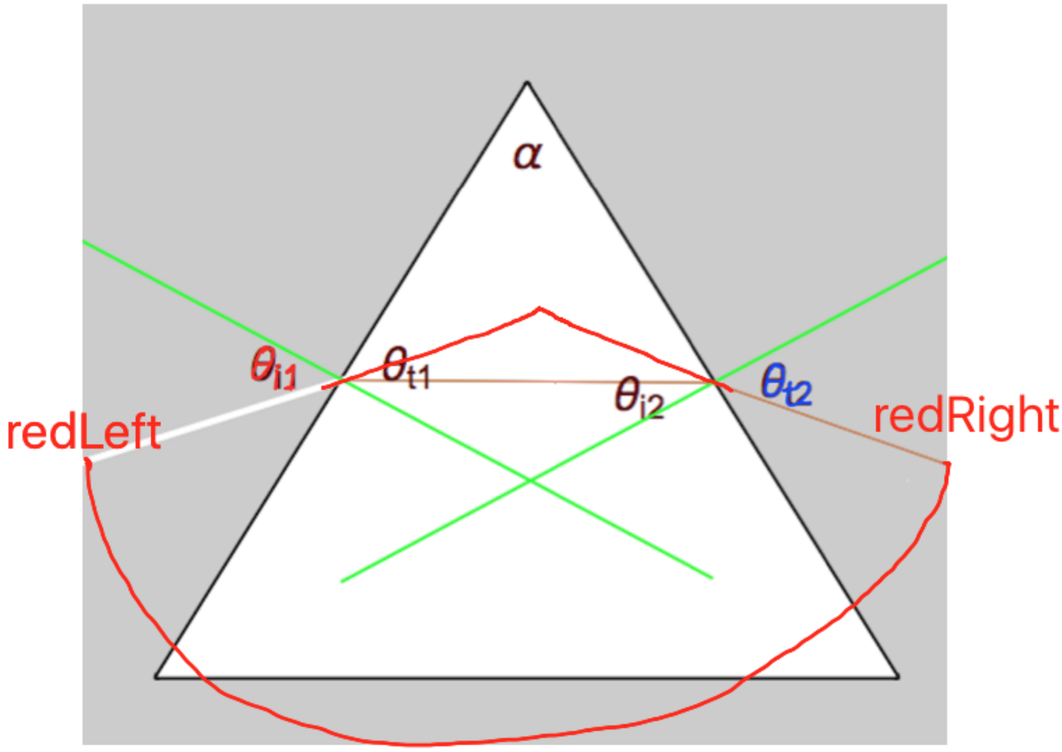
$\delta m = 60.49°$

$\Delta\delta m = 0.04°$

Measuring δm for the blue laser and finding: Error in δ_m for blue laser (λ ~ 405nm)

Here I collect data from the Vernier scale at the positions of minimum deviation at both sides of angle α

In fact at the positions of **incident angles** corresponding to minimum deviation




```
(*Readings of the vernier scale of the instrument.*)
```

$$\text{blueLeft} = \begin{pmatrix} 170 & 20 & 14 \\ 170 & 40 & 6 \\ 170 & 20 & 5 \\ 170 & 0 & 0 \\ 170 & 40 & 5 \\ 171 & 0 & 2 \\ 171 & 0 & 2 \\ 171 & 0 & 0 \\ 171 & 0 & 10 \\ 171 & 0 & 14 \\ 170 & 40 & 19 \\ 171 & 0 & 15 \end{pmatrix};\ \text{blueRight} = \begin{pmatrix} 56 & 0 & 6 \\ 56 & 0 & 11 \\ 56 & 0 & 0 \\ 55 & 40 & 15 \\ 56 & 0 & 8 \\ 55 & 40 & 17 \\ 56 & 0 & 10 \\ 56 & 0 & 13 \\ 55 & 40 & 16 \\ 56 & 40 & 5 \\ 56 & 0 & 5 \\ 56 & 0 & 0 \end{pmatrix};$$

```
convertToArcMinutes[{a_, b_, c_}] :=
  UnitConvert[Quantity[a, "Degrees"] + Quantity[b, "ArcMinutes"] +
     Quantity[c, "ArcMinutes"], "ArcMinutes"] // N;
convertToArcMinutes /@ blueLeft;
convertToArcMinutes /@ blueRight;
LRmatrix =
  Transpose[{convertToArcMinutes /@ blueLeft, convertToArcMinutes /@ blueRight}];
subTractor[{a_, b_}] := Quantity[360, "Degrees"] - (a - b);
DD = subTractor /@ LRmatrix;
```

$$DD = 2\theta + 120^\circ$$

$$\frac{DD - 120^\circ}{2} = \theta$$

$$\text{THETA} = \frac{\text{DD - Quantity[120, "Degrees"]}}{2}$$

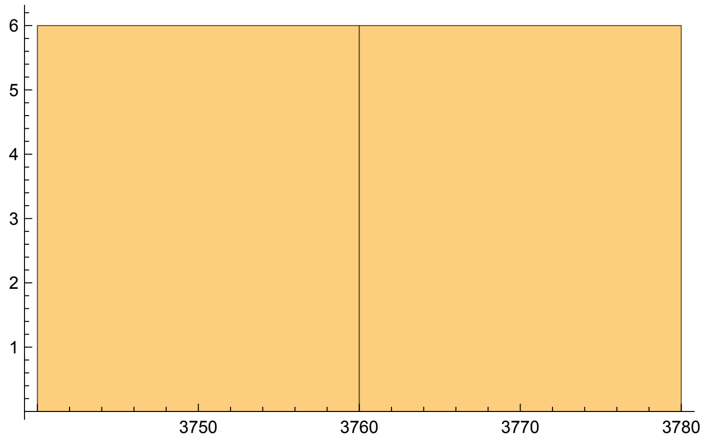
```
{3766.' , 3762.5' , 3767.5' , 3777.5' , 3761.5' ,
 3747.5' , 3754.' , 3756.5' , 3743.' , 3765.5' , 3753.' , 3742.5' }
```

```
Grid[{#} & /@ THETA];
```

```
Insert[ReplacePart[Grid[{#} & /@ THETA],
  1 → Prepend[First[Grid[{#} & /@ THETA]], {"incident angle"}]],
 {Background → {None, {GrayLevel[0.7], {White}}}, Dividers → {Black, {2 → Black}},
  Frame → True, Spacings → {2, {2, {0.7}, 2}}}, 2]
```

incident angle
3766.'
3762.5'
3767.5'
3777.5'
3761.5'
3747.5'
3754.'
3756.5'
3743.'
3765.5'
3753.'
3742.5'

```
Histogram[THETA]
```



lets use the Standard Deviation of the Mean formula.

Standard Deviation of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$\sigma_x =$ standard deviation of all our measurements

$N =$ number of measurements

```
SDOM = StandardDeviation[THETA] / Sqrt[16]
```

2.65638'

```
Δθ = UnitConvert[SDOM, "ArcMinutes"]
```

2.65638'

this is 1 Arc Minute

```
UnitConvert[Δθ, "Degrees"]
```

0.044273°

- The error in θ is 0.02°

```
meanIncidentAngle = Mean[THETA] ~UnitConvert~ "Degrees"
```

62.6347°

δ

$$\delta = \theta_{i1} + \theta_{t2} - \alpha \qquad (5.52)$$

```
Clear[δm, α];
α = Quantity[60.05, "Degrees"];
δm = 2 (meanIncidentAngle) - α
```

65.2194°

- Error propagation rules

Addition or subtraction: If

$$Q = a + b$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2}$$

Measured quantity times exact number:

$$Q = Ax$$

then

$$\delta Q = |A|\,\delta x$$

```
UnitConvert[2 (meanIncidentAngle) - α, "Degrees"]
```

65.2194°

```
Clear[α, Δα, θ, Δθ, δm, Δδm]
Δα = 0.01;
α = 60.05;
θ = 62.63;
Δθ = 0.02;
```

$$\delta_m = 2\theta - \alpha$$

$$\Delta\delta_m = \sqrt{(2\Delta\theta)^2 + (\Delta\alpha)^2}$$

```
δm = 2 θ - α
```

65.21

```
Δδm = √((2 Δθ)^2 + (Δα)^2)
```

0.0412311

- so our angle of minimum deviation for the blue laser is:

$\delta m = 65.21°$

$\Delta\delta m = 0.04°$

Calculating Error in **n**

- We must find the error in the minimum deviation δ_m for each laser as well as the error in the corner angle α

general Error propagation formula

$$q(x, \ldots z)$$

$$\Delta q = \sqrt{\left(\frac{\partial q}{\partial x}\Delta x\right)^2 + \ldots + \left(\frac{\partial q}{\partial z}\Delta z\right)^2}$$

where **Δq** is the absolute error in the function q which depends on **x** and **z**

our formula

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin \alpha/2} \qquad (5.54)$$

```
Clear[n, δm, α];
n[δm_, α_] := Sin[(δm + α) / 2] / Sin[α / 2];
```

$$n(\delta_m, \alpha)$$

$$\Delta n = \sqrt{\left(\frac{\partial n}{\partial \delta_m}\Delta \delta_m\right)^2 + \left(\frac{\partial n}{\partial \alpha}\Delta \alpha\right)^2}$$

In (Tentori and Lerma 1990) they just combine the errors linearly instead of by quadrature

$$\delta N = \left|\frac{\partial N(A,D)}{\partial D}\right| \delta D + \left|\frac{\partial N(A,D)}{\partial A}\right| \delta A$$

lets take the partial derivative of $n(\delta_m, \alpha)$ with respect to δ_m

```
dndδm = D[n[δm, α], δm]
```

$$\frac{1}{2} \operatorname{Cos}\left[\frac{\alpha + \delta m}{2}\right] \operatorname{Csc}\left[\frac{\alpha}{2}\right]$$

Csc[] is the cosecant and is defined as $\frac{1}{\text{Sin[]}}$

$$\frac{\partial n}{\partial \delta_m} = \frac{\operatorname{Cos}[(\alpha + \delta_m)/2]}{2 \sin[\alpha/2]}$$

lets take the partial derivative of $n(\delta_m, \alpha)$ with respect to α

```
dndα = D[n[δm, α], α]
```

$$\frac{1}{2} \operatorname{Cos}\left[\frac{\alpha + \delta m}{2}\right] \operatorname{Csc}\left[\frac{\alpha}{2}\right] - \frac{1}{2} \operatorname{Cot}\left[\frac{\alpha}{2}\right] \operatorname{Csc}\left[\frac{\alpha}{2}\right] \operatorname{Sin}\left[\frac{\alpha + \delta m}{2}\right]$$

```
Simplify[dndα]
```

$$-\frac{1}{2} \operatorname{Csc}\left[\frac{\alpha}{2}\right]^2 \operatorname{Sin}\left[\frac{\delta m}{2}\right]$$

Csc[z] is the cosecant and is defined as $\frac{1}{\text{Sin[z]}}$

$$\frac{\partial n}{\partial \alpha} = \frac{\sin[\delta_m/2]}{2 \sin^2[\alpha/2]}$$

A first attempt at classifying our glass (neglecting error)

```
r = Round[#, 0.01] &;
nRed = Round[1.7215915524089558`, 0.01];
nGreen = Round[1.7367579222216136`, 0.01];
nBlue = Round[1.7760348020706036`, 0.01];
```

```
λRed = 650;
λGreen = 532;
λBlue = 405;
```

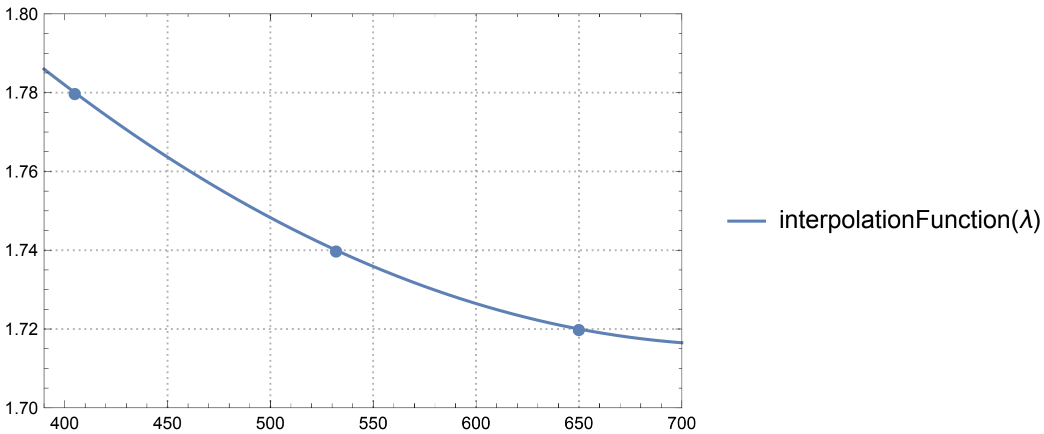
```
interpolationFunction =
 Interpolation[{{λRed, nRed}, {λGreen, nGreen}, {λBlue, nBlue}}]
```

Interpolation: Requested order is too high; order has been reduced to {2}.

```
InterpolatingFunction[ Domain: {{405., 650.}} Output: scalar ]
```

```
Show[
 Plot[interpolationFunction[λ], {λ, 390, 700},
  PlotRange → {{390, 700}, {1.7, 1.8}}, PlotTheme → "Detailed"],
 ListPlot[{{λRed, nRed}, {λGreen, nGreen}, {λBlue, nBlue}},
  PlotMarkers → {Automatic, 10}]
]
```

InterpolatingFunction: Input value {390.006} lies outside the range of data in the interpolating function. Extrapolation will be used.



Abbe Number

The Abbe number,[1][2] V_D, of a material is defined as

$$V_D = \frac{n_D - 1}{n_F - n_C},$$

where n_D, n_F and n_C are the refractive indices of the material at the wavelengths of the Fraunhofer D-, F- and C- spectral lines (589.3 nm, 486.1 nm and 656.3 nm respectively).

interpolating n_D , n_F , n_C

```
fraunhoferD = 589.3;
fraunhoferF = 486.1;
fraunhoferC = 656.3;
nD = interpolationFunction[fraunhoferD] // r
```

1.73

```
nF = interpolationFunction[fraunhoferF] // r
```

1.75

```
nC = interpolationFunction[fraunhoferC] // r
```

InterpolatingFunction: Input value {656.3} lies outside the range of data in the interpolating function. Extrapolation will be used.

1.72

V_D

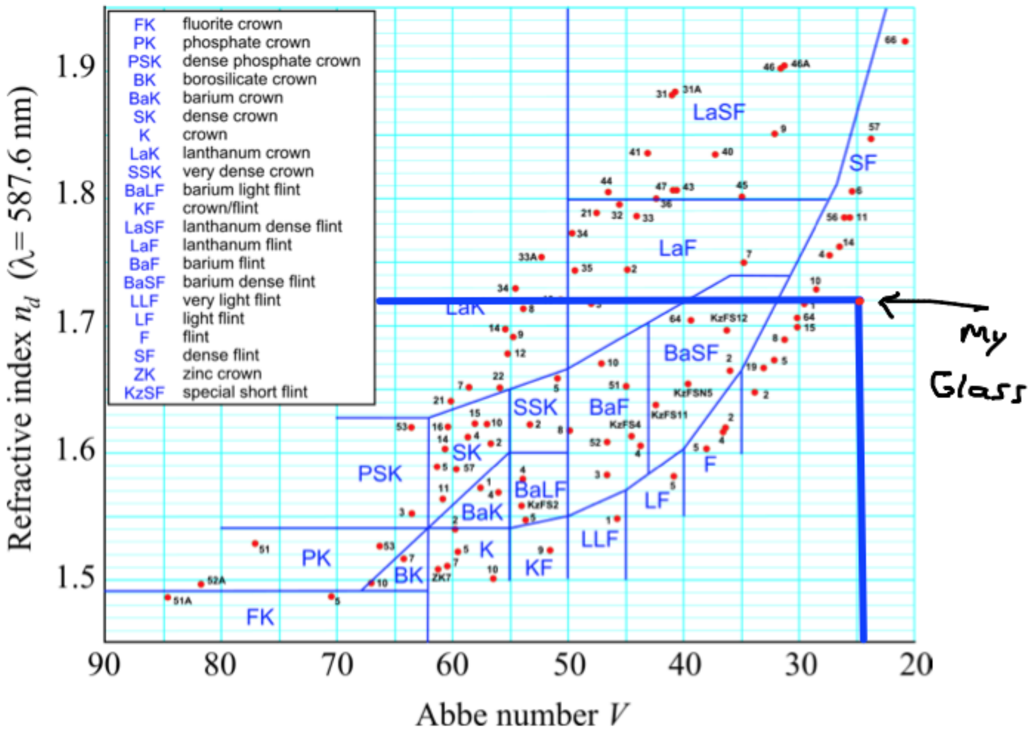
$$V = \frac{nD - 1}{nF - nC}$$

24.3333

So which glass is our glass?

```
λofChart = 587.6;
nd = interpolationFunction[λofChart]
```

1.72852



Conclusion so far... it seems likes its **SF** or some type of **Dense Flint** glass.

$n(\lambda = \text{red})$

$\delta m = 58.73°$

$\Delta\delta m = 0.04°$

$\alpha = 60.05°$

$\Delta\alpha = 0.01°$

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin \alpha/2} \qquad (5.54)$$

calculating **n(red)**

```
Clear[n, δm, α];
δm = Quantity[58.73, "Degrees"];
α = Quantity[60.05, "Degrees"];
n[δm_, α_] := Sin[(δm + α) / 2] / Sin[α / 2];
n[δm, α]
1.72001
```

error in **n(red)**

In (Tentori and Lerma 1990) they just combine the errors linearly instead of by quadrature

$$\delta N = \left|\frac{\partial N(A,D)}{\partial D}\right| \delta D + \left|\frac{\partial N(A,D)}{\partial A}\right| \delta A$$

$$\Delta n = \left|\frac{\partial n}{\partial \delta_m}\right| \Delta \delta_m + \left|\frac{\partial n}{\partial \alpha}\right| \Delta \alpha$$

$$\frac{\partial n}{\partial \delta_m} = \frac{\cos[(\alpha + \delta_m)/2]}{2 \sin[\alpha/2]}$$

$$\frac{\partial n}{\partial \alpha} = \frac{\sin[\delta_m/2]}{2 \sin^2[\alpha/2]}$$

```
δm = 58.73;
Δδm = 0.04;
α = 60.05;
Δα = 0.01;
sin = Sin[Quantity[#, "Degrees"]] &;
cos = Cos[Quantity[#, "Degrees"]] &;
```

$$\Delta n = \text{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2 \sin[\alpha/2]}\right] \Delta\delta m + \text{Abs}\left[\frac{\sin[\delta m/2]}{2 \sin[\alpha/2]^2}\right] \Delta\alpha$$

```
0.0301449
```

which is error has the bigger contribution?

$$\text{Round}[\#, .01] \,\&\, /@ \left\{\text{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2 \sin[\alpha/2]}\right] \Delta\delta m, \text{Abs}\left[\frac{\sin[\delta m/2]}{2 \sin[\alpha/2]^2}\right] \Delta\alpha\right\}$$

```
{0.02, 0.01}
```

- So the error due to the corner contribute half the error due to the measurement of deviation.
- Thus the error due to the measurement of minimum deviation had the largest contribution in my experiment. I suspect this was caused by the the ghost beam getting lost in the cloud.

n(red) = 1.72;
Δn(red) = 0.03;

n(λ = green)

$\delta m = 60.49°$

$\Delta\delta m = 0.04°$

$\alpha = 60.05°$

$\Delta\alpha = 0.01°$

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin \alpha/2} \qquad (5.54)$$

calculating **n(green)**

```
Clear[n, δm, α];
δm = Quantity[60.49, "Degrees"];
α = Quantity[60.05, "Degrees"];
n[δm_, α_] := Sin[(δm + α) / 2] / Sin[α / 2];
Round[n[δm, α], 0.01]
1.74
```

error in **n(red)**

In (Tentori and Lerma 1990) they just combine the errors linearly instead of by quadrature

$$\delta N = \left|\frac{\partial N(A,D)}{\partial D}\right| \delta D + \left|\frac{\partial N(A,D)}{\partial A}\right| \delta A$$

$$\Delta n = \left|\frac{\partial n}{\partial \delta_m}\right| \Delta \delta_m + \left|\frac{\partial n}{\partial \alpha}\right| \Delta \alpha$$

$$\frac{\partial n}{\partial \delta_m} = \frac{\cos[(\alpha + \delta_m)/2]}{2 \sin[\alpha/2]}$$

```
δm = 60.49;
Δδm = 0.04;
α = 60.05;
Δα = 0.01;
sin = Sin[Quantity[#, "Degrees"]] &;
cos = Cos[Quantity[#, "Degrees"]] &;
```

$$\Delta n = \mathrm{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2 \sin[\alpha/2]}\right] \Delta\delta m + \mathrm{Abs}\left[\frac{\sin[\delta m/2]}{2 \sin[\alpha/2]^2}\right] \Delta\alpha$$

0.0298803

which is error has the bigger contribution?

$$\mathrm{Round}[\#, .01]\ \&\ /@\ \left\{\mathrm{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2 \sin[\alpha/2]}\right] \Delta\delta m,\ \mathrm{Abs}\left[\frac{\sin[\delta m/2]}{2 \sin[\alpha/2]^2}\right] \Delta\alpha\right\}$$

{0.02, 0.01}

- So the error due to the corner contribute half the error due to the measurement of deviation.
- Thus the error due to the measurement of minimum deviation had the largest contribution in my experiment. I suspect this was caused by the the ghost beam getting lost in the cloud.

n(green) = 1.74;

Δn(green) = 0.03;

n(λ = blue)

$\delta m = 65.21°$

$\Delta\delta m = 0.04°$

$\alpha = 60.05°$

$\Delta\alpha = 0.01°$

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin \alpha/2} \qquad (5.54)$$

calculating **n(blue)**

```
Clear[n, δm, α];
δm = Quantity[65.21, "Degrees"];
α = Quantity[60.05, "Degrees"];
n[δm_, α_] := Sin[(δm + α) / 2] / Sin[α / 2];
n[δm, α]
1.77477
```

error in **n(blue)**

In (Tentori and Lerma 1990) they just combine the errors linearly instead of by quadrature

$$\delta N = \left|\frac{\partial N(A,D)}{\partial D}\right| \delta D + \left|\frac{\partial N(A,D)}{\partial A}\right| \delta A$$

$$\Delta n = \left|\frac{\partial n}{\partial \delta_m}\right| \Delta \delta_m + \left|\frac{\partial n}{\partial \alpha}\right| \Delta \alpha$$

$$\frac{\partial n}{\partial \delta_m} = \frac{\cos[(\alpha + \delta_m)/2]}{2\sin[\alpha/2]}$$

```
δm = 65.21;
Δδm = 0.04;
α = 60.05;
Δα = 0.01;
sin = Sin[Quantity[#, "Degrees"]] &;
cos = Cos[Quantity[#, "Degrees"]] &;
```

$$\Delta n = \mathrm{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2\sin[\alpha/2]}\right] \Delta\delta m + \mathrm{Abs}\left[\frac{\sin[\delta m/2]}{2\sin[\alpha/2]^2}\right] \Delta\alpha$$

0.0291361

which is error has the bigger contribution?

$$\mathrm{Round}[\#, .01] \;\&\; /@ \left\{\mathrm{Abs}\left[\frac{\cos[(\alpha + \delta m)/2]}{2\sin[\alpha/2]}\right] \Delta\delta m,\ \mathrm{Abs}\left[\frac{\sin[\delta m/2]}{2\sin[\alpha/2]^2}\right] \Delta\alpha\right\}$$

{0.02, 0.01}

- So the error due to the corner contribute half the error due to the measurement of deviation.
- Thus the error due to the measurement of minimum deviation had the largest contribution in my experiment. I suspect this was caused by the the ghost beam getting lost in the cloud.

n(blue) = 1.77;

Δn(blue) = 0.03;

Classifying glass (with error bars in **n**’s)

Classifying our glass with consideration of error in refractive index

We shall neglect error in the laser for now. We will try to include that at the end if we have time. The green laser had no error written on it but the red and blue had an error of **10nm**

```
r = Round[#, 0.01] &;
nRed = 1.72;
ΔnRed = 0.03;
nGreen = 1.74;
ΔnGreen = 0.03;
nBlue = 1.77;
ΔnBlue = 0.03;
λRed = 650;
λGreen = 532;
λBlue = 405;
```

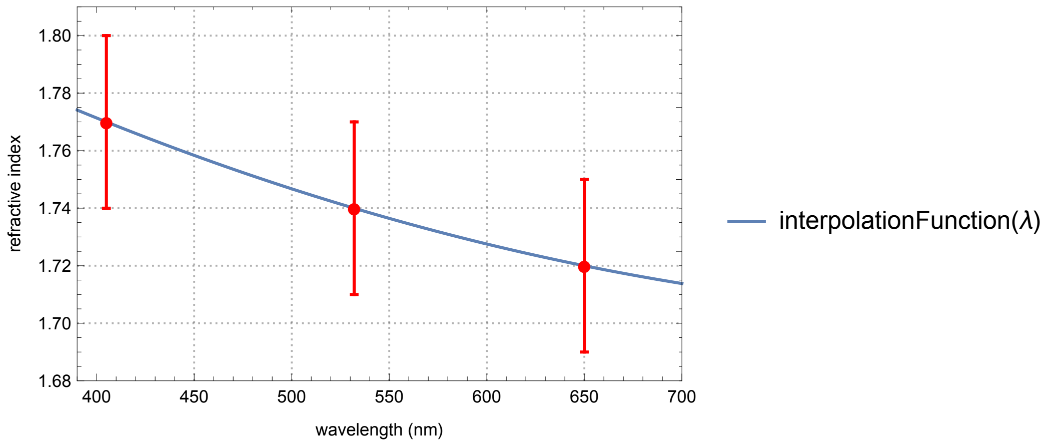
```
(

  interpolationFunctionPlus = Interpolation[
    {{λRed, nRed + ΔnRed}, {λGreen, nGreen + ΔnGreen}, {λBlue, nBlue + ΔnBlue}}];
  interpolationFunction  = Interpolation[
    {{λRed, nRed}, {λGreen, nGreen}, {λBlue, nBlue}}];
  interpolationFunctionMinus = Interpolation[
    {{λRed, nRed - ΔnRed}, {λGreen, nGreen - ΔnGreen}, {λBlue, nBlue - ΔnBlue}}];

 ) // Quiet
```

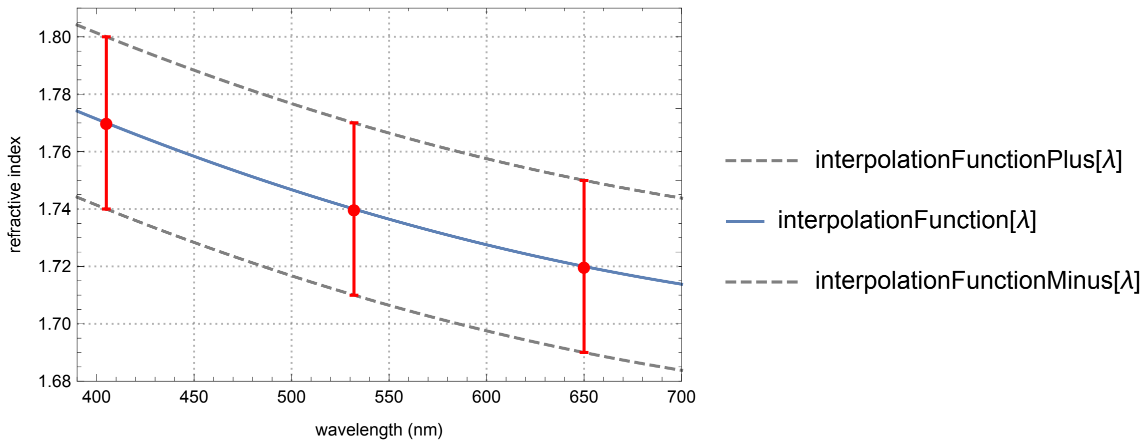
```
Needs["ErrorBarPlots`"]
Quiet@Show[

  Plot[interpolationFunction[λ], {λ, 390, 700},
   PlotRange → {{390, 700}, {1.68, 1.81}}, PlotTheme → "Detailed",
   FrameLabel → {"wavelength (nm)", "refractive index"}],
  ErrorListPlot[{{{λRed, nRed}, ErrorBar[ΔnRed]},
    {{λGreen, nGreen}, ErrorBar[ΔnGreen]}, {{λBlue, nBlue}, ErrorBar[ΔnBlue]}},
   PlotMarkers → {Automatic, 10}, PlotStyle → Red]
 ]
```



```
Needs["ErrorBarPlots`"];
Quiet@Show[
  Plot[interpolationFunctionPlus[λ],
   {λ, 390, 700}, PlotRange → {{390, 700}, {1.68, 1.81}},
   PlotTheme → "Detailed", PlotStyle → {Dashed, Gray},
   FrameLabel → {"wavelength (nm)", "refractive index"}],
  Plot[interpolationFunction[λ], {λ, 390, 700},
   PlotRange → {{390, 700}, {1.68, 1.81}}, PlotTheme → "Detailed"],

  Plot[interpolationFunctionMinus[λ],
   {λ, 390, 700}, PlotRange → {{390, 700}, {1.68, 1.81}},
   PlotTheme → "Detailed", PlotStyle → {Dashed, Gray}],
  ErrorListPlot[{{{λRed, nRed}, ErrorBar[ΔnRed]},
    {{λGreen, nGreen}, ErrorBar[ΔnGreen]}, {{λBlue, nBlue}, ErrorBar[ΔnBlue]}},
   PlotMarkers → {Automatic, 10}, PlotStyle → Red]
 ]
```



Abbe Number

The Abbe number,[1][2] V_D, of a material is defined as

$$V_D = \frac{n_D - 1}{n_F - n_C},$$

where n_D, n_F and n_C are the refractive indices of the material at the wavelengths of the Fraunhofer D-, F- and C- spectral lines (589.3 nm, 486.1 nm and 656.3 nm respectively).

The Abbe number,[1][2] V_D, of a material is defined as

$$V_D = \frac{n_D - 1}{n_F - n_C},$$

where n_D, n_F and n_C are the refractive indices of the material at the wavelengths of the Fraunhofer D-, F- and C- spectral lines (589.3 nm, 486.1 nm and 656.3 nm respectively).

interpolating n_D , n_F , n_C

```
fraunhoferD = 589.3;
fraunhoferF = 486.1;
fraunhoferC = 656.3;
nD = interpolationFunction[fraunhoferD] // r
```

```
1.73
```

```
nF = interpolationFunction[fraunhoferF] // r
```

```
1.75
```

```
nC = interpolationFunction[fraunhoferC] // r
```

```
1.72
```

interpolating error in n_D , n_F , n_C

```
fraunhoferD = 589.3;
fraunhoferF = 486.1;
fraunhoferC = 656.3;
ΔnD = (interpolationFunctionPlus[fraunhoferD] // r) -
   (interpolationFunction[fraunhoferD] // r)
```

```
0.03
```

```
ΔnF = (interpolationFunctionPlus[fraunhoferF] // r) -
   (interpolationFunction[fraunhoferF] // r)
```

```
0.03
```

```
ΔnC = (interpolationFunctionPlus[fraunhoferC] // r) -
    interpolationFunction[fraunhoferC] // r
```

```
0.03
```

V_D

$$\text{VD} = \frac{\text{nD} - 1}{\text{nF} - \text{nC}}$$

```
24.3333
```

Error in V_D using General Error propagation formula

General Error Propagation Formula

$$q(x, \dots z)$$

$$\Delta q = \sqrt{\left(\frac{\partial q}{\partial x}\Delta x\right)^2 + \dots + \left(\frac{\partial q}{\partial z}\Delta z\right)^2}$$

where **Δq** is the absolute error in the function q which depends on **x** and **z**

our formula

$$V_D[n_D, n_F, n_C] = \frac{n_D - 1}{n_F - n_C}$$

$$\Delta V_D = \sqrt{\left(\frac{\partial V_D}{\partial n_D}\Delta n_D\right)^2 + \left(\frac{\partial V_D}{\partial n_f}\Delta n_f\right)^2 + \left(\frac{\partial V_D}{\partial n_c}\Delta n_c\right)^2}$$

```
Clear[nD, nF, nC];
VD[nD_, nF_, nC_] := (nD - 1)/(nF - nC);
{D[VD[nD, nF, nC], nD], D[VD[nD, nF, nC], nF], D[VD[nD, nF, nC], nC]}
```

$$\left\{\frac{1}{-nC + nF}, -\frac{-1 + nD}{(-nC + nF)^2}, \frac{-1 + nD}{(-nC + nF)^2}\right\}$$

```
ΔnD = 0.03;
ΔnF = 0.03;
ΔnC = 0.03;
nD = 1.73;
nF = 1.75;
nC = 1.72;
```

$$\Delta VD = \sqrt{\left(\left(\frac{1}{-nC + nF}\ (\Delta nD)\right)^2 + \left(-\frac{-1 + nD}{(-nC + nF)^2}\ (\Delta nF)\right)^2 + \left(\frac{-1 + nD}{(-nC + nF)^2}\ (\Delta nC)\right)^2\right)}$$

34.4271

This seems too large.

Error in V_D using Percentage error formulas

Addition or subtraction: If

$$Q = a + b$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2}$$

products and quotients

$$Q = \frac{a}{b}$$

$$\frac{\Delta Q}{Q} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$$

Our Formula

$$V_D[n_D, n_F, n_C] = \frac{n_D - 1}{n_F - n_C}$$

First lets calculate the error in the denominator (lets call this ΔQ)

$$Q = n_F - n_C$$

$$\Delta Q = \sqrt{(\Delta n_F)^2 + (\Delta n_C)^2}$$

$$V_D = \frac{n_D - 1}{Q}$$

$$\Rightarrow \frac{\Delta V_D}{V_D} = \sqrt{\left(\frac{\Delta n_D}{n_D}\right)^2 + \left(\frac{\Delta Q}{Q}\right)^2}$$

$$\Rightarrow \Delta V_D = V_D \sqrt{\left(\frac{\Delta n_D}{n_D}\right)^2 + \left(\frac{\sqrt{(\Delta n_F)^2 + (\Delta n_C)^2}}{n_F - n_C}\right)^2}$$

```
ΔnD = 0.03;
ΔnF = 0.03;
ΔnC = 0.03;
nD = 1.73;
nF = 1.75;
nC = 1.72;
```

$$\Delta VD = VD \times \sqrt{\left(\frac{\Delta nD}{nD}\right)^2 + \left(\frac{\sqrt{(\Delta nF)^2 + (\Delta nC)^2}}{nF - nC}\right)^2}$$

```
34.4151
```

Wow it must be correct! why would the math lie.

So which glass is our glass?

```
λofChart = 587.6;
nd = interpolationFunction[λofChart]
```

```
1.72963
```

```
Δnd = interpolationFunctionPlus[λofChart] - interpolationFunction[λofChart]
```

```
0.03
```

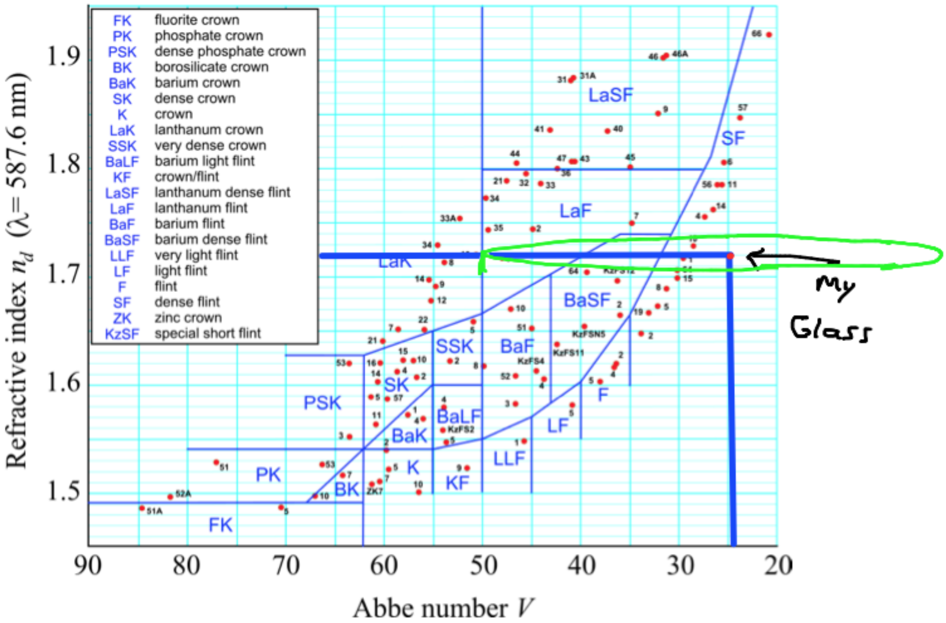
$$V_D = 24$$

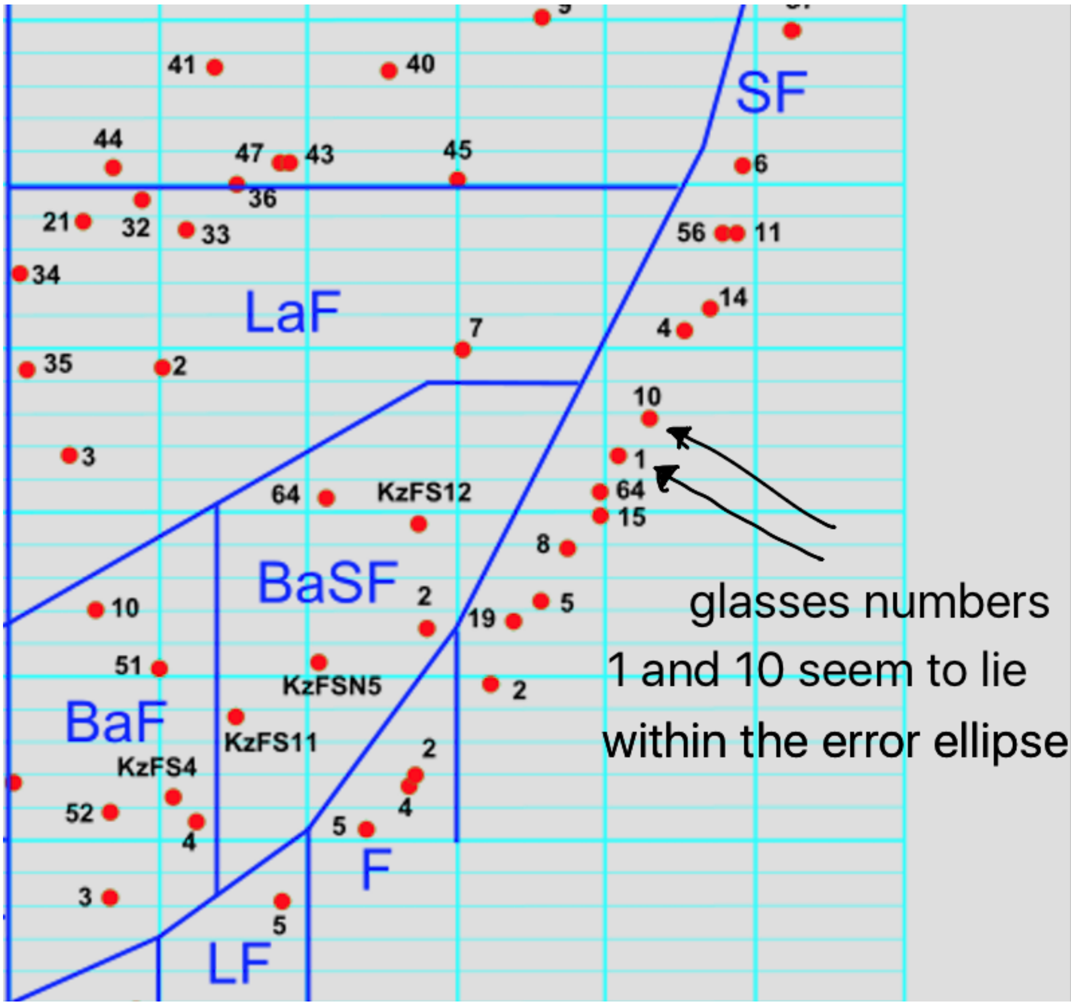
$$\Delta V_D = 34$$

$$n_D = 1.73$$

$$\Delta n_D = 0.03$$

I assume this creates a narrow elliptical region on the graph (roughly represented in green)


FK fluorite crown
PK phosphate crown
PSK dense phosphate crown
BK borosilicate crown
BaK barium crown
SK dense crown
K crown
LaK lanthanum crown
SSK very dense crown
BaLF barium light flint
KF crown/flint
LaSF lanthanum dense flint
LaF lanthanum flint
BaF barium flint
BaSF barium dense flint
LLF very light flint
LF light flint
F flint
SF dense flint
ZK zinc crown
KzSF special short flint
Refractive index n_d (λ= 587.6 nm)
Abbe number V
My Glass


LaF
SF
BaSF
BaF
F
LF
KzFS12
KzFSN5
KzFS11
KzFS4
glasses numbers
1 and 10 seem to lie
within the error ellipse

Discussion and Conclusion

- Our glass still seems to be a type of dense flint glass (even with the error in the Abbe number)
- The error in the Abbe Number seems wrong. (But I will accept it for now). Even though we used two different method to calculate it.

Which was the most critical error?

- Due to the complexity of the error propagation calculations I am uncertain.
- I reckon that the measurement of the corner angle alpha had the most effect as it contributed an absolute error of 0.01 to each refractive index measurement.
- Down stream this resulted in a monstrous error in the abbe number.

How could this experiment be improved?

- If we could reduce the error in the measurements of the refractive index by two orders of magnitude we could significantly reduce the error in the Abbe number

```
ΔnD = 0.0003;
ΔnF = 0.0003;
ΔnC = 0.0003;
nD = 1.73;
nF = 1.75;
nC = 1.72;
```

$$\Delta VD = VD \times \sqrt{\left(\frac{\Delta nD}{nD}\right)^2 + \left(\frac{\sqrt{(\Delta nF)^2 + (\Delta nC)^2}}{nF - nC}\right)^2}$$

```
0.344151
```

- This could be achieved by carefully taking 10000 measurements or by replicating the techniques that Tentori used to get her error in θ down to 0.2 arc seconds.

Appendix A: Error contributions following Tentori’s paper

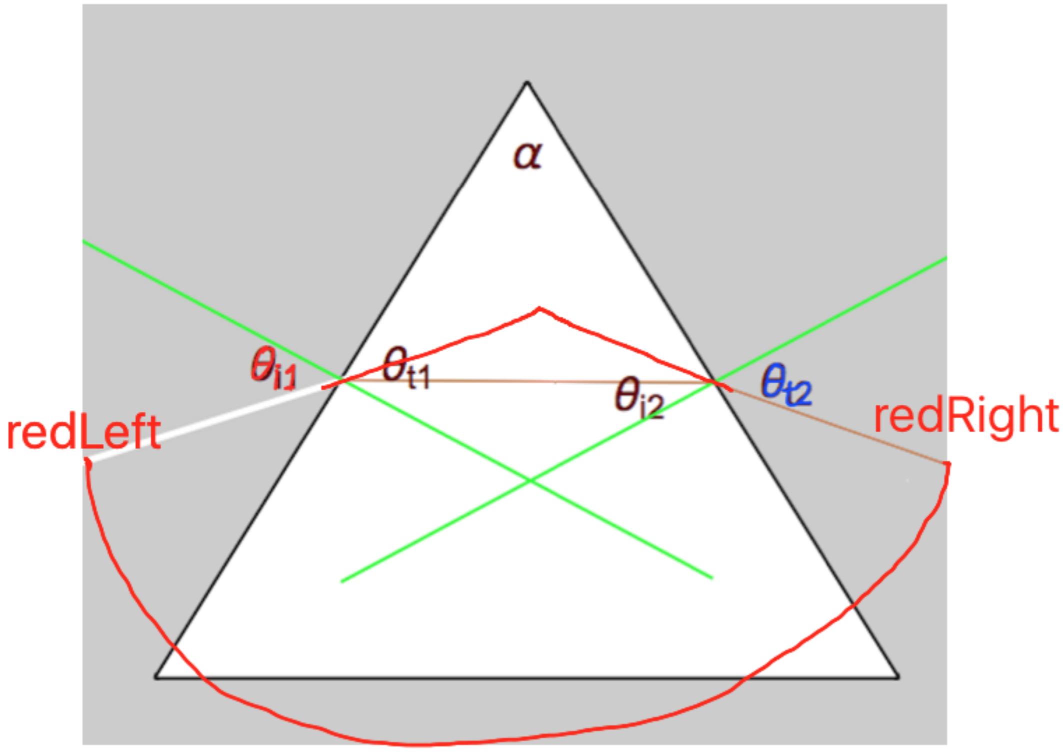
Tentori’s set up

- She wrote this paper in 1990 and at this time it was more convenient to use a bulb and a slit aperture instead of a laser. Hence her experiment is slightly different yet achieves impressively accurate measurements relative to ours.

Instrument contribution to error and error in a measurement of θ_i

Our instrument can measure angles down to 30 arc secondS.

- But our fallible human eyes could not tell at which position, the laser dot reverses direction.
- We shall be conservative and assume we knew this to an accuracy of the 20 arc minutes (the big increments on our instrument)
- Maybe with patience and a simpler technique (measuring the deviation from both sides we could narrow this down to 10 arc minutes). As we eliminate the ambiguity in attaining the measurement when the laser glances of the base (glance angle).
- to try to test our 20 arc minute assumption I recorded the instruments readings all the way to 1 arc minute-resolution, plotted a histogram and took the standard deviation

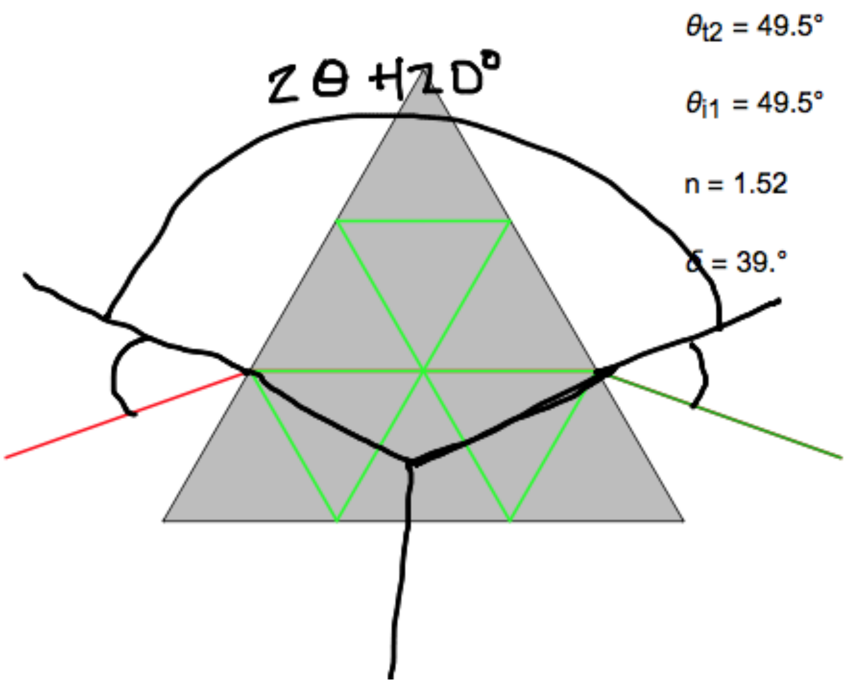


Readings of the vernier scale of the instrument.

$$\text{redLeft} = \begin{pmatrix} 175 & 40 & 18 \\ 176 & 0 & 2 \\ 176 & 0 & 0 \\ 175 & 40 & 10 \\ 176 & 0 & 0 \\ 176 & 0 & 0 \\ 175 & 40 & 12 \\ 176 & 0 & 0 \\ 177 & 0 & 2 \\ 177 & 0 & 17 \\ 177 & 0 & 6 \\ 177 & 0 & 8 \\ 177 & 0 & 18 \\ 176 & 40 & 19 \\ 177 & 0 & 11 \\ 177 & 0 & 12 \end{pmatrix}; \ \text{redRight} = \begin{pmatrix} 54 & 40 & 5 \\ 54 & 20 & 16 \\ 54 & 40 & 7 \\ 54 & 20 & 15 \\ 54 & 40 & 0 \\ 54 & 40 & 4 \\ 54 & 40 & 2 \\ 54 & 40 & 7 \\ 55 & 40 & 7 \\ 56 & 20 & 3 \\ 56 & 0 & 0 \\ 55 & 40 & 18 \\ 56 & 0 & 5 \\ 56 & 0 & 6 \\ 55 & 40 & 4 \\ 55 & 40 & 13 \end{pmatrix};$$

Program which outputs the difference between two corresponding readings in Arc Minutes

```
convertToArcMinutes[{a_, b_, c_}] := UnitConvert[Quantity[a, "Degrees"] +
    Quantity[b, "ArcMinutes"] + Quantity[c, "ArcMinutes"], "ArcMinutes"];
convertToArcMinutes /@ redLeft;
convertToArcMinutes /@ redRight;
LRmatrix =
  Transpose[{convertToArcMinutes /@ redLeft, convertToArcMinutes /@ redRight}];
subTractor[{a_, b_}] :=
  UnitConvert[Quantity[360, "Degrees"], "ArcMinutes"] - (a - b);
DD = subTractor /@ LRmatrix;
```



$$DD = 2\theta + 120^\circ$$

$$\frac{DD - 120^\circ}{2} = \theta$$

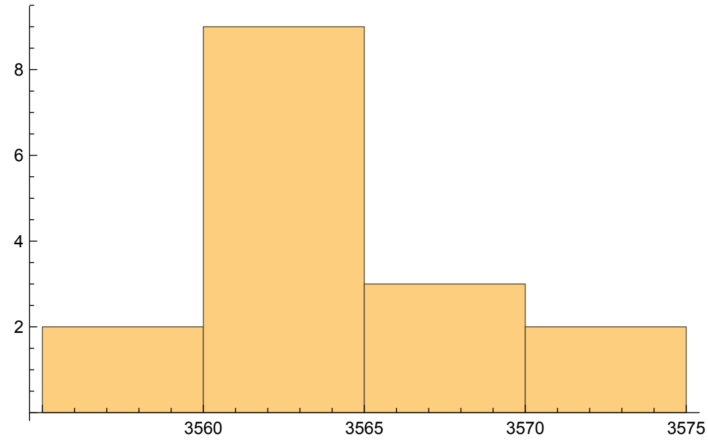
We calculate an incident angle corresponding to the instruments readings.

```
THETA = 1/2 (DD - UnitConvert[Quantity[120, "Degrees"], "ArcMinutes"]) // N;
Grid[{#} & /@ THETA];
```

```
Insert[ReplacePart[Grid[{#} & /@ THETA],
  1 → Prepend[First[Grid[{#} & /@ THETA]], {"incident angle"}]],
 {Background → {None, {GrayLevel[0.7], {White}}}, Dividers → {Black, {2 → Black}},
  Frame → True, Spacings → {2, {2, {0.7}, 2}}}, 2]
```

incident angle
3563.5'
3557.'
3563.5'
3562.5'
3560.'
3562.'
3565.'
3563.5'
3562.5'
3573.'
3567.'
3565.'
3563.5'
3573.5'
3556.5'
3560.5'

```
Histogram[THETA]
```




```
StandardDeviation[THETA]
```

```
4.64298'
```

```
UnitConvert[StandardDeviation[THETA], "ArcSeconds"]
```

```
278.579"
```

My god, 278 Arc seconds!!! wildly inaccurate relative to Tentori's **0.2 arc second**!

```
UnitConvert[StandardDeviation[THETA], "ArcMinutes"]
```

```
4.64298'
```

$$\text{SDOM} = \frac{5}{\text{Sqrt}[16]} \text{ // N}$$

```
1.25
```

So for red light our measurements vary by 5 arc minutes between identical measurements.

I probably should propagate error through my mathematica code, but this is the error of the error, to simplify things I shall assume a 10 arc minute error using my eyes and a red laser.

- Note: the laser dot spreads out into a cloud (about 3 centimeters in diameter) and is not a point. Thus masking where the smaller ghost beam co-aligns with it. **This is a problem we could improve in a future experiment.**
- Utilizing the **SDOM** over 16 measurements we get an error of ~ **2 Arcminutes**

Pyramidal contribution

- *"A prism sample presents a pyramidal error if its faces are not perpendicular to its base plane."*
- I don't know precisely what Tentori means by this as of course its faces are not perpendicular to its base plane they are at 60 degrees to its base!???

Flatness contribution to error.

- We don't know how flat the sides of our prism are.
- Tentori's prism had a flatness contribution relative to the wavelength of light she was using

$\lambda = 0.546\ \mu$m

- She reasons conservatively that when the surface flatness is twice lambda this will change the angle of minimum deviation by 1.2 arc seconds.
- This is negligible relative to 20 arc seconds

Rotational error due to the prism off center

- This causes error in the measurement of the apex angle α

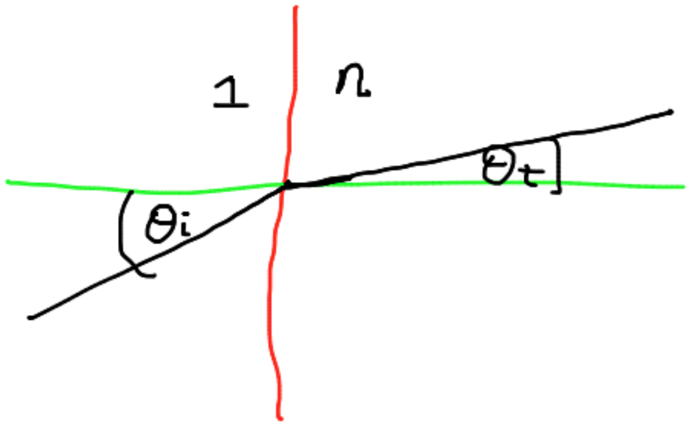
Error due to the laser beam not going through the center of the goniometer wheel.

- This also causes error in the measurement of the apex angle α

Appendix B Deriving

$$\delta(\theta_{i1}, \alpha, n) = \theta_{i1} + \sin^{-1}\left[(\sin\alpha)\sqrt{n^2 - \sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

snells law

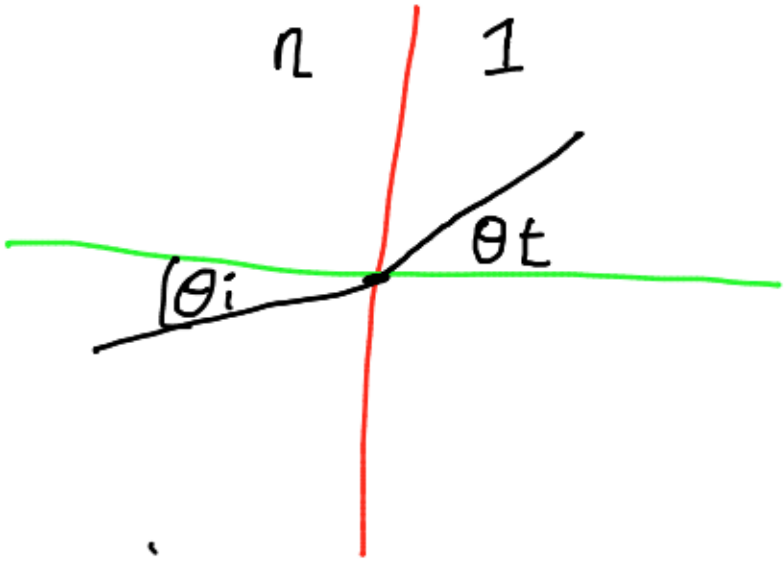


$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{1}$$

$$\sin\theta_i = n\sin\theta_t$$

$$\theta_i = \sin^{-1}(n\sin\theta_t)$$

lets consider $\theta t2$



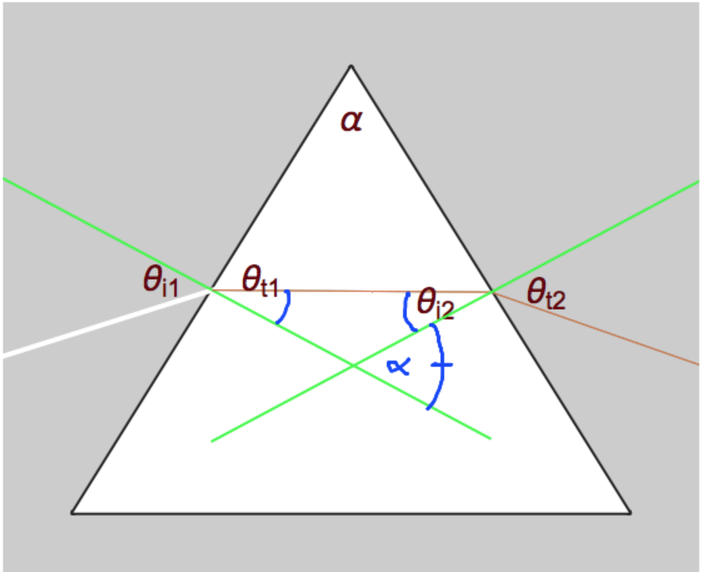
$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{1}{n}$$

$$\sin\theta_t = n\sin\theta_i$$

$$\theta_t = \sin^{-1}(n\sin\theta_i)$$

now lets add the subscripts from Hecht 4th (p187)

$$\theta_{t_2} = \sin^{-1}(n \sin \theta_{i_2})$$



$$\alpha = \theta_{t1} + \theta_{i2}$$

$$\theta_{i2} = (\alpha - \theta_{t1})$$

$$\theta_{t_2} = \sin^{-1}(n \sin \theta_{i2})$$

$$\theta_{t_2} = \sin^{-1}(n \sin (\alpha - \theta_{t_1}))$$

trig identities

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\Rightarrow \sin(\alpha - \theta t1) = \sin\alpha \cos\theta t1 - \cos\alpha \sin\theta t1$$

using

$$\cos^2 B + \sin^2 B = 1$$

$$\cos^2 B = 1 - \sin^2 B$$

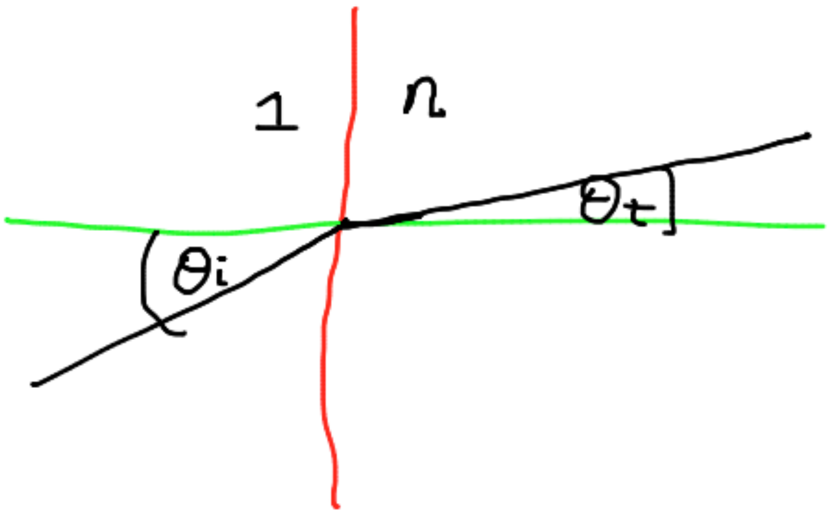
$$\cos B = \sqrt{1-\sin^2 B}$$

$$\sin(\alpha - \theta t1) = \sin\alpha \cos\theta t1 - \cos\alpha \sin\theta t1$$

$$\Rightarrow \sin(\alpha - \theta t1) = \sin\alpha \sqrt{1-\sin^2\theta t1} - \cos\alpha \sin\theta t1$$

using snells law

snells law



$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{1}$$

$$\sin \theta_i = n \sin \theta_t$$

$$\sin \theta_{i1} = n \sin \theta_{t1}$$

$$\frac{\sin \theta_{i1}}{n} = \sin \theta_{t1}$$

applying snells law twice more... we get

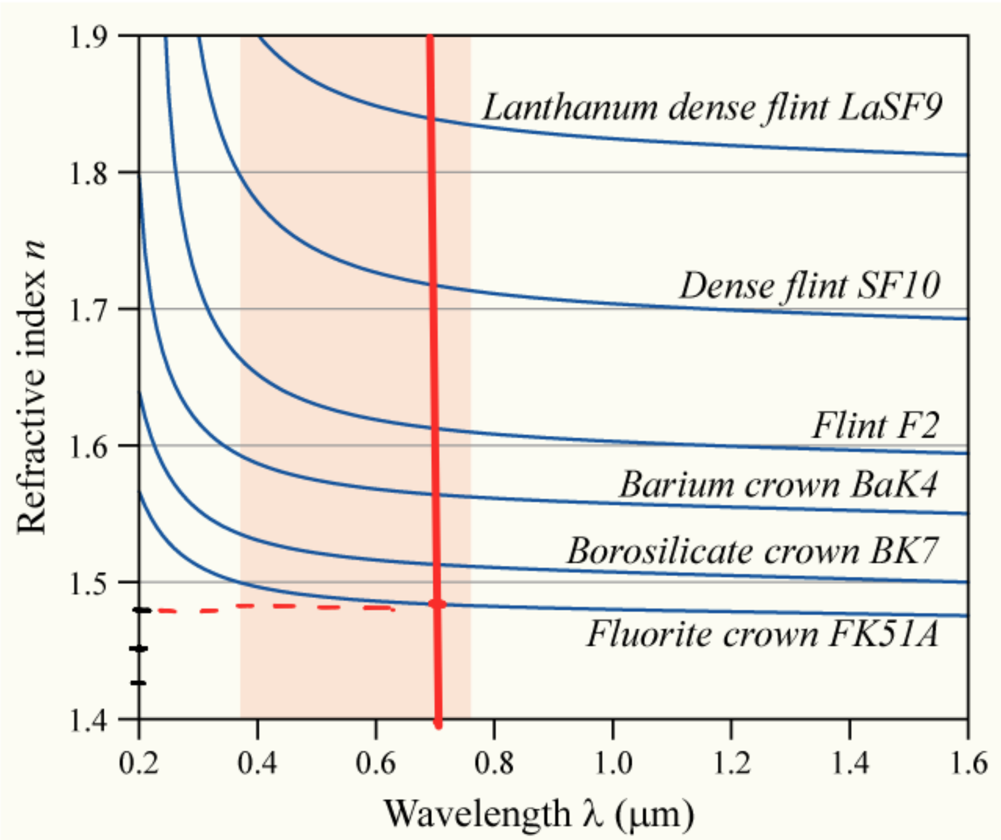
$$\delta(\theta_{i1}, \alpha, n)$$

a function of **angular deviation** δ in terms of the incident angle θ_{i1}, the corner angle α and the refractive index of the prism n

$$\delta(\theta_{i1}, \alpha, n) = \theta_{i1} + \sin^{-1}\left[(\sin\alpha)\sqrt{n^2 - \sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

Appendix C some notes on refractive index (for myself)

n in turn is a function of wavelength and decreases as the wavelength increases



$$n = \frac{c}{v}$$

$$\lambda_0 f = c$$

$$\lambda_m f = v$$

λ_m **is the wavelength in the prism**

λ_0 **is the wavelength in the vacuum**

$$n = \frac{\lambda_0}{\lambda_m}$$

The colors of the visible light spectrum

Color	Wavelength interval	Frequency interval
Red	~ 700–635 nm	~ 430–480 THz
Orange	~ 635–590 nm	~ 480–510 THz
Yellow	~ 590–560 nm	~ 510–540 THz
Green	~ 560–520 nm	~ 540–580 THz
Cyan	~ 520–490 nm	~ 580–610 THz
Blue	~ 490–450 nm	~ 610–670 THz
Violet	~ 450–400 nm	~ 670–750 THz

```
Manipulate[{ColorData["VisibleSpectrum"][n], Quantity[n, "Nanometers"]},
 {{n, 635}, 380, 750, 1}]
```