Application of the General Method of Eurocode 3 by the software GeM

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1 Introduction

EN 1993-1-1 presents an alternative way named by General Method, to assess the stability against buckling of steel members when the application of the expressions for the interaction between lateral-torsional buckling (LTB) and flexural buckling (FB) can't be taken (clauses 6.3.1, 6.3.2 and 6.3.3). This method assumes a full separation of in-plane and out-of-plane behaviours, and the determination of a single generalized slenderness based on the ultimate load amplifier for the elastic or plastic resistance of the cross-sections and critical load amplifier for the out-of-plane instability.

This document explains the way the software *GeM* (**Ge**neral **M**ethod) implements the General Method, for I-shaped web-tapered members that have doubly symmetric cross-sections, that can be subject to axial force and in-plane bending and that can be laterally restrained at any position. The document and implementation of the procedure at normal temperature is based on Part 1-1 of Eurocode 3 (EC3-1-1) [1] and in "Résistance au flambement et au déversement d'un poteau à inertie variable selon l'en 1993-1-1." [2], while at elevated temperatures the procedure was adopted in order to follow the disposed in Part 1-2 of Eurocode 3 (EC3-1-2) [3].

2 General Method according to EC3-1-1

2.1 Object

EC3 proposes a method that, as said in its clause 6.3.4, is based on the concept of a general slenderness and that can be used to check the resistance to lateral and lateral-torsional buckling, for structural components such as:

- Single members, built-up or not, uniform or not, with complex support conditions or not;
- Plane frames or sub-frames composed of such members;

which are subject to compression and/or mono-axial bending in the plane, but which do not contain rotative plastic hinges.

2.2 The Method

In this method, a verification of the overall resistance is carried by ensuring that:

$$\frac{\chi_{op} \cdot \alpha_{ult,k}}{\gamma_{M1}} \geq 1.0 \qquad (1)$$

where:

$\alpha_{ult,k}$ is the minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section of the structural component considering its in plane behavior without taking lateral or lateral torsional buckling into account however accounting for all effects due to in plane geometrical deformation and imperfections, global and local, where relevant;

χ_{op} is the reduction factor for the non-dimensional slenderness, to take account of lateral and lateral-torsional buckling.

Thus, this method consists in assessing the resistance of the most critical cross-section affecting it by a global reduction factor that takes into account the out-of-plane behaviour. The normalized slenderness $\bar{\lambda}_{op}$ is given by:

$$\bar{\lambda}_{op} = \frac{\sqrt{\alpha_{ult,k}}}{\sqrt{\alpha_{cr,op}}} \qquad (2)$$

and where:

$\alpha_{cr,op}$ is the minimum amplifier for the in plane design loads to reach the elastic critical resistance of the structural component with regards to lateral or lateral torsional buckling without accounting for in plane flexural buckling.

With this slenderness one can determine the reduction factor as a minimum or as an interpolation of the values for out of plane flexural buckling, χ_z, and for lateral-torsional buckling, χ_{LT}.

2.3 Notes

As mentioned in [2] , the internal forces and moments must be determined taking into account the global 2nd order effects, global imperfections, local 2nd order effects and local imperfections when necessary.

Generally, the global imperfections and the global second order effects (P-Δ) are considered in the analysis of the whole structure/frame in order to obtain the end forces and end moments to be used in member checks, while the local imperfections and local 2nd order effects (P-δ) can be neglected for the assessment of the member resistance by using the formulae from clause 6.3 of EC3 to estimate them. However, using the General Method requires a 2nd order analysis to be performed.

3 Implementation of the General Method in *GeM*

3.1 Scope of the program

In order to provide a tool that may easily and quickly enable users to run the General Method procedure, the computer program named *GeM* was developed at University of Aveiro. This software makes use of *LTBeamN* (a finite element program developed by CTICM [4]) in order to determine the internal forces and moments and the load amplifiers that are needed.

By using this tool, both designers and researchers have an easy way to either study the General Method or use it in structural design.

3.2 Procedure at normal temperature

Here, a description of each one of the steps involved in the procedure at normal temperature, as carried out by *GeM,* is shown.

This procedure assumes that the member being analysed is isolated from the rest of the structure, after a global analysis has been performed (considering global imperfections and global 2nd order effects (P-Δ)).

Step 1: Discretization of the member

First, there's the need to define a division of the member in a certain number of elements that may be sufficient to emulate the behaviour of the member in a finite element analysis as well as to find the critical cross-section.

Step 2: Determination of the internal forces and properties for each cross-section

For each one of the cross-sections resulting from the discretization made at **step 1,** internal forces are calculated in a 2D analysis performed by *LTBeamN.* Irrespective of the choice for 1st order analysis or 2nd order analysis, *GeM* requests *LTBeamN* to perform in this step a 1st order analysis to evaluate the internal forces to be used in the cross-sectional classification.

Then, as the program reads each one of the values returned by *LTBeamN*, the geometrical properties (area, A, and section modulus about y-y, W_y) for each one of the cross-sections are calculated taking into account the class of the cross-section obtained for that cross-section.

GeM can determine the Class of cross-sections by three different approaches:

Alternative 1: Increasing only $M_{y,Ed}$

One of the options is to reach the ultimate state by increasing only $M_{y,Ed}$. In this case, a direct application of some expressions found in [5] is done (with N_{Ed}), where the formulae depend on N_{Ed} but not on $M_{y,Ed}$.

Alternative 2: Increasing simultaneously and proportionally N_{Ed} and $M_{y,Ed}$ **until the limit state given by Eq. 6.36 of EC3-1-1**

In order to consider a simultaneous and proportional increase of N_{Ed} and $M_{y,Ed}$, the procedure described in Figure 1 can be adopted. Here, the class is determined by other expressions also found in [5] , which require the calculation of μ_{ult}.

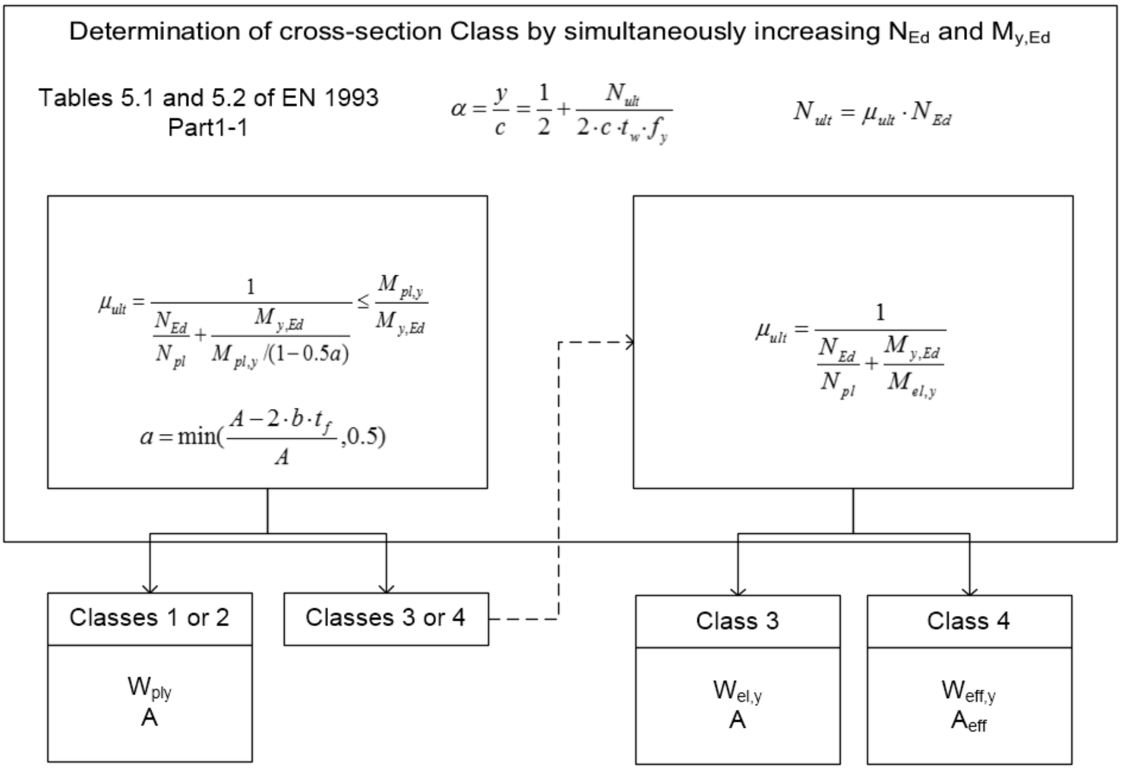

Figure 1. Procedure for the classification and determination of the geometrical properties for each cross-section considering a proportional and simultaneous increase of N_{Ed} and $M_{y,Ed}$

This process starts by considering the cross-section as being a Class 1 (and hence its properties as being $W_{pl,y}$ and A), and the interaction between N and M at the plastic ultimate limit state as following a bilinear curve (see Fig. 2). In case the Class returned is not 1 or 2, a new determination of N_{ult} is carried based on the linear elastic interaction at the elastic ultimate limit state (see Fig. 4) and considering the elastic properties ($W_{el,y}$ and A), establishing if the cross-section is indeed a Class 3 or if it is a Class 4 (and hence the properties to be used are $W_{eff,y}$ and A_{eff}).

Alternative 3: Increasing simultaneously and proportionally N_{Ed} and $M_{y,Ed}$ until the limit state given by theory of plasticity

Another option is to consider the theory of plasticity curve instead of the one given by EC3, and to take the same procedure as in alternative 2, but with μ_{ult} initially given by:

$$\mu_{ult} = \frac{2t_w f_y}{N_{Ed}^2}\left[-M_{Ed} + \sqrt{M_{Ed}^2 + M_{pl,y}\frac{N_{Ed}^2}{t_w f_y}}\right] \tag{3}$$

Step 3 (if e_0 is considered): Initial local imperfection (e_0)

In order to consider the local initial bow imperfection (if this option is activated), the program offers two possibilities for an estimation of this value (along with these two, the program also lets the user set the value of e0).

Alternative 1: Use the value from Table 5.1 of EC3-1-1

One of the ways to determine the value for e_0 is to use the ones given by Table 5.1 of EC3-1-1, which depend on the analysis (elastic or plastic) and on the buckling curve considered. For this purpose, one of the cross-sections has to be chosen in order to determine the corresponding curve, according to Table 6.1.

GeM uses the dimensions of the cross-section at mid length and the steel grade of the profile to find the corresponding buckling curve for the member. If there are any Class 1 or 2 cross-sections among the ones considered, the values for the right column ("plastic analysis") of Table 5.1 are chosen, otherwise it is the values on the left column ("elastic analysis") that are considered by the program.

Alternative 2: Use the value from equation (5.10) of EC3-1-1

The second alternative is to directly estimate the value for e_0 using the equation (5.10) given in Clause 5.3.2 of EC3-1-1. This equation can lead to the local initial bow imperfection:

$$e_0 = \alpha\left(\bar{\lambda}_y - 0.2\right)\cdot\frac{M_{y,Rk}}{N_{Rk}} \qquad \text{for} \qquad \bar{\lambda} \geq 0.2 \tag{4}$$

with α being the imperfection factor that is function of the buckling curve (the one for the cross-section at mid-length, as in alternative 1 of this step). This expression gives more accurate values for the imperfections since it was the basis for the derivation of the European buckling curves. The normalized slenderness $\bar{\lambda}_y$ is given by the following expression:

$$\bar{\lambda}_y = \sqrt{\frac{\alpha_{ult,k,N}}{\alpha_{cr,ip,N}}} = \sqrt{\frac{\frac{N_{Rk}}{N_{Ed}}}{\alpha_{cr,ip,N}}} \tag{5}$$

where $\alpha_{ult,k,N}$ is the minimum amplification factor for the axial force to reach the characteristic resistance N_{Rk}, and where $\alpha_{cr,ip,N}$ is the minimum force amplifier for the axial force to reach the in-plane elastic critical buckling, $N_{cr,y}$.

GeM determines N_{Rk} by multiplying A (or A_{eff} in case of Class 4 cross-sections) by f_y, and uses the value for $\alpha_{cr,ip,N}$ evaluated by *LTBeamN*.

Step 4 (if e_0 is considered): Equivalent load for the initial imperfection (q_0)

GeM considers the initial imperfections by applying one uniformly distributed load q_0 along the length of the member, with its value given by (see 5.3.2(7) from EC3-1-1):

$$q_0 = \frac{8N_{Ed} \cdot e_0}{L^2} \tag{6}$$

where L is the total length of the bar.

The direction of this load is defined according to the maximum value found for the deflection, i.e. positive if the maximum deflection is positive and negative if the maximum deflection is negative.

After having determined q_0, the load is added to the *LTBeamN* input file.

Step 5: Run in-plane analysis (accounting for q_0 if considered)

In this step, *GeM* runs *LTBeamN* again performing a 1st order or a 2nd order analysis depending on the user choice, and stores the new values for the internal forces.

Step 6: Amplification factor for the characteristic resistance $\alpha_{ult,k}$.

The coefficient $\alpha_{ult,k}$ is determined by a cross-section verification. For each one of the cross-sections *i*, *GeM* determines the coefficient $\alpha_{ult,k}$ by proportionally and simultaneously increasing N_{Ed} and $M_{y,Ed}$ until the ultimate limit state is reached, and saves the one with the smallest value as the critical one, as well as its position within the member's length. Depending on the class of each cross-section, $\alpha_{ult,k}$ can be determined:

- **For cross-sections of Class 1 and 2:**

Alternative 1: Plastic interaction (bilinear curve)

The first option when dealing with Class 1 or 2 cross-sections is to consider the bilinear curve that appears in EC3-1-1 as an approximation of the theoretical curve by the Theory of Plasticity (Figure 2).

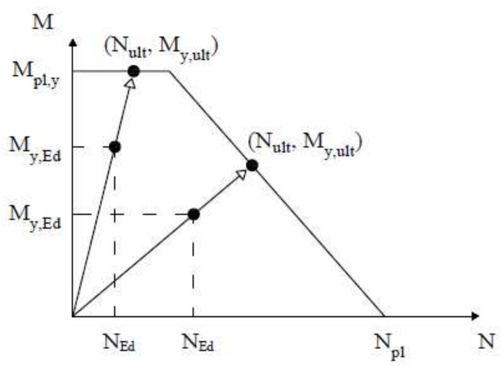

Figure 2. EC3-1-1's bilinear M-N interaction curve for cross-sections with Classes 1 and 2 [5]

In which the amplification factor is given by (7) [5]:

$$\alpha_{ult,k,i} = \frac{1}{\dfrac{N_{Ed,i}}{N_{pl,i}} + \dfrac{M_{y,Ed,i}}{M_{y,pl,i}/(1-0.5a)}} \tag{7}$$

where $N_{pl,i} = A_i \cdot f_y$; $M_{y,pl,i} = W_{y,pl,i} \cdot f_y$ and $a = \min(\frac{A - 2 \cdot b \cdot t_f}{A}, 0.5)$

However, if $\alpha_{ult,k,i} \cdot M_{y,Ed,i} > M_{pl,y,i}$, then:

$$\alpha_{ult,k,i} = \frac{M_{pl,y,i}}{M_{y,Ed,i}} \tag{8}$$

Alternative 2: Linear interaction

The other alternative is to consider a linear interaction between N and M (Figure 3), which is a conservative approach to the real behaviour.

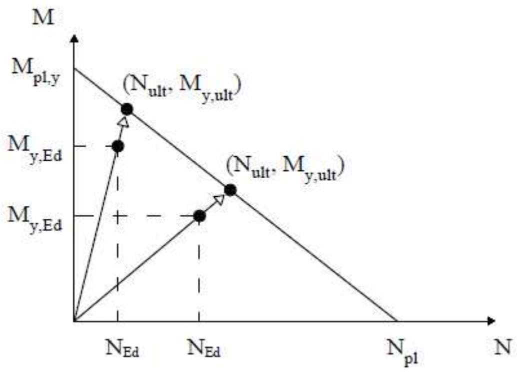

Figure 3. Linear M-N interaction curve for cross-sections with Classes 1 and 2

Thus, the amplification factor is determined by (9):

$$\alpha_{ult,k,i} = \frac{1}{\frac{N_{Ed,i}}{N_{pl,i}} + \frac{M_{y,Ed,i}}{M_{pl,y,i}}} \tag{9}$$

Where $N_{pl,i} = A_i \cdot f_y$ and $M_{pl,y,i} = W_{pl,y,i} \cdot f_y$

- For cross-sections of Class 3:

For cross-sections of Class 3 the reference for the interaction curve is a line that goes from N_{pl} to $M_{el,y}$, as Figure 4 indicates:

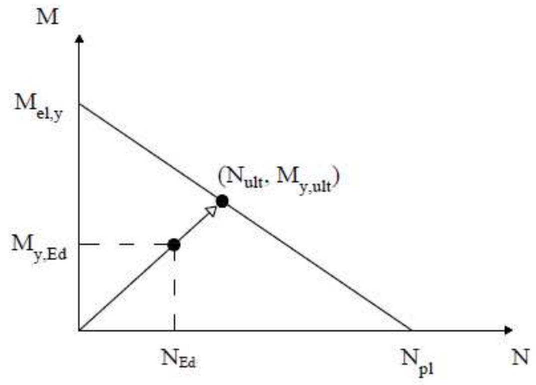

Figure 4. M-N interaction curve for cross-sections with Class 3 [5]

The ultimate load amplifier is given by (10):

$$\alpha_{ult,k,i} = \frac{1}{\frac{N_{Ed,i}}{N_{pl,i}} + \frac{M_{y,Ed,i}}{M_{el,y,i}}} \tag{10}$$

where $N_{pl,i} = A_i \cdot f_y$ and $M_{el,y,i} = W_{el,y,i} \cdot f_y$

- **For cross-sections of Class 4:**

For cross-sections of Class 4 the effective properties are considered and they establish the line that sets the point at which the ultimate cross-section capacity is reached. (Figure 5):

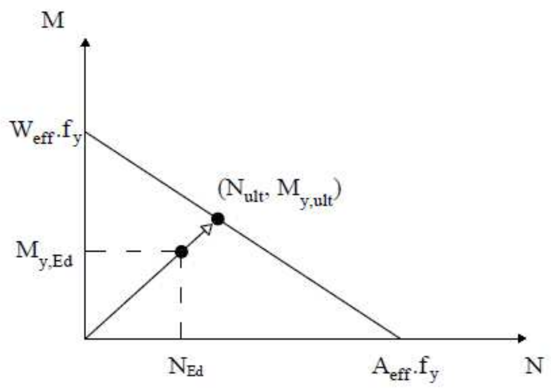

Figure 5. M-N interaction curve for cross-sections with Class 4

In this case $\alpha_{ult,k}$ is given by (11):

$$\alpha_{ult,k,i} = \frac{1}{\frac{N_{Ed,i}}{N_{eff,i}} + \frac{M_{y,Ed,i}}{M_{eff,y,i}}} \tag{11}$$

Where $N_{eff,i} = A_{eff,i} \cdot f_y$ and $M_{eff,y,i} = W_{eff,y,i} \cdot f_y$

Step 7: Amplification factor for out-of-plane instability, $\alpha_{cr,op}$

In case the user has not chosen to insert the value for $\alpha_{cr,op}$ himself, *GeM* reads the value returned by *LTBeamN* in the analysis made at step 2 (first order analysis without global imperfections) and saves it.

Similarly to the previous step, the determination of $\alpha_{cr,op}$ is carried out in *LTBeamN* assuming that neither N nor M are blocked, or in other words, assuming that the multiplier affects both the internal force and bending moment and hence they grow proportionally.

Step 8: Normalized slenderness for the out-of-plane instability, $\bar{\lambda}_{op}$

As mentioned in Clause 6.3.4 of EC3, the normalized slenderness for the out-of-plane instability is calculated by using the following expression:

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \tag{12}$$

where $\alpha_{ult,k} = \min(\alpha_{ult,k,i})$.

Step 9: Reduction factor for the out-of-plane instability, χ_{op}

The reduction factor χ_{op} can be determined by two different ways, according to the clause 6.3.4 (4) of EC3-1-1. For both cases there is the need to determine the reduction factor χ_z for lateral buckling (clause 6.3.1 of EN 1993-1-1) and the reduction factor χ_{LT} for lateral-torsional buckling (clause 6.3.2 of EN 1993-1-1), each one calculated with the out-of-plane slenderness $\bar{\lambda}_{op}$ and by choosing an appropriate buckling curve, considering the dimensions of the cross-section at mid-length or the critical one. For both χ_z and χ_{LT}:

$$\alpha = f(buckling\ curve) \tag{13}$$

$$\phi = 0.5 \cdot \left[1 + \alpha\left(\bar{\lambda}_{op} - 0.2\right) + \bar{\lambda}_{op}^{\,2}\right] \tag{14}$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}_{op}^{\,2}}} \qquad \text{but } \chi \leq 1.0 \tag{15}$$

After having determined χ_z and χ_{LT}, χ_{op} is then determined by one of the two following alternatives:

Alternative 1: Use the minimum value between χ_z and χ_{LT}

In this first alternative, χ_{op} is taken as the minimum value between χ_z and χ_{LT}. This is the same as considering the most adverse curve between the one for flexural buckling and the one for lateral-torsional buckling, and hence returns a more conservative result.

$$\chi_{op} = \min(\chi_z, \chi_{LT}) \tag{16}$$

Alternative 2: Use an interpolated value between χ_z and χ_{LT}

The second option is to find χ_{op} by establishing an interpolation between χ_z and χ_{LT}, as described in [6] and given by:

$$\chi_{op} = \frac{\left(\frac{N_{Ed}}{N_{Rk}} + \frac{M_{y.Ed}}{M_{y,Rk}} \right)}{\left(\frac{N_{Ed}}{\chi_z N_{Rk}} + \frac{M_{y.Ed}}{\chi_{LT} M_{y,Rk}} \right)} \tag{17}$$

where the values for N_{Ed}, M_{Ed}, N_{Rk} and $M_{y,Rk}$ are the ones for the critical cross-section.

Note: ECCS TC8 (2006) recommends the use of the 1st alternative only [7].

If the curve chosen is the same for χ_z and χ_{LT}, both alternative methods will result in the same value for χ_{op}, as the values for χ_z and χ_{LT} are equal.

In case the member is only subjected to N_{Ed} or $M_{y,Ed}$, χ_{op} is equal to χ_z or χ_{LT}, respectively.

Step 10: Resistance criterion

Finally, the expression used by the General Method to check the stability against lateral buckling can be applied:

$$\frac{\chi_{op} \cdot \alpha_{ult,k}}{\gamma_{M1}} \geq 1.0 \tag{18}$$

Flowchart of the procedure

In Figure 6 is shown a flowchart with all the steps involved in the procedure at normal temperature, with all the options related to each step.

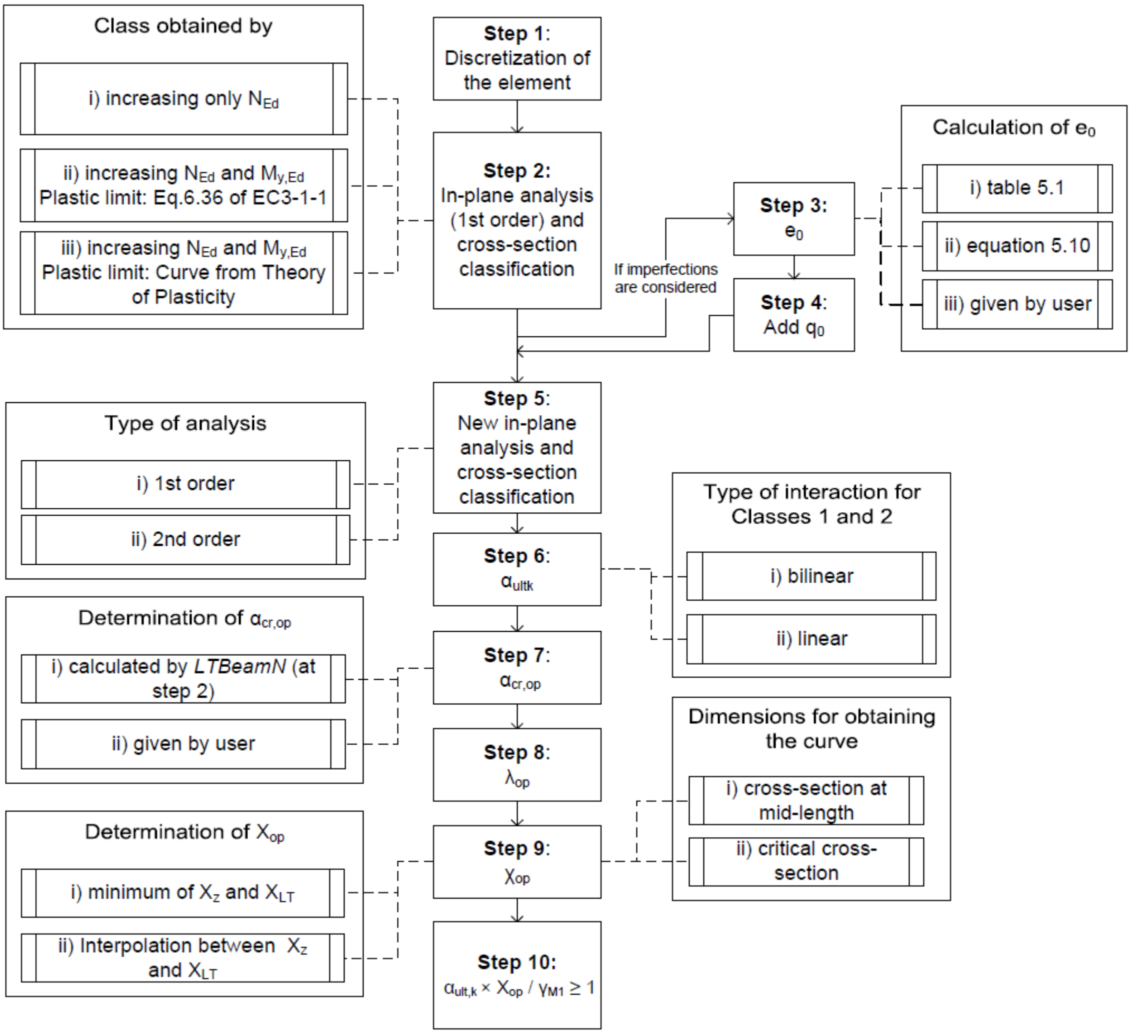

Figure 6. Flowchart of the General Method procedure adopted in GeM for normal temperature

3.3 Procedure at elevated temperature

Under fire conditions, the aforementioned procedure must be adopted in order to accommodate the differences that arise when dealing with elevated temperatures, such as the appearance of the reduction factors for the yield strength, $k_{y,\theta}$ (for classes 1, 2 and 3) or $k_{0.2p}$ (for class 4), and for the elasticity modulus, $k_{E,\theta}$.

Alternative 1 – Adaptation of the procedure for normal temperature

An almost direct application of the procedure for normal temperature is offered to the user, with the changes being the reduction of f_y by $k_{y,\theta}$ or $k_{0.2p}$ and E by $k_{E,\theta}$, the definition of the imperfection factor α, the modification of ε that may alter the class of some cross-sections and hence their properties, and the formulae for determining χ_z and χ_{LT}, that must follow EC3-1-2. The new formulae for ε, α, ϕ, χ_z and χ_{LT} are the following:

$$\varepsilon_\theta = 0.85\sqrt{\frac{235}{f_y}} \tag{19}$$

$$\alpha_{fi} = 0.65 \cdot \sqrt{\frac{235}{f_y}} \tag{20}$$

$$\phi_{fi} = 0.5 \cdot \left[1 + \alpha_{fi} \cdot \bar{\lambda}_{op,fi} + \bar{\lambda}_{op,fi}^{\ 2}\right] \tag{21}$$

$$\chi_{z,fi} = \chi_{LT,fi} = \chi_{op,fi} = \frac{1}{\phi_{fi} + \sqrt{\phi_{fi}^{\ 2} - \bar{\lambda}_{op,fi}^{\ 2}}} \tag{22}$$

It must be noted that $\chi_{z,fi} = \chi_{LT,fi}$ as α_{fi} and $\bar{\lambda}_{op,fi}$ are the same for both cases.

Regarding the value of e0, two options are available which are:

Alternative 1a: same e_0 for normal temperature:

If this first alternative the program uses the same expressions (4) and (5) for normal temperature to estimate the value of e_0.

Alternative 1b: a modification of the e_0 equation:

A modified version of the expressions (4) and (5) for e_0 at elevated temperature may be given by (23) and (24):

$$e_0 = \alpha \cdot \bar{\lambda}_y \cdot \frac{M_{y,Rk}}{N_{Rk}} \tag{23}$$

with

$$\bar{\lambda}_y = \sqrt{\frac{\alpha_{ult,k,N}}{\alpha_{cr,ip,N}}} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = \sqrt{\frac{\frac{N_{Rk}}{N_{Ed}}}{\alpha_{cr,ip,N}}} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} \tag{24}$$

which account for the reduction factors and for the non-existence of a plateau for the buckling curves at elevated temperatures.

Flowchart of the procedure

In Figure 7 the flowchart of the procedure for elevated temperature is shown. Note that the options found in Figure 6 involving the determination of χ_{op} are not shown, as the curve for flexural and lateral-torsional buckling is the same at elevated temperature, and it does not depend on the cross-section properties, only on the steel grade.

Note that the options found in Figure 6 involving the determination of χ_{op} are not shown, as the curve for flexural and lateral-torsional buckling is the same at elevated temperature, and it does not depend on the cross-section properties, only on the steel grade.

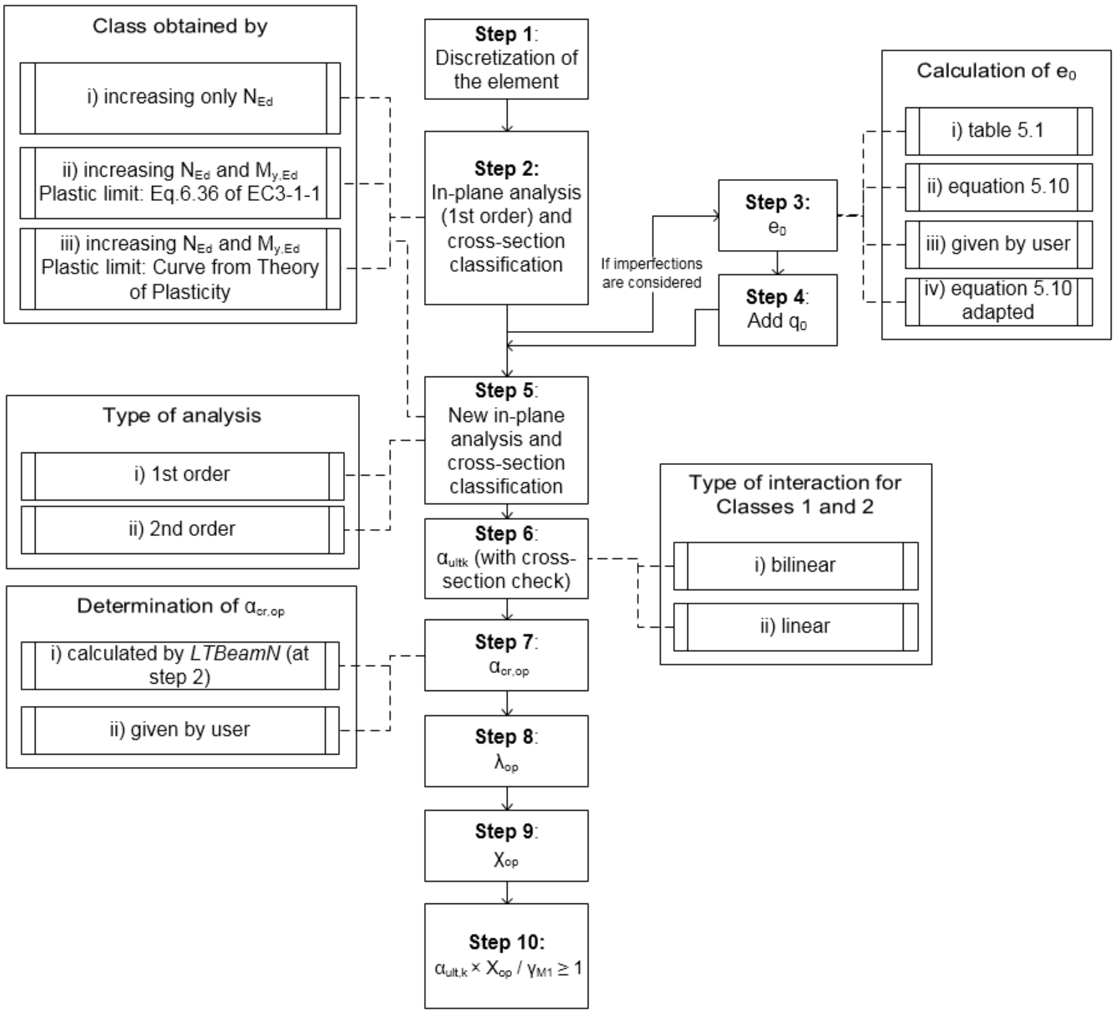

Figure 7. Flowchart of the adaptation of the General Method procedure for elevated temperature

Alternative 2 – Use the interaction formulae

Due to the fact that the consideration of a local 2nd order analysis on *LTBeamN* with a fixed value for E (affected by $k_{E,\theta}$) might not be correct (since the stress-strain relationship at elevated temperature is non-linear), there is another alternative: to account for the reduction factor due to imperfections, 2nd order effects and in-plane buckling, and the interaction coefficients when determining the amplification factor for the characteristic resistance, $\alpha_{ult,k}$.

If this is the case, the procedure will be modified as following:

Step 1: Discretization of the member

This step is the same as in the procedure at normal temperature.

Step 2: Determination of the internal forces and properties for each cross-section

This step remains similar, except for the fact that now two analyses have to be made: one for normal temperature and other for the temperature θ specified by the user, by running *LTBeamN* with the reduced value for E ($k_{E,\theta}\cdot E$) and by calculating the cross-section properties according to $k_{y,\theta}$ and according to the fire classification of the cross-section (it depends on ε_θ).

This analysis at normal temperature is necessary to determine the in-plane normalized slenderness at normal and elevated temperatures, $\overline{\lambda}_{y,20°C}$ e $\overline{\lambda}_{y,\theta}$, that will be needed in **Step 6**.

Steps 3 and 4 (e_0 and q_0):

These 2 steps are not considered since the local imperfections will now be considered indirectly in the formulae for $\alpha_{ult,k,I}$ in **Step 6**.

is not used when determining the internal forces (unlike the procedure at normal temperature, no second order analysis is performed).

Step 5: Run analysis (with LTBeamN)

A new analysis does not need to be carried out as the forces that will be used in **Step 6** come from the 1st order analysis under elevated temperature performed in **Step 2**.

Step 6: Amplification factor for the characteristic resistance, $\alpha_{ult,k}$.

Here is where the big change occurs. Unlike before, where a simple cross-section verification is made, now the in-plane buckling must be considered directly in the determination of $\alpha_{ult,k}$ because no 2nd order analysis is performed nor the local (to the member) imperfections are considered.

For the purpose of determining the most conditioning $\alpha_{ult,k,i}$, Eq. (4.21) of EC3-1-2 is applied to each cross-section:

$$\frac{1}{\alpha_{ult,k,i}} = \frac{N_{fi,Ed,i}}{\chi_{y,fi} \cdot A_i \cdot k_{y,\theta,i} \cdot \dfrac{f_y}{\gamma_{M,fi}}} + k_y \cdot \frac{M_{y,fi,Ed,i}}{W_{y,i} \cdot k_{y,\theta,i} \cdot \dfrac{f_y}{\gamma_{M,fi}}} \leq 1 \qquad (25)$$

where A_i and $W_{y,i}$ are the geometrical properties for each one of the cross-sections that are function of its Class, and where:

$$k_y = 1 - \frac{\mu_y \cdot N_{fi,Ed}}{\chi_{y,fi} \cdot A \cdot k_{y,\theta} \cdot \dfrac{f_y}{\gamma_{M,fi}}} \leq 3 \qquad (26)$$

with A and N_{Ed} the values for cross-section at mid length, and:

$$\mu_y = (2\beta_{M,y} - 5) \cdot (1.1 \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}) + 0.44\beta_{M,y} + 0.29 \leq 0.8 \quad \text{if } \overline{\lambda}_{y,20°C} > 1.1 \qquad (27)$$

$$\mu_y = (2\beta_{M,y} - 5) \cdot \overline{\lambda}_{y,\theta} + 0.44\beta_{M,y} + 0.29 \leq 0.8 \quad \text{if } \overline{\lambda}_{y,20°C} \leq 1.1 \qquad (28)$$

where $\beta_{M,y}$ is given in Table 4.2 of EC3-1-2.

It must be noted that k_y and μ_y do not depend on the cross-section and hence they are calculated only once.

The y-y reduction factor at elevated temperature is given by:

$$N_{cr,y,\theta} = \alpha_{cr,y,\theta} \cdot N_{Ed} \tag{29}$$

$$\bar{\lambda}_{y,\theta} = \sqrt{\frac{A \cdot f_{y,\theta}}{N_{cr,y}}} \tag{30}$$

$$\alpha_{fi} = 0.65 \cdot \varepsilon \tag{31}$$

$$\phi_{y,fi} = 0.5\left[1 + \alpha \cdot \bar{\lambda}_{y,\theta} + \bar{\lambda}_{y,\theta}^{\,2}\right] \tag{32}$$

$$\chi_{y,fi} = \frac{1}{\phi_{y,fi} + \sqrt{\phi_{y,fi}^{\,2} - \bar{\lambda}_{y,\theta}^{\,2}}} \tag{33}$$

where the value for $\alpha_{cr,y,\theta}$ is returned by the in-plane analysis made by *LTBeamN*.

After $\alpha_{ult,k}$ has been calculated for all the considered cross-sections, the minimum value is retained.

Step 7 Amplification factor for out-of-plane instability, $\alpha_{cr,op}$

This step is similar to the procedure at normal temperature, accounting for the fact that E must be affected by $k_{E,\theta}$.

Step 8: Normalized slenderness for the out-of-plane instability, $\bar{\lambda}_{op}$

The normalized slenderness is given by the same expression as in before.

Step 9: Reduction factor for the out-of-plane instability, χ_{op}

The reduction factor is determined as in before by the equations (19), (20) and (21).

Step 10: Resistance criterion

The final step remains the same as stated in [6.3.4] of EC3-1-1.

Flowchart of the procedure

In Figure 8 the flowchart of the adaptation of the procedure for elevated temperature is shown.

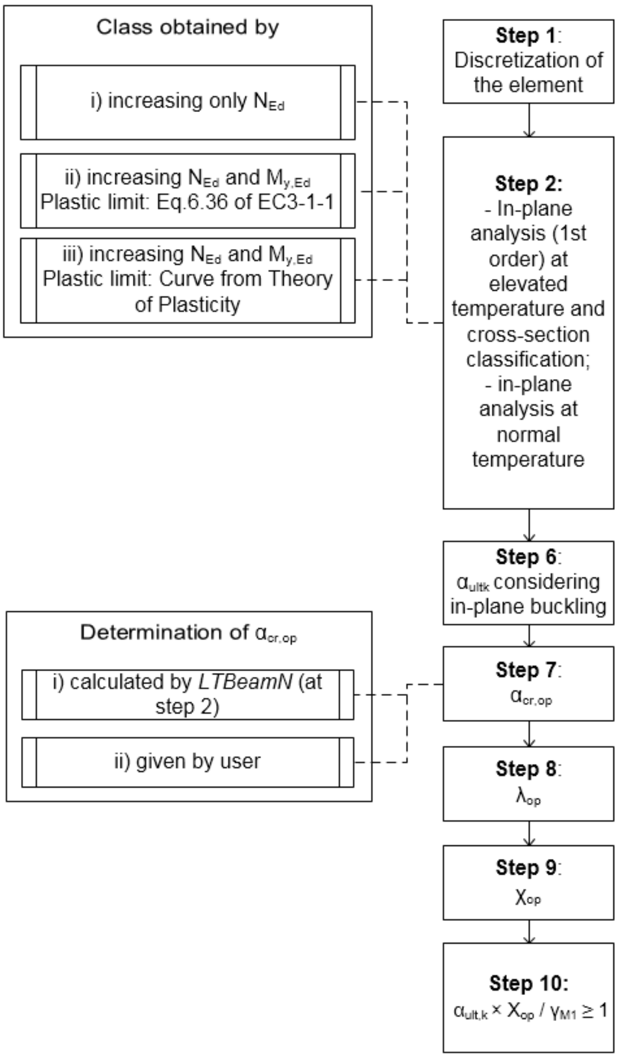

Figure 8. Flowchart of the General Method procedure for elevated temperature: adaptation considering in-plane buckling directly in $\alpha_{ult,k}$

4 Using GeM

4.1 Initiate the program

Once the program is entered a message showing the terms and conditions of use is displayed, as shown in Figure 7. If it's the first time the program is being accessed, a warning message about the need of having *LTBeamN* installed is also displayed.

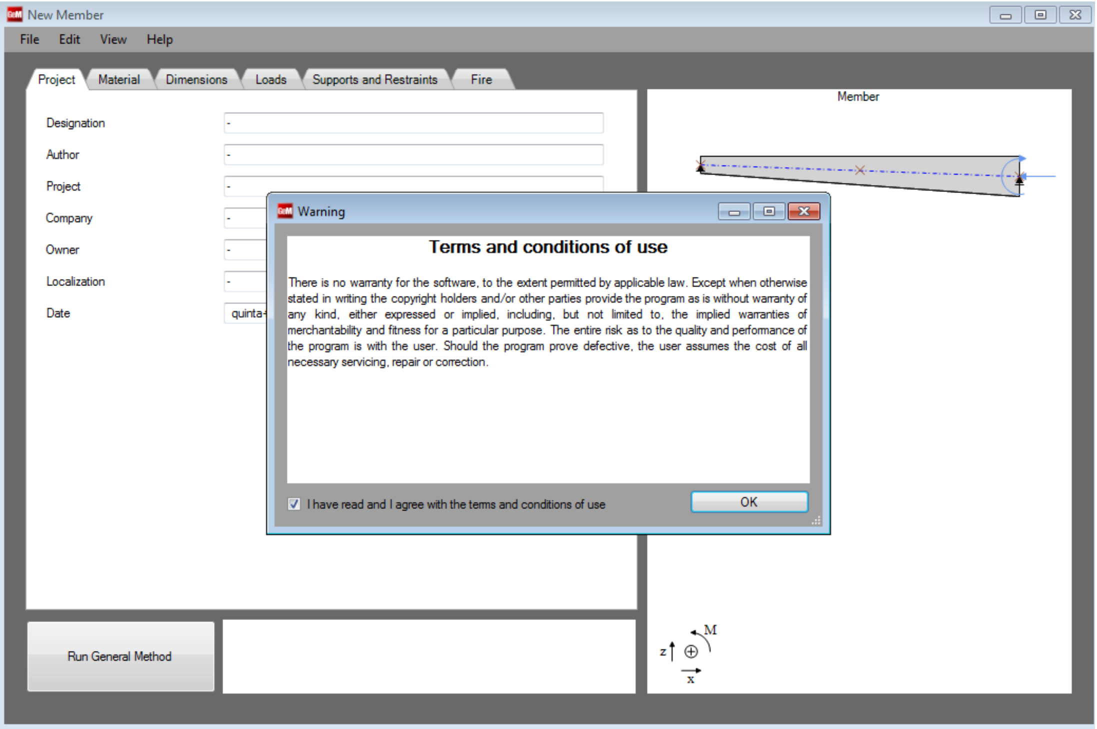

Figure 9. Overview of *GeM* with the dialog box "Terms and Conditions of Use"

4.2 Data entry

The insertion of all the input data is accomplished by going through all tabs ("Project", "Material", "Dimensions", "Loads" and "Supports and Restraints") and inserting them in the fields in the corresponding pages (in Figure 4 a display of the page "Dimensions" is shown). The page "Fire" is used if it is intended to run the procedure under fire conditions.

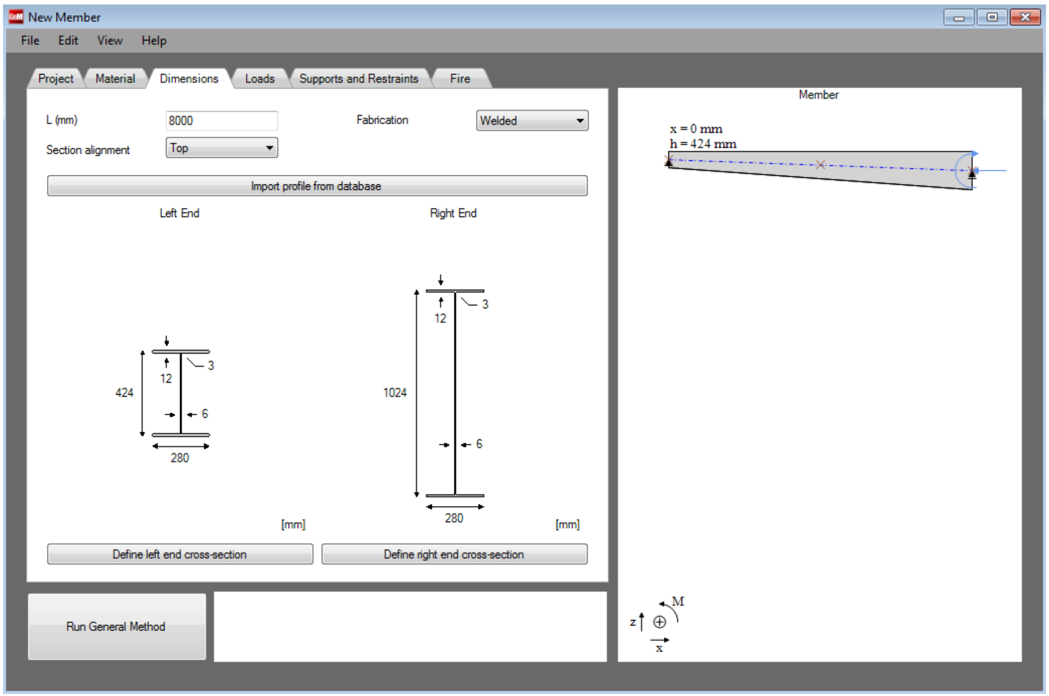

Figure 10. Overview of GeM with the tab page "Dimensions"

4.3 GeM Files

GeM allows users to save cases by writing them in files with the extension ".gem". By going to the menu "File" it is possible to save the current case, to open a previously saved case or to create a new one.

4.4 Program definitions

There are three menu items in "Edit". By clicking on the menu item "LTbeamN Path" it is possible to view and set the path where *LTBeamN* is located.

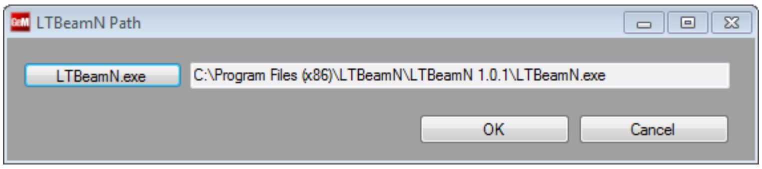

Figure 11. Window "File Paths"

In "Custom Profiles" the user can view and edit the custom profiles database, which contains his own customized profiles (Figure 12), and choose to import one of them to *GeM*.

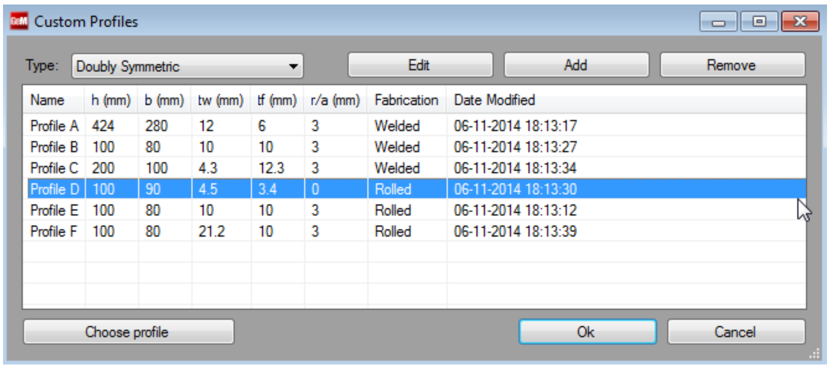

Figure 12. Window "Custom Profiles"

The window "Options" may also be accessed, where it is possible to change the definitions related to the calculation procedure alternatives described in chapters 3 and 4 (Figure 13).

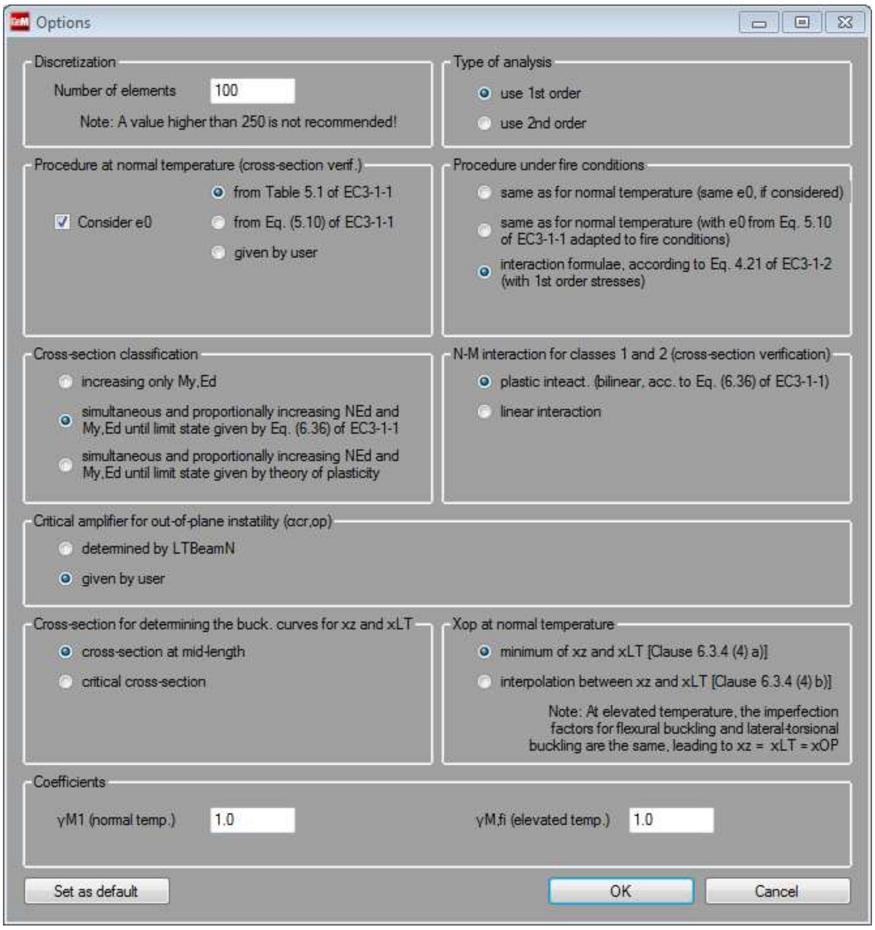

Figure 13. Window "Options"

4.5 Calculation

By clicking on "Run General Method" at the bottom left corner of the main window, *GeM* initiates the process of calculation and displays a window showing that the procedure is being carried out (Figure 14).

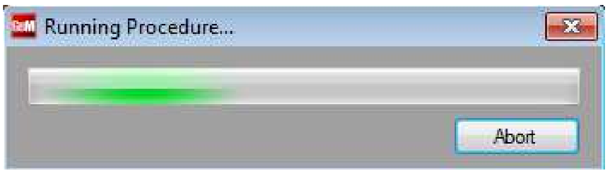

Figure 14. Window indicating that the procedure is running

After having successfully run the procedure, the main window shows one new tab named "Results" with the main outputs that resulted from it, as well as a display of the in-plane analysis with the distribution of the internal forces and bending moments, the classes and the ultimate load amplifiers for each one of the cross-sections considered along the member's length, as shown in Figure 15.

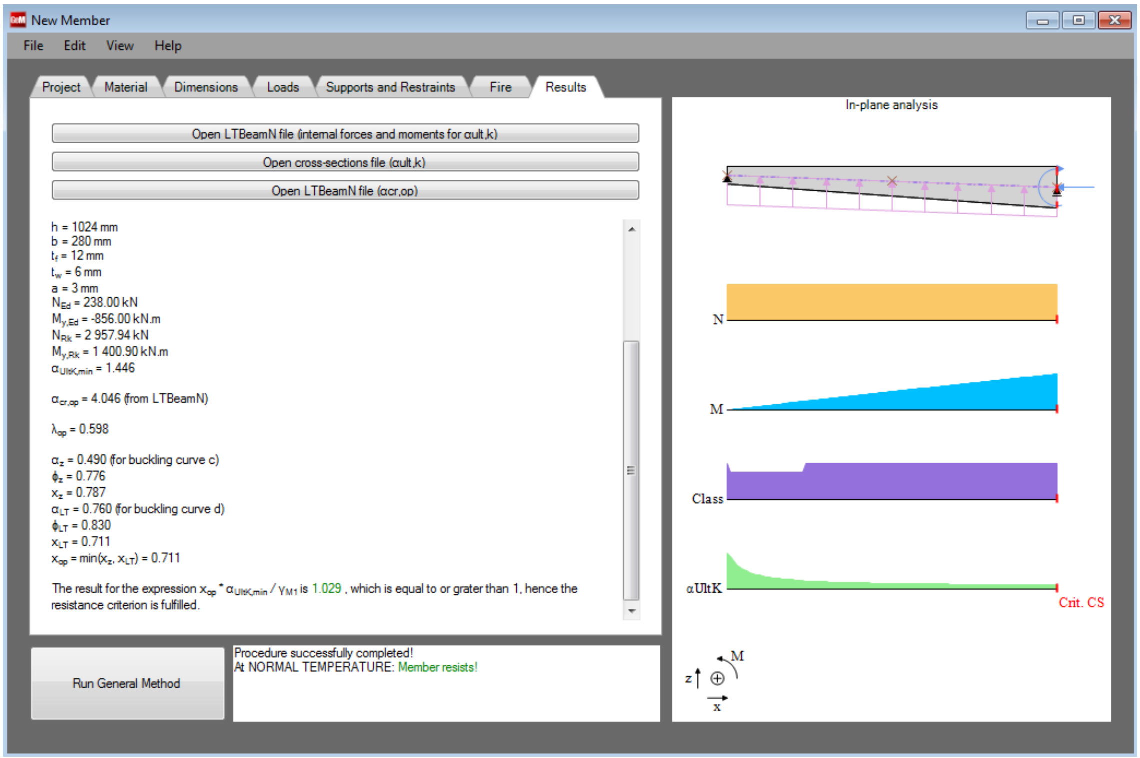

Figure 15. Overview of the main window when the procedure is finished

The *LTBeamN* input files that are created during the procedure and a file containing all the cross-sections considered in the in-plane analysis may be opened by clicking on the three large buttons on the top of the page.

By accessing the menu item “View > Report” in the menu bar, a window containing an extended report can be seen, edited, saved as an .rtf file, sent to a printer or printed into a pdf file (Figure 16).

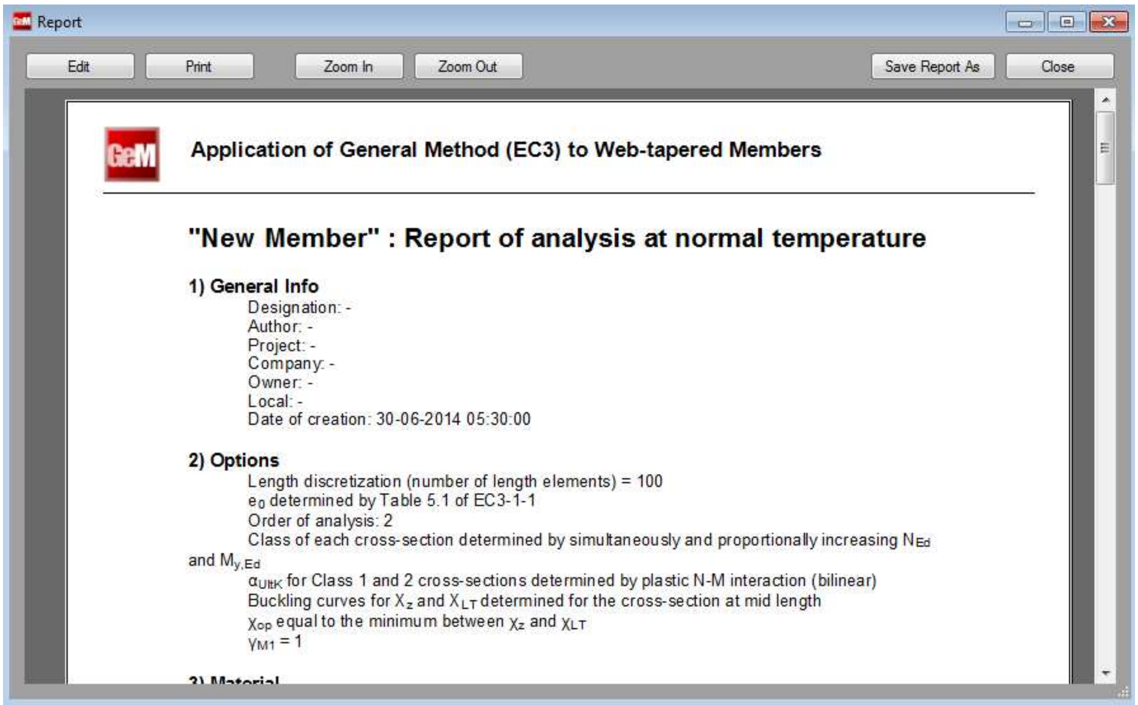

Figure 16. Window “Report”

5 Example of application

In order to better understand how to use *GeM,* an example of its application at normal and elevated temperatures is described in this chapter.

5.1 Characterization

The member that is going to be analysed is a web-tapered column that is part of a frame of an industrial building. In order to apply the General Method a global analysis to the whole frame must be first carried out, considering its initial deformed shape, the applied actions and the position of the lateral restraints (Figure 17).

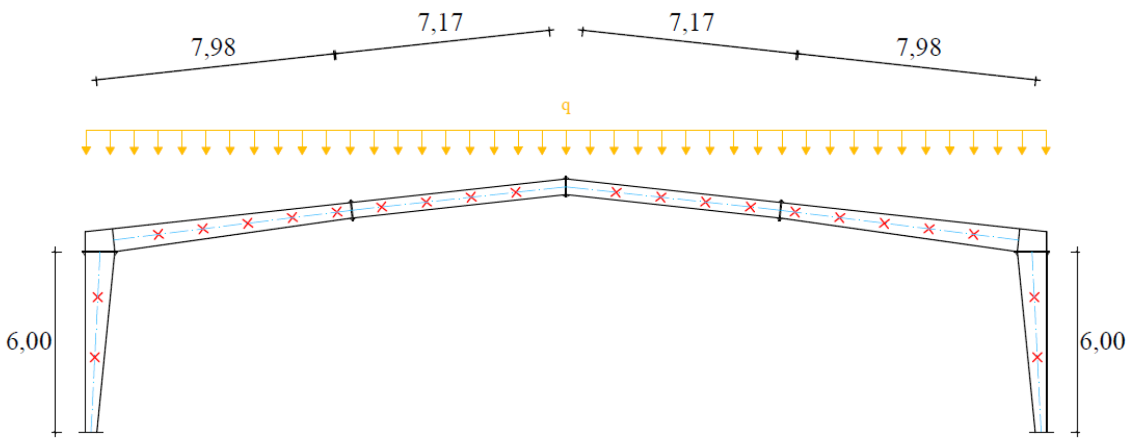

Figure 17. Overview of the studied frame

This first analysis returns the value of the actions in the columns, which can now be isolated from the structure and studied alone.

5.2 Properties

5.2.1 Material

The steel grade is S275. As the maximum thickness of the web and flanges is less than 40mm, the yield strength to be considered is 275 MPa (see Table 3.1 of EC3-1-1). The elasticity modulus, E, is equal to 210 GPa and the value of the Poisson's factor, ν, is 0.3.

5.2.2 Dimensions

The member is a welded I-shaped column with constant web thickness and variable web height. The length of the column is 6000mm with its cross-sections aligned by the top, which have the following dimensions:

h (full height) :	variable from 400 mm (bottom end) to 1000 mm (top end)
b (web width):	220 mm
t_f (flange thickness):	14 mm
t_w (web thickness):	8 mm
a (welding cord thickness):	4 mm

5.2.3 Loads

The loads acting on the column that resulted from the global analysis are:

F_x at the top: 800 kN
M_y at the top: -320 kN.m

5.2.4 Supports and lateral restraints

The column is considered to have fork supports at both ends and has two additional lateral restraints located at 2500mm and 4500mm from the bottom and placed at the centre of gravity of the cross-sections at those positions.

5.3 Options considered

It is considered the value of 200 for the number of finite elements, as well as a 2nd order analysis of the internal forces and bending moments accounting for the imperfection e_0, which is determined according to Table 5.1 of EC3-1-1.

The approach at elevated temperature is the same as at normal temperature, i.e. the characteristic load amplifier is obtained with a cross-section verification of the most critical cross-section.

The classes for the cross-sections are obtained by increasing simultaneously and proportionally N_{Ed} and $M_{y,Ed}$ until the limit state given by Eq. 6.36 of EC3-1-1, and it is assumed a bilinear interaction between N and M in the calculation of $\alpha_{ult,k}$.

The value used for $\alpha_{cr,op}$ is the one returned by *LTBeamN*.

χ_{op} is set to be the minimum value between χ_z and χ_{LT}, and those are determined for the cross-section at mid-length of the column.

For both γ_{M1} and $\gamma_{M,fi}$ the values used are the ones recommended by EC3-1-1 and EC3-1-2, which are both equal to 1.0.

5.4 Calculation at normal temperature

5.4.1 Loads

At normal temperature, the loads considered are the ones obtained from the global analysis, which are depicted in Figure 18.

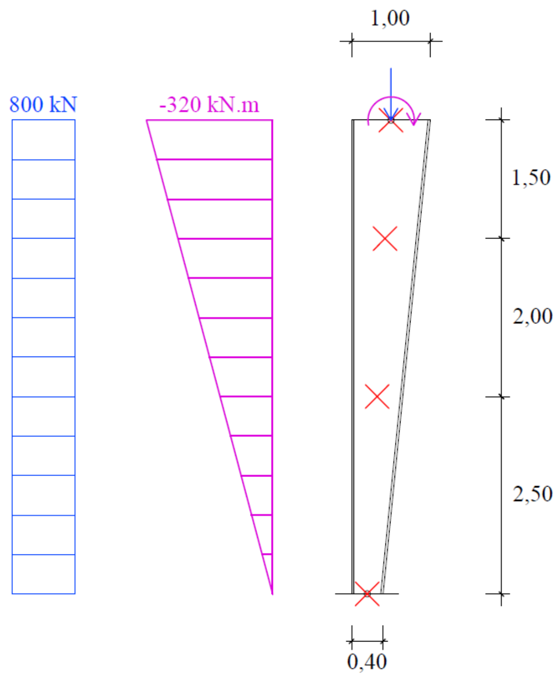

Figure 18. Left column with bending moment and axial force diagrams that result from the global analysis of the frame

5.4.2 Results

The results obtained by *GeM* for the in-plane analysis are shown in Figure 19. The critical cross-section is placed at the top of the column, with the lowest value for $\alpha_{ult,k}$ equal to 1.68.

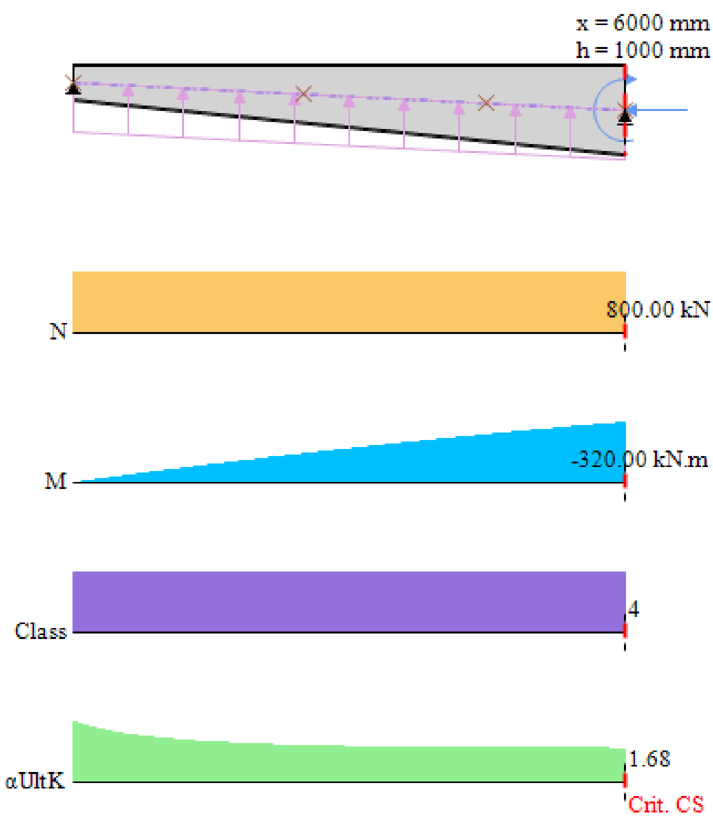

Figure 19. View of the in-plane analysis shown by *GeM* after running the procedure at normal temperature

LTBeamN returns a value for $\alpha_{cr.op}$ = 10.72 that combined with $\alpha_{ult,k}$ results in a normalized slenderness of $\overline{\lambda}_{op}$ = 0.405.

For this slenderness the reduction factors obtained are χ = 0.899 (for buckling curve c) and χ_{LT} = 0.853 (for buckling curve d), with the reduction factor χ_{op} given by the minimum value between these two, 0.853.

Finally, the result obtained for the expression $\frac{\chi_{op} \cdot \alpha_{ult,k}}{\gamma_{M1}}$ is 1.426, which is equal or greater than 1 and hence the resistance condition is fulfilled.

5.5 Calculation at elevated temperature

5.5.1 Loads

The loads for the analysis at elevated temperature are the ones depicted in Figure 20.

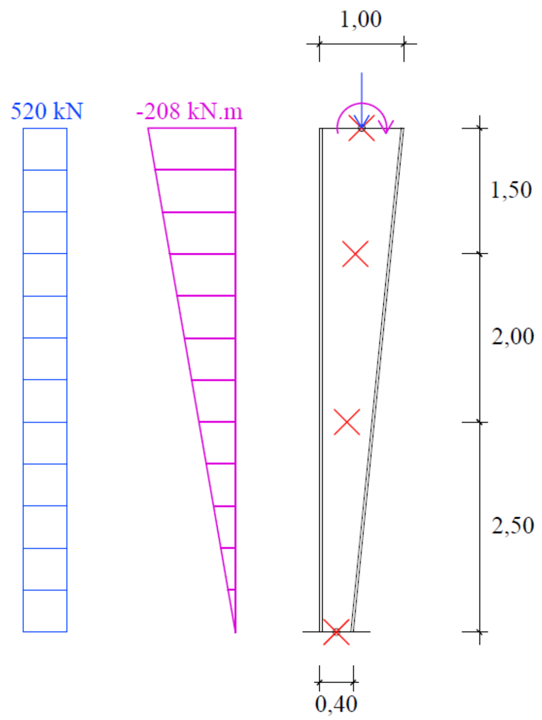

Figure 20. Left column with bending moment and axial force diagrams under fire conditions

5.5.2 Reduction factors for the yield strength and the elastic modulus

It is here considered that the member is subjected to a temperature of 450°. According to Table 3.1 and Table E.1 of EC3-1-2, the reduction factors for that temperature are:

$k_{y,\theta}$ = 0.890
$k_{0.2p,\theta}$ = 0.590
$k_{E,\theta}$ = 0.650

5.5.3 Results

The results obtained with *GeM* for the in-plane analysis at elevated temperature are shown in Figure 21. The critical cross-section is again placed at the top of the column, with the value for $\alpha_{ult,k}$ equal to 1.53.

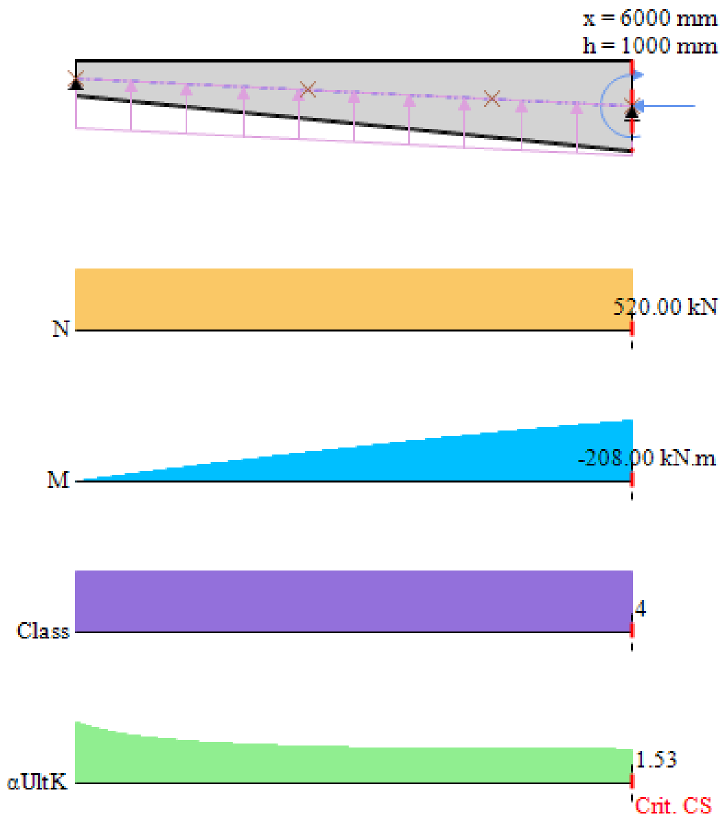

Figure 21. View of the in-plane analysis shown by *GeM* after running the procedure at elevated temperature

For a reduced value of E equal to 136500 MPa, the value returned by *LTBeamN* for $\alpha_{cr.op}$ is 10.65. Thus, the value determined for the normalized slenderness is $\overline{\lambda}_{op}$ = 0.379.

Both reduction factors for flexural and lateral-torsional buckling are the same as they only depend on the yield strength and on the normalized slenderness, which means that the value of the out-of-plane reduction factor is equal to both., i.e. χ_{op} = 0.796.

The result obtained for the resistance condition $\frac{\chi_{op} \cdot \alpha_{ult,k}}{\gamma_{M1}}$ is 1.217, which means that the resistance taking in account out-of-plane buckling is also positively checked at the temperature of 450°.

6 References

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[2] Bureau A., "Résistance au flambement et au déversement d'un poteau à inertie variable selon l'en 1993-1-1." 2007.

[3] CEN, “Eurocode 3: Design of steel structures - Part 1-2: General rules – Structural Fire Design.” Brussels, 2005.

[4] Galéa Y., “Moment critique de déversement élastique de poutres fléchies Présentation du logiciel LTBEAM,” *Revue Construcion Métalique n° 2*, 2003.

[5] Vila Real P. V., “On the classification of cross-sections under bending and axial compression at elevated temperature,” *Commemorative publication on the occasion of Prof. Peter Schaumann’s 60th birthday - Leibniz Universität Hannover*, pp. 221–230, 2014.

[6] Steel Access, “NCCI : General method for out-of-plane buckling in portal frames,” *SN032a-EN-EU*, pp. 1–8, 2007.

[7] Simões da Silva L., R. Simões, H. Gervásio, “Design of Steel Structures.” Wiley-VCH Verlag GmbH, 2010.