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High-Order Numerical Simulations of Incompressible Flow using Correction Procedure via Reconstruction

Christopher Cox, Chunlei Liang and Michael Plesniak
Department of Mechanical & Aerospace Engineering

THE GEORGE WASHINGTON UNIVERSITY
SCHOOL OF ENGINEERING AND APPLIED SCIENCE

Introduction

In computational fluid dynamics, low-order (2nd and below) methods are robust and reliable, but less accurate than high-order (3rd and above) methods. High-order methods, however, are more complicated and difficult to implement. Thus, there is a strong need to simplify high-order methods while maintaining robustness. This motivated the development [1,2] of Correction Procedure via Reconstruction (CPR) and our in-house implementation of this method to simulate incompressible flows.

Artificial Compressibility

Consider the 2-D Navier-Stokes equations with Artificial Compressibility (AC) written in conservation form

$$\frac{\partial Q}{\partial t}+\nabla F_e(Q)-\nabla F_v(\nabla Q)=0$$

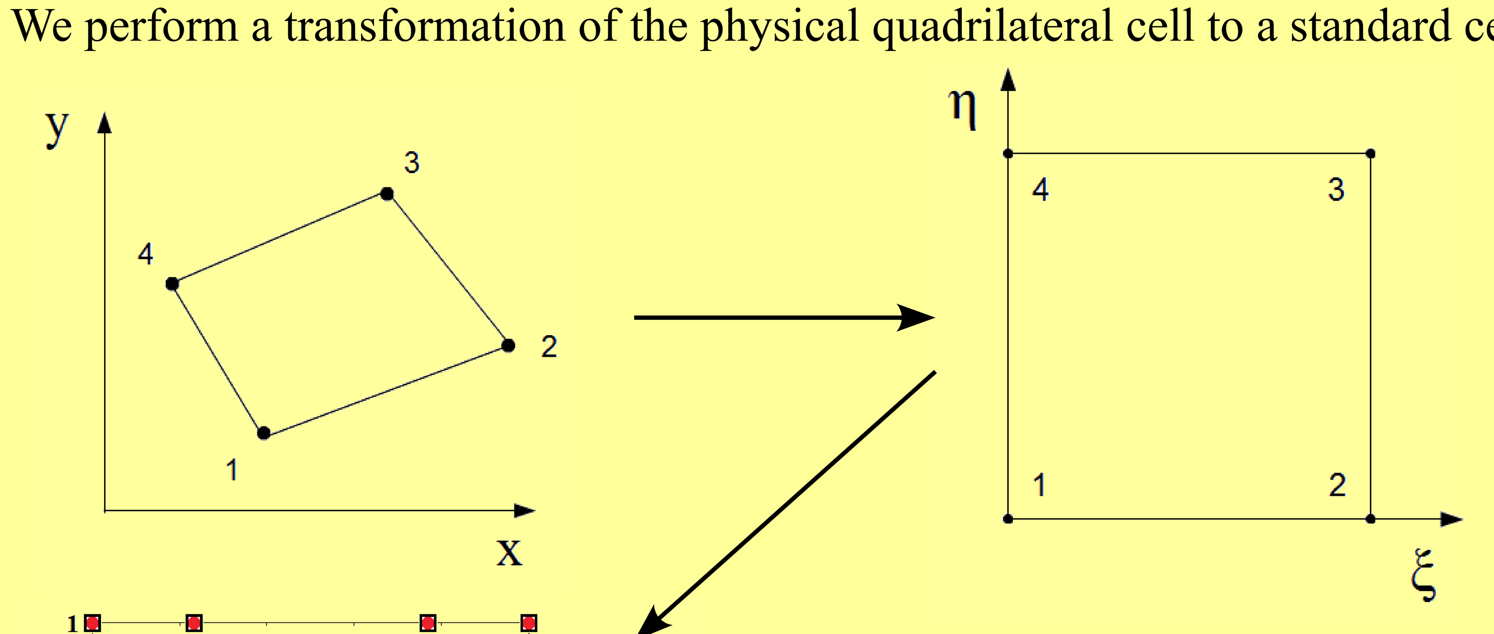
Where $\boldsymbol{Q}$ is the vector of conservative variables for pressure and velocity. The inviscid and viscous flux vectors $\boldsymbol{F}_e$ and $\boldsymbol{F}_v$ in the AC method is written

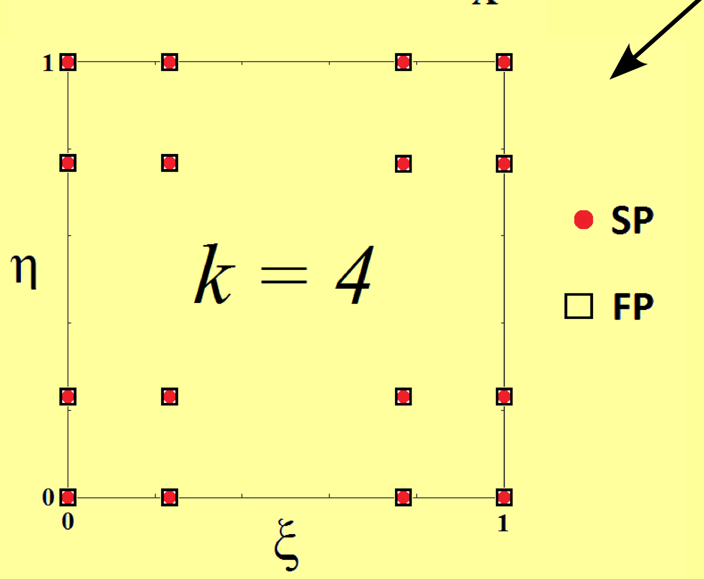
$$F_e(Q)=\begin{bmatrix}\gamma u\\ u^2+p\\ u v\end{bmatrix}+\begin{bmatrix}\gamma v\\ u v\\ v^2+p\end{bmatrix}\qquad F_v(Q)=\begin{bmatrix}0\\ \nu u_x\\ \nu v_x\end{bmatrix}+\begin{bmatrix}0\\ \nu u_y\\ \nu v_y\end{bmatrix}$$

Where γ is the AC relaxation parameter. An artificial temporal term for pressure is introduced to give the incompressible equations hyperbolic character. This allows one to adopt techniques used in compressible solvers to simulate incompressible flows.

High-Order CPR Method

We perform a transformation of the physical quadrilateral cell to a standard cell.





Solutions within each standard cell are approximated by k solution points (in 1-D) that define a polynomial of degree k-1. Within the entire fluid domain, the collection of these polynomials forms a function that is discontinuous across cell interfaces, thus making the method local.

High-Order CPR Method

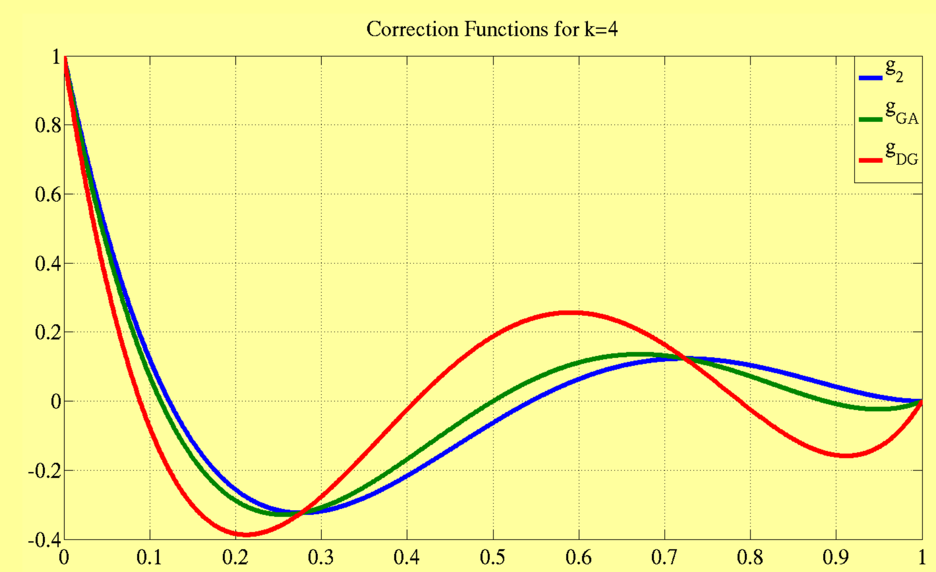
However, the jump up or down in flux across the interface needs a correction to recover a solution of order k. Thus, correction functions $g(\xi)$ are used to adjust the discontinuous flux function $f(\xi)$ within a computational cell in order to ensure flux continuity across cell interfaces. The continuous flux function in the x-direction is given by

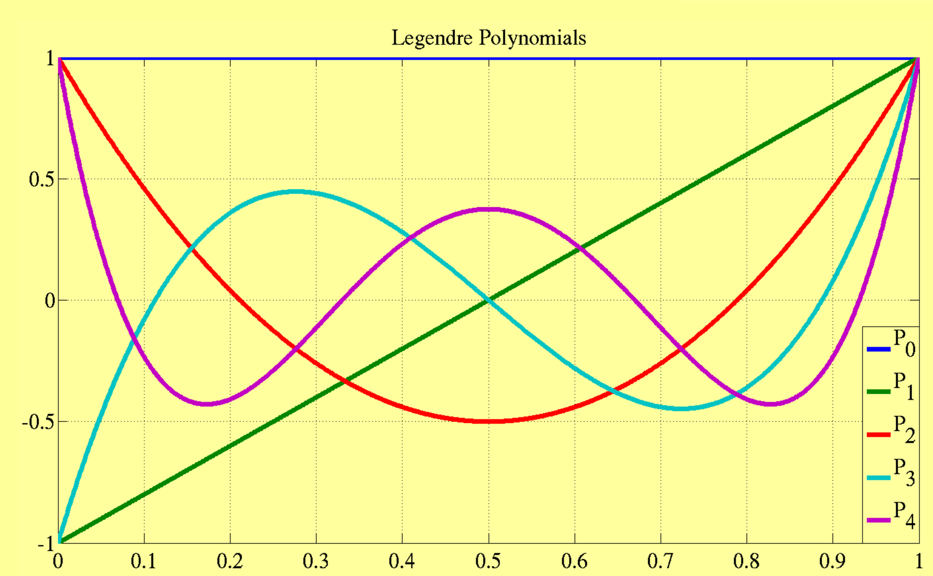
$$F_{j,k}(\xi)=f_{j,k}(\xi)+\left[f^{com}_{j-\frac{1}{2}}-f_j(0)\right]g_{LB}(\xi)+\left[f^{com}_{j+\frac{1}{2}}-f_j(1)\right]g_{RB}(\xi)$$

The correction function chosen here is such that its derivative has zeros that coincide with the Legendre-Lobatto points.

$$g'_{LB}(0)=g'_2(0)=N(1-N)$$

$$g'_{LB}(1)=g'_2(1)=0$$



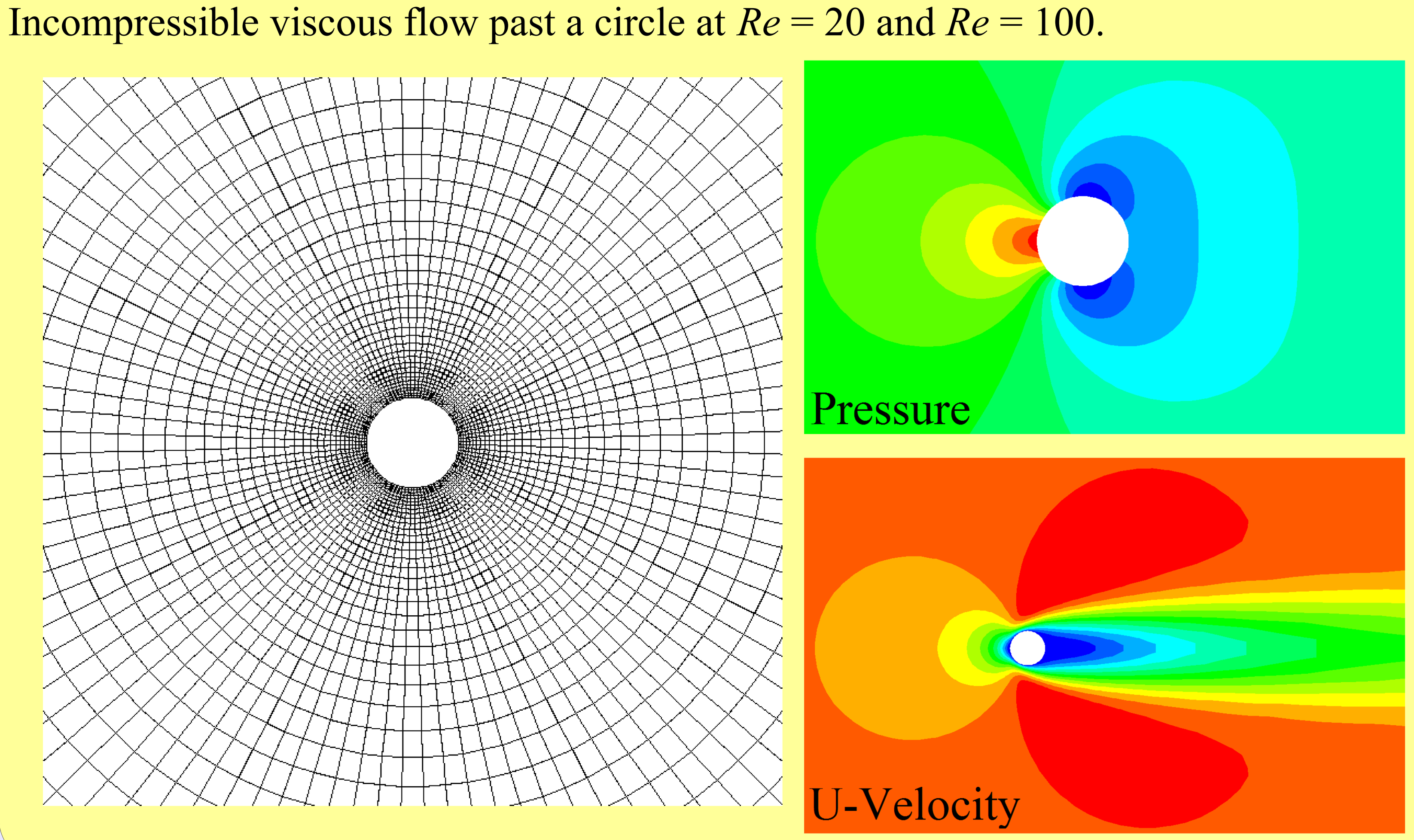


$$g'_{RB}(0)=g'_2(0)=0$$

$$g'_{RB}(1)=g'_2(1)=N(N-1)$$

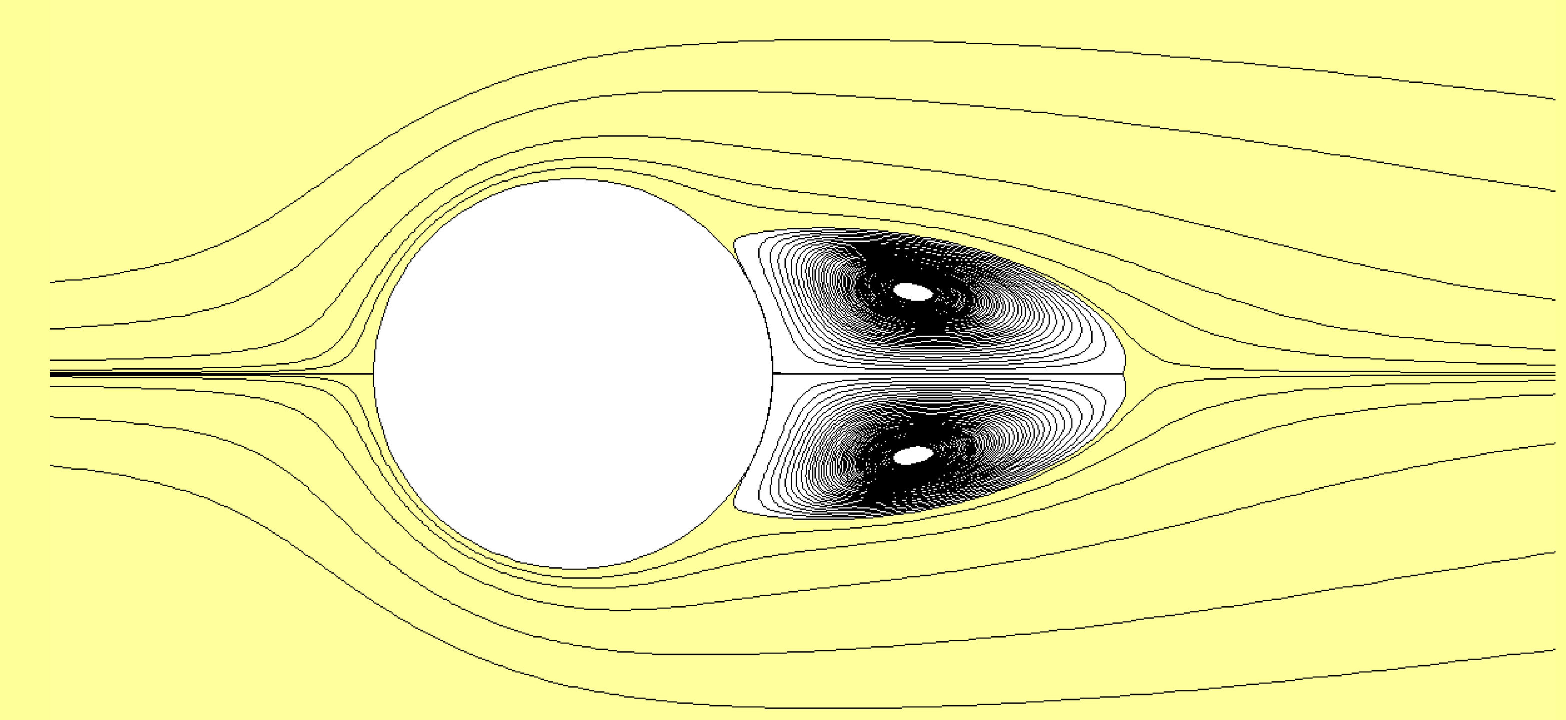
Test Cases

Incompressible viscous flow past a circle at Re = 20 and Re = 100.

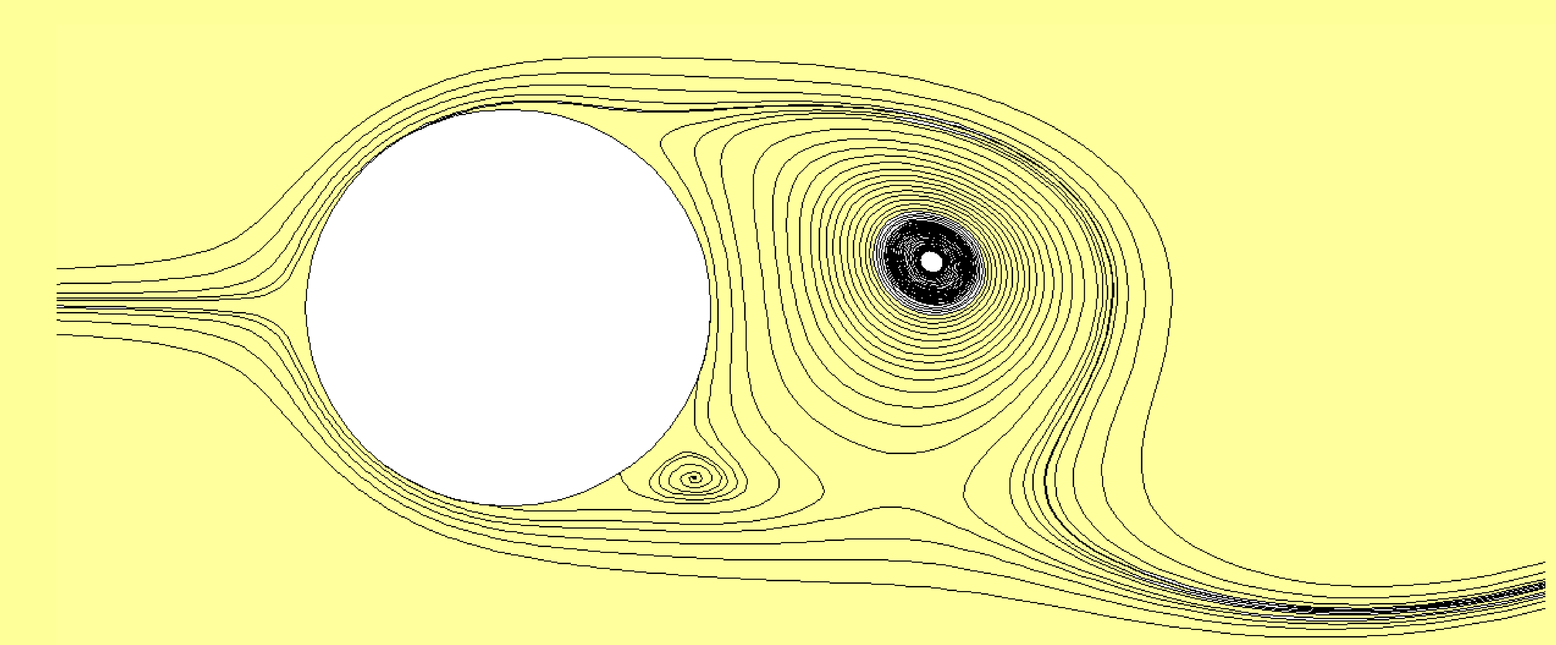


Test Cases

Instantaneous streamlines at Re = 20 (steady state):



Instantaneous streamlines at Re = 100 (vortex shedding):



	Re = 20	Re = 100
C_D	2.041	1.365 ± 0.022
C_L	9.01E-15	± 0.345

Conclusions & Future Work

CPR implementation is straightforward in comparison to other high-order methods. We demonstrate the capability of our solver to simulate 2-D incompressible flows on unstructured grids. Future development will involve expanding the current solver to massively parallel computers to simulate 3-D incompressible flows, particularly those that are viscous and vortex dominated.

References

[1] Huynh, H., "A flux reconstruction approach to high-order schemes including discontinuous Galerkin method," AIAA Paper AIAA-2007-4079, 2007.
[2] Wang, Z.J., Gao, H., "A unifying lifting collocation penalty formulation including the discontinuous Galerkin, spectral volume/difference methods for conservation laws on mixed grids," Journal of Computational Physics, **228**, 21, pp. 8161-8186, 2009.