Development of a Parallel 3D High-order Navier-Stokes Solver for Studying Secondary Flow Structures in a Curved Artery Model

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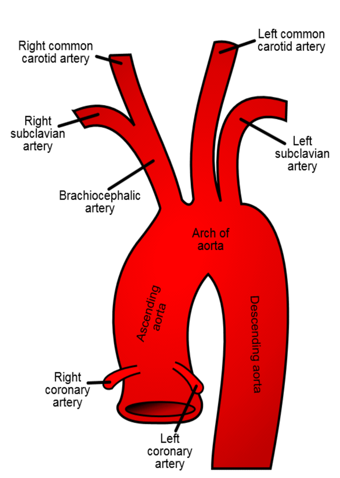
Motivation

The development of this computational fluid dynamics (CFD) solver[1] is motivated by secondary flow structures in curved arteries. Cardiovascular flows are pulsatile, incompressible flows that exist in complex geometries with compliant walls. Together, these factors produce a vortex rich environment that can affect the progression of atherosclerosis by altering wall shear stresses. Unstructured high-order CFD methods (3rd and above) are well-suited for capturing unsteady vortex dominated viscous flows, such as those in curved arteries, and these methods can provide high accuracy for similar cost as low-order methods.



Governing Equations

Consider the unsteady incompressible Navier-Stokes equations with artificial compressibility:

$$\frac{1}{\beta}\frac{\partial p}{\partial \tau} + \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}} = 0 \qquad \nabla\cdot\vec{V} \rightarrow 0$$

$$\frac{\partial u}{\partial \tau} + \frac{\partial u}{\partial t} + \frac{\partial (u^2+p-\upsilon u_x)}{\partial x} + \frac{\partial (vu-\upsilon u_y)}{\partial y} + \frac{\partial (wu-\upsilon u_z)}{\partial z} = 0$$

$$\frac{\partial v}{\partial \tau} + \frac{\partial v}{\partial t} + \frac{\partial (uv-\upsilon v_x)}{\partial x} + \frac{\partial (v^2+p-\upsilon v_y)}{\partial y} + \frac{\partial (wv-\upsilon v_z)}{\partial z} = 0$$

$$\frac{\partial w}{\partial \tau} + \frac{\partial w}{\partial t} + \frac{\partial (uw-\upsilon w_x)}{\partial x} + \frac{\partial (vw-\upsilon w_y)}{\partial y} + \frac{\partial (w^2+p-\upsilon w_z)}{\partial z} = 0$$

$$\boxed{\frac{\partial U}{\partial \tau}} + \boxed{\frac{\partial \hat{U}}{\partial t}} + \boxed{\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}} = 0$$

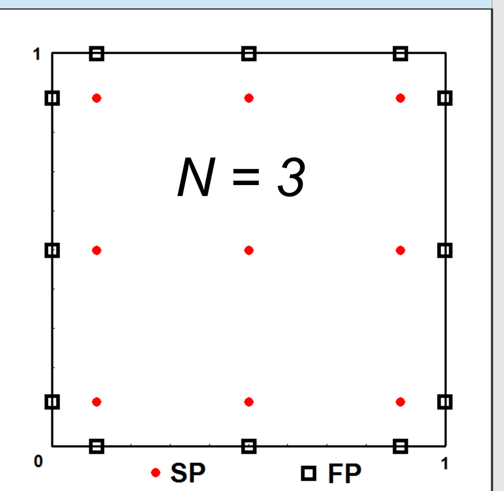
Pseudo Time Stepping
- 1^{st} order backward Euler with LU-SGS

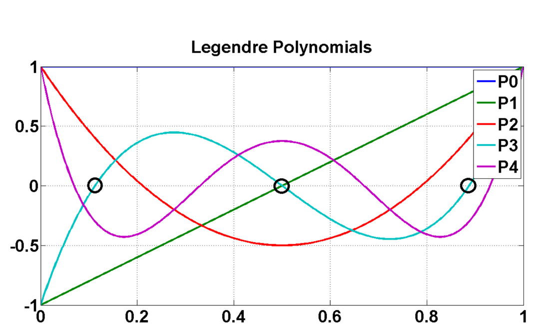
Physical Time Stepping
- 2^{nd} order backward differencing (BDF2)

High-order Method
- Flux Reconstruction

High-order CFD Method

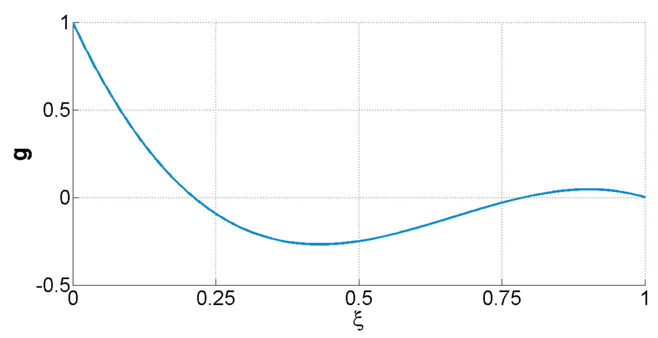
To achieve high order accuracy, each element contains *NxN* solution points (SP) in 2D; these points are Legendre-Gauss values; flux points (FP) located along element boundaries; solution is piecewise continuous across domain.





$$l_i(\xi) = \prod_{s=1, s\neq i}^{N} \left(\frac{\xi - \xi_s}{\xi_i - \xi_s} \right)$$

High-order CFD Method (*cont'd*)



$$U(\xi) = \sum_{i=1}^{N} U_i \; l_i(\xi)$$

$$f_r^D(\xi) = \sum_{i=1}^{N} f_{r|i}^D \; l_i(\xi)$$

$$\frac{\partial f_r}{\partial \xi} = \frac{\partial f_r^D}{\partial \xi} + [f_{r-1/2}^{com} - f_r^D(0)]\frac{d g_r^{LB}}{d\xi} + [f_{r+1/2}^{com} - f_r^D(1)]\frac{d g_r^{RB}}{d\xi}$$

Implicit Time Stepping

Referring back to our mass conservation equation, we can define the residual for the first equation as

$$Residual: \; R_r = \nabla\cdot\vec{V}_r \rightarrow 0$$

and develop an algorithm to drive this residual as close and fast to zero as possible. To solve the governing form with the implicit non-linear LU-SGS scheme, a linearization of the spatial derivatives must be performed:

$$\frac{p_r^{n+1,m+1} - p_r^{n+1,m}}{\Delta\tau} + \nabla\cdot\vec{V}_r^{n+1,m+1} = 0$$

$$\Downarrow$$

$$R_r^{m+1} - R_r^m \approx \frac{\partial R_r}{\partial U_r} + \sum_{nb\neq r}\frac{\partial R_r}{\partial U_{nb}}\Delta U_{nb}$$

$$\Downarrow$$

$$\overbrace{\left[\frac{I}{\Delta\tau} + \frac{\partial R_r}{\partial U_r}\right]}^{A} \delta p_r^{k+1} = -R_r^* - \frac{\Delta p^*}{\Delta\tau}$$

The system of equations is then solved directly using LU decomposition to obtain updated solutions of *p*. Similar expressions can be obtained for *u* and *v*. For higher orders of accuracy, the size of ***A*** matrix renders the solution of x more computationally expensive.

$$\text{Size of } \boldsymbol{A} = [N_{eq}\cdot N^D] \; x \; [N_{eq}\cdot N^D]$$

N	A
2	12x12
3	27x27
4	48x48
5	75x75

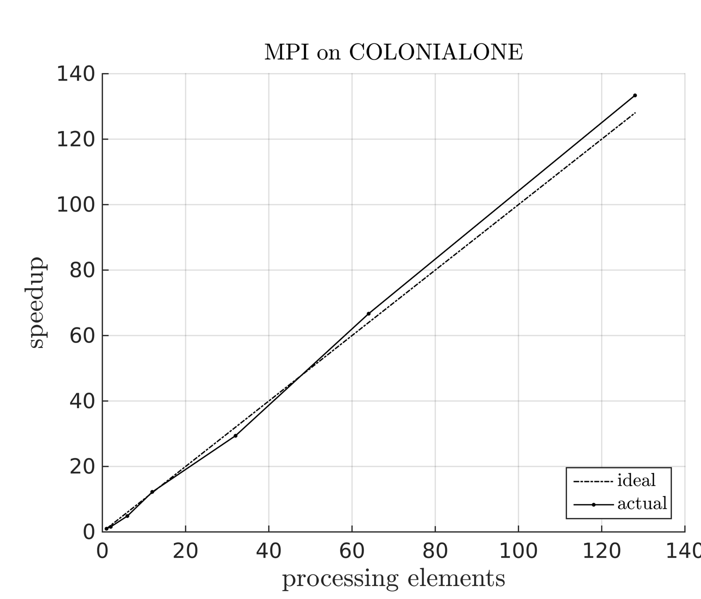
Parallel Scalability

Strong Scaling

Problem size remains constant as the number of processing elements increases.

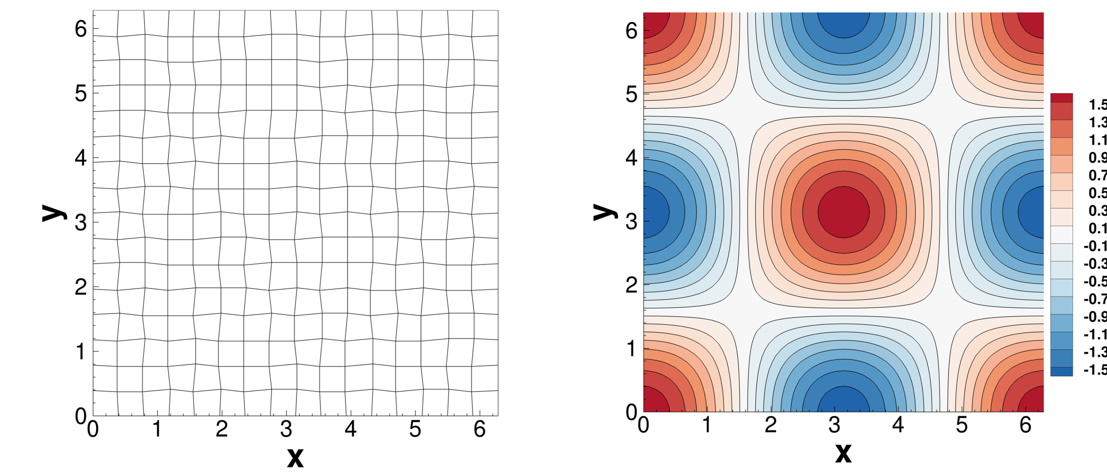
$$S_N = \frac{T_1}{T_N}$$

Parallel implementation uses Message Passing Interface (MPI).



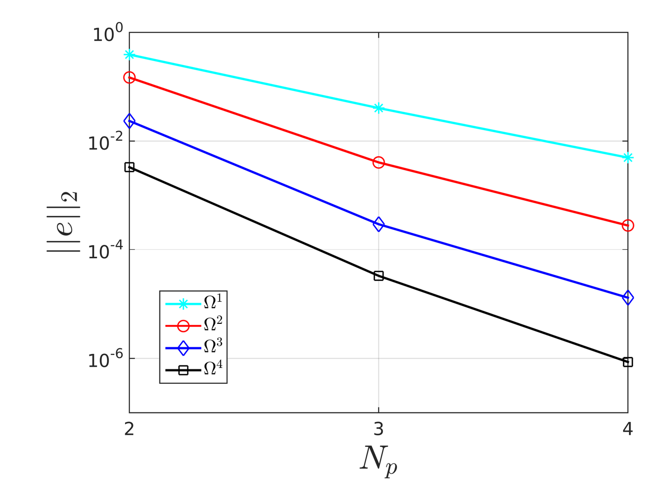
Vortex Decay

Taylor-Green Vortex Decay



$$Reynolds\ Number = 10$$

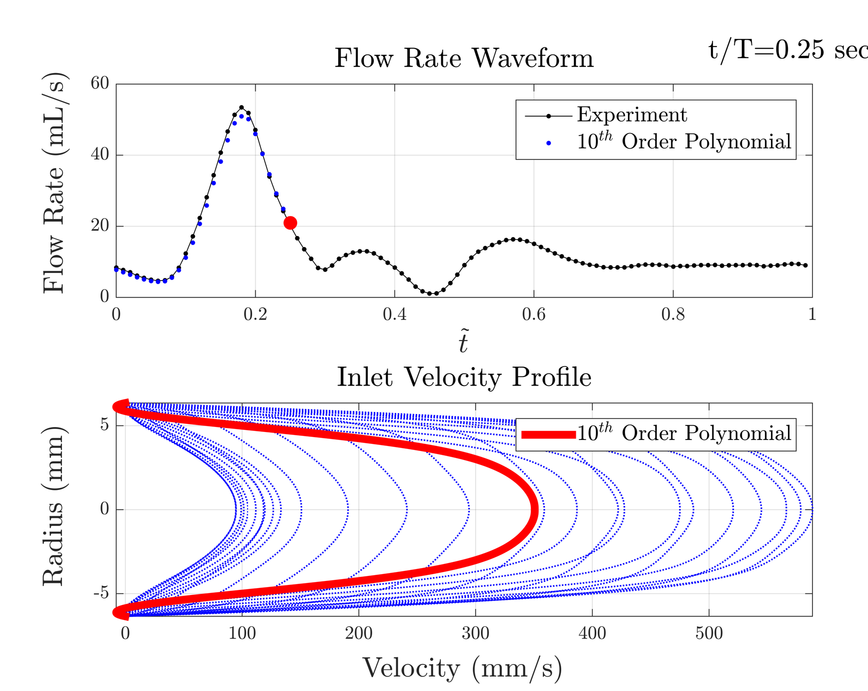
$$L_2 = \left[\frac{\sum_{i=1}^{DOF}(V_{numerical} - V_{exact})^2}{DOF}\right]^{\frac{1}{2}}$$



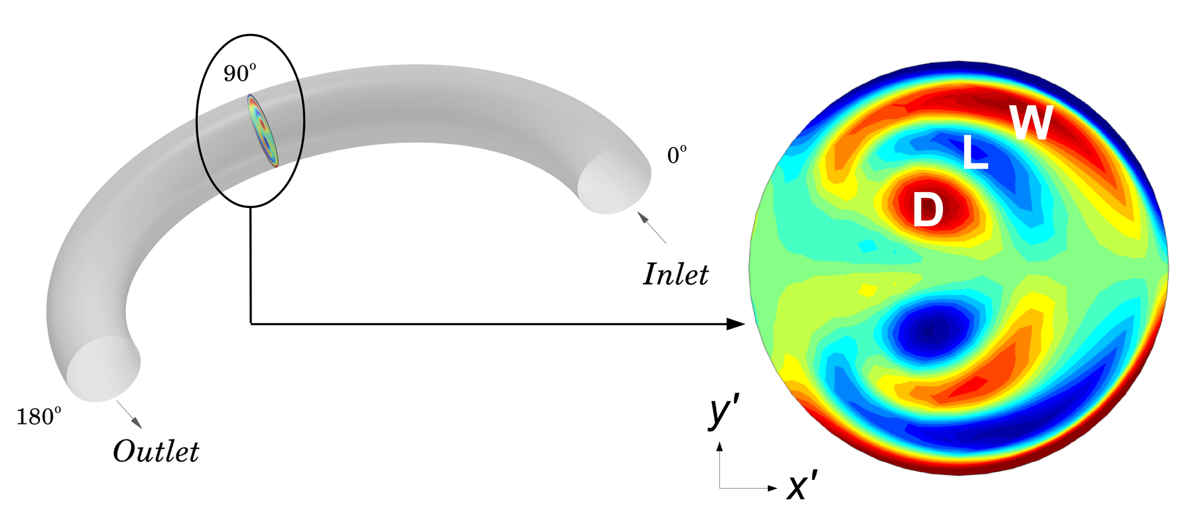
Curved Artery Model

180° Curved Artery Model

The pulsatile waveform, exhibiting rapid acceleration (systole) and deceleration (diastole), is obtained from measurements of the human Carotid artery. The numerical inlet velocity condition is obtained from phase locked experimental PIV data.



GOAL: to simulate newtonian flow through a rigid model and capture ***secondary flow structures*** called Dean (***D***), Lyne (***L***) and Wall (***W***) vortices under these pulsatile conditions and compare numerical results to experimental data.

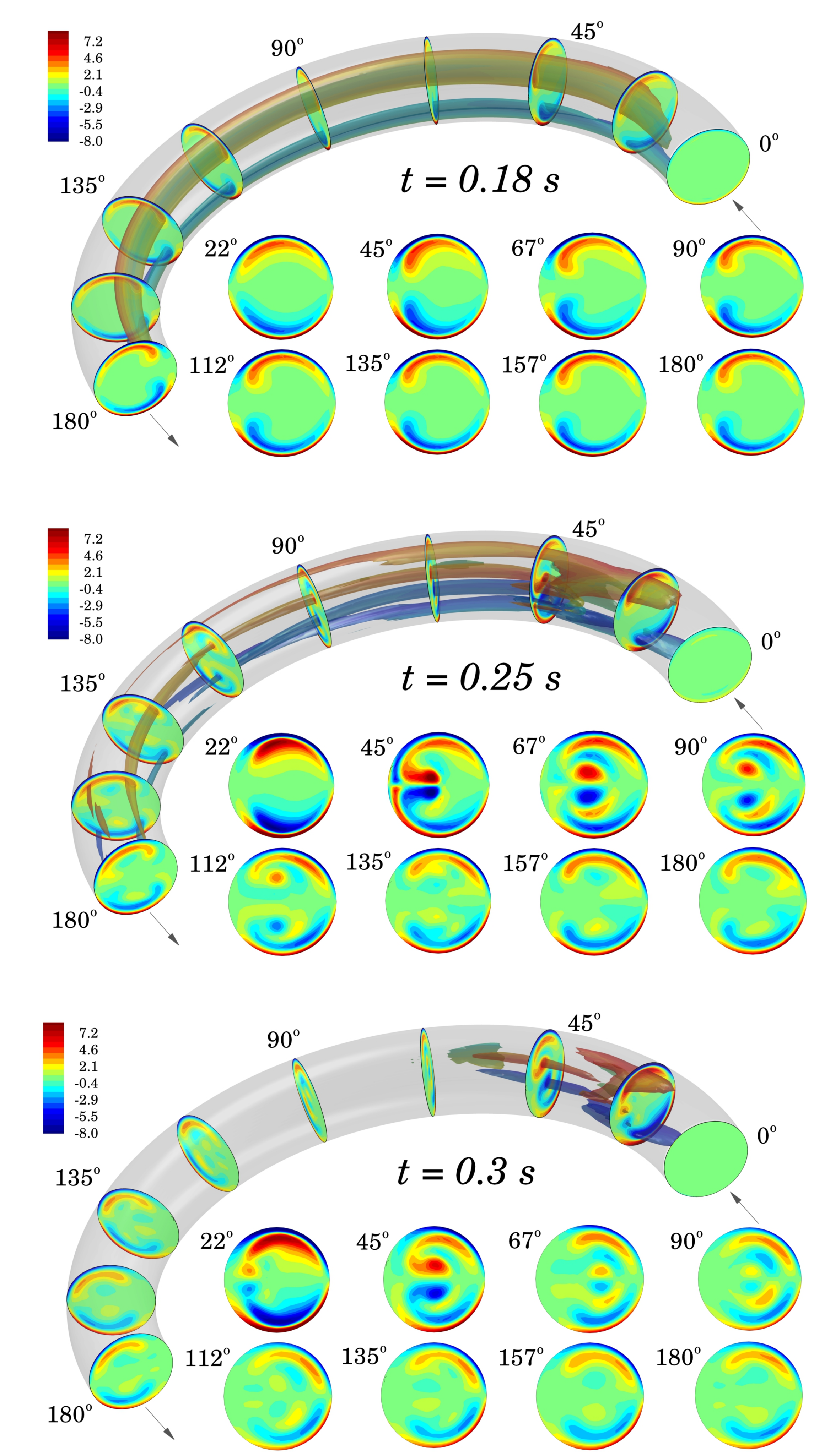


Curved Artery Model (*cont'd*)

Secondary Flow Structures

Vorticity

$$\omega_{z'} = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}$$



Future Research

- Compare numerical results with experimental PIV data.
- Compute wall shear stress and correlate it to the presence of secondary flow structures in a curved artery model.
- Implement moving and deforming mesh capability for fluid structure interaction associated with compliant arterial walls.

References

1) C. Cox, C. Liang and M.W. Plesniak, "A flux reconstruction solver for unsteady incompressible viscous flow using artificial compressibility with implicit dual time stepping," AIAA Paper AIAA-2016-1827, 2016.
2) Huynh, H., "A flux reconstruction approach to high-order schemes including discontinuous Galerkin method," AIAA Paper AIAA-2007-4079, 2007.
3) C. Liang, C. Cox and M.W. Plesniak, "A comparison of computational efficiency of spectral difference method and correction procedure via reconstruction," Journal of Computational Physics, Vol 239, pp 138-146, April 15, 2013.