An implicit time-marching scheme for supersonic flow using MUSCL reconstruction

Christopher Cox[1], Dr. Chunlei Liang[2], Dr. Michael Plesniak[3]
Department of Mechanical & Aerospace Engineering
The George Washington University

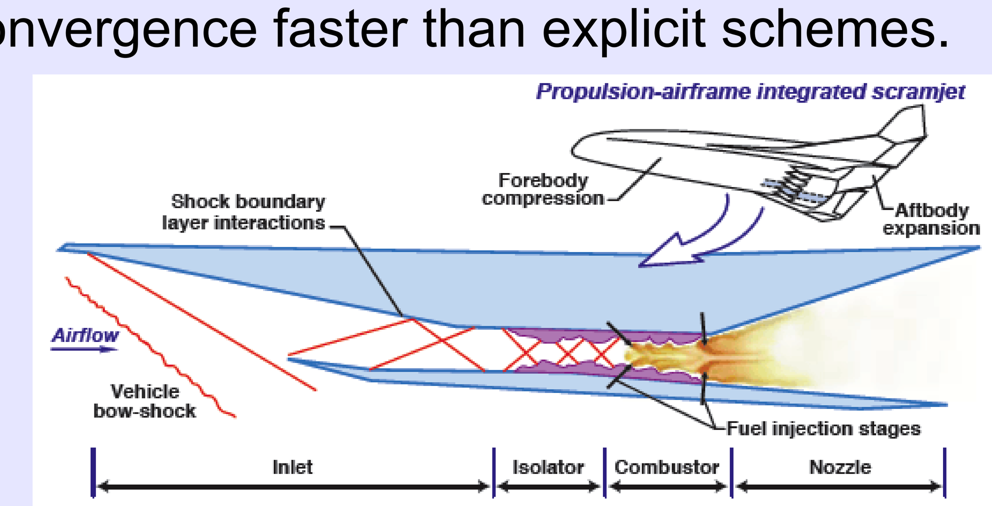
Abstract

An implicit time-marching method has been implemented to solve the 2D Euler equations for supersonic flow on a structured grid using a 2nd order Monotone Upstream-centered Scheme for Conservation Laws (MUSCL). Test simulation results are obtained for the standard Mach 3 wind tunnel setup as described in Woodward & Colella [1].

Introduction

Efficient and accurate simulations of high Mach number flows are needed for the design of supersonic and hypersonic vehicles. During such high speeds, strong discontinuities develop in the flow; as a result, the capability to numerically represent shocks is motivated.

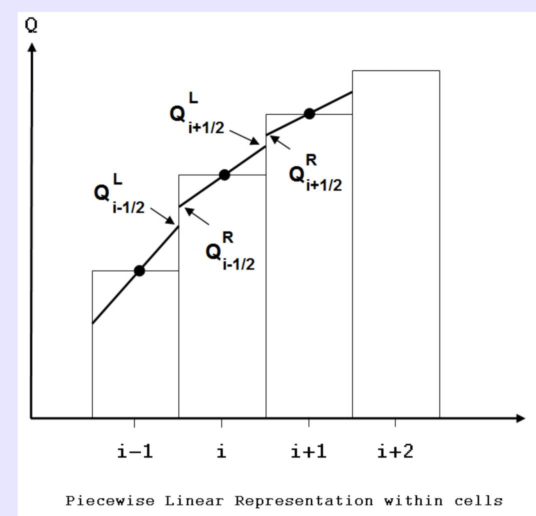
This study focuses on simulating supersonic flow using MUSCL reconstruction and an implicit time-marching scheme. Implicit schemes have the advantage of increased stability at large time steps, thus achieving convergence faster than explicit schemes.



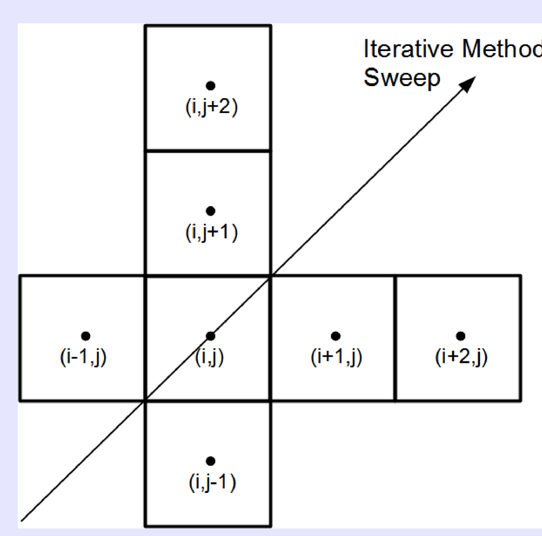
[1] Doctoral Student
[2] Assistant Professor
[3] Professor, Department Chair

Method

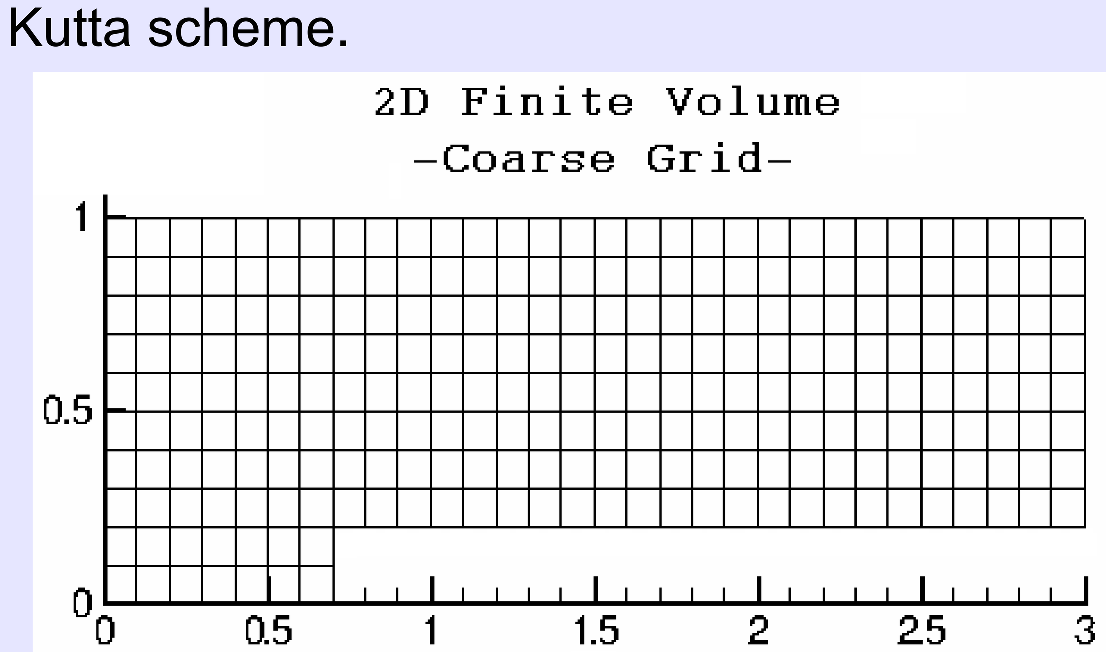
MUSCL schemes reconstruct cell interface left and right states that are used to compute the interface flux for a Riemann solver.



For hyperbolic equations, once the interface fluxes have been computed, a time-integration technique must be employed. This problem is solved implicitly using a formulation for a linearized set of equations, which reduces cost associated with the solution of coupled non-linear equations.



A lower-upper symmetric Gauss-Seidel (LU-SGS) iterative smoother is then used to solve this system of equations. Solutions are also obtained explicitly using a three-stage Runge-Kutta scheme.



Formulation

Governing equations in conservation form:

$$\frac{\partial Q}{\partial t} - R_c(Q) = 0 \qquad R_c(Q) = -\left[\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right]$$

Linearization:

$$\frac{Q_c^{n+1} - Q_c^{n}}{\Delta t} - \left[R_c(Q^{n+1}) - R_c(Q^{n})\right] = R_c(Q^{n})$$

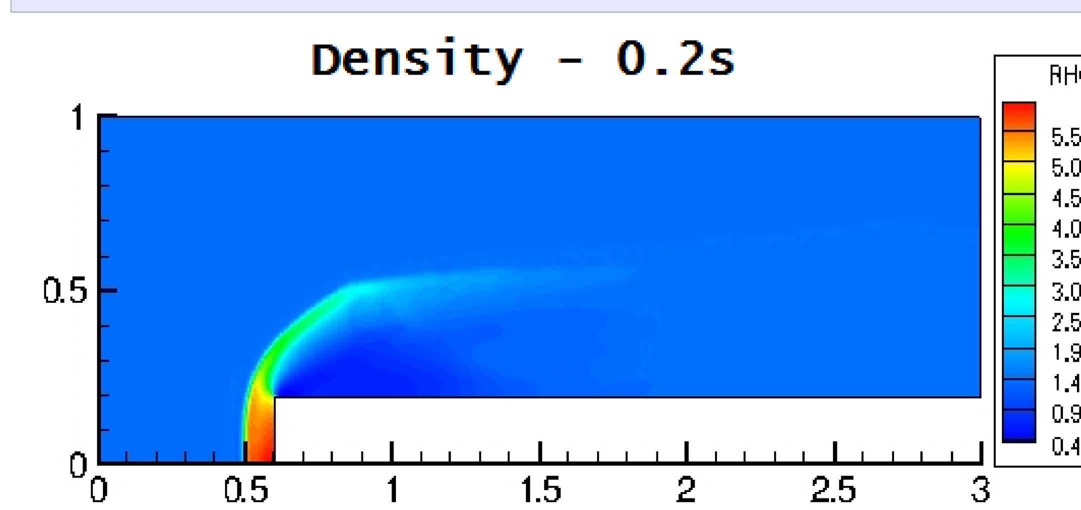
$$R_c(Q^{n+1}) - R_c(Q^{n}) \approx \frac{\partial R_c}{\partial Q_c}\Delta Q_c^{n+1} + \sum \frac{\partial R_c}{\partial Q_{nb}}\Delta Q_{nb}^{n+1}$$

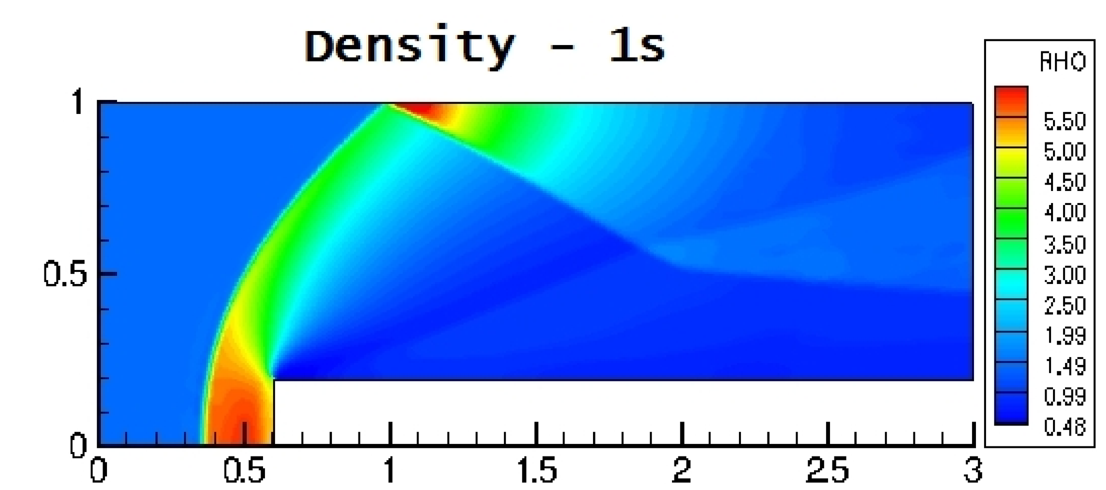
System of linear equations to be solved [2]:

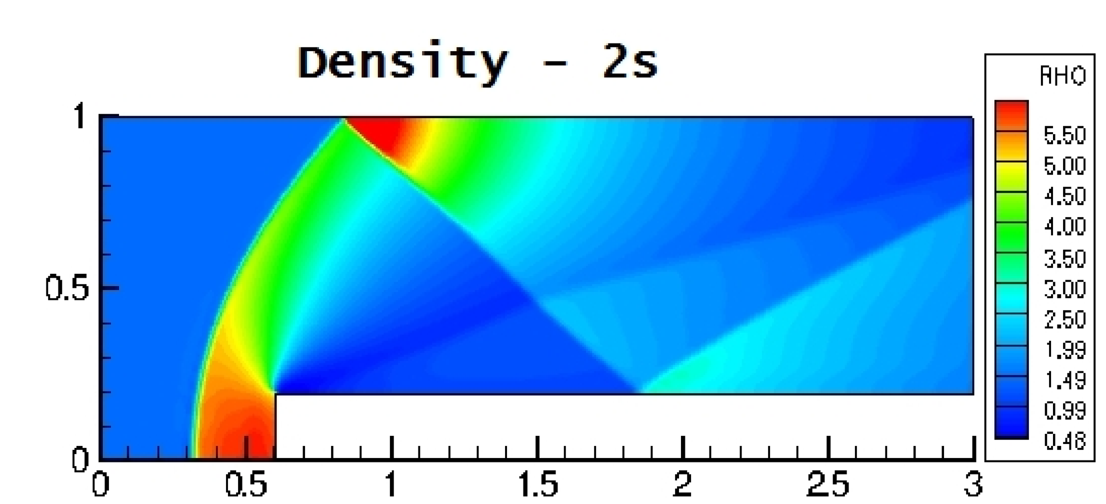
$$\left[\frac{I}{\Delta t} - \frac{\partial R_c}{\partial Q_c}\right]\Delta^2 Q_c^{k+1} = R_c(Q^{star}) - \frac{\Delta Q_c^{star}}{\Delta t}$$

$$Q^{k+1} = Q^{star} + \Delta^2 Q^{k+1}$$

After k iterations, the solution can then be marched to time level $n+1$.

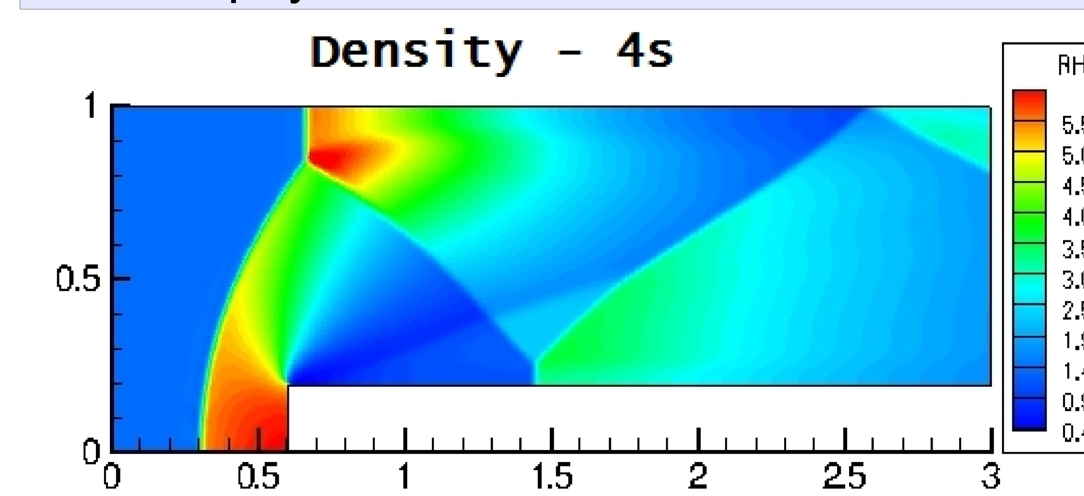


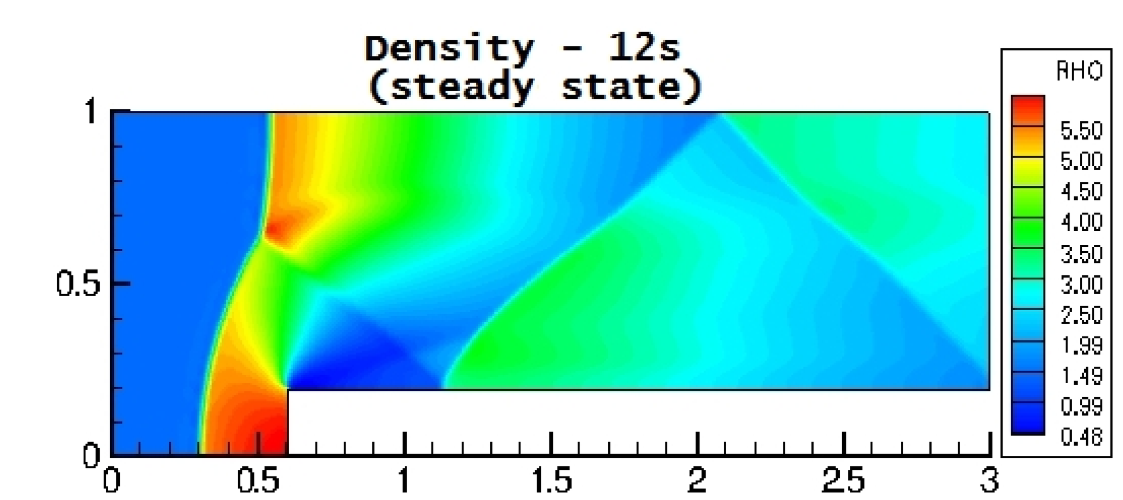




Numerical Simulation

The Mach 3 wind tunnel with a forward facing step is a standard setup that has proven useful in testing numerical methods for supersonic flow. A gamma-law gas is used to initialize the flow field. A steady state solution is reached after 12 physical seconds.





Results

Flow field solutions are obtained for both explicit and implicit techniques (CFL~0.6) over a structured finite volume grid. Convergence times for both time-marching techniques are similar. Future work involves inspection of the MUSCL scheme's flux limiter, particularly in regard to its effect on the gradients of the solution as the iterative smoother operates over the grid. The goal for this work is to have the capability with the implicit method to take sufficiently larger time steps for convergence acceleration over an unstructured mesh containing quadrilateral elements.

Bibliography

[1] P. Woodward, P. Colella, "The numerical simulation of two-dimensional fluid flow with strong shocks," *Journal of Computational Physics*, **54**, 115-173, 1984.

[2] C. Liang, R. Kannan, Z.J. Wang, "A p-multigrid spectral difference method with explicit and implicit smoothers on unstructured triangular grids," *Computers & Fluids*, **38**, 254-265, 2009.