

Question Modification: Since ID=XXXXXXXX9X03 the coefficients are $a = 3$; $b = 0$; $c = 9$

Therefore;

$$y = f(t) = 3t^2 + 9 \text{ in interval of } 0, 1$$

Solution:

$$T = 1 \rightarrow \omega = 2\pi/T = 2\pi$$

$$a_0 = \frac{2}{1} \int_0^1 f(t) dt = 2 \int_0^1 (3t^2 + 9) dt = 2 [t^3 + 9t]_0^1 = 20$$

$$a_n = \frac{2}{1} \int_0^1 (3t^2 + 9) \cos(n\omega t) dt$$

$$b_n = \frac{2}{1} \int_0^1 (3t^2 + 9) \sin(n\omega t) dt$$

Maxima is used to integrate the $f(t) * \cos(2\pi n)$ and $f(t) * \sin(2\pi n)$ to get cosine and sine compliance. The commands are listed below.

```
ratsimp(integrate((3*t^2+9)*cos(w*t*n),t,0,1));
ratsimp(integrate((3*t^2+9)*sin(w*t*n),t,0,1));
```

$$a_n = 2 \times \frac{6 \cdot n \cdot w \cdot \cos(n \cdot w) + (12 \cdot n^2 \cdot w^2 - 6) \cdot \sin(n \cdot w)}{n^3 \cdot w^3}$$

$$b_n = 2 \times \frac{-6 + 9 \cdot n^2 \cdot w^2 + (6 - 12 \cdot n^2 \cdot w^2) \cdot \cos(n \cdot w) + 6 \cdot n \cdot w \cdot \sin(n \cdot w)}{n^3 \cdot w^3}$$

Therefore;

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{100} a_n \cos(\omega n t) + b_n \sin(\omega n t)$$

To simplify the output; π is set to 3.14 therefore $\omega = 6.28$ since $\omega = 2\pi$

$$y(t) = \frac{20}{2} + \sum_{n=1}^{100} a_n \cos(6.28 n t) + b_n \sin(6.28 n t)$$

```
1 sum((2*6*n*(2*3.14)*cos(n*(2*3.14))+(12*n^2*(2*3.14)^2-6)*sin(n*(2*3.14)))/(n^3*(2*3.14)^3)*cos
((2*3.14)*n*t)+2*(-6+9*n^2*(2*3.14)^2+(6-12*n^2*(2*3.14)^2)*cos(n*(2*3.14))+6*n*(2*3.14)*sin(n
*(2*3.14)))/(n^3*(2*3.14)^3)*sin((2*3.14)*t*n), n, 1, 100), simpsum;
```

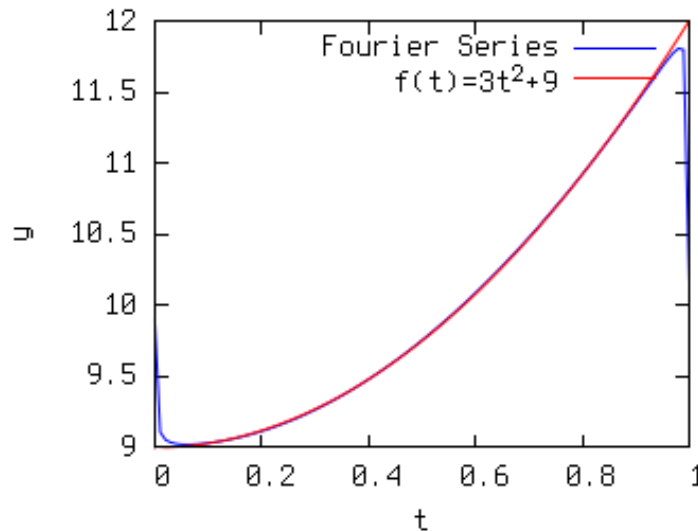


Figure 1: $f(t)$ vs. Fourier Series

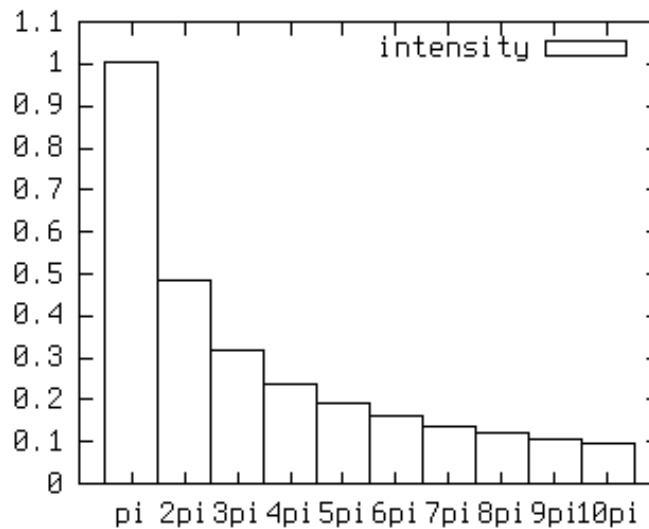


Figure 2: Intensity vs. frequency

Following Octave code is used to determine intensity of each sine/cosine component for the frequency from π to 10π

```

1 function ycos=an(n)
2     ycos=2*(6*n*2*pi*cos(n*2*pi)+(12*n^2*(2*pi)^2-6)*sin(n*2*pi))/(n^3*(2*pi)^3);
3
4 endfunction
5
6 function ysin=bn(n)
7     ysin=2*(-6+9*n^2*(2*pi)^2+(6-12*n^2*(2*pi)^2)*cos(n*(2*pi))+6*n*(2*pi)*...
8         sin(n*(2*pi)))/(n^3*(2*pi)^3);
9
10 endfunction
11
12
13 for i=1:1:10
14     m(i,1)=sqrt(an(i)^2+bn(i)^2);
15 endfor
16
17
18 f1 = fopen('~\Yukseklisans/muzik/hw3_m.dat','w');
19 for i = 1:length(m)
20     fprintf(f1,'%d\n',m(i));
21 end
22 fclose(f1);

```