Basics of Neural Network Programming

Logistic Regression

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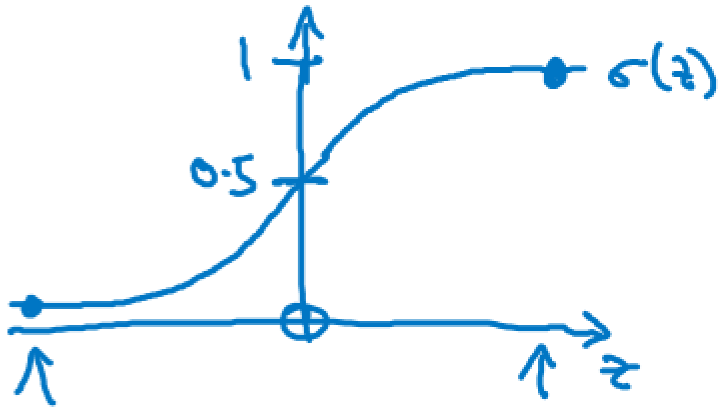
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\quad 0 \le \hat{y} \le 1$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1, \quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\underbrace{\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix}}$$

$\theta_0 \} b \leftarrow$

$\theta_1, \dots, \theta_{n_x} \} w \leftarrow$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Logistic Regression cost function

Logistic Regression cost function

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

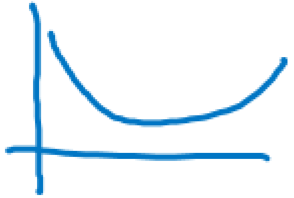
$z^{(i)} = w^T x^{(i)} + b$

Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$ $y^{(i)}$ $z^{(i)}$ i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$

$$\boxed{\mathcal{L}(\hat{y}, y)} = -\left(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y})\right)$$



If $y=1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← Want $\log \hat{y}$ large, want $\hat{y}$ large

If $y=0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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