UNIVERSITY OF HELSINKI
DEPARTMENT OF PHYSICS

BASICS OF MONTE CARLO SIMULATIONS

Exercise 2

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Note. Please use the Makefile in order to compile the programs. See all images in the figures folder.

Problem 1

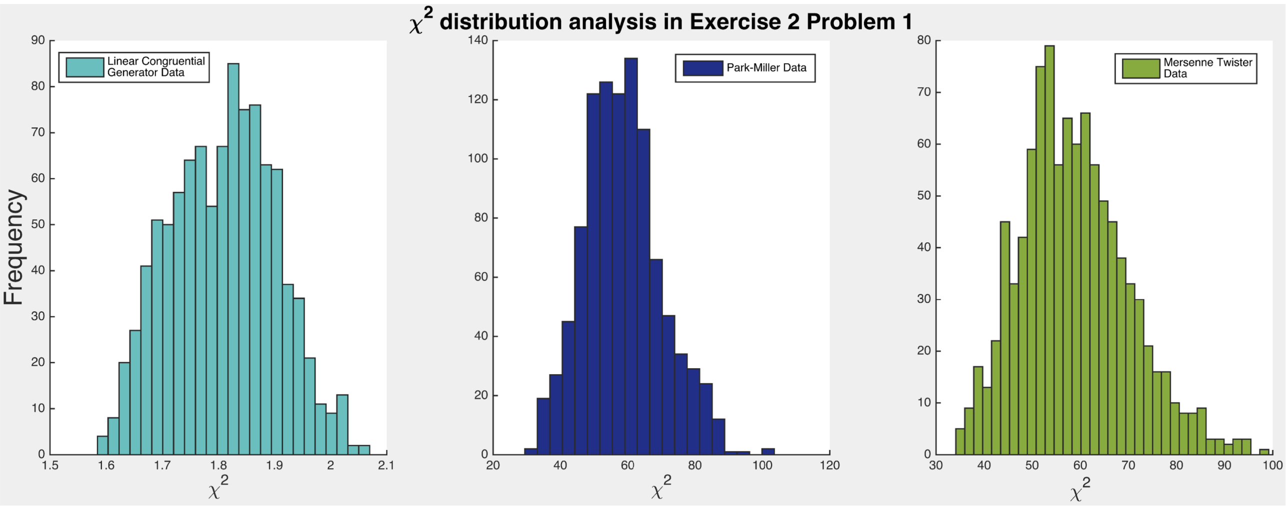

Figure 1: χ^2 histograms.

LCG

See program's output below for the LCG case:

```
+------------------------------------------------------------+
| NUMBER OF RANDOM NUM:                                      |
|                              1000000                       |
| NUMBER OF BINS:                                            |
|                              60                            |
| DEGREES OF FREEDOM:                                        |
|                              59                            |
| EXPECTED VALUE:                                            |
```

```
|                                     16666.666                    |
|    CHI-SQUARED MEAN:                                             |
|                                          1.809                    |
|    CHI-SQUARED MEDIAN:                                           |
|                                          1.815                    |
|    LOWER ONE-SIDED:                                              |
|                                  <          1.658                 |
|    UPPER ONE-SIDED:                                              |
|                                  >          1.959                 |
+------------------------------------------------------------------+
```

We can see clearly that the LCG fails the χ^2 since even the mean, or the median, values are far from the degree of freedom (in our case 60-1).

Park-Miller

See program's output below for the PMG case:

```
+------------------------------------------------------------------+
|    NUMBER OF RANDOM NUM:                                         |
|                                  1000000                         |
|    NUMBER OF BINS:                                               |
|                                  60                              |
|    DEGREES OF FREEDOM:                                           |
|                                  59                              |
|    EXPECTED VALUE:                                               |
|                                     16666.666                    |
|    CHI-SQUARED MEAN:                                             |
|                                         58.533                    |
|    CHI-SQUARED MEDIAN:                                           |
|                                         57.973                    |
|    LOWER ONE-SIDED:                                              |
|                                  <        40.673                 |
|    UPPER ONE-SIDED:                                              |
|                                  >        80.249                 |
+------------------------------------------------------------------+
```

Now, we can see clearly that the PMG has much better χ^2 values, since the mean, or the median, values are close to the degree of freedom. However, its median is not k-2/3, where k is the degree of freedom. The difference between the median and its expected value (k-2/3), however, is negligible depending on the desired precision.

In order to do the one-sided tests with $\alpha = 0.05$, we need to compare our lower one-sided value with 42.339, and our upper one-sided value with 77.931. It is unfortunate but our lower one-sided value (40.673) is less than the critical value, and our upper one-sided value (80.249) is greater than the critical value. Therefore, according to the test rules, this random number generator fails both one-sided tests.

Mersenne twister

See program's output below for the MTG case:

```
+------------------------------------------------------------------+
|    NUMBER OF RANDOM NUM:                                         |
|                                  1000000                         |
|    NUMBER OF BINS:                                               |
|                                  60                              |
|    DEGREES OF FREEDOM:                                           |
|                                  59                              |
|    EXPECTED VALUE:                                               |
|                                     16666.666                    |
|    CHI-SQUARED MEAN:                                             |
|                                         58.684                    |
|    CHI-SQUARED MEDIAN:                                           |
|                                         57.784                    |
|    LOWER ONE-SIDED:                                              |
```

	UPPER ONE–SIDED :	<	41.972
		>	78.710

As in the previous example, we can see clearly that the MTG has much better χ^2 values, since the mean, or the median, values are close to the degree of freedom. However, its median is not k-2/3, where k is the degree of freedom. The difference between the median and its expected value (k-2/3), however, is negligible depending on the desired precision.

In order to do the one-sided tests with $\alpha = 0.05$, we need to compare our lower one-sided value with 42.339, and our upper one-sided value with 77.931. It is unfortunate but our lower one-sided value (41.972) is less than the critical value, and our upper one-sided value (78.710) is greater than the critical value. Therefore, according to the test rules, this random number generator fails both one-sided tests.

Problem 2

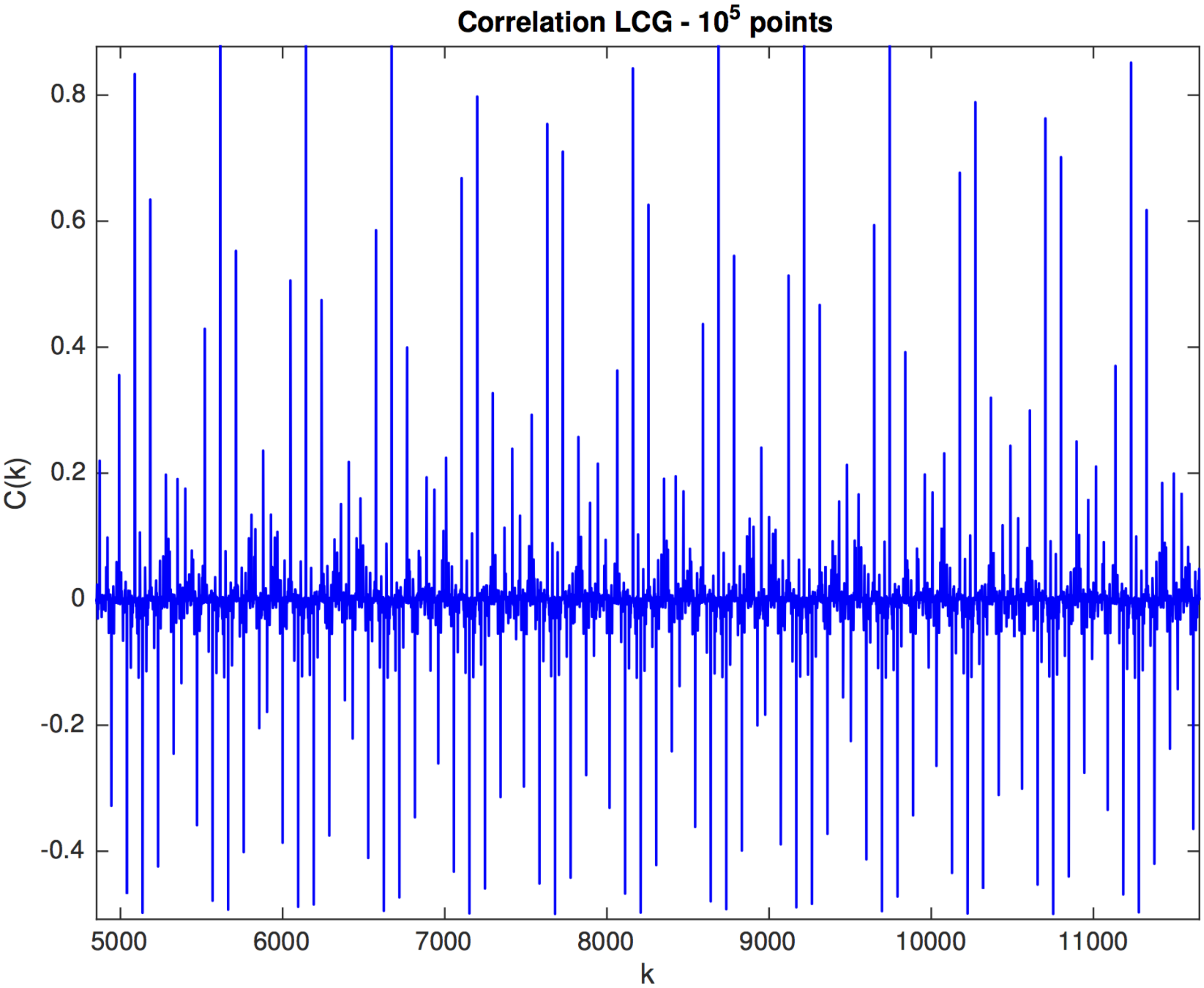

Figure 2: LCG correlation test. We can clearly see sudden peaks.

According to the theory, the correlation values C_k fluctuates around zero with sudden peaks if the generator fails the test. Therefore, we can conclude that our LCG fails the test (you can see the peaks).

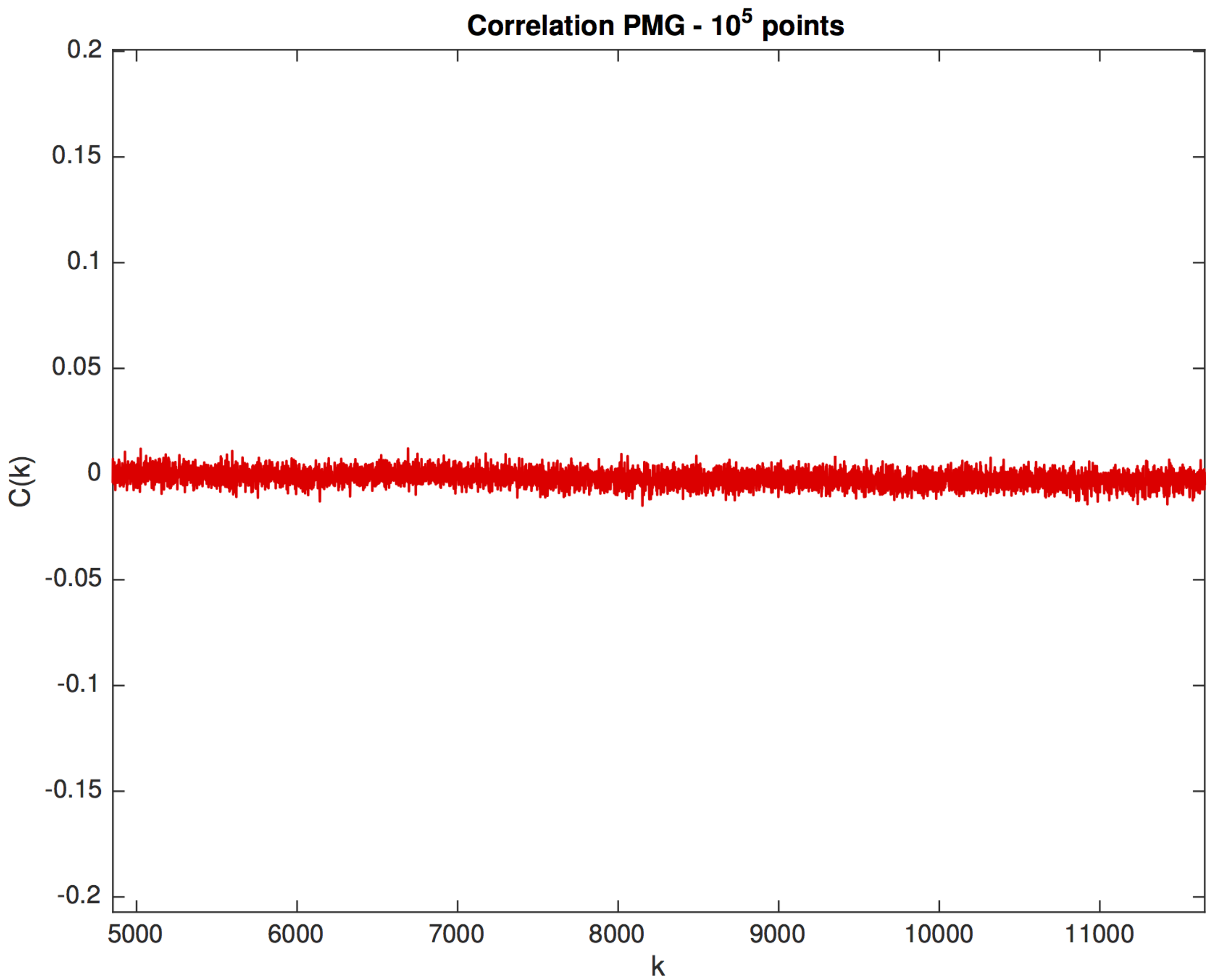

Figure 3: PMG correlation test.

According to the theory, the correlation values C_k fluctuates around zero with sudden peaks if the generator fails the test. Therefore, we can conclude that our Park-Miller generator passes the test (you can not see any peaks).

After updating our LCG with the 32-element table trick, the correlation test becomes much better, allowing our generator to pass the test (see figure 4).

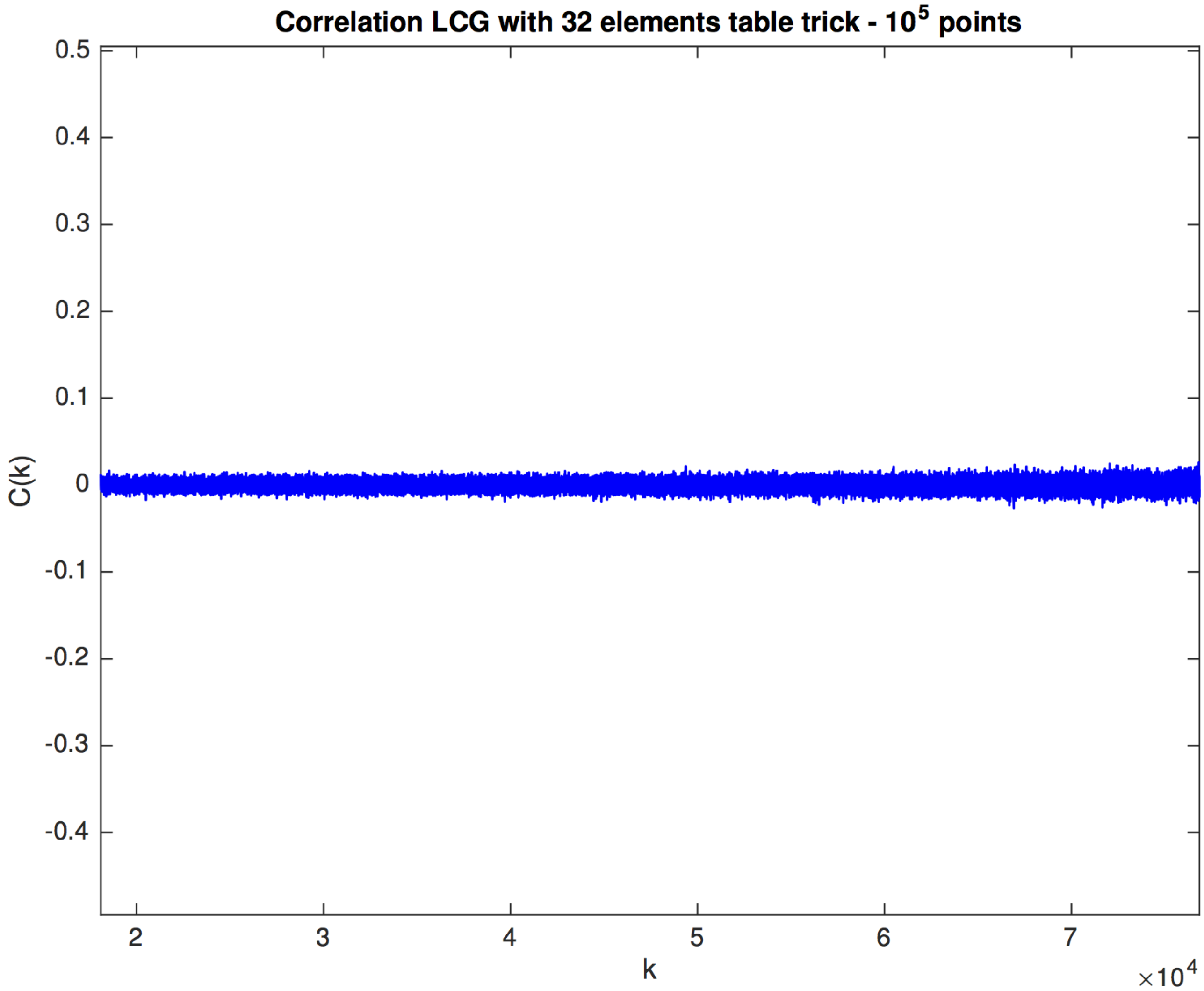

Figure 4: Fixed LCG correlation test.

Problem 3

Hit-and-miss method

See histograms below. For b=10 we have a ratio of hit/miss of 99961/1000000.

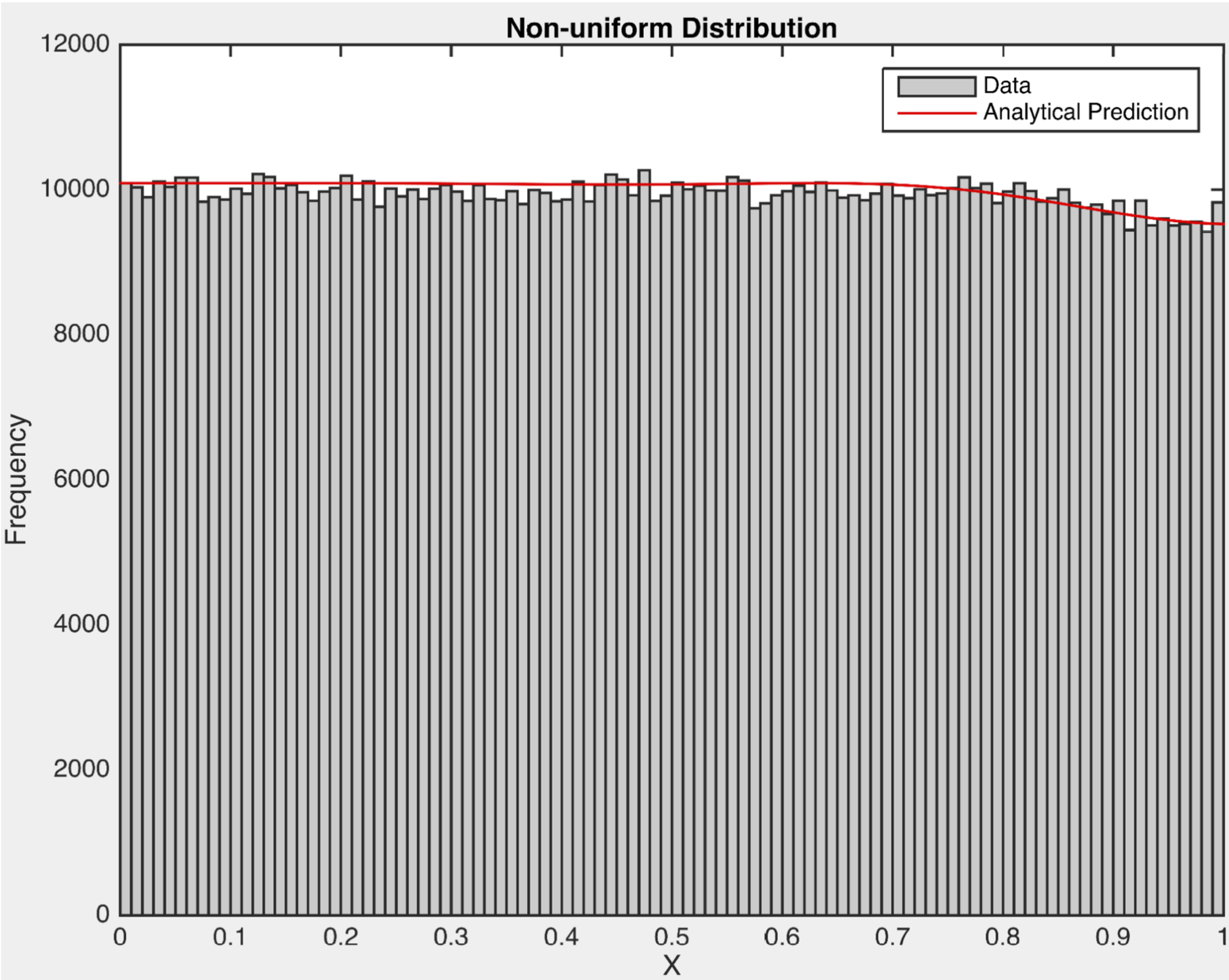

Figure 5: Hit-and-miss method: b=1.

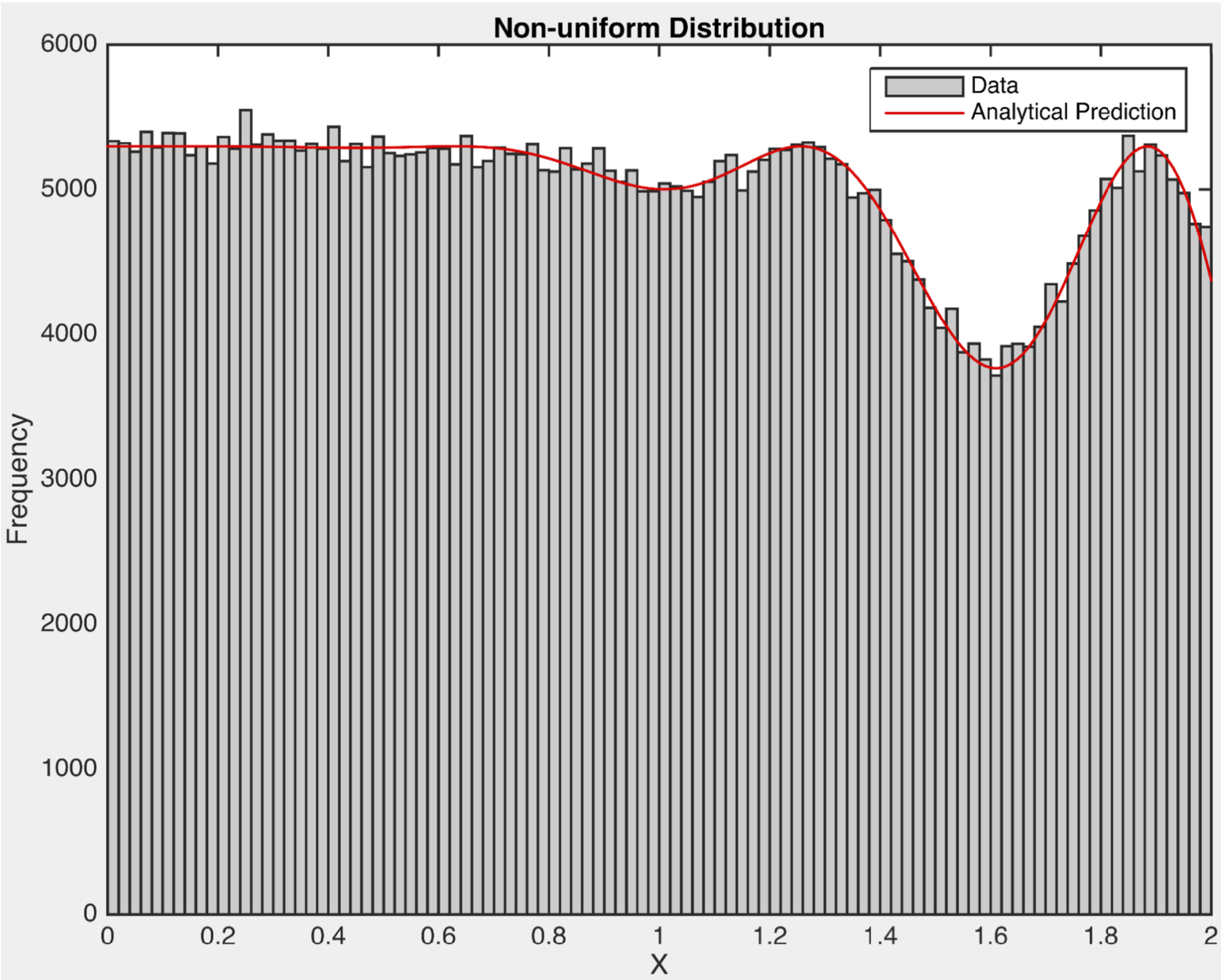

Figure 6: Hit-and-miss method: b=2.

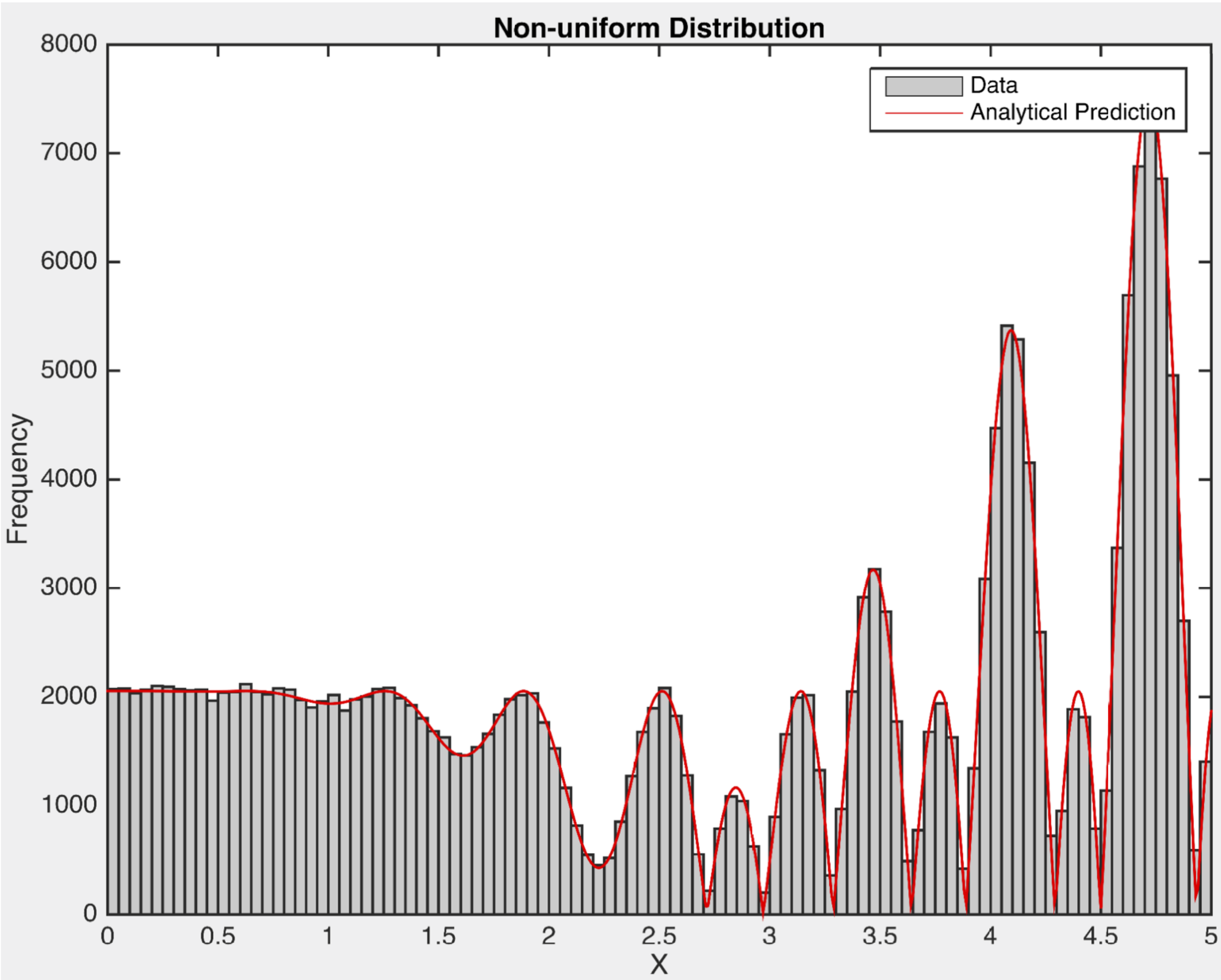

Figure 7: Hit-and-miss method: b=5.

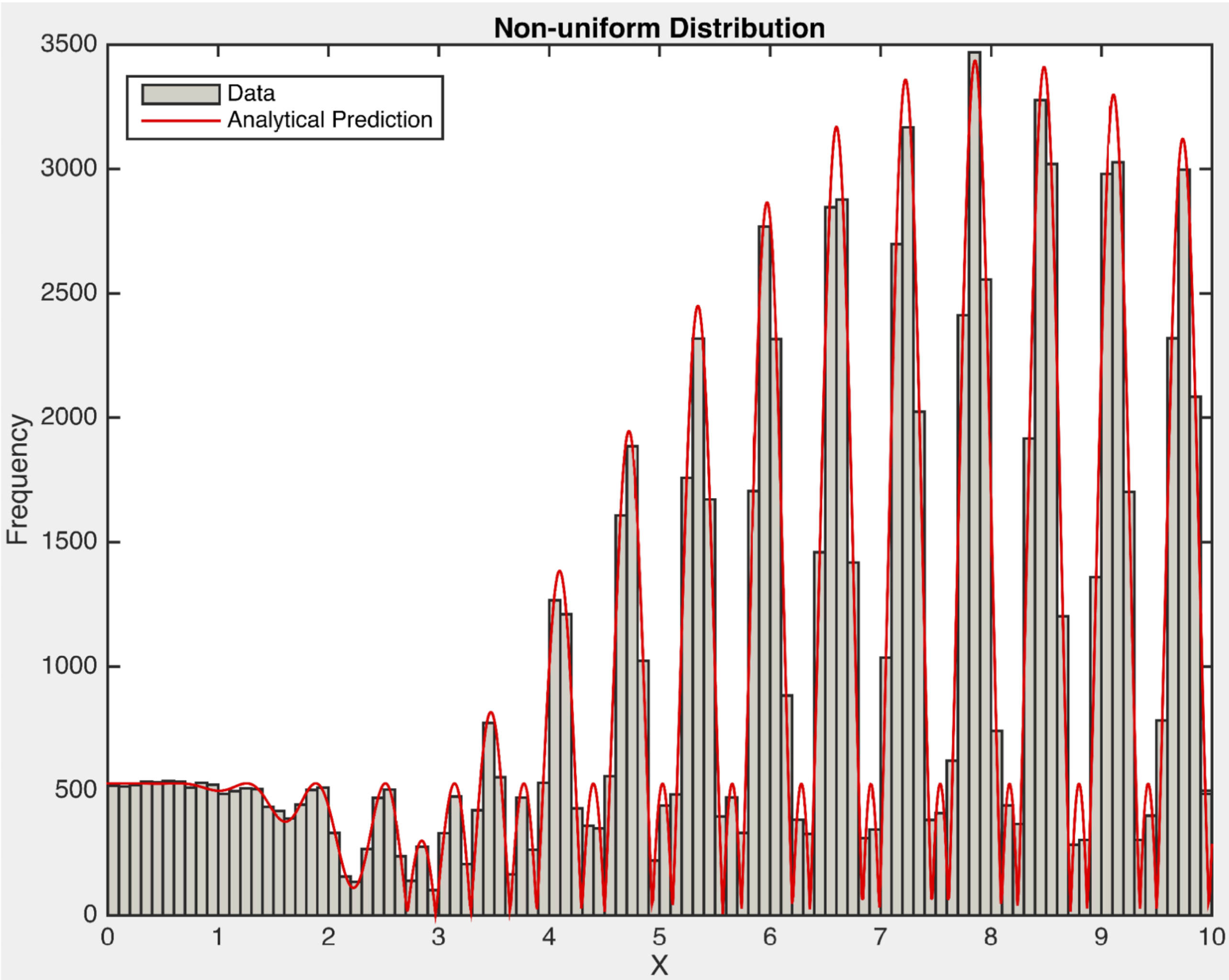

Figure 8: Hit-and-miss method: b=10.

Analytical-rejection method

See the p.d.f. used in this method ($f(x)$ and $g(x)$) in figure 9.

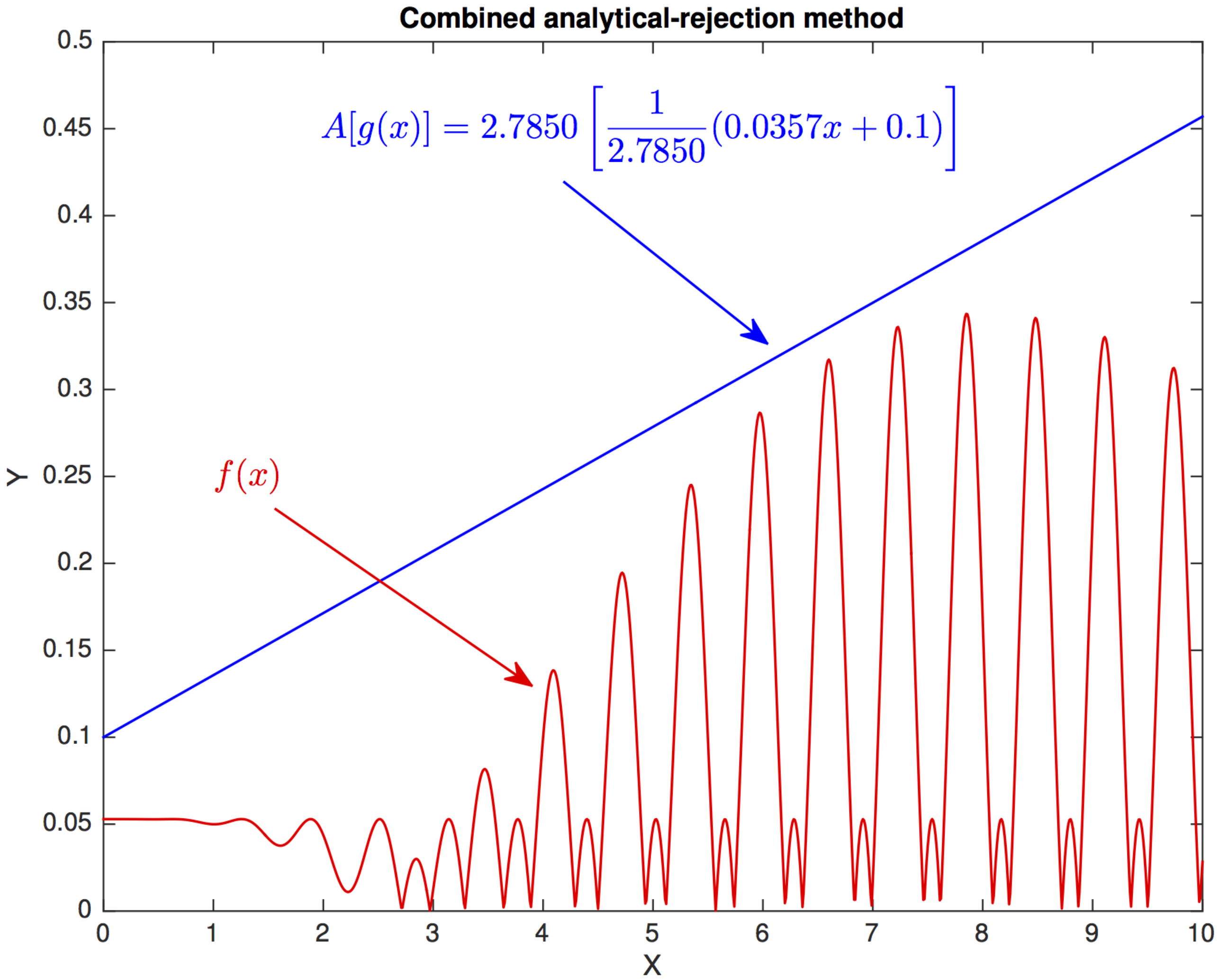

Figure 9: Analytical-rejection method functions.

See the histogram for b=10 in figure 10. For b=10 we have a ratio of hit/miss of 359354/1000000. Note that this ratio is much bigger than the one seen in the hit-and-miss method.

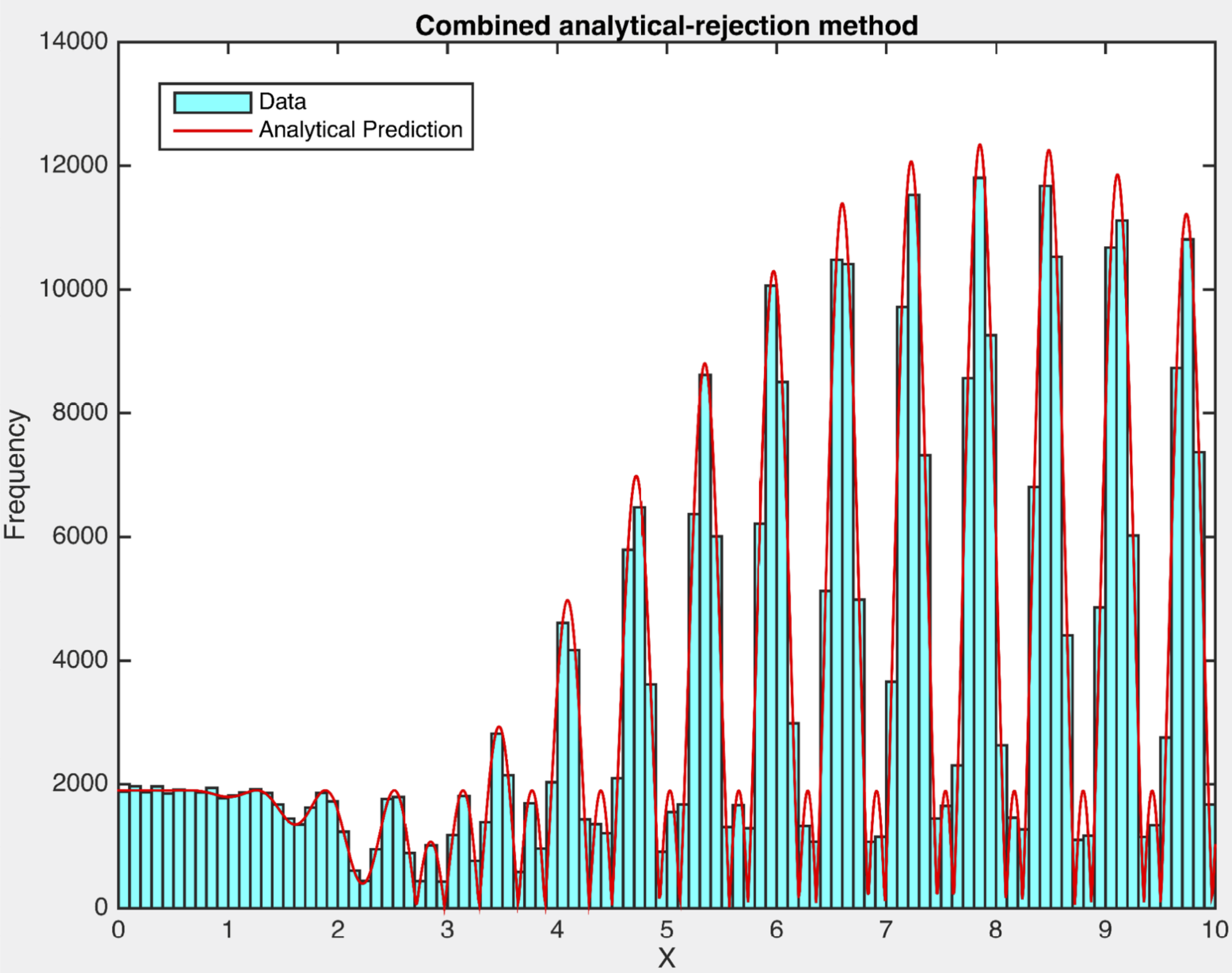

Figure 10: Analytical-rejection method histogram for b=10.

Problem 4

See the distribution created from the RNG using the irrational number files in figure 11. It is clearly a uniform distribution.

See the Random Walker results in figure 12.

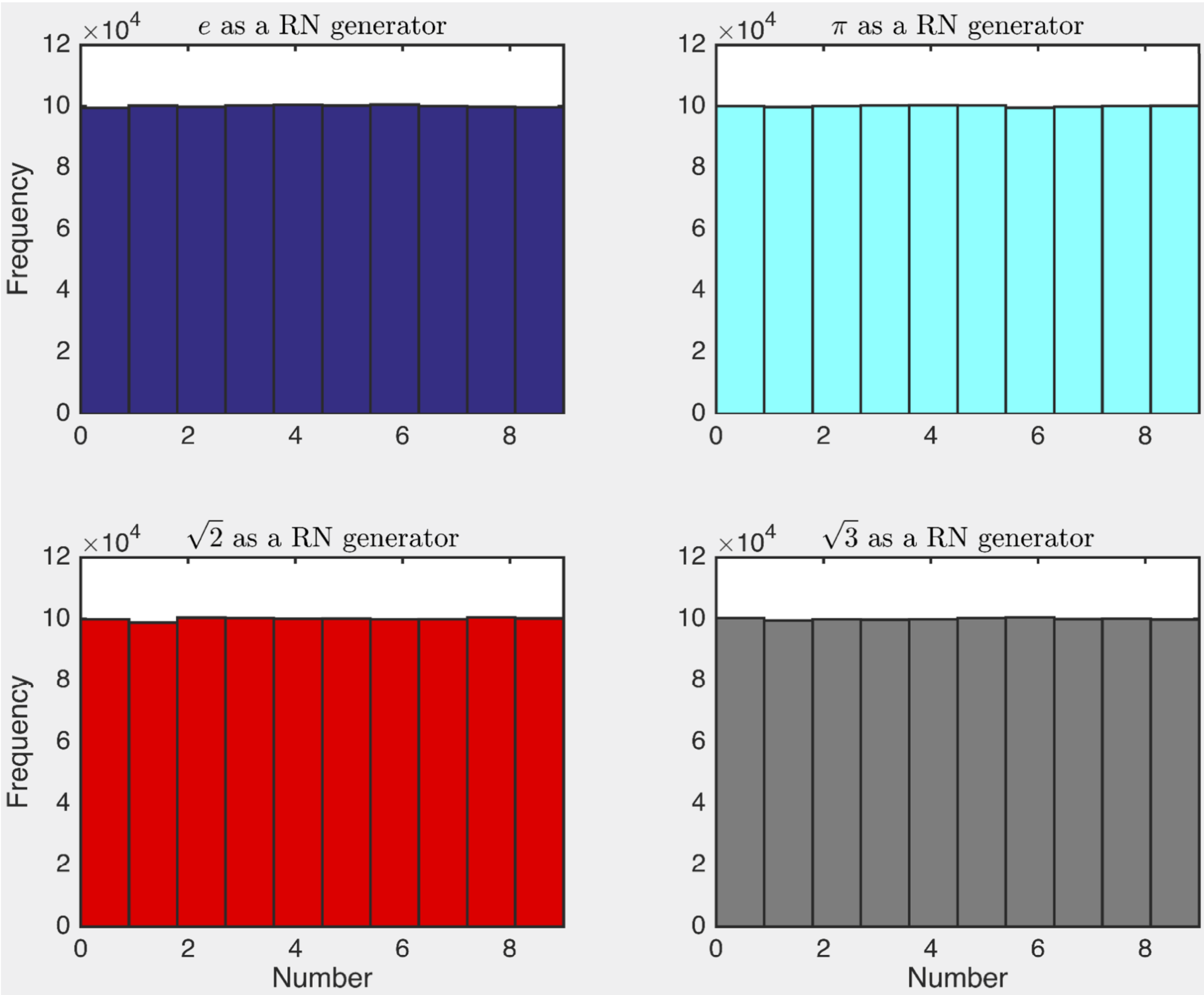

Figure 11: Uniform Distribuition.

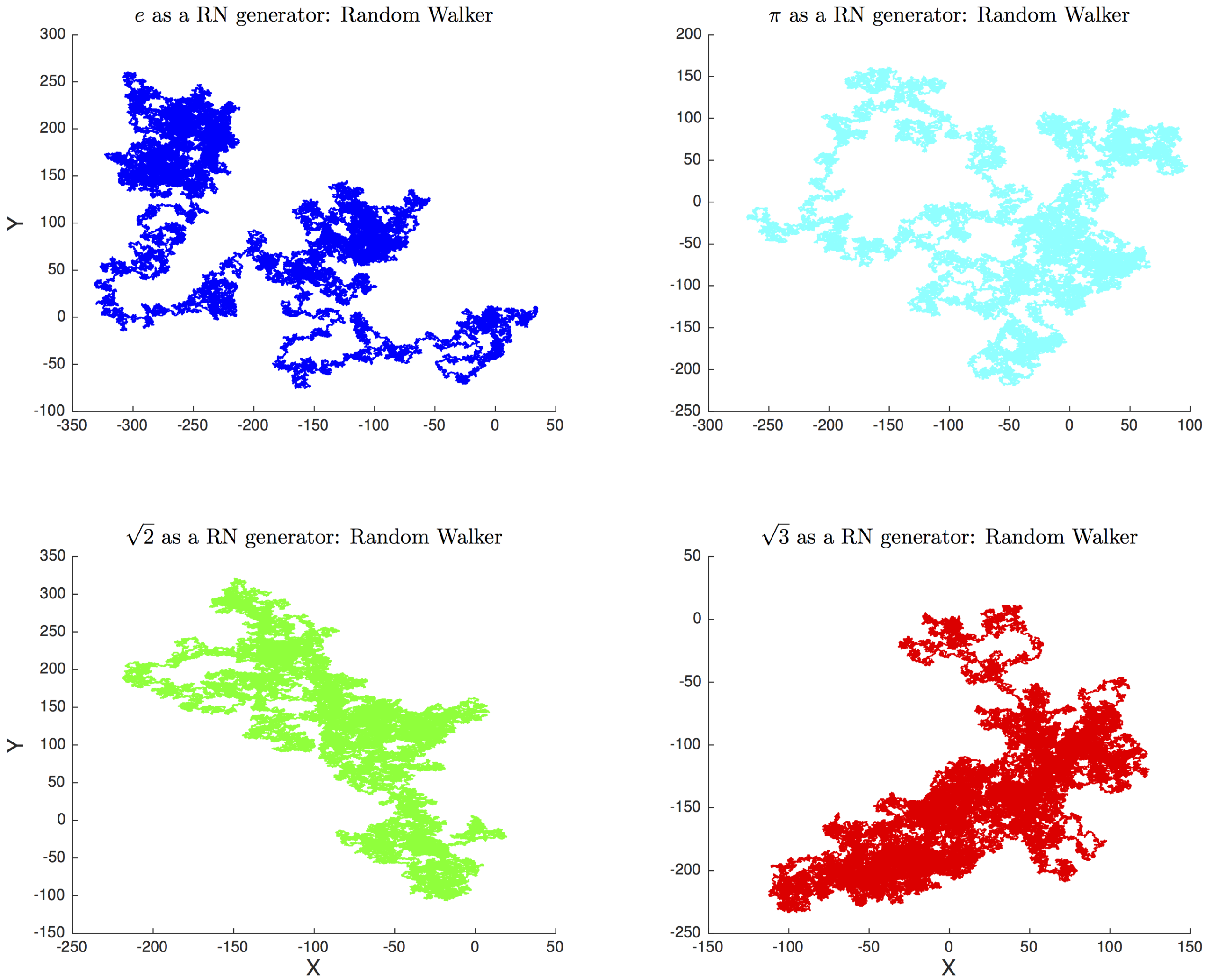

Figure 12: Random walker paths.