# Tug of War

## The Heterogeneous Effects of Outbidding between Terrorist Groups

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October 2022

# What is outbidding?

Explanation for violence in civil conflict with multiple anti-gov groups

1. Groups use violence to increase public support at other groups' expense
2. Terrorism signals resolve to a population that is uncertain about which group is strongest

Unique story in literature: enemy tangentially related to interaction

- Kydd & Walter 2006: "it provides a potential explanation for terrorist attacks that continue even when they seem unable to produce any real results"

Popularity entails instrumental benefits: recruitment, financing, information

- Crenshaw 1981; Acosta 2014; Fortna 2015

# The outbidding mechanism

Group competition should generally exasperate violence (Bloom 2004)

- Eg: Rise of Hamas encouraging Fatah to use violence

Conrad and Greene (2015) summarize logic:

> *Since competition directly and indirectly threatens the resource base necessary to sustain the organization and ensure its effectiveness, it follows that terrorist organizations should make tactical choices in an effort to increase their share of resources within a competitive environment*

Mixed empirical evidence

- No effects: Findley & Young 2015
- Mixed effects: Nemeth 2014; Conrad & Greene 2015
- Positive effects: Chenoweth 2010; Wood & Kathman 2015

# How to study outbidding?

Mixed results are not surprising: Outbidding is consistent with both encouragement *but also deterrence*

- If a side becomes more effective, others may fight harder to keep up
- If a side becomes more effective, others may give up, creating a feedback loop

If deterrence exists, previous (reduced form) approaches are ambiguous:

- Focus on overall average effects, but countervailing/asymmetric effects may exist
- As such, *any* reduced form output is consistent with outbidding (non-falsifiable)

Measurement challenges

- Most proxy competitive incentives with number of groups
- Hard to measure popularity with cross-sectional data

# Our approach

In this paper, we . . .

1. Model outbidding as a dynamic contest between two groups
   - Competitive incentives: value of popularity, effectiveness of terrorism at boosting public opinion, cost of attacking
2. Measure monthly relative popularity of Fatah and Hamas, 1994-2018
3. Fit model to data on popularity and terrorist attacks (GTD)
4. Use the fitted model to quantify how much competition affects violence

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- For Hamas we find heterogeneous effects. During the Oslo era, competition *decreases* Hamas's propensity for violence, but encourages after.

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Asymmetric effects of competitive incentives:

- Increasing Hamas's incentives, increases violence by both groups
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Results emerge from asymmetric contests with a Fatah advantage

- Fatah is more effective at shifting public opinion, although Hamas has smaller attack costs

# Contributions

## Outbidding literature

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## Policy

- Many recent presidential administrations provided support to build Fatah's capabilities
- Our model suggests countervailing effects: Lowers the costs of violence for Fatah (pro-peace), but may make Fatah value public support less (anti-peace)
- More work required to determine how U.S. and Israeli actions directly affect specific competitive incentives

# Setup

Actors: Fatah and Hamas ($i = F, H$)

Time: $t = 1, 2, \ldots$

Relative popularity of the groups: $s^t \in \mathcal{S} = \{s_1, \ldots, s_K\} \subset \mathbb{R}$

- Public state variable observed in period $t$
- $s_k < s_l$ denotes Hamas is more popular relative to Fatah in state $s_k$ than $s_l$
- equally spaced: $s_k - s_{k-1} = 2d$

Private State Variable: $\varepsilon_i^t \in \mathbb{R}^2$

- Private information, action-specific shocks
- $\varepsilon_i^t(a_i)$ indexes shock to action $a_i$ for player $i$ in period $t$

# Strategic interaction

Timing in period $t$:

1. $i$ observes relative popularity level $s^t$ and private information $\varepsilon_i^t$
2. Simultaneously choose to commit terrorism ($a_i^t = 1$) or not ($a_i^t = 0$)
3. Payoffs are accrued
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- Hypothesis: $\beta_H < 0$, $\beta_F > 0$, and $\kappa_i < 0$

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- Private information $\varepsilon_i^{t+1}$ is drawn i.i.d. T1EV

# Popularity follows AR-1 process

Tomorrow's $s^{t+1}$ drawn from a discretized normal with mean:

$$\mu[a^t, s^t; \gamma] = \gamma_0 + \gamma_1 \cdot s^t + \underbrace{(\gamma_{H,1} + \gamma_{H,2} \cdot s^t)}_{\text{effect of } H\text{'s attacks}} \cdot a_H^t + \underbrace{(\gamma_{F,1} + \gamma_{F,2} \cdot s^t)}_{\text{effect of } F\text{'s attacks}} \cdot a_F^t$$

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- $\gamma_{i,1} + \gamma_{i,2} \cdot s^t$ is effectiveness at using terrorism to gain support
- Hypothesis: $\gamma_{H,1} < 0$ and $\gamma_{F,1} > 0$
- No expectations over how states affect effectiveness, $\gamma_{i,2}$

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Transition probabilities take the form:

$$f(s'; a^t, s^t, \gamma) = \begin{cases} \Phi\left(\frac{s_1 + d - \mu[a^t, s^t; \gamma]}{\sigma}\right) & s' = s_1 \\ \Phi\left(\frac{s' + d - \mu[a^t, s^t; \gamma]}{\sigma}\right) - \Phi\left(\frac{s' - d - \mu[a^t, s^t; \gamma]}{\sigma}\right) & s' \notin \{s_1, s_K\} \\ 1 - \Phi\left(\frac{s_K - d - \mu[a^t, s^t; \gamma]}{\sigma}\right) & s' = s_K. \end{cases}$$

# Equilibria

Maximize expected utility with common discount factor $\delta$

- $\delta$ not uniquely identified (Abbring and Daljord 2020)
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Aguirregabiria and Mira (2007) characterize Markov Perfect equilibria

- Net-of-shock expected utility: $v_i(a_i, s)$
- $v$ is an equilibrium if and only if $\mathcal{V}(v) = v$
- Eq. Choice probabilities: $P(a_i, s; v) = \frac{\exp\{v_i(a_i,s)\}}{\exp\{v_i(0,s)\} + \exp\{v_i(1,s)\}}$
- $\mathcal{V}$ depends on $\beta$, $\kappa$, $\gamma$, $\sigma$, estimated below

More info Numerical Example

# Data

Treat a period as a month and need measures of actions and states

Monthly indicator of attacks using Global Terrorist Database

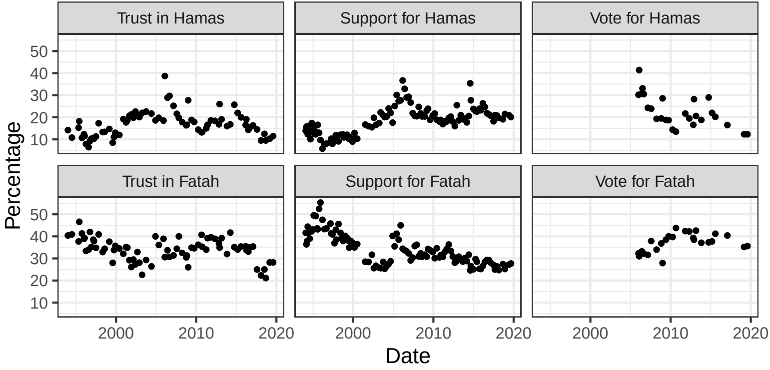
- Hamas: Islamic Resistance Movement, *Al-Qassam* Brigades
- Fatah: *Al-Fatah*, Palestinian Liberation Organization

Public opinion polls from JMCC and PSR

- Rolling cross sections; access to polling totals, but not individual data
- Face-to-face interviews, representative sample of West Bank and Gaza Strip
- Appear in Bloom 2004; Clauset et al. 2010; Jaeger et al. 2012
- 1994–2018 gives us 300 months

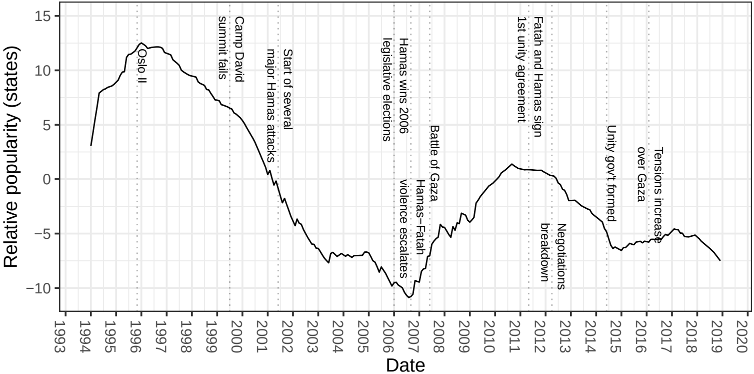
▸ Example

# Six public opinion indicators




▸ Correlations

# Our measure of relative popularity




- Dynamic factor analysis backs out a continuous state variable $\tilde{s}^t$
- All indicators load onto $\tilde{s}^t$ in the expected direction
- Timeline suggests that $\tilde{s}^t$ is likely measuring relative popularity

More info

# Two-step estimation

Step 1: Estimate effectiveness ($\gamma_{i,1}$) and transition probs

- AR-1 models with continuous $\tilde{s}^{t+1}$ as the dependent variable:

$$\tilde{s}^{t+1} = \gamma_0 + \gamma_1 \tilde{s}^t + \gamma_{H1} a_H^t + \gamma_{H2}(\tilde{s}^t \times a_H^t) + \gamma_{F1} a_F^t + \gamma_{F2}(\tilde{s}^t \times a_F^t)$$

- Unit root problems: point estimates are super-consistent but SEs are incorrect
- Use error correction methods for inference/robustness

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Step 2: Estimate value of public support and cost of terrorism ($\beta_i, \kappa_i$) and equilibrium expected utilities $v$

- Discretize state space and use $\hat{\gamma}$ and $\hat{\sigma}$ from step 1
- Use CMLE a la Su and Judd (2012)
- Murphy and Topel (1985) SE correction

More info

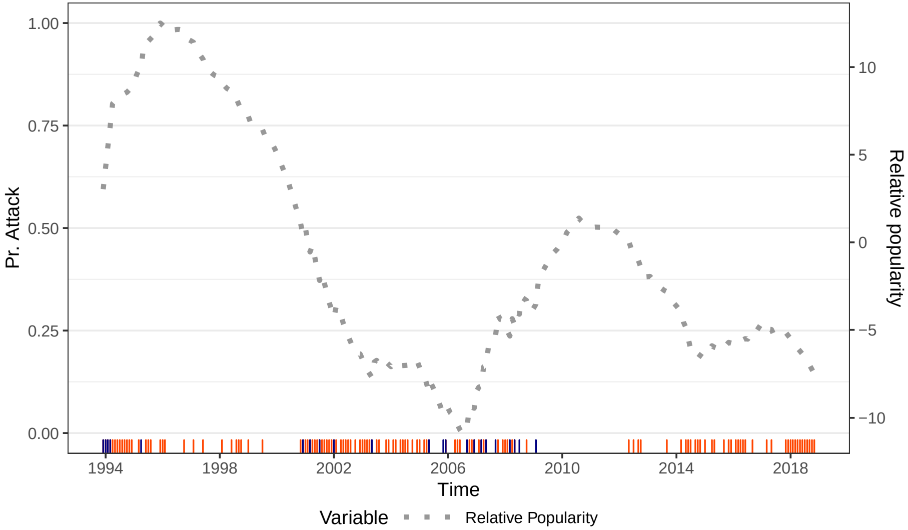
# Effectiveness

| *Model:* | AR(1) | ECM |
|---|---|---|
| *Dependent variable:* | $\tilde{s}^{t+1}$ | $\Delta\ \tilde{s}^{t+1}$ |
| Hamas attacks ($\gamma_{H,1}$) | $-0.21$ | $-0.21$ |
| | | (0.04) |
| Fatah attacks ($\gamma_{F,1}$) | 1.12 | 1.03 |
| | | (0.05) |
| Hamas attacks $\times$ Lag states ($\gamma_{H,2}$) | 0.01 | 0.002 |
| | | (0.01) |
| Fatah attacks $\times$ Lag states ($\gamma_{F,2}$) | 0.03 | 0.01 |
| | | (0.01) |
| Lag state ($\gamma_1$) | 1.00 | |
| $\Delta$ Lag state | | 0.33 |
| | | (0.04) |
| Constant ($\gamma_0$) | $-0.02$ | $-0.01$ |
| | | (0.02) |
| $\sigma$ | 0.22 | 0.18 |

# Values and costs

| | | Estimate | Std. Error |
|---|---|---|---|
| Value of public support , $\beta_i$ | Hamas | $-0.007$ | 0.005 |
| | Fatah | 0.0005 | 0.0004 |
| Attack costs, $\kappa_i$ | Hamas | $-0.95$ | 0.28 |
| | Fatah | $-2.45$ | 0.39 |
| Log-Likelihood | | -278.09 | |

# Estimated EQ choice probabilities



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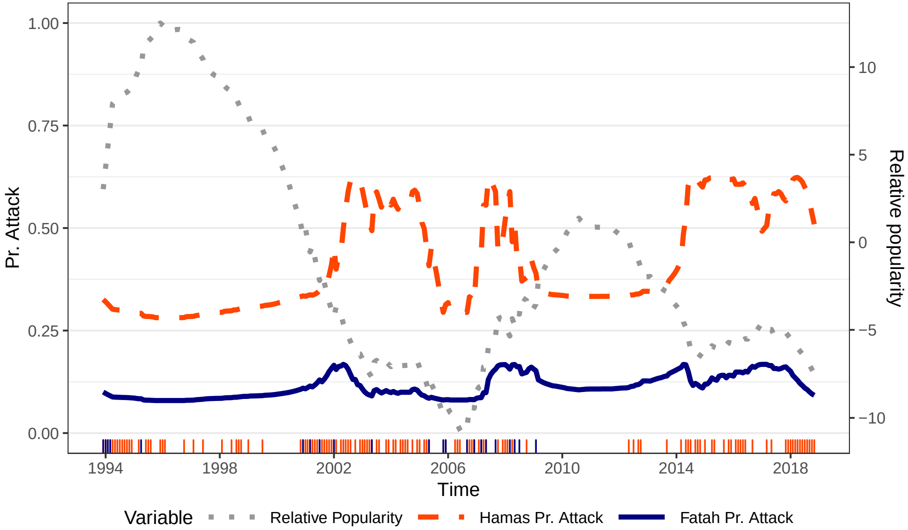


# Quantifying Effects of Competition

*Single-agent comparison:* How do $i$'s equilibrium attack probabilities compare to $i$'s single agent problem?

- Quantifies the total effect of competition on violence

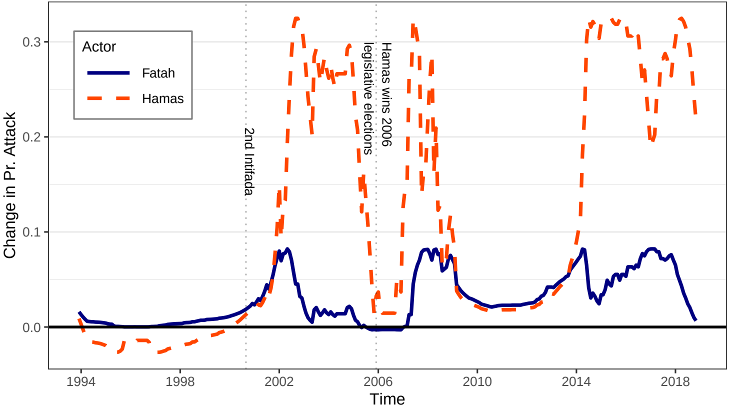
*Comparative statics:* How do equilibrium attack probabilities change with different competitive incentives? (Deferred for time)

Direction of effects are not obvious even with model

# Effects of competition

Subtract single-agent attack probabilities from estimated EQ ones

- EQ probs capture violence *with* competition
- Single-agent probs capture violence *without* competition



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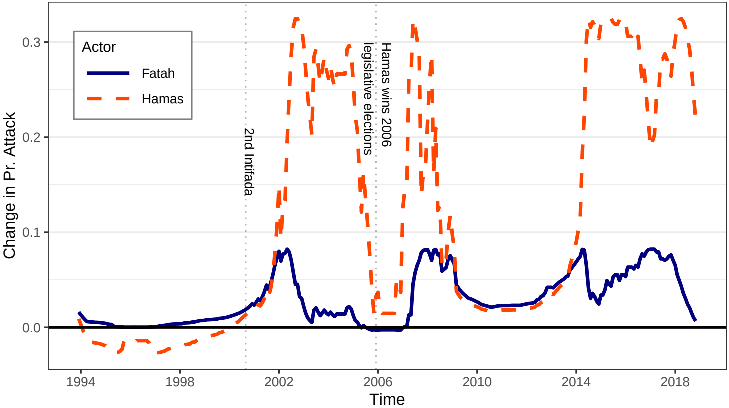


**Takeaway 1:** Competition from Hamas encourages Fatah to use more violence

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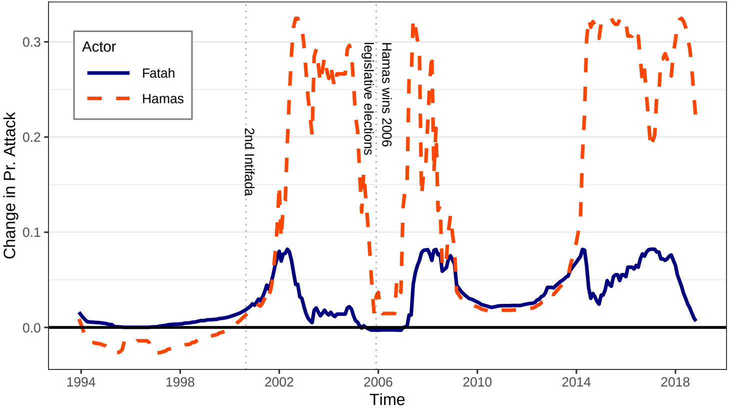


**Takeaway 2:** Averaging over time, Fatah uses 34% more violence than it would without Hamas attacks (expected)

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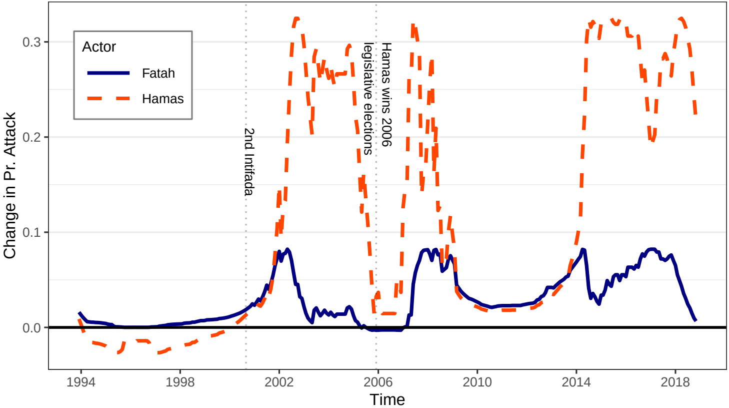


**Takeaway 3:** Averaging over time (post-Oslo), Hamas would use 37% more violence than it would without Fatah attacks (expected)

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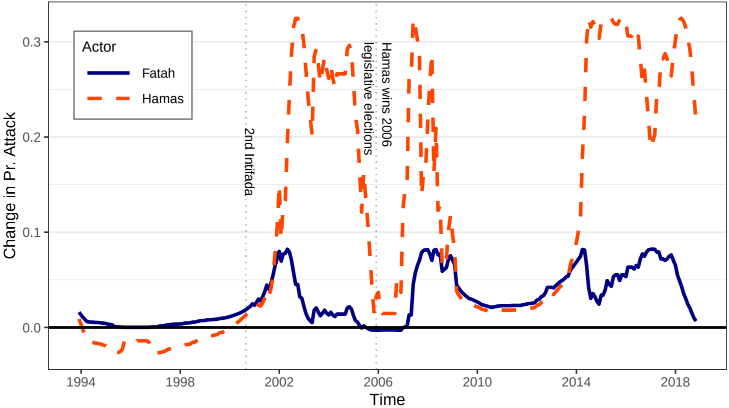


**Takeaway 4:** Average masks heterogeneity: when Fatah is popular (during Oslo), competition depresses Hamas attacks by about 4% (not expected)

# Effects of competition: By era

Subtract single-agent attack probabilities from estimated EQ ones

- EQ probs capture violence *with* competition
- Single-agent probs capture violence *without* competition

| | Oslo era | 2nd Intifada | post-2006 election |
|---|---|---|---|
| Hamas | −0.014 | 0.177 | 0.148 |
| | (0.001) | (0.010) | (0.007) |
| Fatah | 0.005 | 0.027 | 0.043 |
| | (0.0003) | (0.002) | (0.001) |

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**Takeaway 1:** Heterogeneous effects over time *and* across groups

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**Takeaway 2:** Competition from Hamas is making Fatah more violent over time

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**Takeaway 3:** Competition from Fatah was only deterring during the Oslo era

# Additional results

## Robustness checks

1. Vary the end of the time frame: 2000, 2006, 2011, 2015
2. Grid search for best $\delta$
3. Specification of measurement model
4. Specification of first-step model

## Comparative statics

1. Changes to the effectiveness of terrorism $\gamma$
2. Changes to the value of support $\beta$
3. Changes to the cost of terrorism $\kappa$

## Model fit

1. Choice probabilities and attack decisions
2. Comparison to “no competition” model

# Concluding implications

For the literature: group competition has countervailing effects on Hamas and Fatah's use of violence

Stronger incentives to compete can decrease violence

- Fatah is effective at shifting support, so when its incentives to compete increase, it deters Hamas from attacking, creating a feedback loop
- Explanation does not require free riding or government behavior

Even in canonical outbidding example, competition has a complicated relationship with violence

We help to explain why it is difficult to find evidence of outbidding

# Concluding implications

For policy: US often supporting Fatah to counterbalance Hamas

- Kalman 2006

Could be effective depending on dominant effect:

1. Peace enhancing if foreign support increases Fatah's effectiveness or decreases its cost
2. Peace diminishing if foreign support decreases Fatah's value of local support

Uniformly raising costs of terrorism is not necessarily best

- Focus on increasing Hamas's costs *and* decreasing Fatah's.

Thank you!

# Appendix

# Equilibrium characterization

$i$'s net-of-shock expected utility: $v_i(a_i, s)$

- $i$ chooses $a_i$ in state $s$ if and only if

$$a_i \in \operatorname*{argmax}_{a_i \in \{0,1\}} \{v_i(a_i, s) + \varepsilon_i(a_i)\}.$$

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$v$ is an EQ if and only if $v_i(a_i, s) = \mathcal{V}_i^v(a_i, s)$ for all $(i, a_i, s)$, where:

$$\mathcal{V}_i^v(a_i, s) = u_i(a_i, s; \theta) + \sum_{a_j} P_j^v(a_j, s) \left[ \delta \sum_{s'} f_l(s'; a_i, a_j, s, \gamma) V_i^v(s') \right]$$

Appendix EQ: Presentation

# CMLE: Su and Judd 2012

Data: $Y = \{(a^t, s^t)\}_{t=1}^{T}$.

- Month observations between 1994 and 2018: $T = 300$
- Actions from GTD. $s^t$ is discretized version of our estimated relative popularity

Maximize logit log-likelihood $(v, \theta)$:

$$\mathcal{L}(\mathbf{v} \mid Y) = \sum_{t=1}^{T} \left[ \log P_H^v(a_H^t, s^t) + \log P_F^v(a_F^t, s^t) \right]$$

Subject to the constraint:

$$\mathcal{V}_i^v(a_i, s; \theta, \hat{\gamma}) = v_i(a_i, s), \text{ for all } i, a_i, s$$

This produces estimates of $\theta = (\beta_H, \beta_F, \kappa_H, \kappa_F)$.

- Note: $\hat{\gamma}$ estimated in step 1 via AR-1 model

Appendix · back to talk

# Symmetric Setup

- States: $\mathcal{S} = \{-50, -49, \ldots, 49, 50\}$
- Value: $\beta_F = -\beta_H = \frac{1}{500}$
- Cost: $\kappa_i = -2$
- Effectiveness: $\gamma_{F,1} = -\gamma_{H,1} = 1$ and $\gamma_{i,2} = 0$
- Mean of $s^{t+1}$: $\mu[a^t, s^t; \gamma] = s^t - a_H^t + a_F^t$
- Standard deviation of $s^{t+1}$: $\sigma = 2$

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# Single-Agent Problem

What is the probability that $i$ attacks at relative popularity level $s$ when expecting $-i$ to never attack?

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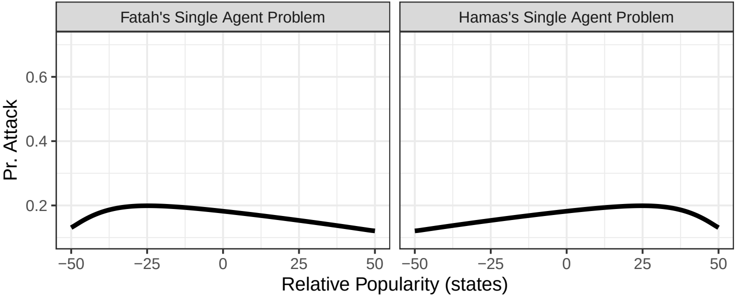
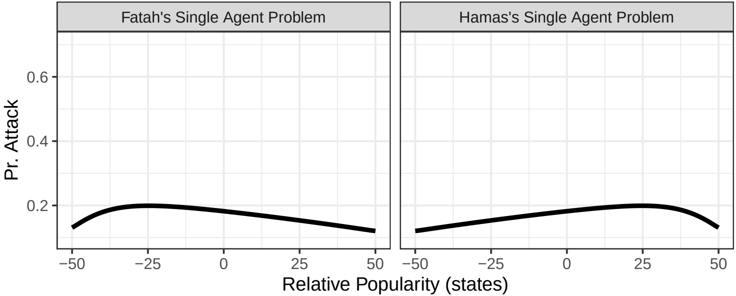



Appendix back to talk

# Single-Agent Problem

What is the probability that $i$ attacks at relative popularity level $s$ when expecting $-i$ to never attack?




**Takeaway 1:** $i$ attacks in least favorable states (with end point effects)

Appendix | back to talk

# Single-Agent Problem

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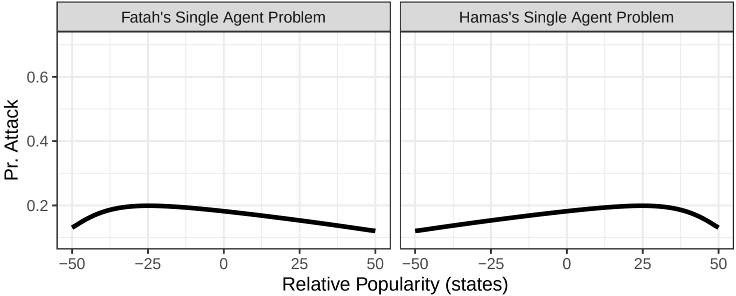


**Takeaway 2:** Attack probabilities are small, from 0.1 to 0.2

Appendix back to talk

# Single-Agent Problem

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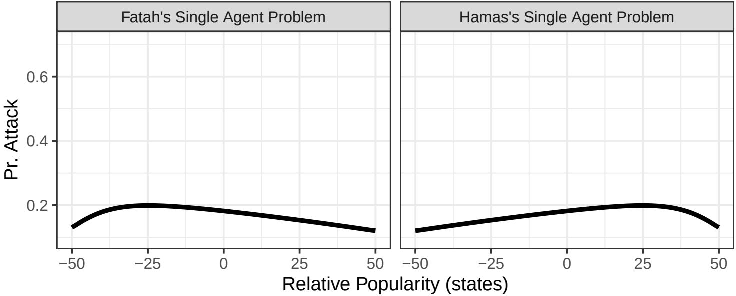


**Takeaway 3:** Attack probabilities are negatively correlated across states

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# Equilibria of the game

What is the equilibrium probability that $i$ attacks in state $s$?

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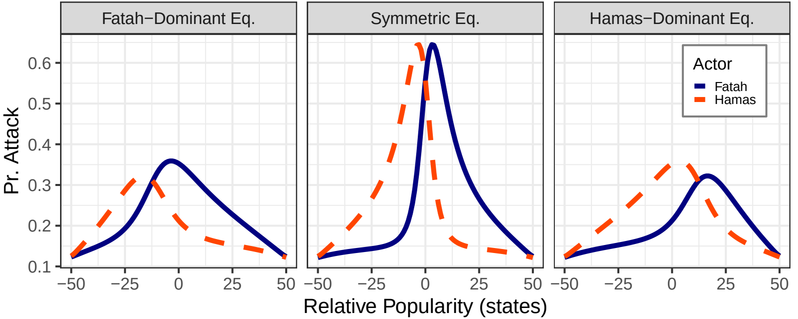


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# Equilibria of the game

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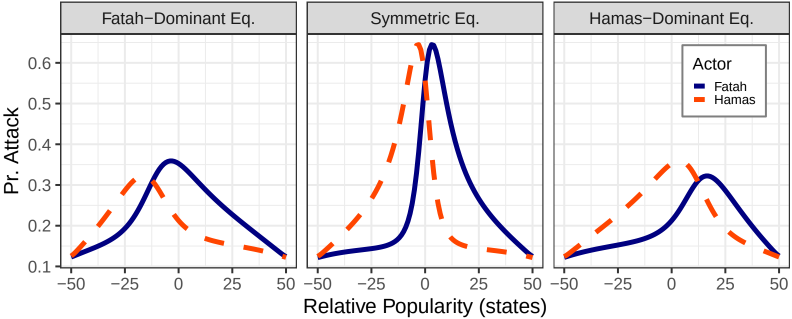
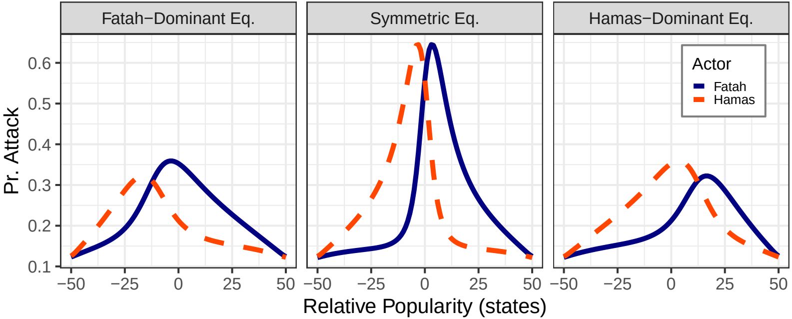



**Takeaway 1:** Attack probabilities are larger, from 0.1 to 0.4 or 0.6

Appendix back to talk

# Equilibria of the game

What is the equilibrium probability that $i$ attacks in state $s$?

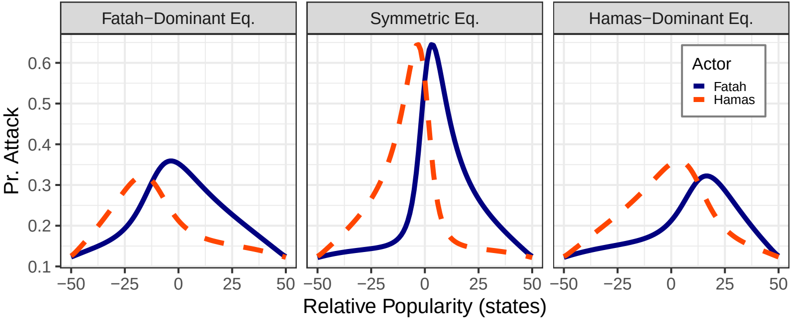



**Takeaway 2:** Attack probabilities are positively correlated across states

Appendix back to talk

# Equilibria of the game

What is the equilibrium probability that $i$ attacks in state $s$?




**Takeaway 3:** $i$ is most likely to attack the most when improving $i$'s popularity decreases $-i$'s attack probability the most

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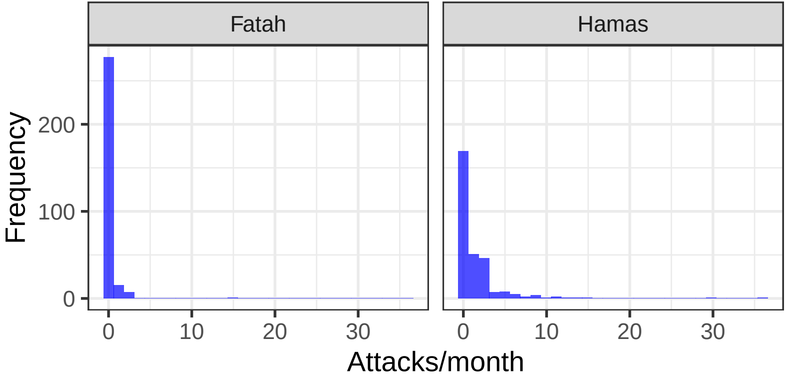
# Observed terrorism




Hamas Mean: 1.5 attacks/month. Median: 0. Range: 0-36.

Fatah Mean: $<1$ attacks/month. Median: 0. Range: 0-15.

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# JMCC Example

JMCC: "Which Political or religious faction do you trust the most"

- Open ended question. Sample sizes around 1200.
- We record percent answering Fatah, percent answering Hamas.

Q31- Which Political or religious faction do you trust the most?

| | Total | West Bank | Gaza |
|---|---|---|---|
| | **n=1199** | **n= 749** | **n=450** |
| Fatah | 35.4 | 34.8 | 36.2 |
| Hamas | 20.0 | 18.6 | 22.4 |
| PFLP | 3.2 | 3.1 | 3.3 |
| Other Islamic factions | 3.1 | 1.9 | 5.1 |
| Others | 3.5 | 4.0 | 2.7 |
| I don't trust anyone | 29.2 | 31.1 | 26.0 |
| No answer | 5.6 | 6.5 | 4.3 |

***This was an open-ended question no options were read to the interviewee**

- August 2015 poll. 52.5% from West Bank, 10.0% from Jerusalem, 37.5% from the Gaza Strip. 50.5% were male. Average age 37.

Appendix back to talk Summary of indicators

# PSR Example

PSR: "Which of the following political parties do you support?"

- Sample sizes around 1200. We record percent answering Fatah, percent answering Hamas.

| **Which of the following political parties do you support?** | | | |
|---|---|---|---|
| 1) PPP | 1.4 | 1.6 | 0.9 |
| 2) PFLP | 3.1 | 3.2 | 3.0 |
| 3) Fateh | 32.3 | 31.7 | 33.3 |
| 4) Hamas | 29.3 | 25.4 | 36.0 |
| 5) DFLP | 1.0 | 1.4 | 0.2 |
| 6) Islamic Jihad | 1.6 | 1.8 | 1.1 |
| 7) Fida | 0.2 | | 0.5 |
| 8) National Initiative (Mubadara) | 1.0 | 1.2 | 0.7 |
| 9) Independent Islamists | 3.3 | 3.9 | 2.3 |
| 10) Independent Nationalists | 4.3 | 4.2 | 4.6 |
| 11) None of the above | 22.4 | 25.3 | 17.4 |
| 12) Other, specify | 0.2 | 0.4 | 0.0 |

- December 2006 poll. 1270 adults: 830 in the West Bank and 440 in the Gaza Strip. Columns are total, West Bank, Gaza Strip.

▸ Appendix ▸ Summary of indicators

# Correlation of public opinion indicators

| | Trust in Hamas | Trust in Fatah | Support for Hamas | Support for Fatah | Vote for Hamas | Vote for Fatah |
|---|---|---|---|---|---|---|
| Trust in Hamas | 1.00 | −0.19 | 0.77 | −0.44 | 0.98 | −0.72 |
| Trust in Fatah | −0.19 | 1.00 | −0.26 | 0.69 | −0.54 | 0.92 |
| Support for Hamas | 0.77 | −0.26 | 1.00 | −0.57 | 0.89 | −0.83 |
| Support for Fatah | −0.44 | 0.69 | −0.57 | 1.00 | 0.56 | −0.32 |
| Vote for Hamas | 0.98 | −0.54 | 0.89 | 0.56 | 1.00 | −0.73 |
| Vote for Fatah | −0.72 | 0.92 | −0.83 | −0.32 | −0.73 | 1.00 |

- each variable correlates as expected with at least three of the five other indicators suggesting that they can be collapsed onto one dimension.

Appendix Summary of indicators

# Measurement Model: Setup

Recall our 6 public opinion measures:

$$y^t = \begin{bmatrix} \text{\% of Population that trusts Fatah} \\ \text{\% of Population that trusts Hamas} \\ \text{\% of Population that supports Fatah} \\ \text{\% of Population that supports Hamas} \\ \text{\% of Population that plans to vote for Fatah} \\ \text{\% of Population that plans to vote for Hamas} \end{bmatrix}^t,$$

Define $z^t$ as $z_j^t = \frac{y_j^t - \bar{y}_j}{\sqrt{\text{Var}(y_j)}}$, $j = 1, \ldots, 6$.

Our **dynamic factor analysis** is comprised of :

$$z^t = \tilde{s}^t \cdot \omega + \xi^t$$
$$\tilde{s}^t = \rho \tilde{s}^{t-1} + \alpha_0 + a_H^{t-1} \cdot \alpha_H + a_F^{t-1} \cdot \alpha_F + \eta^t$$
$$\xi^t \sim N(0, \mathbf{1})$$
$$\eta^t \sim N(0, 1)$$

Appendix back to talk Additional results

# Measurement Model: Results

Parameters ($\omega$, $\alpha$) estimated using MLE and EM using the `MARSS` package for R

| Equation | Variable | Estimate |
|---|---|---|
| Factor Weights ($\omega$) | Trusts Hamas | -0.09 |
| | Trust Fatah | 0.06 |
| | Supports Hamas | -0.11 |
| | Supports Fatah | 0.10 |
| | Votes Hamas | -0.07 |
| | Votes Fatah | 0.07 |
| AR(1) term ($\rho$) | Lagged DV | 0.99 |
| Additional inputs ($\alpha$) | Constant | -0.01 |
| | Hamas attack | -0.28 |
| | Fatah attack | 1.05 |

Appendix | back to talk | Additional results

# Measurement Model: Robustness

1. Main specification (described above)
2. Fixed AR(1) term for $\tilde{s}^t$.
3. Estimated and homoskedastic variance in $\xi^t$.
4. Estimated and heteroskedastic variance in $\xi^t$.
5. A model with no $\alpha$s (setting $\alpha = 0$)
6. Remove the "plans to vote for" surveys

# Measurement Model: Robustness

We correlate $\tilde{s}^t$ across models:

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| Model 1 | 1.00 | 1.00 | 0.99 | 0.87 | 0.99 | 0.99 |
| Model 2 | 1.00 | 1.00 | 1.00 | 0.88 | 1.00 | 0.99 |
| Model 3 | 0.99 | 1.00 | 1.00 | 0.88 | 1.00 | 0.98 |
| Model 4 | 0.87 | 0.88 | 0.88 | 1.00 | 0.89 | 0.91 |
| Model 5 | 0.99 | 1.00 | 1.00 | 0.89 | 1.00 | 0.99 |
| Model 6 | 0.99 | 0.99 | 0.98 | 0.91 | 0.99 | 1.00 |

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# Equilibrium choice probabilities

What is the equilibrium probability that $i$ attacks in state $s$?

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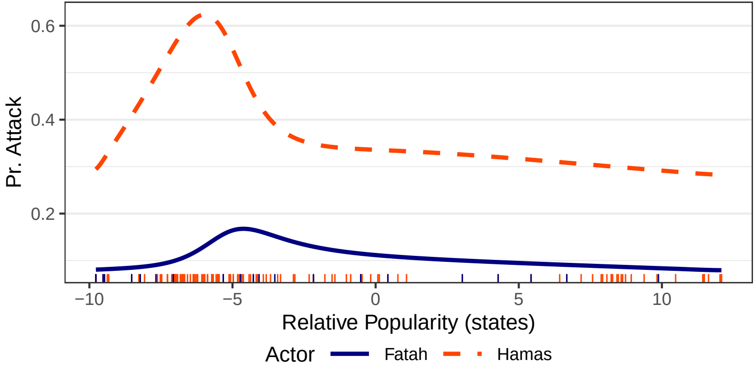


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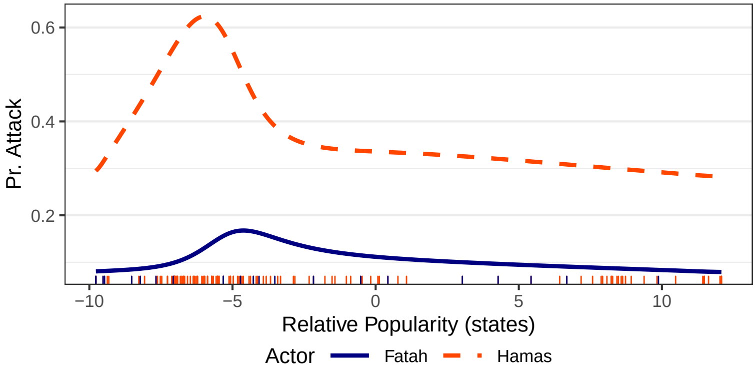


**Takeaway 1:** Attack probabilities are positively correlated across states

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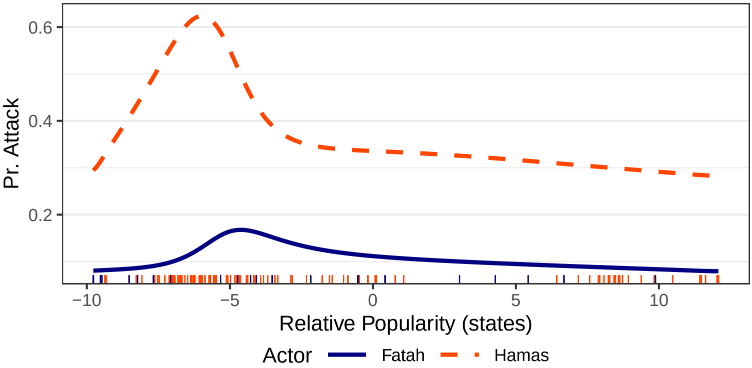


**Takeaway 2:** $i$ is most likely to attack the most when improving $i$'s popularity decreases $-i$'s attack probability the most

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# Equilibrium choice probabilities

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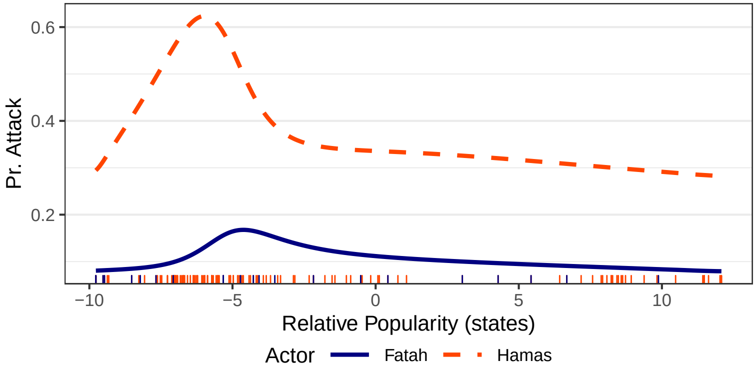


**Takeaway 3:** There is a set of attacks that are poorly explained by the model. Even with this, we find evidence of competition

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# Model comparison

Two model fit exercises to compare the fitted model to a "no competition" model where neither side can use violence to move public opinion

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# Model comparison

Two model fit exercises to compare the fitted model to a "no competition" model where neither side can use violence to move public opinion

- Expected number of attacks correctly predicted

$$\text{ePCP}_i = \frac{1}{T}\sum_{t=1}^{T} P(a_i^t; s^t, \hat{v}_i),$$

- Likelihood ratio test to compare the models

Appendix back to talk

# Model comparison

Two model fit exercises to compare the fitted model to a "no competition" model where neither side can use violence to move public opinion

| | Main model | No Competition |
|---|---|---|
| ePCP–Hamas | 0.54 | 0.51 |
| ePCP–Fatah | 0.83 | 0.86 |
| ePCP–Overall | 0.46 | 0.44 |
| LR test (d.f. = 6) | $\chi^2 = 275$ ($p < 0.01$) | |

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# Robustness: First stage

| *Controlling for* | Fatah | Hamas |
|---|---|---|
| Base Model | $(0.94, 1.14)$ | $(-0.23, -0.19)$ |
| No interaction | $(1.00, 1.00)$ | $(-0.21, -0.21)$ |
| Unemployment | $(0.94, 1.21)$ | $(-0.21, -0.18)$ |
| Support for violence | $(0.96, 1.14)$ | $(-0.24, -0.22)$ |
| 2nd Intifada | $(0.94, 1.16)$ | $(-0.21, -0.23)$ |
| Time since last election | $(0.95, 1.16)$ | $(-0.21, -0.19)$ |
| Palestinians killed | $(0.87, 1.44)$ | $(-0.12, -0.60)$ |

Values are the estimated marginal effects of a terrorist attack on relative popularity at the extremes $\tilde{s}_{t-1} = \{-10, 12\}$. All estimates are from an ECM model.

Appendix

# Robustness: Time frame

| | Full Sample | 2014 Agreement | 2011 Agreement | 2006 Elections | Second Intifada† |
|---|---|---|---|---|---|
| $\beta_H$ | −0.007 | −0.007 | −0.007 | −0.021 | −0.007 |
| | (0.004) | (0.005) | (0.004) | (0.013) | (0.004) |
| $\beta_F$ | 0.0005 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| | (0.0003) | (0.0003) | (0.0003) | (0.0002) | (0.0152) |
| $\kappa_H$ | −0.95 | −0.90 | −0.69 | −0.74 | −0.89 |
| | (0.23) | (0.23) | (0.22) | (0.26) | (0.10) |
| $\kappa_F$ | −2.45 | −2.47 | −2.31 | −2.34 | −2.89 |
| | (0.28) | (0.31) | (0.28) | (0.26) | (0.23) |
| $T$ | 300 | 243 | 208 | 144 | 80 |
| LL | −278.09 | −230.98 | −206.84 | −125.27 | −71.82 |

Note: †CMLE did not converge, estimates from nested-pseudo-likelihood (NPL) estimator. Samples begin at Jan. 1994 and end in one month prior to the event listed.

▸ Appendix

# Robustness: Discount factor

| $\delta$ | Log-Likelihood |
|---|---|
| 0 | -284.18 |
| 0.9 | -280.98 |
| 0.925 | -281.54 |
| 0.95 | -282.43 |
| 0.975 | -283.67 |
| 0.99 | -280.35 |
| 0.999 | -278.09 |
| 0.9999* | -288.14 |

Note: *Model failed to converge.

Appendix

# Homotopy Algorithm

**Algorithm 1:** COMPARATIVE STATICS (CS) using a homotopy

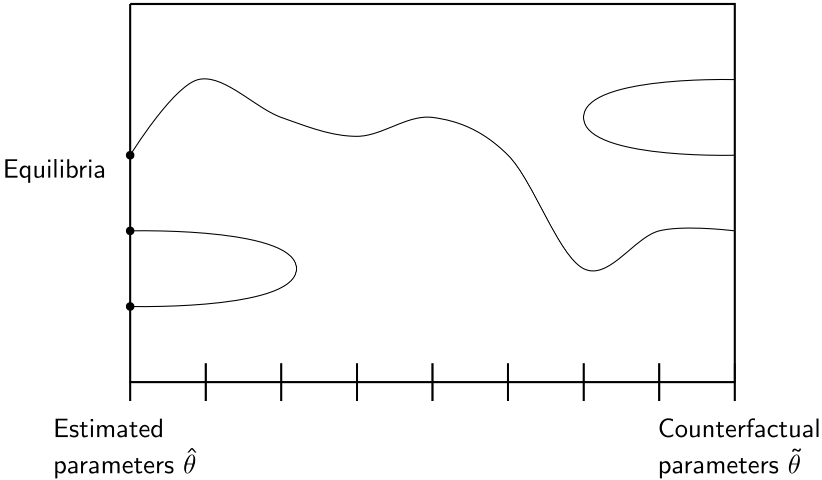
**Input:** An initial coefficient vector $\theta_0$ and equilibrium $v_0$; new values $\tilde{\theta}$; and a tuning parameter $n \in \mathbb{N}$. For NEWTON solver, a convergence tolerance $\varepsilon > 0$ and maximum number iterations $m \in \mathbb{N}$.

**Output:** An equilibrium $\tilde{v}$ under new parameters $\tilde{\theta}$

$\theta_{\texttt{old}} \leftarrow \theta_0$
$f_{\texttt{old}} \leftarrow f(\theta_0)$
$v_{\texttt{old}} \leftarrow v_0$
**for** $i \leftarrow 1$ **to** $n$ **do**
  $\lambda \leftarrow \frac{i}{n}$
  $\theta_{\texttt{new}} \leftarrow (1-\lambda)\theta_0 + \lambda\tilde{\theta}$
  $f_{\texttt{new}} \leftarrow f((1-\lambda)\theta_0) + f(\lambda\tilde{\theta})$
  $\texttt{slope} \leftarrow -\left(D_\theta \mathcal{V}(v_{\texttt{old}} \mid \theta_{\texttt{old}}, f_{\texttt{old}})\right)' \left(D_v \mathcal{V}(v_{\texttt{old}} \mid \theta_{\texttt{old}}, f_{\texttt{old}})\right)^{-1}$
  $\texttt{start} \leftarrow v_{\texttt{old}} + [\theta_{\texttt{new}} - \theta_{\texttt{old}}]\,\texttt{slope}$
  $(v_{\texttt{new}}, \texttt{success}) \leftarrow \text{NEWTON}(\texttt{start}, \mathcal{V}(v \mid \theta_{\texttt{new}}, f_{\texttt{new}}) - v, \varepsilon, m)$
  **if** *success* **then**
    $\theta_{\texttt{old}} \leftarrow \theta_{\texttt{new}}$
    $f_{\texttt{old}} \leftarrow f(\theta_{\texttt{new}})$
    $v_{\texttt{old}} \leftarrow v_{\texttt{new}}$
  **else**
    $\tilde{v} \leftarrow$ "Warning: Convergence Problems."
    **break**
$\tilde{v} \leftarrow v_{\texttt{new}}$
**return** $\tilde{v}$

Appendix

# Homotopy method for comparative statics




Appendix

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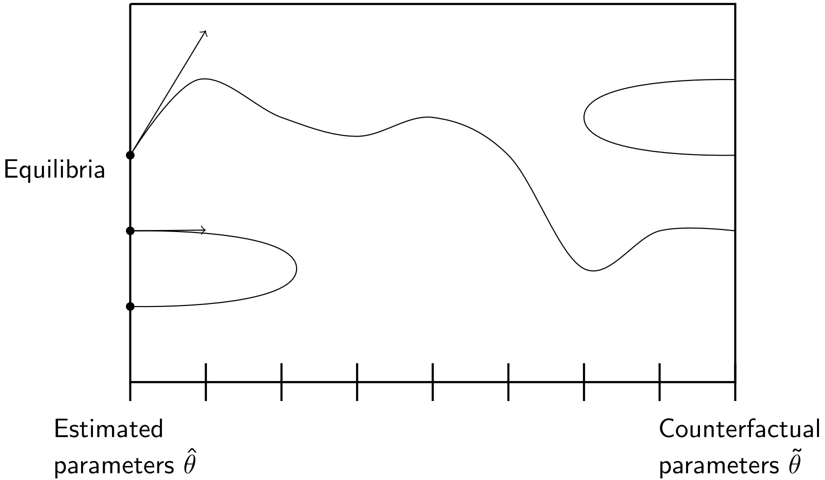


Appendix

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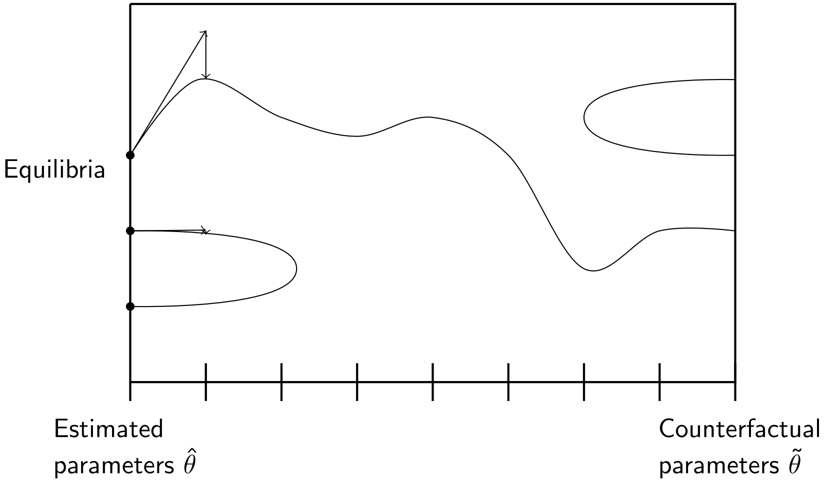


Appendix

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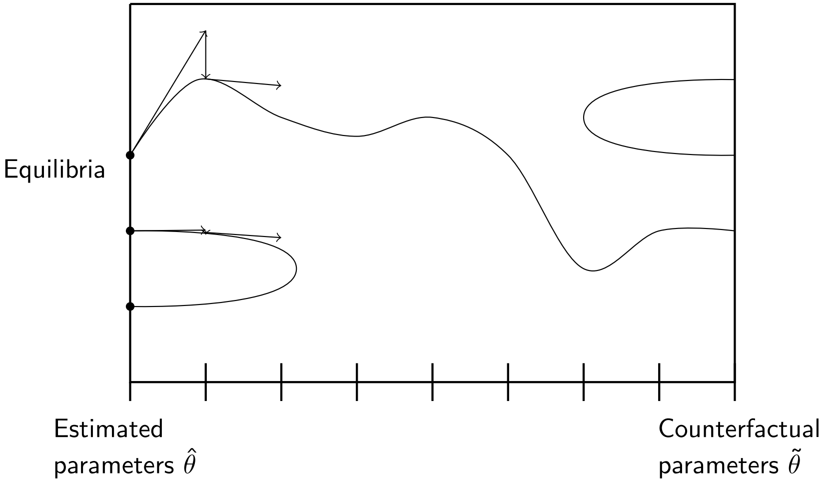


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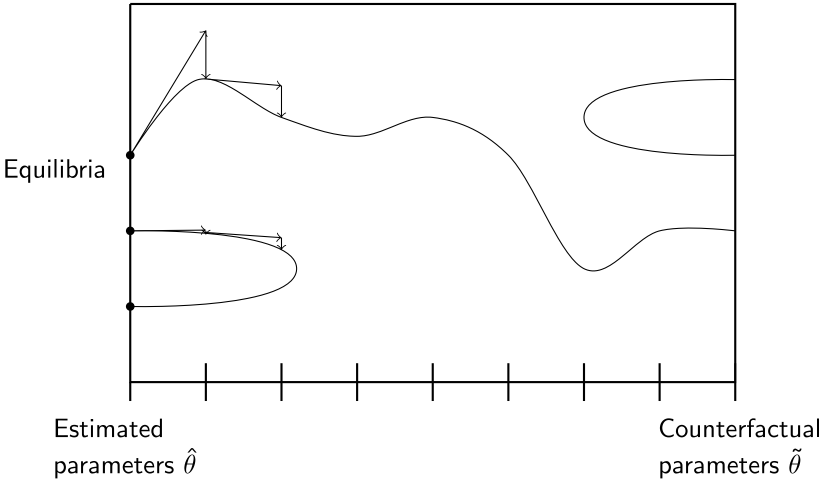


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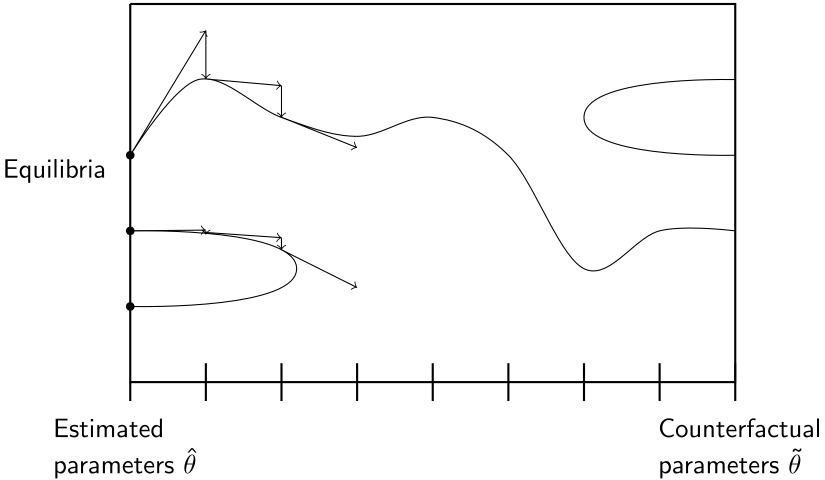
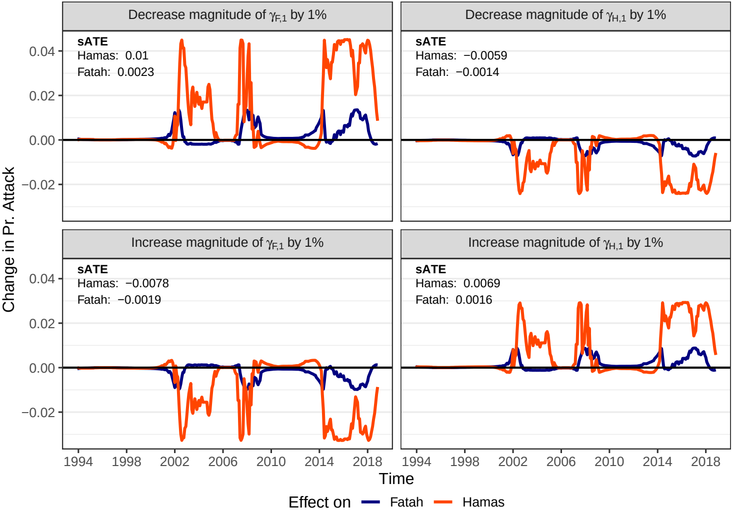
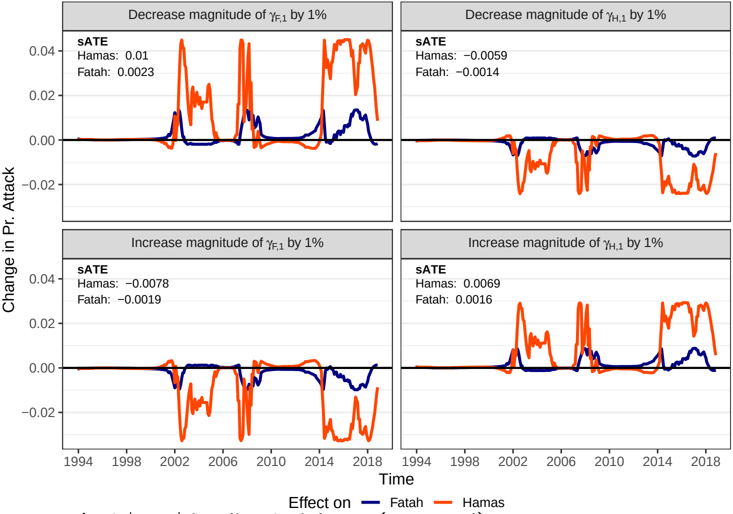



Appendix

# Changing effectiveness



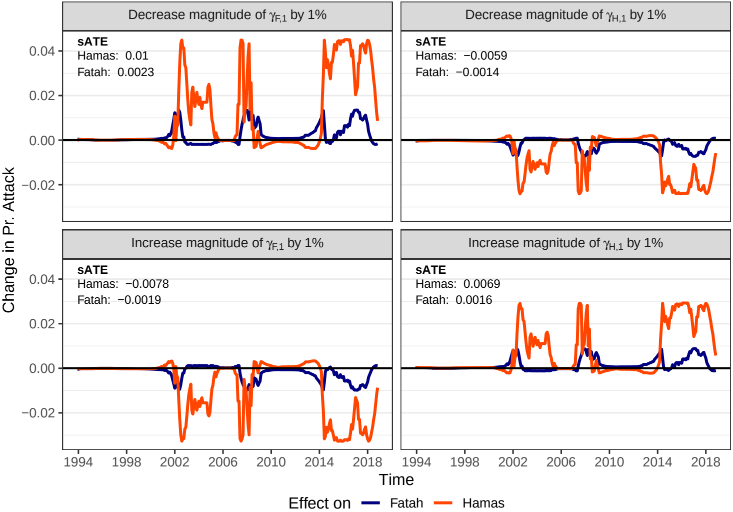
# Changing effectiveness




**Takeaway 1:** $\uparrow |\gamma_{H,1}|$ implies $\uparrow$ violence (expected)

▸ Appendix ▸ Additional results ▸ Return to talk

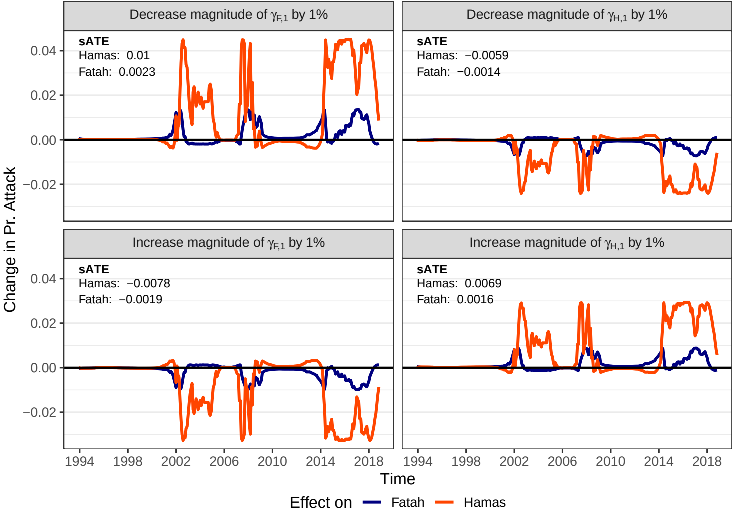
# Changing effectiveness




**Takeaway 2:** $\uparrow |\gamma_{F,1}|$ implies $\downarrow$ violence (not expected)

Appendix | Additional results | Return to talk

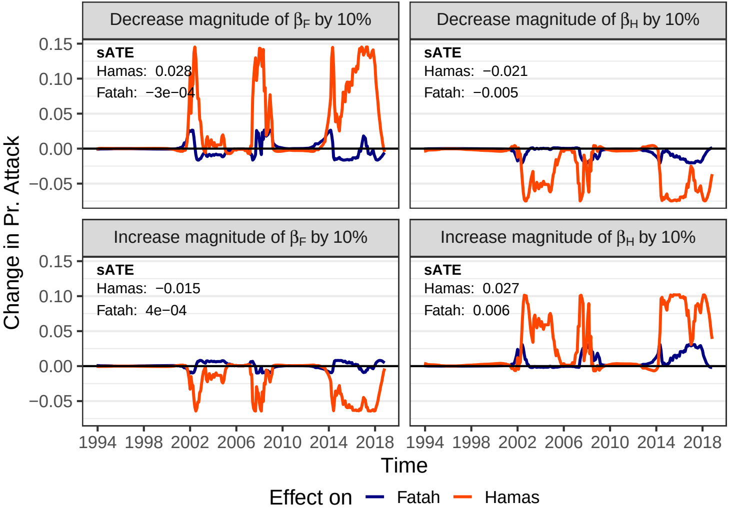
# Changing effectiveness




**Takeaway 3:** Same conclusions when changing values or costs

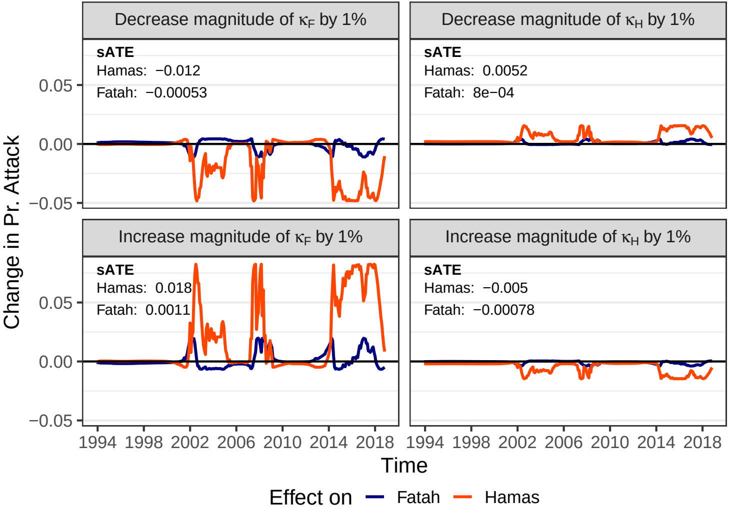
Appendix Additional results Return to talk

# Changing values




▸ Appendix ▸ Additional results

# Changing attack costs




Appendix Additional results