Online Appendix for "Tug of War: The Heterogeneous Effects of Outbidding between Terrorist Groups"

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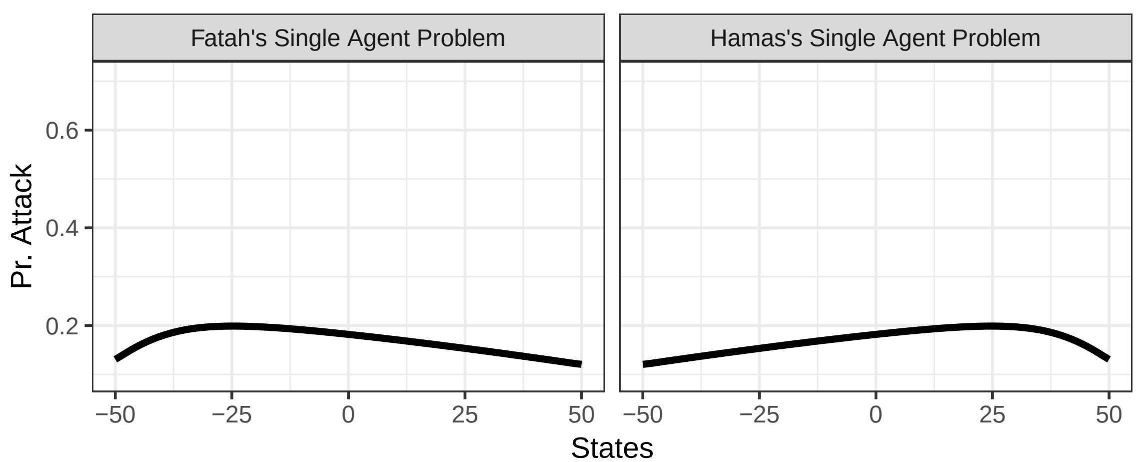
A Numerical example

To illustrate the strategic tensions in the model, we pick hypothetical values for the parameters and study the equilibria that arise. The specification is symmetric to aid in interpretation, but the model is more general, allowing the groups to have different competitive incentives. Although the selected values are similar in scale to our estimates below, they are hypothetical and are used for illustrative purposes. The popularity levels are $\mathcal{S} = \{-S, -S+1, \ldots, S-1, S\}$ where $S = 50$. For the payoff parameters, we set $\beta_F = \frac{1}{500} = -\beta_H$ and $\kappa_i = -2$. For the transitions, we assume $\mu[a^t, s^t; \gamma] = s^t - a_H^t + a_F^t$ and $\sigma = 2$. In other words, group i's attacks shift the mean of tomorrow's expected relative popularity by one in its preferred direction. Note that the current popularity level does not change the effectiveness of attacks ($\gamma_{i,2} = 0$).

To build intuition, Figure A.1 first presents group i's optimal attack probabilities when its rival never attacks, i.e., when i is the only relevant group. The probabilities range from 0.1 to 0.2. Notice i is most likely to attack when its relative popularity is weak (small s for F and large s for H), although end-point effects emerge because $\mathcal{S}$ is bounded. When the current state is at the boundary, a group's popularity cannot get worse or better tomorrow, which decreases the groups' incentives to attack. When a group is relatively unpopular, it has stronger dynamic benefits from using costly attacks to increase its popularity: attacking increases i's future payoffs for some time and decreases its need to use costly attacks in the future. Thus, comparing across the two single-agent problems, the groups generally attack in different states—the correlation coefficient of their attack probabilities is $\rho = -0.13$.

Figure A.1: Attack probabilities without competition in the numerical example.



Notes: Left panel graphs the probability that Fatah attacks (y-axis) as a function of the states (x-axis) in its single-agent dynamic programming problem, i.e., when Hamas never attacks. The right panel graphs the attack probabilities for Hamas's single-agent dynamic programming problem.

Turning to the strategic setting, we investigate equilibrium probabilities of attacking. To find equilibria, we repeatedly compute solutions to Equations 4–7 (with the fixed parameters specified in this example) using the Newton-Raphson method and different starting values. All computed solutions corresponded to one of three equilibria. Figure A.2 graphs the corresponding attack probabilities for each equilibrium. In the symmetric equilibrium, terrorism is most fierce when the groups are equally popular, and group i attacks with the highest probability once it begins to be slightly less popular than its rival. The other two equilibria are asymmetric but are essentially the same with the actors and states flipped. In these equilibria, one actor (the one labeled dominant) is using violence with higher probability than the remaining actor for the majority of the state space.

Figure A.2: Equilibria in the numerical example.

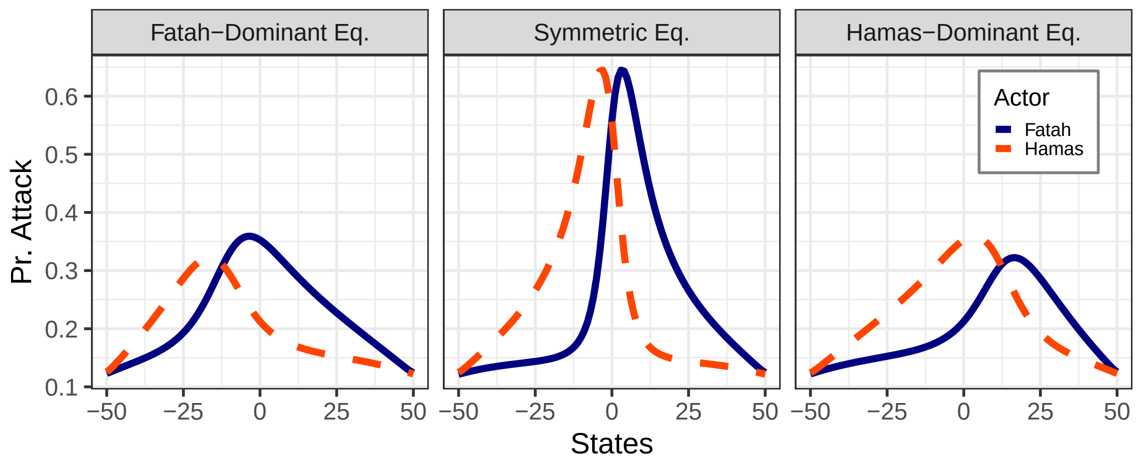
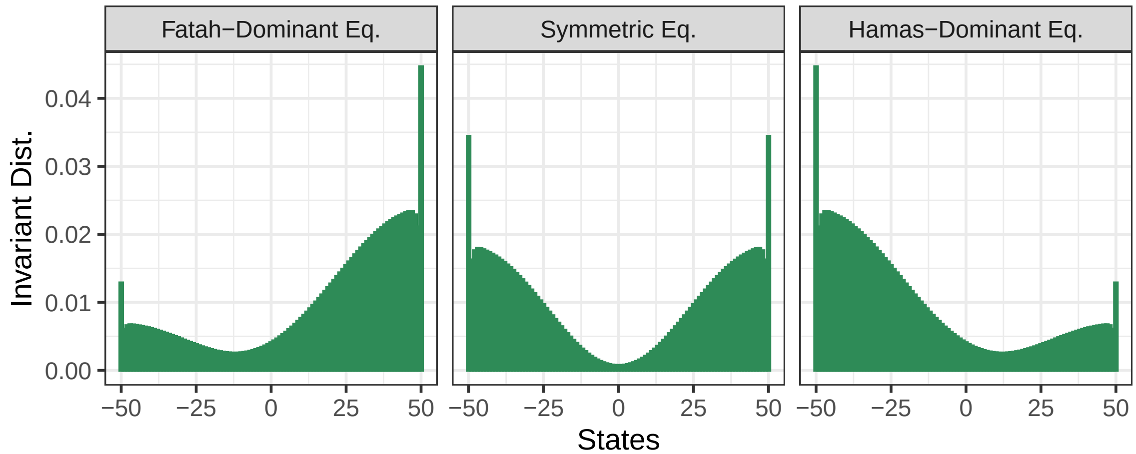


Figure A.3 graphs the invariant distribution for each of the three equilibria. In words, the invariant distribution tell us how likely it is for the equilibrium path to visit each of the relative popularity levels *in the long run*. Across the three equilibria, the equilibrium path is not likely to frequent states with high levels of violence. As expected, we find that the equilibrium path in an asymmetric equilibria is more likely to visit popularity levels that are favorable to the dominant (more violent) actor. In the symmetric equilibrium, the invariant distribution is symmetric around zero. Finally, there are spikes in the invariant distribution at the end points of the state space which reflects the fact that the interaction can bunch at high and low values due to relative popularity levels being bounded.

The example illustrates three features. First, violence between groups exhibits some strategic complementarities: attack probabilities are positively correlated across states. In the asymmetric equilibria, the correlation coefficient is $\rho = 0.41$, and in the symmetric equilibrium it is $\rho = 0.28$. These complementarities do not arise through the group's per-period payoffs in Equation 1, because i's per-period payoff does not depend on $-i$'s action. In addition, the groups do not become more effective in certain states as $\gamma_{i,2} = 0$.

Figure A.3: Equilibrium invariant distributions in the numerical example.



Instead, the complementarities arise endogenously through tug-of-war dynamics in which competition can increase violence. Indeed, group i's attack probabilities in any of the three equilibria are larger than those in its single-agent problem. Thus, our dynamic model endogenizes the strategic complementarities for violence found in previous analyses using static games (Gibilisco, Kenkel and Rueda 2019).

Second, these complementarities are moderate, i.e., the attack probabilities are not perfectly correlated. In all equilibria, the state in which Hamas is most likely to attack is strictly less than the state in which Fatah is most likely to attack. This arises because, all else equal, Hamas wants to exert costly effort to attack at popularity levels where becoming more popular reduces Fatah's likelihood to attack. In contrast, Fatah wants to attack at levels where becoming more popular reduces Hamas's likelihood to attack. These incentives not only create but also temper the potential strategic complimentarities in the model. To better see this, define the effect of increasing the state on group i's equilibrium attack probability in state $s \notin \{-S, S\}$ as

$$\Delta(s, v_i) = P(1; s+1, v_i) - P(1; s-1, v_i).$$

Table A.1 summarizes how $P(1; s, v_i)$ correlates with $\Delta(s, v_{-i})$.[1] In the state where Fatah is attacking with its highest probability, a local increase in Hamas's popularity (smaller s), will not greatly reduce Fatah's probability of attacking. That is, $\Delta(s, v_F)$ is close to zero. There is a similar story in states where Fatah is attacking with low probability. In states where $\Delta(s, v_F)$ is large and positive, making Hamas more popular (decreasing the

[1] Of course, $P(1; s, v_i)$ and $\Delta(s, v_{-i})$ are not perfectly correlated because relative popularity can change by more than one level between periods. The value $\Delta(s, v_{-i})$ is an approximation of i dynamic incentives of attacking in state s.

state variable) greatly reduces Fatah's probability of attacking. As such, Hamas has greater dynamic incentives to use terrorism.

Table A.1: Correlation between $P(1; s, v_i)$ and $\Delta(s, v_{-i})$ across states.

	Fatah-Dominant Eq.	Symmetric Eq.	Hamas-Dominant Eq.
$i =$ Hamas	0.86	0.79	0.78
$i =$ Fatah	-0.78	-0.79	-0.86

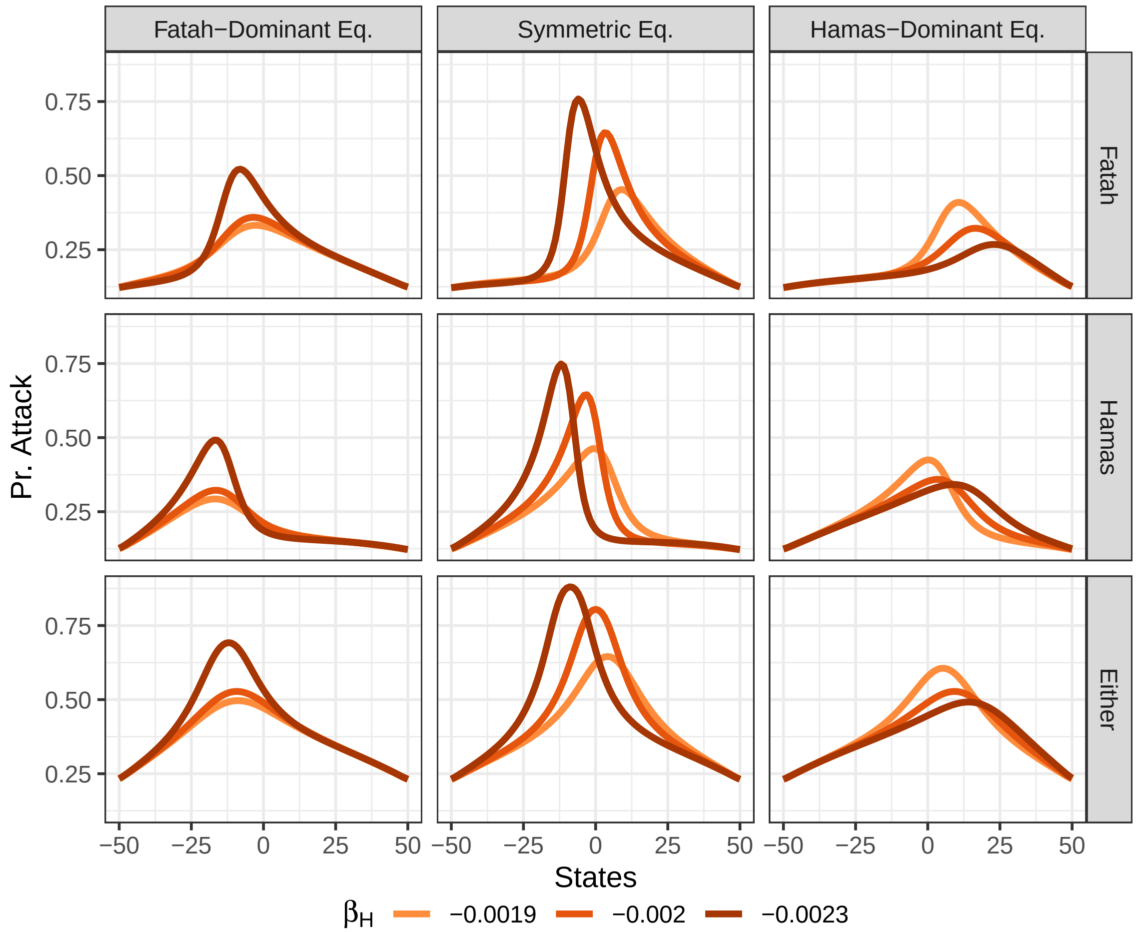
Third, equilibrium rates of attacks are not perfect measures of competitive incentives. Consider the Fatah-dominant equilibrium. At the majority of popularity levels, Fatah is attacking with greater probability than Hamas, so one might conclude that Fatah has smaller attack costs or a greater value of popularity. The example is symmetric, however, and both groups have identical competitive incentives. Thus, incentives to compete do not directly map onto observed rates of violence as the relationship is mediated by a strategic interaction. As a result, reduced-form regressions using observed terrorism as the dependent variable may obscure some aspects of the outbidding process. Directly estimating the model's parameters allows for a deeper exploration of how competition affects violence.

Finally, we conduct a comparative statics exercise to illustrate the potential effects of how exogenous changes in competitive incentives can affect the groups' use violence.[2] In Figure A.4, we graph how the three equilibria change as Hamas's value of popularity, β_H, varies. In this figure, more intense competitive incentives correspond to β_H moving toward negative infinity away from the baseline value of $-\frac{1}{500}$. Less intense incentives to compete correspond to β_H moving toward zero. In the Fatah-dominant equilibrium, larger competitive incentives generally increase both actors' propensity for violence. This effect is expected in the outbidding literature. The result does not hold in the other two equilibria, however. In the symmetric equilibrium, larger competitive incentives for Hamas increase violence in pro-Hamas states ($s < 0$) but can decrease violence in pro-Fatah states ($s > 0$). In the Hamas-dominant equilibrium, greater competitive incentives generally lead to less terrorism when looking at attacks by either group. We also repeated similar exercises when we vary Hamas's effectiveness, $\gamma_{H,1}$, and its cost of attacking, κ_H. Overall, the results largely mirror Figure A.4.

These comparative statics help to motivate the structural exercise. Even with a strong qualitative understanding of the case, it is not clear what values of the parameters and what equilibrium is empirically relevant and should be prioritized when discussing the effects of competition on violence. Furthermore, the example illustrates that enhanced incentives to

[2] We use the homotopy method in Aguirregabiria (2012) to compute counterfactuals throughout the analysis. See Section 6.

Figure A.4: Comparative statics and Hamas's value of popularity, β_H, in the example.



Notes: We increase and decrease β_H from the baseline numerical example where $\beta_H = -0.002$. The horizontal axis is the relative popularity levels (smaller values are more favorable to Hamas) and the vertical axis is the probability of an attack. Columns correspond to the equilibrium and rows denote which group's probability of attacking is graphed. Darker red denotes stronger incentives to compete, β_H is more negative.

compete can either increase or decrease overall violence levels. This raises two questions. First, what effect dominates in the data? Second, do these effects vary by actor when the model is not symmetric?

B More details on the survey data and dynamic factor model

Table B.1 shows the raw correlations among the survey questions. Within each group trust and support are highly correlated (0.69 and 0.77 for Hamas and Fatah, respectively). Likewise, support for one group is negatively correlated with support for the other. The remaining correlations are almost all in the expected directions, suggesting that the population does in fact trade off among supporting these two leading actors. The only exceptions is the negative correlation between voting for and supporting Fatah and the positive correlation between supporting Fatah and voting for Hamas. However, trust in Fatah correlates highly with both supporting and voting for Fatah, while voting for Hamas correlates highly with support and trust in Hamas.

Table B.1: Correlations among survey responses

	Trust in Hamas	Trust in Fatah	Support for Hamas	Support for Fatah	Vote for Hamas	Vote for Fatah
Trust in Hamas	1.00	−0.19	0.77	−0.44	0.98	−0.72
Trust in Fatah	−0.19	1.00	−0.26	0.69	−0.54	0.92
Support for Hamas	0.77	−0.26	1.00	−0.57	0.89	−0.83
Support for Fatah	−0.44	0.69	−0.57	1.00	0.56	−0.32
Vote for Hamas	0.98	−0.54	0.89	0.56	1.00	−0.73
Vote for Fatah	−0.72	0.92	−0.83	−0.32	−0.73	1.00

To produce the latent state variable, we first let y^t be the column vector denoting the 6 survey values at time t such that

$$y^t = \begin{bmatrix} \text{\% of Population that trusts Fatah} \\ \text{\% of Population that trusts Hamas} \\ \text{\% of Population that supports Fatah} \\ \text{\% of Population that supports Hamas} \\ \text{\% of Population that plans to vote for Fatah} \\ \text{\% of Population that plans to vote for Hamas} \end{bmatrix}^t . \tag{B.1}$$

Let z^t denote the vector of z-transformed surveys, where for survey $j = 1, \ldots, 6$, $z_j^t = \frac{y_j^t - \bar{y}_j}{\sqrt{\mathrm{Var}(y_j)}}$. We construct a continuous state variable $\tilde{s}^t$ as a function of past terrorist attacks and past population support using the dynamic factor model given by

$$z^t = \tilde{s}^t \omega + \xi^t, \tag{B.2}$$

and

$$\tilde{s}^t = \tilde{s}^{t-1} + \alpha_0 + a_H^{t-1} \cdot \alpha_H + a_F^{t-1} \cdot \alpha_F + \eta^t. \tag{B.3}$$

Here, a_F^{t-1} and a_H^{t-1} record attacks by Fatah and Hamas, respectively while the α_F and α_H weights the impact of those attacks. Including the attacks in the measurement of s^t reflects the strategic interdependence between the states and actions. Note that simply including attacks in the measurement model does not presuppose their relationship in the first-stage regressions below. The α_i can take on any value, including zero. Likewise, α_0 is a constant term and ω is a length-6 column vector of factor weights. Finally, $\eta^t \sim N(0,1)$ and $\xi^t \sim N(0,\mathbf{1})$ are random perturbations, where $\mathbf{1}$ is the identity matrix.

The parameter vector $\Theta = (\omega, \alpha)$ can be estimated using maximum likelihood using the `MARSS` package for R (Holmes, Ward and Scheuerell 2018). Starting with an initial guess of the parameters $\hat{\Theta}_1$, the estimator relies on the following EM for iteration k:

1. **Expectation step:** Generate expected values of $\tilde{s}^t$ using a Kalman filter and current givens $\hat{\Theta}_k$, z_t, and a^{t-1}. During this step missing values in z^t are also imputed by a Kalman filter.

2. **Maximization step:** Using the generated values of $\tilde{s}^t$ and imputed z^t, maximize the multivariate normal log-likelihood. This step outputs $\hat{\Theta}_{k+1}$

3. Repeat the EM steps until no improvement in the log-likelihood is gained.

The estimates of (ω, α) are reported in Table B.2, while the estimates of $\tilde{s}^t$ are presented in Figure 2 (main text). Notice that the variables all load onto the dynamic factor in the expected direction: pro-Hamas responses have negative weights and pro-Fatah responses have positive weights.

Table B.2: ML estimates for the factor model

Equation	Variable	Estimate
Factor Weights (ω)	Trusts Hamas	-0.08
	Trust Fatah	0.06
	Supports Hamas	-0.10
	Supports Fatah	0.09
	Votes Hamas	-0.06
	Votes Fatah	0.06
Additional inputs (α)	Constant	0.00
	Hamas attack	-0.41
	Fatah attack	1.61

We also consider the robustness of this measurement model by comparing the estimated $\tilde{s}^t$ from the following six models.

1. Main specification (described above)

2. Estimated AR(1) term for $\tilde{s}^t$. Modify Eq. B.3 s.t.

$$\tilde{s}^t = \rho \tilde{s}^{t-1} + \alpha_0 + a_H^{t-1} \cdot \alpha_H + a_F^{t-1} \cdot \alpha_F + \eta^t.$$

3. Estimated and homoskedastic variance in ξ^t. Modify Eq. B.2 s.t. $\xi^t \sim N(0, \sigma_\xi^2 \mathbf{1})$.
4. Estimated and heteroskedastic variance in ξ^t. Modify Eq. B.2 s.t. $\xi^t \sim N(0, \sigma_{\xi,j}^2 \mathbf{1})$, where $j = 1, \ldots, 6$ indexes surveys
5. A model with $\alpha = 0$. Modify Eq. B.3 s.t.

$$\tilde{s}^t = \tilde{s}^{t-1} + \eta^t.$$

6. Remove the "plans to vote for" surveys (which start later than the other four). Modify y^t and z^t to only contain the first four survey responses.

Note that each of the robustness checks considers one change to the main specification (i.e., these are not cumulative changes to the factor analysis). Fitting these models gives us six specifications, each of which produces its own estimate of $\tilde{s}^t$. In Table B.3 we present the correlation matrix of these different approaches. Overall, we see that these methods all produce remarkably similar estimates. The biggest difference from the main model comes from heteroskedastic version, where a separate variance term is estimated for each of the six surveys. However, the correlation here is still roughly 0.9. As such, we conclude that these deviations from the main specification result in little change to $\tilde{s}^t$.

Table B.3: Correlations across measurement specifications

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Model 1	1.00	1.00	0.99	0.87	0.99	0.99
Model 2	1.00	1.00	1.00	0.88	1.00	0.99
Model 3	0.99	1.00	1.00	0.88	1.00	0.98
Model 4	0.87	0.88	0.88	1.00	0.89	0.91
Model 5	0.99	1.00	1.00	0.89	1.00	0.99
Model 6	0.99	0.99	0.98	0.91	0.99	1.00

C First stage robustness

In this appendix, we consider the robustness of the first-stage model and the estimates of γ. Specifically, we want to be sure that the relationship between attacks and relative popularity is not being driven or confounded by other factors that may affect each group's decision to attack and their relative popularity. In particular, we will consider economic factors, the conflict environment, and the public's underlying support for violence against Israel.

To measure the latent variables that describe attitudes toward violence and the economic situation we will again aggregate survey data using dynamic factor models. Both models use the same basic specification described in Equations B.2 and B.3, but with different surveys forming z^t and as such a different latent variable output (i.e., $\tilde{v}^t$ and $\tilde{u}^t$ for attitudes for violence and unemployment, respectively rather than $\tilde{s}^t$). For attitudes towards violence we use survey questions that record 26 different responses to various aspects of the conflict/peace process. From the surveys run by Jerusalem Media and Communication Centre (JMCC) (N.d) we include:

- 4 responses about attitudes to a two-state solution
- 2 responses about attitudes towards peace negotiations
- 2 responses about attitudes towards military operations against Israeli targets
- 2 responses about attitudes towards suicide bombings against Israeli civilians
- 4 responses about optimism/pessimism regarding a peaceful settlement with Israel
- 3 responses about attitudes towards the Oslo peace process
- 2 responses about attitudes towards the 2nd *Intifada*
- 3 responses about whether the current peace process is alive, dead, or unclear.

Additionally, we add some 4 responses from surveys by Palestinian Center for Policy and Survey Research (PCPSR) (N.d) that record support for armed attacks against

- Israel generally
- Israeli civilians
- Israeli soldiers
- Israeli settlers in the West Bank.

Many of these variable are correlated. We avoid perfect correlations between combinations of factors by the virtue of "don't know," "no answer," and similar non-answers. The high correlations across answers and across questions provides us with strong evidence that these responses can be reduced into a latent measure. The factor weights are reported in Table C.1. All of the surveys load in the expected way where surveys that should correlate with approval towards violent tactics load positively and surveys that correlate with approval of peaceful tactics and negotiated settlement load negatively. Additionally, we see that the latent support for violence reaches a minimum during Oslo and a maximum during the second *Intifada*. Overall, this give us strong assurance that the latent variable captures the Palestinian public's underlying attitudes toward violence against Israel at any given month.

Table C.1: ML estimates for latent support for violence

Variable	Est.
% Supporting two-state solution	-0.14
% Supporting a one shared state solution	0.02
% Supporting a one Islamic state solution	0.14
% Saying there is no solution	0.13
% Supporting a peace process	-0.16
% Opposing a peace process	0.16
% Supporting military action against Israel	0.16
% Opposing military action against Israel	-0.16
% Supporting suicide bombings	0.16
% Opposing suicide bombings	-0.16
% Very optimistic about peace	-0.15
% Optimistic about peace	-0.15
% Pessimistic about peace	0.12
% Very pessimistic about peace	0.16
% Strongly support Oslo	-0.11
% Support Oslo	-0.10
% Oppose Oslo	0.13
% Support the *Intifada*	0.09
% Oppose the *Intifada*	-0.08
% Who think peace is dead	0.09
% Who think the peace process is stalled	-0.02
% Who think peace is alive	-0.12
% Support armed attacks generally	0.16
% Support armed attacks against civilians	0.15
% Support armed attacks against soldiers	0.12
% Support armed attacks against settlers	0.11

For the unemployment latent variable we combine four survey responses

1. % of respondents telling pollster they are unemployed to Jerusalem Media and Communication Centre (JMCC) (N.d) pollsters

2. % of respondents telling pollster they are unemployed to Palestinian Center for Policy and Survey Research (PCPSR) (N.d) pollsters

3. Estimated true unemployment by Palestinian Center for Policy and Survey Research (PCPSR) (N.d)

4. Unemployment rate reported in Labor Force Surveys published by the Palestinian Central Bureau of Statistics (PCBS) (N.d)

These results are reported in Table C.2. Here we see that all the unemployment rates load onto the latent dimension in the same direction, but with different weightings.

Table C.2: ML estimates for latent unemployment conditions

Variable	Est.
Self reported unemployment rate (JMCC)	0.15
Self reported unemployment rate (PCPSR)	0.16
Estimated unemployment rate (PCPSR)	0.06
Estimated unemployment rate (PCBS)	0.17

With these measures in hand, we can check the robustness of the first stage relations. As seen in Table C.3 (which should be compared to Table 1 in the main text), the effect of attacks on relative popularity are similar across all specifications, and the new control variables are all related to popularity in expected directions. As the economy gets worse (changes in latent unemployment increase), the balance of support tends to shift toward Hamas, although this not significant in every model. Further as changes in the latent support for violence among the population increase, support moves towards Hamas. Finally, during the Second Intifada support was stronger for Hamas relative to other stages in the conflict.

Looking at the final model, we see that switch from a binary measure of terrorism to a count of the number of terrorist attacks in month t does not change the main result. This consistency across measurement is reassuring as we do not want the first stage to be dependent on the level of aggregation used.[3] Additionally, we see that the decision to include the interaction between the lagged state and the actions does not affect the main result either. Overall, these various robustness checks of the AR(1) model provide confidence in our using it in the first stage of the analysis.

D Standard errors and sensitivity analysis

In this appendix, we describe the standard errors reported for the two-step CMLE estimates and consider how sensitive the structural estimates of β and κ are to the first-stage estimates γ. The standard result on two-step estimation involving a maximum likelihood estimator comes from Murphy and Topel (1985). Aguirregabiria and Mira (2007) use this result to describe the asympotic distribution of the two-step pseudo-likelihood estimator from Hotz and Miller (1993) and we follow the same logic here. Specifically, let $\theta_2 = (\beta, \kappa, v)$

[3]The only circumstance where we fail to reject the null of equally effective groups is in Model 7, but only when $\tilde{s}^t < -11$. So if Hamas is *extremely* popular, the two groups might be equally effective at moving the states space according to one of the seven models.

Table C.3: Robustness checks for the first-stage model

	Dependent variable:				
	Δ State				
	(1)	(2)	(3)	(4)	(5)
Hamas attacks (binary)	−0.33*	−0.31*	−0.36*	−0.34*	
	(0.04)	(0.04)	(0.03)	(0.03)	
Fatah attacks (binary)	1.54*	1.63*	1.60*	1.60*	
	(0.08)	(0.04)	(0.04)	(0.04)	
Hamas attacks (count)					−0.05*
					(0.01)
Fatah attacks (count)					0.64*
					(0.06)
Δ Lag state	0.19*	0.16*	0.12*	0.09*	0.02
	(0.03)	(0.03)	(0.03)	(0.03)	(0.05)
Δ Lag unemployment		−0.30*	−0.15	−0.10	−0.14
		(0.13)	(0.10)	(0.07)	(0.08)
Δ Lag support for violence			−0.31*	−0.35*	−0.27*
			(0.08)	(0.07)	(0.07)
Second Intifada				−0.15*	−0.21*
				(0.05)	(0.06)
Constant	−0.01	−0.01	0.01	0.03	0.001
	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)
Interactions	No	Yes	Yes	Yes	Yes
T	298	298	298	298	298
adj. R^2	0.805	0.831	0.864	0.876	0.475
$\hat{\sigma}$	0.207	0.193	0.173	0.165	0.339

Note: Newey-West standard errors in parenthesis.

be the set of parameters estimated in the second stage, then the two-step correction gives the variance of $\hat{\theta}_2$ as

$$\widehat{\text{var}}(\hat{\theta}_2) = \hat{\Sigma}_{\theta_2} + \hat{\Sigma}_{\theta_2}\left(\hat{\Omega}\hat{\Sigma}_{\gamma}\hat{\Omega}^{\mathrm{T}}\right)\hat{\Sigma}_{\theta_2},$$

as described in Aguirregabiria and Mira (2007, Proposition 1). Here $\hat{\Sigma}_{\theta_2}$ is the ordinary CMLE covariance matrix described by Silvey (1959) which is

$$\hat{\Sigma}_{\theta_2} = \begin{bmatrix} H_{\theta_2}L(\hat{v}|Y) + J_{\theta_2}\mathcal{V}(\hat{\theta}_2;\hat{\gamma})^{\mathrm{T}} J_{\theta_2}\mathcal{V}(\hat{\theta}_2;\hat{\gamma}) & -J_{\theta_2}\mathcal{V}(\hat{\theta}_2;\hat{\gamma})^{\mathrm{T}} \\ -J_{\theta_2}\mathcal{V}(\hat{\theta}_2;\hat{\gamma}) & \mathbf{0} \end{bmatrix}^{-1},$$

where H_x and J_x respectively denote the Hessian and the Jacobian of a function with respect to x.

The remaining two matrices are related to the first-stage estimates $\hat{\gamma}$. The matrix $\hat{\Omega}$ describes how the CMLE's Lagrangian changes with respect to γ and θ_2 and is given by

$$\hat{\Omega} = \begin{bmatrix} J_{\theta_2}L^*(\hat{v}|Y,\hat{\gamma})^{\mathrm{T}} J_{\gamma}L^*(\hat{v}|Y,\hat{\gamma}) + J_{\theta_2}\mathcal{V}(\hat{\theta}_2;\hat{\gamma})^{\mathrm{T}} J_{\gamma}\mathcal{V}(\hat{\theta}_2;\hat{\gamma}) \\ \mathbf{0} \end{bmatrix}.$$

Here, L^* is the vector-valued log-likelihood of the entire data:

$$L^*(v|Y,\gamma) = \left(\log P(a_H^t; s^t, v_H) + \log P(a_H^t; s^t, v_H) + \log f(s^t; a^{t-1}, s^{t-1}, \gamma)\right)_{t=1}^{T}.$$

Note that for a given estimate of γ, the transition probabilities are fixed and so using either $L(v|Y)$ or $\sum_{t=1}^{T} L^*(v|Y,\hat{\gamma})$ as the CMLE's objective function will return the same constrained maximum likelihood estimates of θ_2.[4] The final piece is the first-stage covariance matrix $\hat{\Sigma}_{\gamma}$, which we construct using a parametric bootstrap.

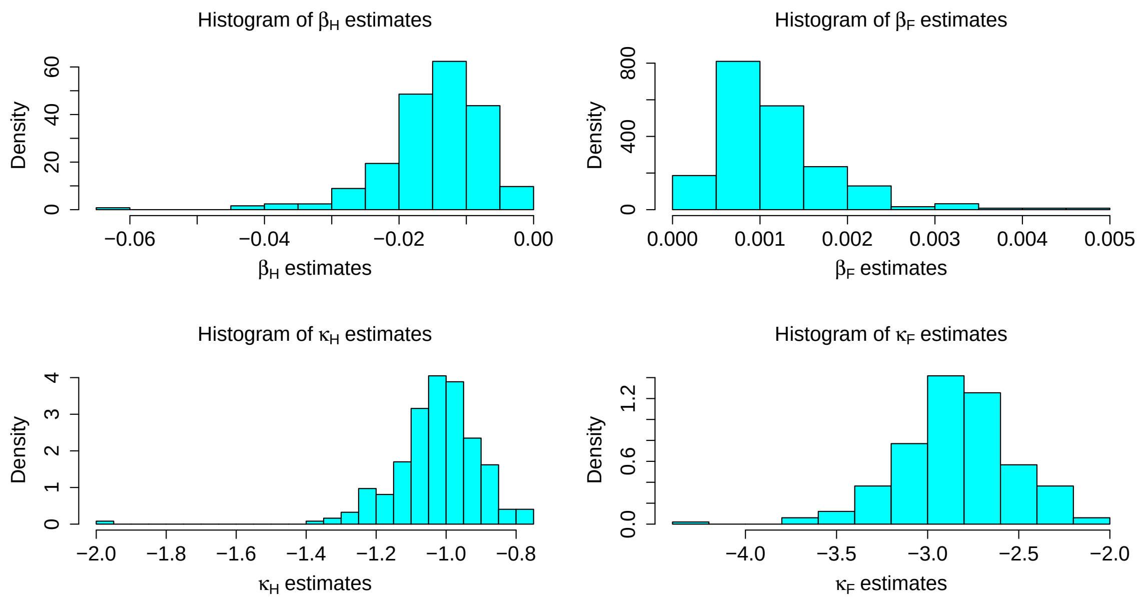
Furthermore, we also want to know how sensitive the second-stage estimates are to changes in the first-stage estimates. To consider this we conduct a sensitivity analysis where for each iteration $b = 1, 2, \ldots, B$ we conduct the following exercise:

1. Draw new values for continuous state variable $\tilde{s}_b^t$ using the parameters from the first-stage model
2. Re-fit the first-stage model to produce new estimates $\hat{\gamma}_b$.
3. Re-fit the second-stage model using $\hat{\gamma}_b$, s^t, and a^t. Save $\hat{\beta}_b$, $\hat{\kappa}_b$.
4. Repeat steps 1-3, B times.

This analysis allows us to consider how much variation there is in the second-stage estimates under a range of plausible values of $\hat{\gamma}$. If the analysis is highly sensitive to the first-

[4]For completeness in L^* we impose $f(s^1; a^0, s^0, \gamma) = 1$ or $\log(f(s^1; a^0, s^0, \gamma)) = 0$

Figure D.1: Sensitivity of structural estimates to first-stage results



stage values, then we should see a large range of second-stage values. We are particularly interested in seeing how frequently the signs on are estimates change. We run the analysis for 500 iterations and the results are reported as histograms in Figure D.1.

There are few points of interest in Figure D.1. Most importantly the signs on the estimates never change over the course of this experiment and are always in the expected direction. Interestingly, there appears to be a long tail on the distributions of some estimates, but it is always in the direction away from zero. Overall, these histograms all peak around the point estimates reported in Table 2, which is a good sign that the exact estimates of γ used in the main model are not driving the main results.

E Robustness to different time spans

In this appendix, we consider three different alternative time frames for the main model. We consider these different time spans because they all represent plausible break points in the Fatah-Hamas relationship, such that the underlying competition between the groups may have changed. As such we want to be sure that our point estimates are robust to the exclusion of some of these later observations. Specifically, we consider the following three cutpoints:

1. The formation of the Fatah-Hamas unity government (end the data in 2015),
2. The signing of the first Fatah-Hamas unity agreement (end the data in 2011),
3. Hamas wins the 2006 legislative elections (end the data in 2005).

The results of these short-T robustness checks are presented in Table E.1 along with a reproduction of the main model (1994-2018) for comparison. Overall we see very few changes across the four models. In general the estimates appear to shrink as the time frame shortens, but the differences are relatively minor. The only notable change is in Hamas's cost of terrorism, where terrorism is about half as costly if we only consider the period before the 2006 election.

Table E.1: Robustness to different time periods

	1994–2018	1994–2015	1994–2011	1994–2005
β_H	−0.009	−0.008	−0.009	−0.008
	(0.006)	(0.005)	(0.006)	(0.003)
β_F	0.0007	0.0005	0.0007	0.0004
	(0.0003)	(0.0003)	(0.0004)	(0.0003)
κ_H	−0.952	−0.884	−0.620	−0.497
	(0.284)	(0.264)	(0.257)	(0.258)
κ_F	−2.572	−2.548	−2.390	−2.414
	(0.349)	(0.336)	(0.354)	(0.306)
T	300	252	204	132
Log-Likelihood	−281.90	−240.56	−206.17	−115.87

F Choice of discount factor

In this appendix we consider how our choice of discount factor affects our results. Specifically we fix δ to different values in the interval $[0, 1)$ and then reestimate the second-stage model at each value. We can then compare how the likelihood, payoff parameter estimates, and equilibrium attack probabilities vary at different values of the discount factor. Note that the transition parameter estimates in Table 1 will not depend on the the discount factor, so our choice of δ only affects our second-stage estimates.

Table F.1 shows the log-likelihood of the second-stage model under different fixed values of the discount factor δ. Over 16 values the best fit (maximum likelihood value) was found at $\delta = 0.999$ although the model did not converge with $\delta = 0.9999$. As such we use this value in both the main model specification and the numerical examples.

Table F.1 suggests that $\delta = 0.999$ should be prioritized for the baseline analysis, but how sensitive are the results to this choice? Table F.2 displays the payoff parameter estimates given the discount factors above 0.8. For the most part we see that the estimates are relatively stable. The costs are fairly constant across specifications and the value parameters β tend to decrease in magnitude as δ gets closer to one. The one outlier here is $\delta = 0.95$ where the signs on β_H and β_F flip and κ_H is notably smaller. These results appear to

Table F.1: Discount factors and model fit

δ	Log-Likelihood
0	-286.70
0.1	-282.77
0.2	-282.88
0.3	-283.01
0.4	-283.16
0.5	-283.36
0.6	-283.63
0.7	-284.02
0.8	-284.67
0.9	-285.93
0.925	-286.39
0.95	-286.69
0.975†	-285.43
0.99	-283.72
0.999*	-281.90
0.9999†	-284.25

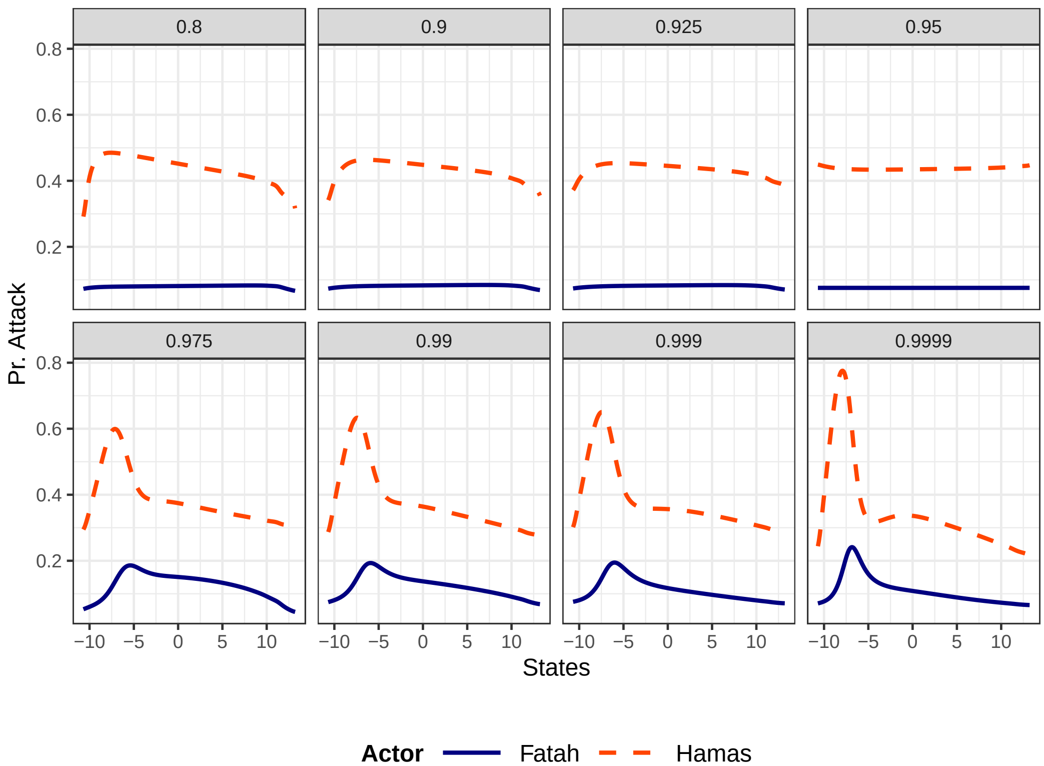
Note: *Best fitting model; †Model failed to converge.

Table F.2: Estimates at different discount factors

δ	0.8	0.9	0.925	0.95	0.975	0.99	0.999	0.9999
β_H	-0.66	-0.17	-0.08	0.01	-0.04	-0.02	-0.01	-0.01
	(0.42)	(0.16)	(0.12)	(0.07)	(0.02)	(0.01)	(0.01)	(0.01)
β_F	0.03	0.01	0.01	-0.00	0.02	0.00	0.00	0.00
	(0.13)	(0.05)	(0.04)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)
κ_H	-1.06	-0.73	-0.57	-0.20	-0.92	-1.03	-0.95	-1.34
	(0.48)	(0.41)	(0.39)	(0.36)	(0.27)	(0.29)	(0.28)	(0.35)
κ_F	-2.65	-2.61	-2.59	-2.50	-3.08	-2.62	-2.57	-2.65
	(0.80)	(0.73)	(0.70)	(0.66)	(0.52)	(0.40)	(0.35)	(0.33)
LL	-284.67	-285.93	-286.39	-286.69	$-285.43^{\dagger}$	-283.72	-281.90	$-284.25^{\dagger}$

Note: †Model failed to converge. Two-step corrected standard error in parenthesis

Figure F.1: Discount factors and equilibrium attack probabilities



Notes: Graphs of the estimated equilibrium choice probabilities at eight different discount factors. Setting $\delta = 0.999$ is identical to Figure 3.

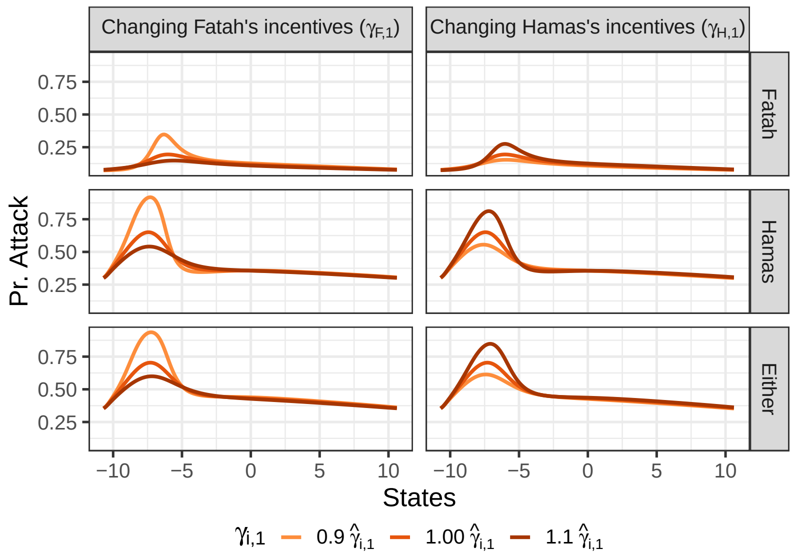
mostly be an anomaly, and we note that this model has the worst model fit among this set (and indeed it performs worse than every non-zero value of δ we tried).

In a similar vein, Figure F.1 shows how the choice of discount factor affects the estimated equilibrium attack probabilities. Notice that when $\delta = 0.999$, the probabilities are identical to those in Figure 3 from the baseline analysis. The graphs in Figure F.1 demonstrate that for $\delta \in \{0.975, 0.99, 0.999\}$ the attack probabilities are almost identical. Comparing across these three estimated models, the average difference in attack probabilities is 1.5 percentage points for Hamas and 1.6 for Fatah across the states. The maximum difference is 7 percentage points for Hamas and 4 for Fatah. Technically, at a tolerance of 0.1 (i.e., 10 percentage points), the equilibrium choice probabilities are identical across specifications of the discount factor. Substantively, the predictions in the estimated model are roughly invariant to choosing a discount factor between $[0.975, 0.999]$. These similarities likely arise because discount factors are not identified in dynamic discrete choice models generally (Magnac and Thesmar 2002). Even with suitable exclusion restrictions, the discount factor is not even guaranteed to be point identified (Abbring and Daljord 2020). When δ is not identified, we would expect several values of δ to return essentially identical equilibrium attack probabilities, which is what we see in Figure F.1.

G Changing effectiveness and costs of terrorism

In this, appendix we consider how changes to group i's effectiveness ($\gamma_{i,1}$) or costs of terrorism (κ_i) affect the likelihood of terrorism. These results are presented in Figures G.1 and G.2, respectively. Overall, these results match the results from the main text and continue to show that changes that make Fatah more competitive tend to reduce violence, but changes that make Hamas more competitive tend to increase violence.

Figure G.1: Effectiveness of attacks and its relationship to terrorism



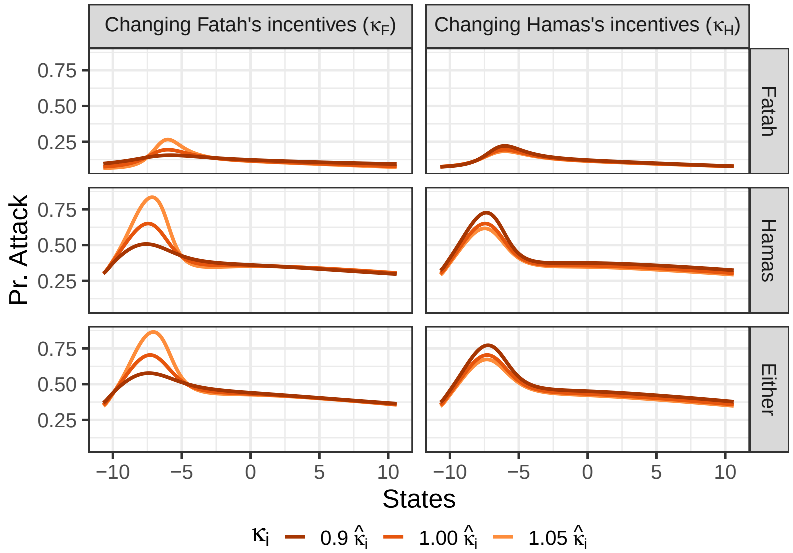
Notes: We increase and decrease $\gamma_{i,1}$ from its estimated by 10% for each group. The horizontal axis is the relative popularity levels (smaller values are more favorable to Hamas) and the vertical axis is the probability of an attack. Columns correspond to which group's incentives are changing in the counterfactuals, and rows denote which group's probability of attacking is graphed. Darker reds denote stronger incentives to compete, $|\gamma_{i,1}|$ becoming larger. All other parameters are held constant at their estimated values.

H Additional counterfactual: Shocks to public opinion

We can also use the model to study how the likelihood of terrorist attacks change after a group experience a upswing in public opinion. For example, Israeli military operations in the Gaza strip tend to create "surges" in public support for Hamas (Taylor 2014; Wike 2009). We might consider an unexpected surge in public support as making a group stronger in the contest of public opinion.

Figure 3 provides some insight into this question. Namely, both groups are more likely to use terrorism when Hamas is moderately favored, i.e., when the state variable is between -8 and -4, roughly one standard deviation below the mean of our relative popularity measure. In contrast, when Hamas is very popular ($s < -8$) or when Fatah is more favored ($s > 0$), there is relatively less violence. Based on this, we suspect that shocks in favor of Fatah will result in an immediate decrease in violence, while shocks in favor of Hamas will have the

Figure G.2: Costs of attacking and their relationship to terrorism



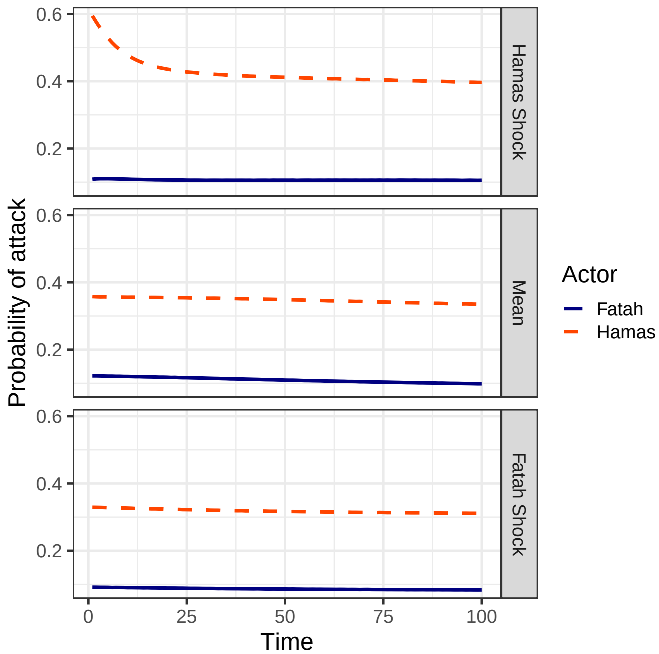
Notes: We decrease κ_i from its estimated by 10% for each group and increase it by 5%—increasing κ_F by more makes the equilibrium unstable. The horizontal axis is the relative popularity levels (smaller values are more favorable to Hamas) and the vertical axis is the probability of an attack. Columns correspond to which group's incentives are changing in the counterfactuals, and rows denote which group's probability of attacking is graphed. Darker reds denote stronger incentives to compete, κ_i moving closer to zero. All other parameters are held constant at their estimated values.

opposite effect. The probabilities in Figure 3 are the per-period probabilities of terrorist attacks conditional on the current state, however. As such, they cannot answer how long the effects persist as relative popularity continues to evolve after the initial shock to public opinion.

To answer these questions more fully, we use the estimated model to repeatedly simulate the equilibrium path of play when the interaction begins at one of three popularity levels: the mean level ($s \approx -0.89$), one standard deviation above the mean ($s \approx 6.51$), and one standard deviation below the mean ($s \approx -8.39$). We interpret the latter two states as those that are pro-Fatah and pro-Hamas, respectively. In the data, the mean level appears in 2009, the summer after the Gaza War (Operation Cast Lead). The pro-Hamas state appears in the summer of 2004 during which Hamas launches several suicide bombings on Israeli buses. The pro-Fatah state appears in the fall of 1999, a year before the Second Intifada. Using the simulations, we can compute the probability that group i attacks after $t \geq 0$ months have passed given the initial popularity level. These probabilities are reported in Figure H.1, where the panels labeled Fatah and Hamas shocks correspond to the estimated attack probabilities when the equilibrium path begins in the state one standard deviation above and below the mean, respectively.

To then quantify the effect of a pro-Fatah popularity shock, we subtract the probability

Figure H.1: Attack probabilities over time at three different initial values.



Notes: The horizontal axis denotes time measured in periods/months, and the vertical axis is the probability of attack. The panels correspond to initial relative popularity level at the beginning of the interaction: middle panel has the mean level, top panel has the mean plus one standard deviation level (i.e., Fatah is favored), and bottom panel has the mean minus one standard deviation level (i.e., Hamas is favored).

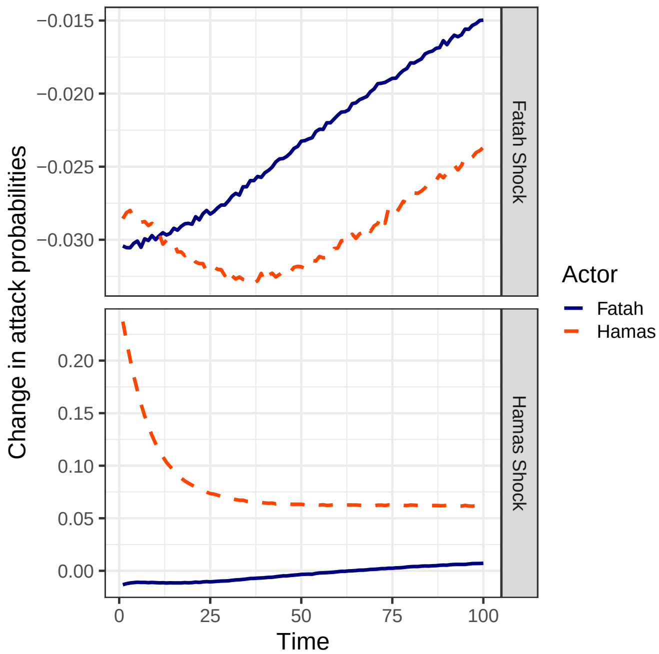
that group i attacks at time t when the initial state is at the mean from the same probability when the initial state is one standard deviation above the mean. If this difference is positive it means that pro-Fatah shocks lead to more terrorist attacks from group i at time t. If it is negative, then pro-Fatah shocks lead to less violence. We repeat this process to quantify the effects of pro-Hamas shocks, but use one standard deviation below the mean as the relevant initial state.

Figure H.2 summarizes the results. As expected, pro-Hamas shocks substantially increase the rate of Hamas violence. Initially, the probability of Hamas attacking increases by 25 percentage points over the counterfactual scenario where the shock does not happen. This large effect tapers off over time but remains larger than 5 percentage points after roughly eight years.[5] In contrast, Fatah's use of terrorism remains largely unaffected by pro-Hamas public opinion jumps. The effects of pro-Fatah shocks are more complicated. Generally, once public opinion becomes more favorable to Fatah, the probability of terrorist attacks from either group decreases by roughly 3–4 percentage points. For Fatah, these

[5] One concern is that our shocks to public opinion are unrealistic in magnitude, but our relative popularity measure reports such shifts. Over the first six months of the time series there is a large rapid swing in favor of Fatah that more than matches the magnitude of the shock considered here. See also the time frame from 2000-2001.

effects diminish over time. For Hamas, however, their propensity to use terrorism first decreases then increases over time. In fact, following a pro-Fatah shock, Hamas's probability of attacking reaches a minimum after 30 months. After roughly eight years, the effects of a pro-Fatah jump in public opinion persist: Hamas's probability of attacking is depressed by 2.5 percentage points.

Figure H.2: Public opinion shocks and their effects on terrorism



Notes: Top panel is the difference between attack probabilities after a pro-Fatah popularity shock relative to the mean popularity level. Bottom panel is the difference between attack probabilities after a pro-Hamas shock. Positive values indicate that the shock increases violence for a group and negative values indicate that the shock decreases violence for the respective group. Time is measured in periods/months.

Substantively, these results match those from Section 6. When Hamas becomes more competitive in the sense that they have relatively large public support, violence increases although only Hamas becomes more likely to attack. When Fatah becomes more popular, however, violence decreases. These effects persist even after several years. This result matches the observed record, with the so-called "Oslo Lull" (1993-2000) emerging after a sudden rise in Fatah's popularity (Dugan and Chenoweth 2012).

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