# Distributions and some linear algebra

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## 1 Distributions

**Normal** The PDF of the normal distribution is given as

$$\phi(x|\mu, \sigma^2) = \frac{1}{\sqrt{\sigma^2 2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right). \tag{1}$$

While Eq. 1 is a nice expression, CDF does not have a closed form solution and is given as

$$\Phi(x|\mu, \sigma^2) = \int_{-\infty}^{x} \phi(y|\mu, \sigma^2) dy.$$

**Logistic** The standard logistic CDF is given as

$$\Lambda(x) = \frac{1}{1+e^{-x}}.$$

We will take the derivative to find the pdf.

$$\begin{aligned} \lambda(x) &= \frac{d}{dx}\Lambda(x) \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \end{aligned}$$

## 2 Matrices and linear algebra

Example: prove that the transpose of $AB$ is equal to $B'A'$.

**Claim 1.** *Let $A$ be a matrix of dimensions $m \times n$ and let $B$ be a matrix of size $n \times p$, then $(AB)' = B'A'$.*

*Proof.* Let $AB = C$ and let $B'A' = D$ we will show that $C' = D$. Consider element $c_{ji} \in C$, we need to show that $c'_{ji} = d_{ji}$.To see this consider

$$c'_{ji} = c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} = \sum_{k=1}^{n} a'_{ki}b'_{jk} = \sum_{k=1}^{n} b'_{jk}a'_{ki} = d_{ji}.$$

This is what we had to show. □

In a typical game of matching pennies the expected utilities for each player and each action are given by

$$\begin{aligned} U_1(H|\sigma_2) &= \sigma_2 - (1-\sigma_2) \\ U_1(T|\sigma_2) &= -\sigma_2 + (1-\sigma_2) \\ U_2(H|\sigma_1) &= -\sigma_1 + (1-\sigma_1) \\ U_2(T|\sigma_1) &= \sigma_1 - (1-\sigma_1). \end{aligned}$$

To find the mixed strategy equilibrium we setup a system of equations

$$\begin{aligned}\sigma_2 - (1-\sigma_2) - (-\sigma_2 + (1-\sigma_2)) &= 0\\ \sigma_1 - (1-\sigma_1) - (-\sigma_1 + (1-\sigma_1)) &= 0,\end{aligned}$$

which simplify to

$$\begin{aligned}4\sigma_2 &= 2\\ 4\sigma_1 &= 2.\end{aligned}$$

Now obviously we can just solve this, but if it was a hard system of equations we would write this in matrix form and solve it that way. In matrix form we get

$$\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

Solving this we get our solution $\sigma = \left(\frac{1}{2}, \frac{1}{2}\right)$.