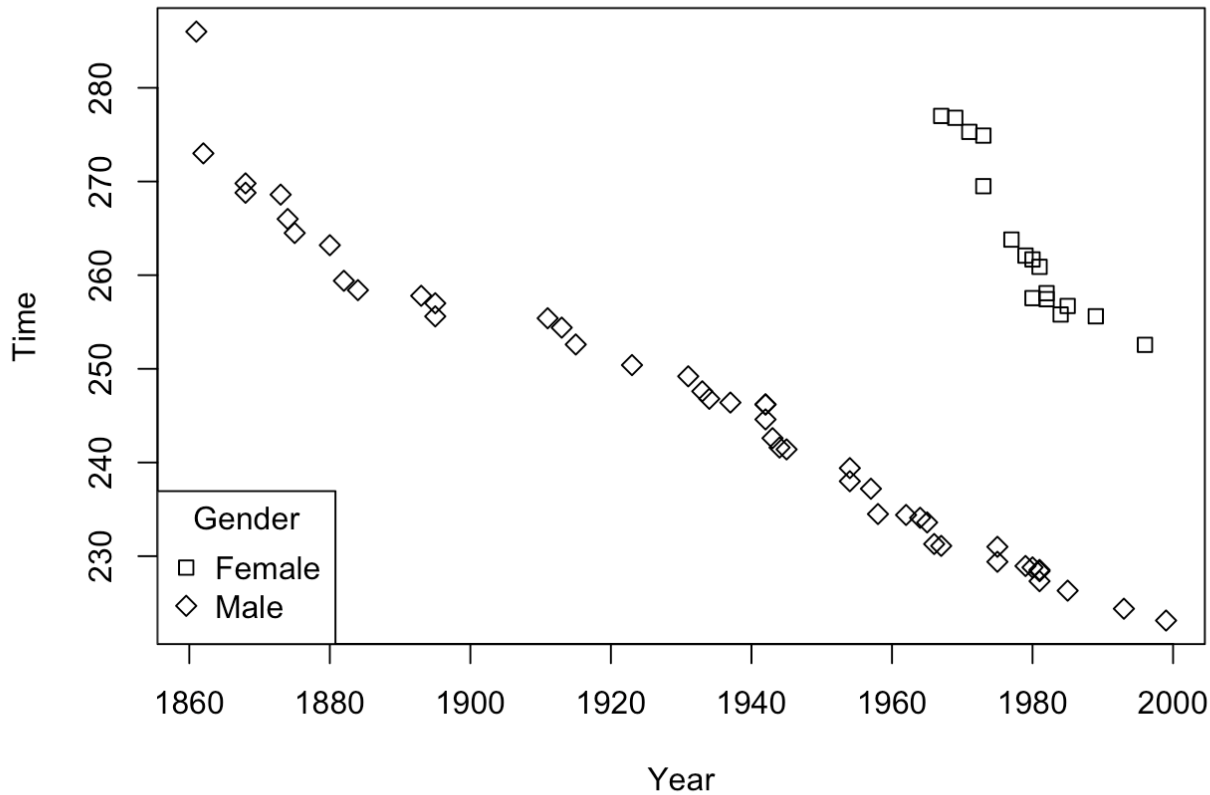
7.12

7.12.1

The graph below shows the male record mile time since 1861 and the female record time since 1967. It appears that the female mile time record has decreased at a faster rate than the male mile time record.




```
attach(mile)
mile$symb <- ifelse(mile$Gender == 'Female',0,5)
attach(mile)
plot(Year,Time, pch = mile$symb)
legend("bottomleft", title="Gender", legend = c("Female", "Male"),
    pch = c(0,5))
```

7.12.2

For every Olympic Games since 1861, the average decrease in mile time has been .3662 for males. For females, for every Olympic Games since 1967, the average decrease in mile time has been 1.034.

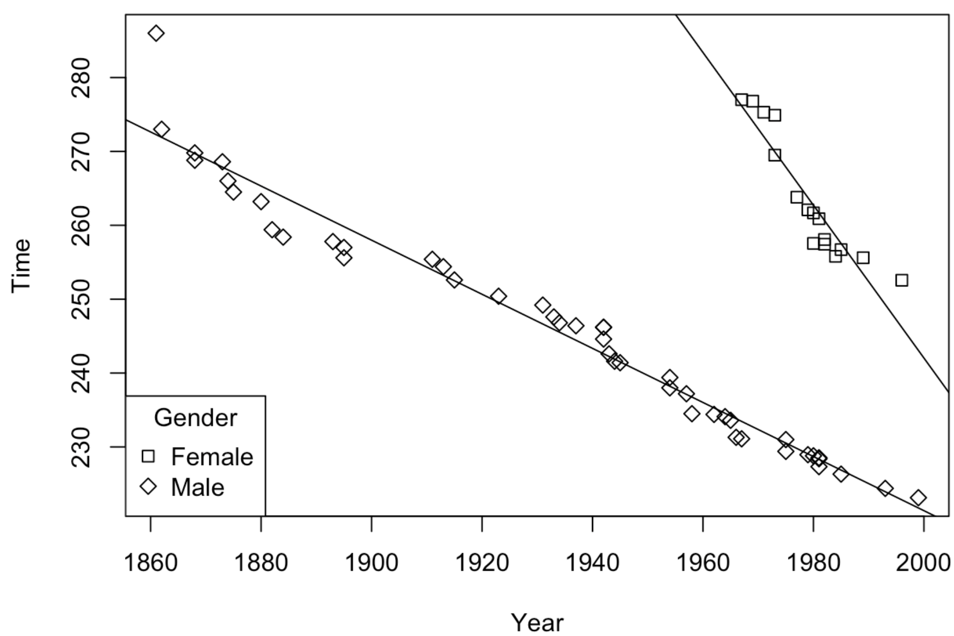
```
> m

Call:
lm(formula = Time ~ Year, data = male)

Coefficients:
(Intercept)         Year
   953.7470      -0.3662
> f

Call:
lm(formula = Time ~ Year, data = female)

Coefficients:
(Intercept)         Year
   2309.425       -1.034
```

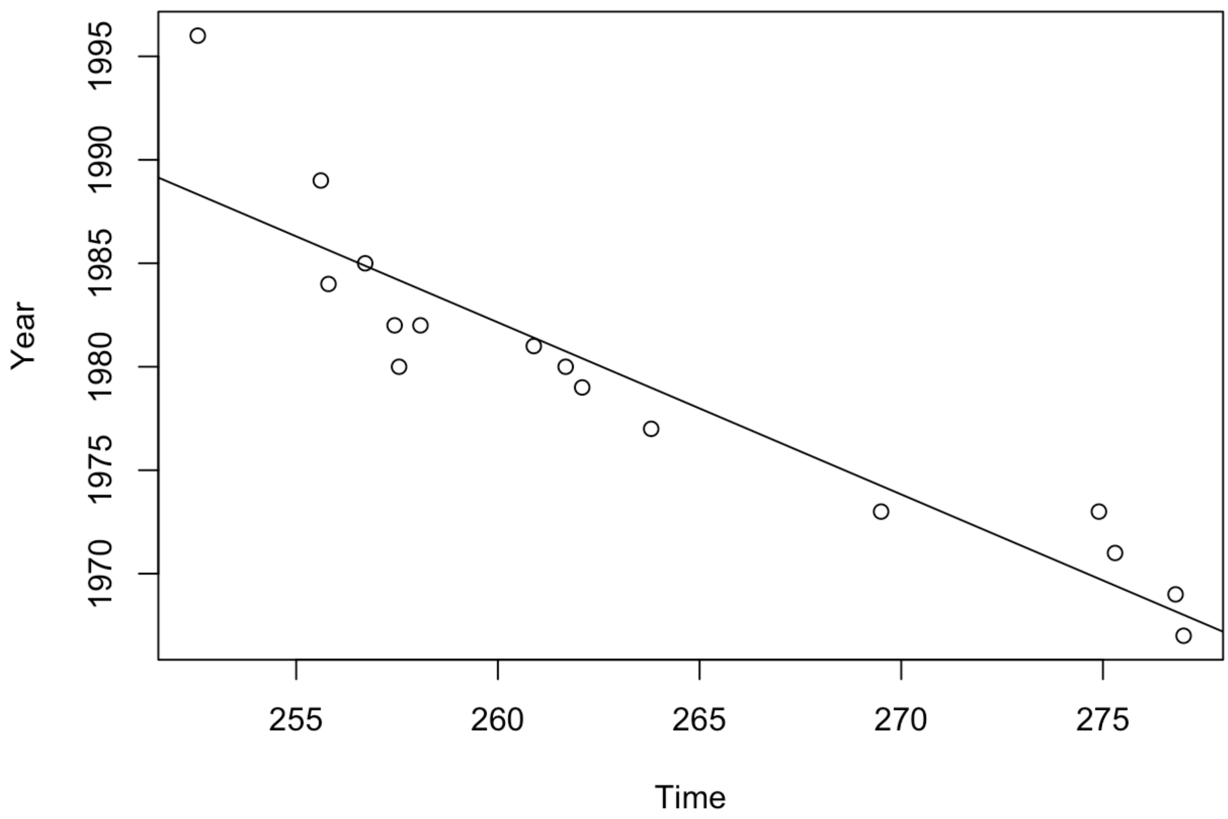


male = subset(mile, Gender == 'Male')
female = subset(mile, Gender == 'Female')
m = lm(Time~Year, male)

f = lm(Time~Year, female)
abline(m)
abline(f)

7.12.3
The predicted year that the female record for the mile reaches 240 seconds is calculated to be 1999 (rounded up from 1998.776
inv = lm(Year~Time,female)
inv
plot(Year~Time)
abline(inv)
predict(inv, newdata=data.frame(Time=c(240)))

```
> plot(Year~Time)
> abline(inv)
> predict(inv, newdata=data.frame(Time=c(240)))
       1
1998.776
```

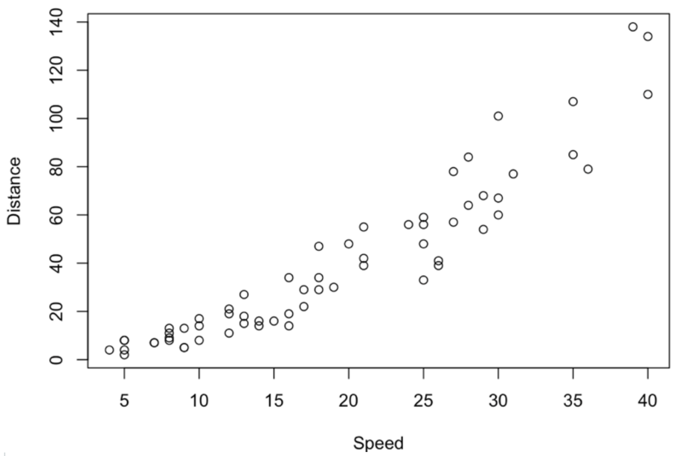


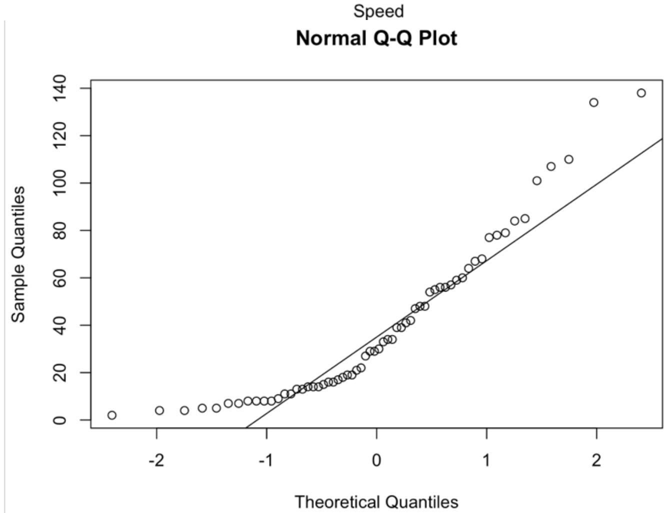
Using the delta method, we get a predicted standard error of 2.23
grad = c(1,240)
vb <- vcov(inv)
vb
vG <- t(grad) %% vb %*% grad*

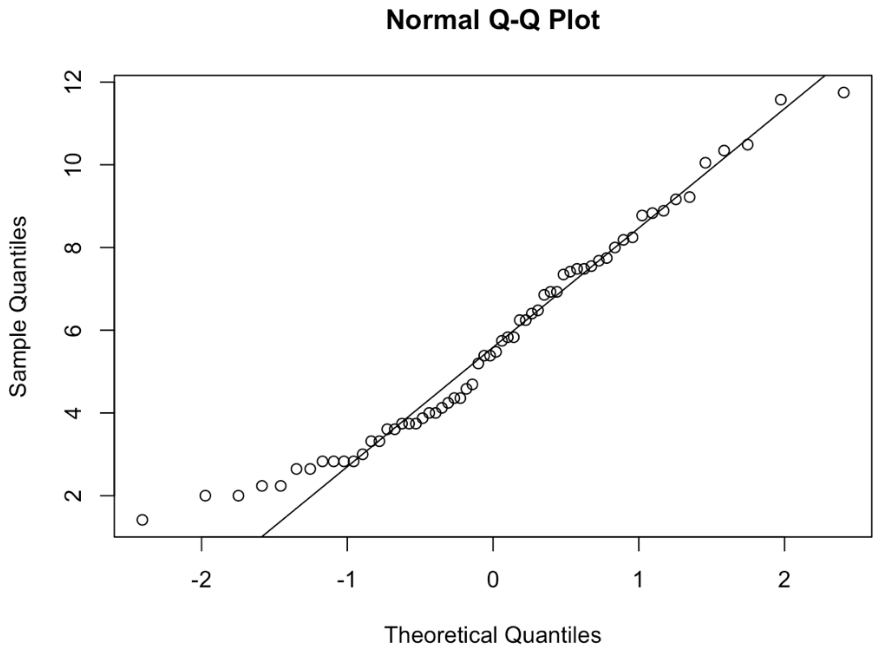
sqrt(vG)

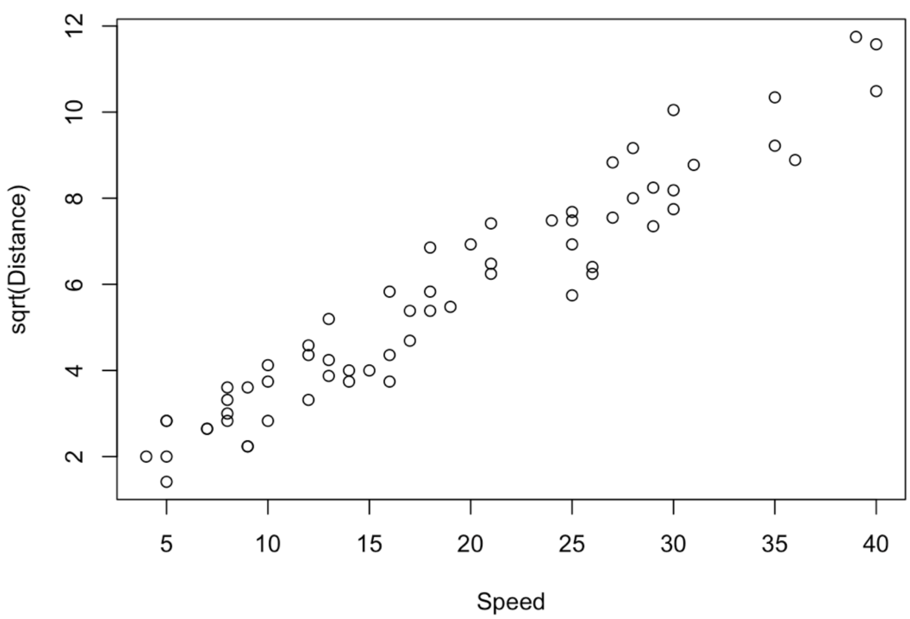
```
> sqrt(vG)
         [,1]
[1,] 2.233268
```

8.2
8.2.1
Taking the square root seems to appropriately linearize the regression by way of scatter plots and qq plots
plot(Distance~Speed)
qqnorm(Distance)
qqline(Distance)
qqnorm(sqrt(Distance))
qqline(sqrt(Distance))
plot(sqrt(Distance)~Speed)






Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
2
4
6
8
10
12
-2
-1
0
1
2


sqrt(Distance)
Speed
2
4
6
8
10
12
5
10
15
20
25
30
35
40

8.2.2

The likelihood ratios are all very high except for lambda=.5, showing that none of these transformations are sufficient except for .5. The p-values also indicate to reject the null hypothesis of using the model with respective lambdas in favor of the original for each transformation except for .5.

summary(a3 <- powerTransform(Distance~Speed+Speed2),stopping)
testTransform(a3, c(1))
testTransform(a3, c(0))
testTransform(a3, c(-1))
testTransform(a3, c(1/2))

```
                           LRT df        pval
LR test, lambda = (1) 60.09488  1 8.9928e-15
> testTransform(a3, c(0))
                           LRT df        pval
LR test, lambda = (0) 33.67953  1 6.4981e-09
> testTransform(a3, c(-1))
                            LRT df        pval
LR test, lambda = (-1) 214.5843  1 < 2.22e-16
> testTransform(a3, c(1/2))
                             LRT df    pval
LR test, lambda = (0.5) 1.641002  1 0.20019
```

8.2.3

The absolute likelihood ratio for fm2 is smaller than for fm1, showing that including Speed^2 matches the data better.

fm1 <- lm(Distance~Speed, stopping)
fm2 <- lm(Distance~Speed+Speed2,stopping)
logLik(fm2)
logLik(fm1)

```
> logLik(fm2)
'log Lik.' -228.7425 (df=4)
> logLik(fm1)
'log Lik.' -239.8153 (df=3)
```

9.6.1

$U = [1\ 0\ 0\ \ldots\ 0]^T$

The hat matrix maps the vector of responses to the vector of fitted responses. Since U is a one-dimensional matrix of Nx1, each of the fitted responses will correlate with H_{1j} where j is the jth response.

9.6.2

Since we have already shown that the vector of fitted values from the regression of U on X and we know that the vector of residuals is Y – Yhat or in this case $U_j - h_{ij}$, the first residual will be $U_1 - h_{11} = 1 - h_{11}$ and the rest of the residuals will be $0 - h_{1j}$ or $-h_{1j}$ since every element in U except for U_1 is 0.