2023.9.15 P8 T1-3.6.7.9.

1. (1) $\{x \mid x \neq 1, x \in R\}$

(2) $\{x \mid x \geq -2$ 且 $x \neq 1\}$

(3) $[-4, -\pi] \cup [0, \pi]$

(4) $[-1, 2]$

3. $f(1) = 1 - 3 + 2 = 0$

$f(x-1) = (x-1)^2 - 3(x-1) + 2$

$= x^2 + 1 - 2x - 3x + 3 + 2$

$= x^2 - 5x + 6$

6. $f(-2) = -2 - 1 = -2$

$f(0) = 0 + 1 = 1$

$f(1) = 1 + 1 = 2$

$f(x-1) = \begin{cases} x - 2, & -1 \leq x < 1 \\ x, & 1 \leq x \leq 3 \end{cases}$

7. (1) $y = \frac{x^2 - 4}{x + 2}$

(2) $y = 1 - |x|$

(3)

6. $f(2x+1) = \begin{cases} 2x + 3, & x \leq -1 \\ -2x - 1, & -1 \leq x \leq 0 \\ 2x - 1, & x > 0 \end{cases}$

P24. T1.2

1. (1) $|x + 3| x - 2 \leq 1$

(2) $-1 \leq 3x - 1 \leq 1$

(3) $-1 \leq \frac{2x-1}{3} \leq 1$

4. $f(x) = f(x+1)$ 赋值的思想

$f(2(x+\frac{1}{2}))$

即 $f(2(x+\frac{1}{2})) = f(2x+1)$

∴ 周期为 $\frac{1}{2}$

5.

6. 当 $x \in (0, 1)$

当 $x \in (1, +\infty)$, $f(x) < 1$

∴ $f(x) = \frac{1}{x}$ … 無界

P17. T1.3.6.

1. (1) $y = \frac{1-x}{1+x}$

$1 - x = y + xy$

$1 - y = x(1 + y)$

反: $x = \frac{1-y}{1+y}$ $(y \neq -1)$

$y = \frac{1-x}{1+x}$

(2) $y = 2^{2x+1}$

$\log_2 y = 2x + 1$

$x = \frac{1}{2}(\log_2 y - 1)$

$x = \frac{1}{2}\log_2 y - \frac{1}{2}$ → $y = \frac{1}{2}\log_2 x - \frac{1}{2}$

3. R: $[-1, 1]$

4. $f(x-1) = \begin{cases} 1 & 0 \leq x < 1 \\ x & 1 \leq x \leq 3 \end{cases}$

P33. (B)

1. $\arcsin(1 - x^2)$

3. D

2023.9.24. P39.

2-3 P51

P55 2-4

P62 2-5

(3) $\left(\lim_{x\to 0}(1+2\tan x)^{\frac{1}{\tan x}}\right)^{\frac{x}{2}}$

$= e^{\frac{1}{2}}\ e^{\frac{1}{x}}$ ✓

(4) $\left(\lim_{x\to\frac{\pi}{2}}(1+\cos x)^{3\sec x}\right)^{3} = e^{3}$ ✓

P66. 2-6

1. $\lim_{x\to 0}\frac{2x-x^2}{x^2-x^3} = \lim_{x\to 0}\frac{2-x}{x-x^2} = \infty$

则 $\lim_{x\to 0}\frac{x^2-x^3}{2x-x^2} = \lim_{x\to 0}\frac{x-x^2}{2-x} = 0$

则 x^2-x^3 是比 $2x-x^2$ 高阶的无穷小。 ✓

2. (1) $\lim_{x\to 0^+}\frac{\sin ax}{\sqrt{1-\cos x}} = \lim_{x\to 0^+}\frac{ax}{\frac{\sqrt{2}}{2}x} = \sqrt{2}a$ ✓

(2) $\lim_{x\to 0}\frac{\cos ax-\cos bx}{x^2} = \lim_{x\to 0}\frac{-2\sin\frac{a+b}{2}x\sin\frac{a-b}{2}x}{x^2} = \lim_{x\to 0}\frac{-2\cdot\frac{a+b}{2}x\cdot\frac{a-b}{2}x}{x^2} = \frac{b^2-a^2}{2}$

(3) $\lim_{x\to 0}\frac{\arctan x^2}{\sin\frac{x}{2}\arcsin x} = \lim_{x\to 0}\frac{x^2}{\frac{x}{2}\cdot x} = 2$ ✓

(4) $\lim_{x\to 0}\frac{\frac{x}{\sqrt{1-x^2}}}{\ln(1-x)} = \lim_{x\to 0}\frac{x}{-x\sqrt{1-x^2}} = \lim_{x\to 0}\frac{1}{-\sqrt{1-x^2}} = -1$ ✓

(5) $\lim_{x\to 0}\frac{1-\cos 4x}{2\sin^2 x+x\tan^2 x} = \lim_{x\to 0}\frac{\frac{1}{2}(4x)^2}{2\sin^2 x+x\tan^2 x} = \dots = \lim_{x\to 0}\frac{8x^2}{2x^2} = 4$ ✓

(6) $\lim_{x\to 0}\frac{\ln(\sin^2 x+e^x)-x}{\ln(x^2+e^{2x})-2x} = \dots$

P75. 2-7

4. (1) $\lim_{x\to\infty}x\cdot\ln\left(1+\frac{a}{x}\right) = \lim_{x\to\infty}x\cdot\frac{a}{x} = a$ ✓

(2) $\lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4}}} = 2$

(3) $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{2x} = \lim_{x\to 0}\frac{x^2+x}{2x(\sqrt{1+x+x^2}+1)} = \lim_{x\to 0}\frac{x+1}{2(\sqrt{1+x+x^2}+1)} = \frac{1}{4}$ ✓

(4) $\lim_{x\to 0}\frac{\tan x}{1-\sqrt{1+\tan x}} = \lim_{x\to 0}\frac{x(1+\sqrt{1+\tan x})}{-\tan x} = \lim_{x\to 0}-(1+\sqrt{1+\tan x}) = -2$ ✓

(5) $\lim_{x\to+\infty}(x)(\mathrm{arccot}\,x) = 1$ ✓

(6) $\lim_{x\to 0}\frac{\ln\frac{1+x}{1-x}}{x}$

$\lim_{x\to 0}\frac{\ln(1+x)-\ln(1-x)}{x} = \lim_{x\to 0}\frac{x-(-x)}{x} = 2$ ✓

5. 令 $y = 2^x - x^2 = 0$，

当 $x=-1$，$y = 2^{-1}-(-1)^2 < 0$

当 $x=1$，$y = 2-1^2 > 0$，

∴ 在 $(-1,1)$ 中有实根

(1) $y = \frac{1-\cos 2x}{x^2}$ 在 $x=0$ 处无定义，

$\lim_{x\to 0}\frac{1-\cos 2x}{x^2} = \lim_{x\to 0}\frac{\frac{1}{2}(2x)^2}{x^2} = 2$，

∴ $x=0$ 是 $y=\dots$ 的可去间断点，

若令 $y|_{x=0}=2$，则在 $x=0$ 处连续 ✓

(2) $y = \arctan\frac{1}{x}$ 在 $x=0$ 处无定义，

$\lim_{x\to 0}\arctan\frac{1}{x} = \frac{1}{x} = \infty$ 不存在极限，

$\lim_{x\to 0^+}\arctan\frac{1}{x} = \frac{\pi}{2}$

$\lim_{x\to 0^-}\arctan\frac{1}{x} = -\frac{\pi}{2}$

∴ $x=0$ 是 $y=\arctan\frac{1}{x}$ 的第二类间断点，

跳跃

(3) $y = e^{-\frac{1}{x}}$ 在 $x=0$ 处无定义，

~~$\lim_{x\to 0}e^{-\frac{1}{x}} = \dots$ 不存在~~

$\lim_{x\to 0^+}e^{-\frac{1}{x}} = 0$

$\lim_{x\to 0^-}e^{-\frac{1}{x}} = +\infty$

$\lim_{x\to 0}e^{-\frac{1}{x}} = 0$，

~~∴ $x=0$ 是 $y=e^{-\frac{1}{x}}$ 的可去间断点，~~

~~若 $y|_{x=0}=0$，则在 $x=0$ 处连续~~

第二类间断点

5. $\lim_{x\to 0}\frac{f(2x)}{x} = 2\lim_{x\to 0}\frac{f(2x)-f(0)}{2x} = 2f'(0) = 2$

P78 (A)

1. (1) B (2) C

2. (1) $\lim_{x\to 1}(x+1) = 2$

(3) $\lim_{n\to\infty}\frac{3n^2}{n^2} = 3$

(4) $\lim_{x\to 0}\frac{x-\tan x}{x^3}$

6. $\lim_{n\to\infty}\left(1-\frac{1}{n\alpha}\right)^n = e^{-1}$

7.

8. D ✓

9. $\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\cot x} = \lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\frac{1}{x}} = e^{\frac{1}{2}}$ ✓

P94. 3-2

1.

(1) $y' = 2x + \frac{3}{x^2} - \cos x$ ✓

(2) $y = x^3 + \frac{2\sqrt{x}}{x^3} - \frac{1}{x^3}$，$y' = 3x^2 + \frac{\frac{1}{\sqrt{x}}x^3 - 2\sqrt{x}\cdot 3x^2}{x^6} - (-3)\cdot\frac{1}{x^4}$

$= 3x^2 + \frac{x^3\cdot\sqrt{x} - 6x^3\cdot\sqrt{x}}{x^6} + \frac{3}{x^4}$

$= 3x^2 - \frac{5\sqrt{x}+3}{x^4}$ ✓

(3) $s' = \frac{\pi-1}{2\sqrt{t}}\sin t + \sqrt{t}\cos t$ ✓

(4) $y' = (x\cos x)'\ln x + x\cos x(\ln x)'$

$= (\cos x - x\sin x)\ln x + \cos x$ ✓

(5) $y = \frac{x-1+2}{x-1} = 1+\frac{2}{x-1}$

$y' = \frac{-2}{(x-1)^2}$ ✓

(6) $y' = \frac{e^x(x^2+1) - e^x\cdot 2x}{(x^2+1)^2}$ ✓

2.

(1) $y' = \arccos x - x\frac{1}{\sqrt{1-x^2}}$

$y'|_{x=\frac{1}{2}} = \arccos\frac{1}{2} - \frac{\sqrt{3}}{3}$

$\frac{\pi}{3}$

(2) $\rho' = \tan\theta + \theta\sec^2\theta + \sec\theta\tan\theta$

$\frac{d\rho}{d\theta}\Big|_{\theta=\frac{\pi}{4}} = 1 + \frac{\pi}{4}\cdot\frac{1}{\frac{1}{2}} + \frac{1}{\frac{\sqrt{2}}{2}}\cdot 1$

$= 1 + \frac{\pi}{2} + \sqrt{2}$ ✓

(3) $f'(x) = \frac{1}{\sqrt{\frac{e^{3x}}{e^{3x}+1}}}\cdot\frac{1}{2}\frac{1}{\sqrt{\frac{e^{3x}}{e^{3x}+1}}}\cdot\frac{3e^{3x}(e^{3x}+1) - e^{3x}\cdot 3\cdot e^{3x}}{(e^{3x}+1)^2}$

$= \frac{e^{3x}+1}{2e^{3x}}\cdot\frac{3e^{6x}+3e^{3x}-3e^{6x}}{(e^{3x}+1)^2}$

$= \frac{3}{2(e^{3x}+1)}$

$f'(0) = \frac{3}{4}$

3. $y' = 3x^2 - 1$ $\quad k = 2$

当 $y' = 2$ 时，$x = \pm 1$ $\quad (1,2)$ 或 $(-1,2)$ ✓

4.

(1) $y' = \frac{\cos x}{\sin x} = \cot x$ ✓

(2) $y' = 10(x^3-1)^9\cdot 3x^2$ ✓

(3) $y' = 3(x+\cos^2 x)^2(1 - 2\cos x\sin x)$

(4) $y' = \frac{\sqrt{x^2+1}}{\sqrt[3]{x-2}}\cdot\dots$

(5) $y' = 2\sin x\cdot\sin x^2 + \sin^2 x\cos x^2\cdot 2x$

(6) $y' = \sec^2[\ln(1+x^2)]\cdot\frac{1}{1+x^2}\cdot 2x$

$= \frac{2x\sec^2[\ln(1+x^2)]}{1+x^2}$ ✓

(7) $y' = -2^{\cos\frac{1}{x}}\ln 2\cdot\cos\frac{1}{x}\cdot\frac{1}{x^2}$ ✓

(8) $y' = e^{\frac{x}{\ln x}}\cdot\frac{\ln x-1}{(\ln x)^2}$ ✓

(9) $y' = \frac{1}{x+\sqrt{x^2+a^2}}\left[1+\frac{1}{2}(x^2+a^2)^{-\frac{1}{2}}\cdot 2x\right]$

$= \frac{1+x\frac{1}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$

$= \frac{x+\sqrt{x^2+a^2}}{x\sqrt{x^2+a^2}+x^2+a^2}$

P99. 3-3

1. (1) $x^4 - y^4 - y + 4xy = 0$

$\Rightarrow 4x^3 - 4y^3y' + (4x)'y + 4xy' = 0$

$\frac{dy}{dx} = y' = \frac{4y+4x^3}{4y^3-4x} = \frac{y+x^3}{y^3-x}$ ✓

(2) $\Rightarrow y'\sin x + y\cos x - \sin(x-y)(1-y') = 0$

$y'\sin x + y\cos x - \sin(x-y) + y'\sin(x-y) = 0$

$\frac{dy}{dx} = y' = \frac{\sin(x-y) - y\cos x}{\sin x + \sin(x-y)}$ ✓

(3) $\Rightarrow e^x - e^y\cdot y' - \cos xy\cdot(y+xy') = 0$

$e^x - y'e^y - y\cos xy - xy'\cos xy = 0$

$\frac{dy}{dx} = y' = \frac{e^x - y\cos xy}{e^y + x\cos xy}$ ✓

(4) $\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{y'x-y}{x^2}\right) = \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x+2y\cdot y') = 0$

$\left(\frac{x}{x^2+y^2}\right)y' - \frac{y}{x^2+y^2} - \frac{x}{x^2+y^2} - \left(\frac{y}{x^2+y^2}\right)y' = 0$

$\frac{dy}{dx} = y' = \frac{\frac{x+y}{x^2+y^2}}{\frac{x-y}{x^2+y^2}} = \frac{x+y}{x-y}$ ✓

3. $x^2+3xy+y^3-3=0$

$\Rightarrow 3x^2+3y+3xy'+3y^2\cdot y'=0$

$\therefore y'=-\frac{3x^2+3y}{3x+3y^2}=-\frac{x^2+y}{x+y^2}$ 当 $x=1$, $1+3y+y^3=3$, 则 $y=1$

$y'(x=1)=-\frac{1+1}{1+1}=-1$

切线方程: $y-1=-(x-1)$ $\Rightarrow$ 即 $x+y-2=0$

法线方程: $y-1=x-1$ $\Rightarrow$ $x-y=0$

2.(1) $\ln y=\sin x\ln x$

$\Rightarrow \frac{1}{y}\cdot y'=\cos x\ln x+\frac{\sin x}{x}$

$y'=y\cos x\ln x+\frac{y}{x}\sin x$ $\quad y=x^{\sin x}$ 代入

(3) $\ln y=\frac{1}{2}[\ln(x-1)+\ln(x-2)-\ln(x-3)-\ln(x-4)]$

$\frac{1}{y}\cdot y'=\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}$

$y'=\frac{y}{2(x-1)}+\frac{y}{2(x-2)}-\frac{y}{2(x-3)}-\frac{y}{2(x-4)}$ $\quad y=\cdots$ 代入

(4) $y\ln\sin x=x\ln\cos y$

$y'\ln\sin x+y\frac{1}{\sin x}\cos x=\ln\cos y+x\frac{1}{\cos y}(-\sin y)\cdot y'$

$y'=\frac{\ln\cos y-x\tan x-y\cot x}{\ln\sin x}$

(2) $y'=\cdots+ax^{a-1}+a^x\ln a+(x^x)'$ $=ax^{a-1}+a^x\ln a+(\ln x+1)x^x$

令 $f(x)=x^x$, 则 $\ln y=x\ln x$

$\frac{1}{y}\cdot y'=\ln x+x\cdot\frac{1}{x}$

$f'=(\ln x+1)x^x$

4.(1) $\frac{dy}{dx}=\frac{y'(t)}{x'(t)}=\frac{-2t}{1-2t}$

(2) $\frac{dy}{dx}=\frac{y'(\theta)}{x'(\theta)}=\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta}=\tan\theta$

5. $\frac{dy}{dx}=\frac{y'(t)}{x'(t)}=\frac{4\cos t}{-6\sin t}=-\frac{2}{3}\cot t$

$\frac{dy}{dx}\Big|_{t=\frac{\pi}{4}}=-\frac{2}{3}\times1=-\frac{2}{3}$

当 $t=\frac{\pi}{4}$: $x=6\cdot\frac{\sqrt2}{2}=3\sqrt2$, $y=4\cdot\frac{\sqrt2}{2}=2\sqrt2$

切线方程: $y-2\sqrt2=-\frac{2}{3}(x-3\sqrt2)$

$\Rightarrow 2x+3y-12\sqrt2=0$

P.123. 3-4

1. $dy=y'dx=2xdx$ $\quad dx=1.005-1=0.005$ $\quad x=1$

$dy=2\times1\times0.005=0.01$

2. $dy=y'dx=2\cos2x\,dx$

$dy|_{x=0}=2dx$

3.(1) $dy=y'dx$

$=(2e^{2x}\cos x+e^{2x}(-\sin x))dx$

$=e^{2x}(2\cos x-\sin x)dx$

(2) $dy=y'dx$

$=\left(\frac{2x\cos2x\cdot x^2-\sin2x\cdot2x}{x^4}\right)dx$

$=\left(\frac{2x\cos2x-2\sin2x}{x^3}\right)dx$

(3) $dy=y'dx$

$=\left(\frac{e^{-x^2}\cdot-2x}{1+e^{-x^2}}\right)dx$

(4) $dy=y'dx$

$=\left(\frac{1}{1+\ln x^2}\cdots\right)dx$

$=\left(\frac{x}{(1+x^2)\sqrt{1+x^2}}\right)dx$

(5) $(e^{2x})'$ $e^x dx$

(6) $xdy^2+y^2dx+x^2dy+ydx^2=0$

4.(1) $e^{0.03}\approx1+0.03=1.03$

(2) $\sqrt[5]{30}=\cdots=2(1-\frac{1}{5}\cdot\frac{1}{32})=1.9875$

5.(1) x^2+C

(2) $\frac{2}{3}x^{\frac{3}{2}}+C$

(3) $-\frac{1}{\omega}\cos\omega t$

(4) $-\frac{1}{2}\cos^2 x$

P.127. 3-5.

1.(1) $y'=3x^2+8+\sin x$

$y''=6x+\cos x$

(2) $y'=2x\arctan x+(1+x^2)\frac{1}{1+x^2}$

$=2x\arctan x+1$

$y''=2\arctan x+\frac{2x}{1+x^2}$

(3) $y'=e^{x^2}+x\cdot e^{x^2}\cdot2x$

$=e^{x^2}+2x^2e^{x^2}$

$y''=2xe^{x^2}+4xe^{x^2}+2x^2\cdot e^{x^2}\cdot2x$

$=2xe^{x^2}+4xe^{x^2}+4x^3e^{x^2}$

$=4x^3e^{x^2}+6xe^{x^2}$

(4) 令 $\ln y=x\ln x$

$\frac{1}{y}\cdot y'=\ln x+1$

$y'=y(\ln x+1)=x^x(\ln x+1)$

$y''=y'(\ln x+1)+y(\frac{1}{x})$

$y''=x^{x-1}+x^x(\ln^2x+2\ln x+1)$

P.132. 4-3

(1) $\lim\limits_{x\to0}\frac{e^x+e^{-x}}{\cos x}=\lim\limits_{x\to0}2$

(2) $\lim\limits_{x\to\pi}\cdots=-2$

(3) $\lim\limits_{x\to0}\frac{e^x+e^{-x}-2}{1-\cos x}=\lim\limits_{x\to0}\frac{e^x-e^{-x}}{\sin x}=\lim\frac{e^x+e^{-x}}{\cos x}=2$

(4) $\lim\limits_{x\to+\infty}\cdots=\lim\limits_{x\to+\infty}\frac{2}{2}=1$

(5) $\lim\limits_{x\to\pi}\cdots=\lim\limits_{x\to\pi}\frac{\sin^2 3x}{3\sin^2x}=\lim\frac{6\sin3x\cos3x}{6\sin x\cos x}=\lim\frac{\sin6x}{\sin2x}=\lim\frac{6\cos6x}{2\cos2x}=3$

(6) $\lim\limits_{x\to0^+}\cdots=\lim\limits_{x\to0^+}\frac{\sin^2x}{-\tan x\cdot x}=\lim\limits_{x\to0^+}\frac{x^2}{-x\cdot x}=-1$

(7) $\lim\limits_{x\to0}\cdots=\lim\frac{x^3}{x-\tan x}=\lim\frac{3x^2}{1-\sec^2x}=\cdots=3$

(8) $\lim\limits_{x\to0}\frac{2xe^{-x^2}+2x}{\sin^2 2x+\cdots}=\lim\frac{1-e^{-x^2}}{-6x^2}=\lim\frac{2xe^{-x^2}}{12x}=\frac{1}{6}$

(11) … (15) $\lim e^{\cdots}=e^{\cdots}$

(16) $e^{\lim\limits_{x\to1}\frac{1}{x-1}\ln(\ln x)}=e^{\lim\frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}}=\cdots$

$e^{\lim\limits_{x\to1}\frac{\ln(\ln x)}{x-1}}=e^{\lim\frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}}=\cdots=1$

(17) $\lim\limits_{x\to+\infty}\frac{e^{2x}-1}{e^{2x}+1}=\lim\limits_{x\to+\infty}\frac{2e^{2x}}{2e^{2x}}=1$

(18) $\lim\limits_{x\to\infty}\frac{1-\frac{\sin x}{x}}{1+\frac{\sin x}{x}}=1$

2. $\lim\limits_{x\to0}\frac{f'(x)-2}{2x}=\lim\limits_{x\to0}\frac{f''(x)}{2}=\frac{3}{2}$

P.137. 4-4

1. $y=e^x-x-1$ 在 $(-\infty,+\infty)$ 上连续

$y'=e^x-1$

当 $y'>0$, $e^x>1$, 即 $x>0$, $y\uparrow$

当 $y'<0$, $e^x<1$, 即 $x<0$, $y\downarrow$

2. $y'=-\sin x+\sin x+x\cos x=x\cos x$

当 $x\in(\frac{\pi}{2},\frac{3\pi}{2})$ 时, $y'<0$, $y\downarrow$

3.(1) $f'(x)=6x^2-18x+12$

$=6(x^2-3x+2)$

$=6(x-1)(x-2)$

$(-\infty,1]$, $[2,+\infty)\uparrow$, $[1,2]\downarrow$

(2) $f'(x)=4x-\frac{1}{x}$ $(x>0)$

若 $f'(x)>0$, $4x>\frac{1}{x}$, $x>\frac{1}{2}$

若 $f'(x)<0$, $0<x<\frac{1}{2}$ $\downarrow$

(3) $f(x)=(2-x)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}$

$f'(x)=\frac{2}{3}(2-x)^{-\frac{1}{3}}(x-1)^{\frac{1}{3}}+\frac{1}{3}(2-x)^{\frac{2}{3}}(x-1)^{-\frac{2}{3}}$

若 $f'(x)>0$ …

$x<1$ $\downarrow$; $1<x<2$ $\uparrow$; $x>2$ $\uparrow$

(4) $x<-1$ $\downarrow$; $x>-1$ $\uparrow$

4.(1) 令 $f(x)=2+x-2\sqrt{1+x}$ $(x>0)$

$f'(x)=1-2\cdot\frac{1}{2}\frac{1}{\sqrt{1+x}}$

$=1-\frac{1}{\sqrt{1+x}}>0$

$f(x)\uparrow$, $f(0)=0$

$\therefore f(x)>0$

(2) $f(x)=x-\ln(1+x)$

$f'(x)=1-\frac{1}{1+x}>0$, $f(0)=0$, $f(x)>0$

$g(x)=\ln(1+x)-x+\frac{1}{2}x^2$

$g'(x)=\frac{1}{1+x}-1+x$

$=\frac{1+x^2-1}{1+x}=\frac{x^2}{1+x}>0$

$g(0)=0$, $g(x)>0$

5. $f(x)=x^5+2x^3+x-1$

$f'(x)=5x^4+6x^2+1>0$, $f(x)\uparrow$

$f(1)>0$, $f(0)<0$

6.(1) $y'=12x^3-12x^2$

$y''=36x^2-24x$

$=12(3x^2-2x)$

当 $y''=0$, $x=0$ 或 $x=\frac{2}{3}$ 拐点

$(-\infty,0]$, $[\frac{2}{3},+\infty)$ 凹

$[0,\frac{2}{3}]$ 凸

(2) $y'=-\frac{1}{3}(x-1)^{-\frac{2}{3}}$

$y''=-\frac{1}{3}(-\frac{2}{3})(x-1)^{-\frac{5}{3}}$

$=\frac{2}{9}(x-1)^{-\frac{5}{3}}$

$[1,+\infty)$ 凹

$(-\infty,1]$ 凸

(3) $y'=\frac{-4\cdot2x}{(1+x^2)^2}=\frac{-8x}{(1+x^2)^2}$

$y''=\frac{-8(1+x^2)^2+8x\cdot2(1+x^2)\cdot2x}{(1+x^2)^4}$

$=\frac{-8(1+x^2)+32x^2}{(1+x^2)^3}$

凹 $(-\infty,-\frac{1}{\sqrt3}]$, $[\frac{1}{\sqrt3},+\infty)$

凸 $[-\frac{1}{\sqrt3},\frac{1}{\sqrt3}]$

拐点 $(-\frac{1}{\sqrt3},3)$, $(\frac{1}{\sqrt3},3)$

(4) $y'=\cdots+\frac{2}{3}(x-1)x^{-\frac{1}{3}}$

$y''=\cdots$

$y''=0$ 时, …

7. 证:

令 $f(t)=te^t$

$\Rightarrow\frac{f(x)+f(y)}{2}>f(\frac{x+y}{2})$

$\Rightarrow$ 证 $f(t)$ 在 $(0,+\infty)$ 内为凹

$\because f'(t)=te^t+e^t$, $f''(t)=te^t+2e^t$

又 $t>0$, $f''(t)>0$

$\therefore f(t)$ 在 $(0,+\infty)$ 凹 … 得证

P.143. 4-5

1.(1) $f'(x)=3x^2-6x-9=3(x^2-2x-3)=3(x+1)(x-3)$

令 $f'(x)=0$, $x=-1$ 或 $x=3$

极大值 $f(-1)=-1-3+9+3=8$

极小值 $f(3)=27-27-27+3=-24$

(2) $f'(x)=\frac{1+x^2-2x^2}{(1+x^2)^2}=\frac{-x^2+1}{(1+x^2)^2}$

极大值 $f(1)=\frac{1}{2}$

极小值 $f(-1)=-\frac{1}{2}$

(3) $f'(x)=4x-\frac{1}{x}$ $(x>0)$

极大值 …

极小值 $f(\frac{1}{2})$ …

(4) $f'(x)=\frac{2}{3}[\cdots]^{-\frac{1}{3}}\cdot[\cdots]$

极大值 f …

极小值 …

(5) $f'(x)=3(x^2-1)^2\cdot2x=6x(x^2-1)^2$

极大值 ~~$f(x)=f(-1)=\cdots$~~

极小值 $f(0)=-1$

2. $f'(x)=a\cos x+\cos3x$

$f'(\frac{\pi}{3})=\frac{a}{2}-1=0$ $\quad a=2$

$f''(x)=-a\sin x-3\sin3x$

$f''(\frac{\pi}{3})=\cdots=-\sqrt3<0$

$f(\frac{\pi}{3})=\sqrt3$

3.(1) $f'(x)=4x^3-4x$

$f(x)_{min}=f(1)=1-2+3=2$

$f(x)_{max}=\cdots$

(2) $f'(x)=1-\frac{1}{2}\frac{1}{\sqrt{1-x}}$

$f(x)_{min}=-1$

$f(x)_{max}=\frac{5}{4}$

(3) $f'(x)=\sin x+x\cos x-\sin x$

$=x\cos x$

$f(x)_{min}=-1$

$f_{max}=\frac{\pi}{2}$

1.(1) $\lim\limits_{x\to\infty}\frac{x}{x}+\frac{\sin x}{x}=\lim\limits_{x\to\infty}1+0=1$ $\quad y=1$

(2) $\lim e^{\frac{1}{x}}+1=e+1$ … $x=1$

$\lim\limits_{x\to\infty}e^{\frac{1}{x}}+1=2$ $\quad y=2$

P.174. 5-3

1.(1) $\frac{1}{a}$, $\frac{2}{3}$ (2) $xdx=\frac{1}{2}dx^2=-\frac{1}{2}d(-x^2+1)$ $\Rightarrow-\frac{1}{2}$

(3) $x^3dx=\frac{1}{4}dx^4=\frac{1}{12}d(3x^4+1)$ $\Rightarrow\frac{1}{12}$ (4) $-\frac{1}{3}e^{-3x}d(-3x)=\cdots$ $\Rightarrow-\frac{1}{3}$

(5) $\frac{2}{3}$ (6) $\frac{1}{1+(\sqrt x)^2}dx$ $\Rightarrow\frac{1}{2}$

(7) $(3x^2+2)dx=d(x^3+2x)$ $\Rightarrow-1$ (8) $\ln|x|$ $\Rightarrow\frac{1}{3}$

(9) $-d\arcsin x$ $\Rightarrow-1$ (10) $d(\sqrt{1-x^2})=\frac{-2x}{2\sqrt{1-x^2}}dx$

$=\frac{1}{-4x\sqrt{1-x^2}}dx$

$\Rightarrow x^2\quad-1$

4-6. P.152

1.(1) $C(100)=\frac{1}{40}\times100^2+200\times100+1000$

$=250+20000+1000$

$=21250$

$\bar{C}(100)=\frac{C(100)}{100}=212.5$

(2) $\frac{C(Q)}{Q}=\frac{1}{40}Q+200+\frac{1000}{Q}\geq200+2\sqrt{\frac{Q}{40}\cdot\frac{1000}{Q}}$

$=200+5\times2$

$=210$

当且仅当 $\frac{Q}{40}=\frac{1000}{Q}$ 即 $Q=200$ 时取等

(3) 利润: $400Q-\frac{1}{40}Q^2-200Q-1000$

$=200Q-\frac{1}{40}Q^2-1000$

$Q=\cdots=4000$

$200\cdot4000-\frac{1}{40}\cdot4000\cdot4000-1000$

$=800000-400000-1000$

$=399000$

$\frac{399000}{4000}=99.75$

$L(200) = -0.1Q^2 + 60Q - 5000$

$Q = -\frac{60}{-0.2} = 300$

5. (1) $\eta(p) = p \cdot \frac{f'(p)}{f(p)}$

(2) $\eta(p) = -\frac{1}{4} \cdot 100 = -25$

6. (1) $\eta(p) = p \cdot \frac{f'(p)}{f(p)} = -\frac{p}{3}$

$f'(p) = -\frac{p}{3} e^{-\frac{p}{3}}$

$\eta(3) = -1$

$\eta(4) = -\frac{4}{3}$

7. (1) $f'(p) = -2p$

$f'(5) = -10$

(2) $\eta(p) = p \cdot \frac{f'(p)}{f(p)}$

(3) $R' = f(p)(1+\eta)$

$R = pf(p)$

(4)

$\frac{\partial R}{\partial p} = R' = 1+\eta = -0.85$

P103 3-1

1. (1) $\frac{4}{3}x^{3}$ (2) $-\sin x$ (3) $e^{\frac{x}{2}}$

(4) $2\arccos x$

3. (1) $f'(x) = \frac{1}{\sqrt{1+x^2}}$

(2) $f'(x) = -\frac{1}{2}(1+x^2)^{-\frac{3}{2}} \cdot 2x = -x(1+x^2)^{-\frac{3}{2}}$

4. $f(0) = 1$

$f'(x) = e^{-x} \to f(x) = -e^{-x} + C$

$f(x) = -e^{-x} + 2$

2. (1) $\left(-\frac{1}{2}x^{-2}\right)' = -\frac{1}{2}(-2)x^{-3} = \frac{1}{x^3}$

(2) $(-\cos x + \sin x)' = \sin x + \cos x$

P105 5-2

1. (1) $\int \sqrt{x}(x^2-4)dx = \int (x^{\frac{5}{2}} - 4x^{\frac{1}{2}})dx = \frac{2}{7}x^{\frac{7}{2}} - \frac{8}{3}x^{\frac{3}{2}} + C$

(10) $\int \sin^2\frac{x}{2}dx = \int \frac{1}{2} - \frac{\cos x}{2}dx = \frac{1}{2}x - \frac{1}{2}\sin x + C$

P178 5-3

2. (1) $\frac{1}{3}\int a^{3x}d3x = \frac{1}{3}\frac{1}{\ln a}a^{3x} + C$

(9) $\int \frac{1}{x+1}d(x+1) = \ln|x+1| + C$

(11) $\int \frac{e^x}{e^{2x}+1}dx = \int \frac{1}{e^{2x}+1}de^x = \arctan e^x + C$

(18) $\int \frac{1}{(x+2)^2+1}d(x+2) = \arctan(x+2) + C$

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P180 5-4

(1) $\int x d\sin x = x\sin x - \int \sin x dx = x\sin x + \cos x + C$

(11) $\frac{1}{2}\int \ln(x-1)\,dx^2 = \frac{1}{2}x^2\ln(x-1) - \frac{1}{2}\int\frac{x^2}{x-1}dx$

$= \frac{1}{2}x^2\ln(x-1) - \frac{1}{2}\int(x+1)\,d(x+1) - \frac{1}{2}\int\frac{1}{x-1}\,d(x-1)$

$= \frac{1}{2}x^2\ln(x-1) - \frac{1}{4}(x+1)^2 - \frac{1}{2}\ln|x-1| + C$

(12) $\frac{1}{2}\int x^2(1+\cos x)\,dx = \frac{1}{2}\int x^2dx + \frac{1}{2}\int x^2\cos x\,dx$ $= \frac{1}{2}\int x^2 d\sin x$

$= \frac{1}{6}x^3 + \frac{1}{2}x^2\sin x - \frac{1}{2}\int\sin x\,dx^2$

$- \int x\sin x\,dx$

$+ \int x\,d\cos x$

$+ x\cos x - \int\cos x\,dx$

$\Rightarrow \frac{1}{6}x^3 + \frac{1}{2}x^2\sin x + x\cos x - \sin x + C$

(13) $-\int \ln^3x\,d\frac{1}{x} = -\frac{1}{x}\ln^3x + \int\frac{1}{x}\,d\ln^3x$

$= -\frac{1}{x}\ln^3x + \int\frac{3\ln^2x}{x^2}dx$

$= -\frac{1}{x}\ln^3x - \frac{3\ln^2x}{x} + 3\int\frac{1}{x}\,d\ln^2x$

$+ 3\int\frac{2\ln x}{x^2}dx$

$- 6\frac{\ln x}{x} + 6\int\frac{1}{x^2}dx$

$\Rightarrow -\frac{1}{x}\ln^3x - \frac{3\ln^2x}{x} - \frac{6\ln x}{x} - \frac{6}{x} + C$

(14) $\frac{1}{4}\int x\sin 2x\,dx = -\frac{1}{8}\int x\,d\cos 2x = -\frac{1}{8}x\cos 2x + \frac{1}{8}\int\cos 2x\,dx$

$= -\frac{1}{8}x\cos 2x + \frac{1}{16}\sin 2x + C$

(15) $\int\tan^2x\sec x\,dx = \int\tan x\,d\sec x = \tan x\sec x - \int\sec^3x\,dx$

$\int\frac{\sin^2x}{\cos^3x}dx$

$= \tan x\sec x - \int\sec x\,dx - \int\frac{\sin^2x}{\cos^3x}dx$

$= \int\sec x\,dx - \int\frac{\sin^2x}{\cos^3x}dx$

$= \ln|\sec x+\tan x| - \int\frac{\sin^2x}{\cos^3x}dx$

(16) $\int\frac{xe^x}{(1+x)^2}dx$

$= -\int xe^x\,d\frac{1}{x+1}$

$= -\frac{xe^x}{x+1} + \int\frac{1}{x+1}\,dxe^x$

$+ \int\frac{e^x+xe^x}{x+1}dx$

$+ \int e^x dx$

$\Rightarrow -\frac{xe^x}{x+1} + e^x + C$