NOTES AND LETTERS

• *ABSTRACT*

Social determinists have argued that the occurrence of independent discoveries and inventions demonstrates the inevitability of techno-scientific progress. Yet the frequency of such multiples may be adequately predicted by a probabilistic model, especially the Poisson model suggested by Price. A detailed inquiry reveals that the Poisson distribution can predict almost all the observed variation in the frequency distribution of multiples collected by Merton, and by Ogburn and Thomas. This study further indicates that: (a) the number of observed multiples may be greatly underestimated, particularly those involving few independent contributors; (b) discoveries and inventions are not sufficiently probable to avoid a large proportion of total failures, and hence techno-scientific advance is to a large measure indeterminate; (c) chance or 'luck' seems to play such a major part that the 'great genius' theory is no more tenable than the social deterministic theory.

Independent Discovery in Science and Technology: A Closer Look at the Poisson Distribution

Dean Keith Simonton

One of the more curious and significant phenomena in the history of science and technology is the appearance of independent and often simultaneous discoveries or inventions. Classic cases of such 'multiples' abound: calculus was independently devised by Newton and Leibniz; the theory of evolution was separately proposed by Darwin and Wallace; and oxygen was independently prepared by Priestley and Scheele. Such multiples are so striking, particularly when considered in isolation from all the numerous 'singletons', that many researchers have maintained that such occurrences support a social deterministic outlook.[1] Thus Ogburn and Thomas compiled a list of almost 150 multiples in order to provide an affirmative answer to the question 'Are inventions inevitable?'[2] In a similar vein, Kroeber employed a smaller list of these events to show that personal genius is epiphenomenal to the onward advance of culture.[3] But perhaps the social scientist most strongly associated with the 'social deterministic' interpretation of this phenomenon is Merton.[4] Besides gathering a collection of some 264 independent discoveries and inventions, Merton has closely scrutinized their sociological foundation. Although willing to grant a more prominent place to creative genius that most pure social determinists, Merton nontheless held that multiples provide convincing testimony for the inevitability of

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techno-scientific contributions. Once the necessary cultural base has accumulated, the advent of calculus was largely predetermined: if neither Newton nor Leibniz had developed the technique, someone else certainly would have quickly done so.

Though the social deterministic perspective has been endorsed by a great many investigators, a few have voiced dissent. Perhaps the most potent critique has come from Schmookler, who claimed that the arguments favouring the inevitability hypothesis are weak on several counts.[5] For our current purposes the most significant criticism is Schmookler's observation that the appearance of multiples does not necessarily support a social deterministic theory. Using a simple probabilistic model, he demonstrated that a large percentage of duplicate inventions can occur even if inventions have a probability less than unit. Briefly expressed, multiples can just as well illustrate a probabilistic rather than deterministic model of techno-scientific achievements.

Schmookler's criticism is compatible with another nondeterministic framework suggested by Price.[6] Taking the 264 instances of multiples cited by Merton, Price outlined a 'ripe apple' model based on the Poisson distribution. If there are 1000 discoveries waiting to be made and 1000 scientists ready to make them, and if the average scientist makes one discovery, the Poisson distribution can be used to predict the number of singletons, doublets, triplets, quadruplets, quintuplets, and so on. The resulting frequency distribution is quite similar to the figures offered by Merton. Thus Price has indicated how a probabilistic model might account for the phenomenon. What renders this application intriguing is that the Poisson distribution tends to characterize *rare*, rather than inevitable, events.

Unfortunately, Price mentioned the Poisson model only in passing, and therefore he did not develop its full implications. Yet I think Price's suggestion deserves detailed inspection. In particular, I would like to address three questions. One, what is the theoretical basis for the Poisson distribution which might render it singularly suitable for predicting the occurrence of multiples? Two, how well does the Poisson model actually describe the empirical data? Three, assuming that the Poisson distribution provides an adequate fit, what are the implications for our understanding of scientific and technological creativity?

The Model

Suppose for any given invention or discovery there are n individuals independently capable of making the contribution, where the integer $n \geqslant 1$. Suppose further that each of these individuals has the same probability p of being successful, where $0 < p \leqslant 1$. Then clearly the odds are that any given contribution will be independently made by np individuals. Thus if $n = 10$ and $p = 0.1$, then the typical invention or discovery will be the work of a single individual. However, this overall expectancy is merely an average, and accordingly any given contribution may not be the independent work of np contributors. In some instances all n researchers may utterly fail, whereas in other instances all n may succeed. To be more precise, therefore, we must deal with a frequency distribution characterizing the number of successes, from zero all the way to n. So the problem now becomes how to use these parameters to define the probability function describing the distribution.

One obvious possibility is to employ the binomial expansion for n trials of events having the probability p of success. That is, we may use the binomial theorem to expand $(q + p)^n$, where the probability of failure $q = (1 - p)$. But one major difficulty prevents this application: we must be able to provide an a priori specification of the two parameters p and n. To be sure, in theory we might be able to estimate these two free parameters from empirical observations. Yet the available data are inadequate to the task. Especially problematic is our ignorance regarding the total number of complete failures. Histories seldom recount discoveries and inventions which no one ever made, and those null contributions which are recorded invariably have zero probabilities (for example, the *perpetuum mobile*, the squaring of the circle, the doubling of the square, the trisection of the angle, the proof of the parallel postulate, and the like). In addition, for reasons to be discussed later, we cannot even be confident regarding the proportion of singletons to multiples. In all, there exists insufficient information to identify the parameters of a binomial distribution given the observed distribution of multiples.

The cause is not entirely hopeless, nonetheless. The binomial expansion has an exponential approximation known as the Poisson distribution. The chief asset of this latter distribution is that it contains only one free parameter, μ. Because μ represents both the mean and the variance of the distribution, it usually can be estimated from less adequate data than can the binomial distribution. In fact, we can fit a Poisson model while remaining ignorant of the number of total failures and single successes. At the same time, since the two distributions are closely related, they can be interpreted in much the same way. This similarity becomes manifest when we recognize that $\mu = np$. In other words, the Poisson approximation reduces the number of parameters from two to one by making their *product* a constant, and by supposing that p is very small (thus equating the mean np and variance npq). Making these simplifying assumptions, the probability of x successes is given as:[7]

$$P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(np)^x e^{-np}}{x!} \quad \text{where } x = 0, 1, 2, \ldots\ldots n$$

Of course, the assumption that np equals a constant μ is certainly implausible in detail. Essentially we are assuming that those techno-scientific problems having the least probability of solution are just those most likely to have the most investigators working towards their solution. Alternatively, we may hold that n and p are approximately constant across all potential contributions. But in neither case can we plausibly affirm that np is an exact constant.

Still, if we grant the assumption $np = \mu$ as a rough approximation, then the determination of the parameter μ can help us evaluate the significance of multiples. If a Poisson distribution holds and $\mu < 1$, then techno-scientific advances are achieved, on the average, less than once. Under such specifications, discoveries and inventions could hardly be deemed inevitable. In contrast, if $\mu > 1$, techno-scientific accomplishments are typically the result of more than one independent worker. Yet even in this latter case it does not follow that any given contribution is inevitable. For example, if $\mu = 1$, then 37% of all possible contributions will never see the light of day, even though a full 37% will be brought to light by a single individual and 26% by more than one person. Only when μ is fairly large can invention and discoveries have some claim to inevitability. Thus if $\mu = 4.6$, only 1% of the potential contributions will fail to materialize, and a mere 5% will be the unique contributions of single individuals, while an impressive 94% will be the brainchildren of two or more researchers. Hence only if μ is fairly large can techno-scientific achievements be deemed inevitable in even a probabilistic sense. Accordingly, if the distribution of observed successes can be adequately described by a Poisson distribution, then the estimated parameter μ can inform us as to the probabilistic inevitability of techno-scientific advances.

Empirical Test

At first glance it would still seem impossible to test whether the Poisson model can be used to predict the distribution of independent contributions. Merton has persuasively argued that the occurrence of multiples is much more extensive than history records — that many singletons are really disguised multiples.[8] Therefore it may be completely misguided to fit a Poisson distribution to both singletons and multiples. Nevertheless, the situation can be saved if we assume that the relative proportion of doublets, triplets, and higher grade multiples is not appreciably altered by the erroneous conversion of multiples to pseudo-singletons. Thus if in the real world the ratio of doublets to triplets equals three, the ratio may still be three in the history books. In a while we will remove this assumption, so for the moment we will need it only to proceed.

(a) *Goodness of Fit: General Samples*

Let us begin with the same data set Price used, namely the data cited by Merton.[9] These figures are given in Table 1. On the far left are the observed frequencies of doublets, triplets, on through nonets. To the right are predicted values using the Poisson distribution with μ varying from 0.8 to 1.6. The column under $\mu = 1.0$ is identical to that calculated by Price. Although this particular column of predicted frequencies is a fairly good fit, the fit is not optimal by a χ^2 criterion. As μ is increased the fit improves until $\mu = 1.4$, after which further increases in μ diminishes the goodness of fit. This can be seen from the χ^2 values calculated for each column which attain a minimum at $\mu = 1.4$. The Pearson product-moment correlation coefficient between the predicted and observed frequencies for $\mu = 1.4$ is 0.980, which signifies that 96% of the variance in the observed and predicted distributions is shared. Thus the Poisson model offers a highly accurate prediction of the empirical distribution of multiples.

Table 2 shows the result of applying the same comparison in Table 1 to the multiples listed by Ogburn and Thomas.[10] Even though this sample is less comprehensive than Merton's, the two samples correlate 0.992 with one another, and therefore can be said to have the same general form. This near identity notwithstanding, in Table 2 we observe that the optimal μ is 1.2, a value slightly smaller than that in Table 1. Still, the two values are close enough to conclude that the true μ is probably somewhat more than unity and definitely less than two. Hence, so far techno-scientific contributions hardly appear inevitable.

But one problem confronts any further conjectures. Even though Tables 1 and 2 show that the Poisson distributions offer fairly good fits, the match is perfect in neither case. Indeed, if we employ the χ^2 test on both fits, we learn that the odds are less than one out of a thousand that any discrepancies are due to mere chance or random noise in the data.[11] Thus the departures cannot be ignored. At least three ex-

Table 1.
Fit of Poisson Distribution to Frequencies of Multiples
Merton's General Sample

x	Observed Frequency	Expected Frequency for μ = 0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6
0	—	(620)	(472)	(368)	(292)	(236)	(193)	(159)	(133)	(112)
1	—	(496)	(424)	(368)	(321)	(283)	(251)	(223)	(200)	(180)
2	179	198	191	184	177	170	163	156	150	144
3	51	53	57	61	65	68	70	73	75	76
4	17	10	13	15	18	20	23	26	28	31
5	6	2	2	3	4	5	6	7	8	10
6	8	0	0	0	1	1	1	2	2	3
7	1	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0	0	0
9	2	0	0	0	0	0	0	0	0	0
χ^2	—	119	115	67	30	25	22	20	22	24

Note: Chi-squares are all based on the same degrees of freedom (d.f. = 2), and all significant at the 0.001 level.

Table 2.
Fit of Poisson Distribution to Frequencies of Multiples:
Ogburn-Thomas General Sample

x	Observed Frequency	Expected Frequency	μ	r	χ^2	df	p	Reconstructed Frequency
0	—	(132)						5,500
1	—	(158)						6,600
2	90	95						3,960
3	36	38						1,584
4	9	11	1.2	0.999	31.07	3	< 0.001	475
5	7	3						114
6	2	1						23
7	2	0						4
8	1	0						1
9	1	0						0

planations can be put forward. First, the Poisson distribution may be only a very rough approximation to reality. For instance, the assumption of a constant np may be crude at best. Second, it is feasible that the bias against reporting multiples is not constant across all grades of multiples. As an example, a doublet may be more easily overlooked than a nonet, the latter representing a much more outstanding coincidence of a 'believe it or not' variety. Third, the scientific and technological enterprise may not be so homogeneous as to justify the use of a single μ for all disciplines.[12] To illustrate, the evolution of highly codified or paradigmatic disciplines such as mathematics, astronomy, or physics may be more determined than the evolution of uncodified or pre-paradigmatic disciplines such as medicine, biology, or geology. Consequently, this difference should be reflected in the respective values of μ. Thus the μ for mathematics may be appreciably larger than the μ for biology, due to the greater inevitability of discoveries in the former discipline. So let us examine whether μ changes from discipline to discipline.

(b) *Goodness of Fit: Specific Disciplines*

Merton has not published a detailed breakdown of his multiples, but Ogburn and Thomas did provide a rudimentary classification into eight research areas.[13] Because one of these groups has few entries and is closely related to another, two groups were collapsed to yield a total of seven groups. These groups cover multiples in astronomy, mathematics, chemistry, physics, electricity, biology-medicine, and technology. Separate Poisson distributions were then fitted to each group. The results are shown in Table 3. Observe the following three points:—

1. The parameter μ varies a great deal across the seven groups, ranging from 0.6 for biology-medicine to 2.5 for technology. Although there exists some relationship between the size of μ and a discipline's codification, exceptions exist. Thus while mathematics has more apparent inevitability than the physical sciences, and the physical sciences more inevitability than the biological sciences, technological endeavours (including electricity) appear the most determined of all. The typical invention is made more than twice! If this is true, it may be due to the greater extrinsic urgency behind technological growth. However, such inferences must remain tentative. If we hold that the Ogburn-Thomas sample constitutes solely a small fraction of the total number of actual multiples, then the differences in the μ may be attributable to sampling error. In fact, a test for the homogeneity of the Poisson distribution[14] reveals that we cannot reject the null hypothesis that the μ is the same for all seven disciplines. Moreover, we will see shortly that there is good cause for concluding that the Ogburn-Thomas sample indeed comprises but a small subset of the probable supply of actual multiples. Therefore, even though the observed interdisciplinary differences do make substantive sense, a more complete data set would be required for a secure interpretation.

2. Although the correlations between the predicted and observed frequencies range from 0.961 to 1.000, the fit is seldom perfect. In the cases of physics and biology-medicine, in fact, departures from the Poisson model still exceed what could be expected on chance alone. Perhaps these two groups are not sufficiently homogeneous to permit the use of a single μ Yet even in these two cases the correlation between prediction and observation exceeds 0.98, and so at least 96% of the variance in the frequency distribution of multiples can still be predicted via a

Poisson model. Hence it seems that the model provides a very reasonable approximation to reality. Particularly remarkable is the absolutely perfect fit for astronomy.

3. Where departures occur from predicted values, the discrepancies seem to entail high grade multiples — that is, contributions engaging four or more individuals. Price observed the same fact, and added the explanation that certain inventions or discoveries may attract more researchers than others. I would like to advance an alternative position. As remarked in the previous section, we may not be completely justified in presuming that any bias against recording multiples operates uniformly on all grades. On the contrary, we may propose that the probability of recording a multiple as such (rather than as a singleton) is a positive function of its grade. The fewer the contributors independently engaged in making the same contribution, the larger the odds that a multiple will be recorded in history as a singleton. This proposition would be testable if we could figure out some way of reconstructing the actual distribution of multiples.

(c) *Reconstructed Frequencies*

Given the parameter μ and the total number of discoveries or inventions actually achieved in a given discipline, it is an easy matter to reconstruct the probable frequencies prior to any selective bias. Since we have already estimated values of μ, all we need are estimates of the total population of contributions. Such estimates have been compiled from Darmstaedter's exhaustive chronology,[15] and the results published in Sorokin's work on sociocultural dynamics.[16] Thus Table 5 in the second volume of Sorokin's work lists 478 contributions to astronomy, 329 to mathematics, 2469 to chemistry, 1511 to physics, 3737 to biology and medicine, and 4830 to technology. The total number for nine disciplines is 12,761.

In the extreme right column of Table 2 we see the outcome of applying the Poisson distribution with $\mu = 1.2$ to 12,761 techno-scientific discoveries and inventions. Notice that the ratio of the reconstructed to the observed frequencies tends to get appreciably larger as we proceed from high to low grade multiples. Thus multiples involving seven or more contributors are at least in the same order of magnitude. Multiples involving six independent contributors are under-reported by a factor of 10, while doublets are under-reported by a factor of 44. Similar conclusions can be drawn if the reconstructed frequencies in Table 2 are compared to the observed frequencies inTable 1 (making due allowances for the slightly different μ). Hence, the overall picture indicates some selective bias in reporting multiples.

The extreme right column of Table 3 displays reconstructed frequencies for all groups except electricity (since no figures are available). In most cases it seems as if both multiples and singletons are under-represented; if this is so, it may be partly attributable to the greater selectivity of the Ogburn-Thomas compilation. Yet this general shift aside, it again appears that low grade multiples are less likely to be reported than high grade multiples.

Three main inferences can be made from the reconstructed frequencies:—

1. The number of multiples in reality probably immensely surpasses the number in the historical record. In all instances the number of reconstructed multiples greatly exceeds the number of observed multiples. Again, Merton has offered several reasons why this might happen.[17] To underline the potential extent of these

Table 3.
Fit of Poisson Distribution to Frequencies of Multiples: Ogburn-Thomas Subgrouped Sample

Group	x	Observed Frequency	Expected Frequency	μ	r	χ^2	df	p	Reconstructed Frequency
I	0	—	(18)						238
	1	—	(19)						262
Astronomy	2	11	11	1.1	1.000	0.00	1	>0.995	144
	3	4	4						53
	4	1	1						14
	5	0	0						3
	6	0	0						1
II	0	—	(16)						108
	1	—	(23)						150
Mathematics	2	15	16						106
	3	7	7	1.4	0.993	4.40	2	>0.1	49
	4	2	3						17
	5	2	1						5
	6	0	0						1
	7	1	0						0
III	0	—	(20)						1,064
	1	—	(24)						1,277
Chemistry	2	15	14						766
	3	6	6	1.2	0.997	0.57	1	>0.75	306
	4	1	2						92
	5	0	0						22
	6	0	0						4
	7	0	0						1
IV	0	—	(27)						651
	1	—	(32)						782
Physics	2	18	19						469
	3	7	8						188
	4	1	2	1.2	0.994	9.68	2	<0.01	56
	5	2	1						14
	6	0	0						3
	7	0	0						0
	8	1	0						0
	9	1	0						0

Table 3 (continued)

Group	x	Observed Frequency	Expected Frequency	μ	r	χ^2	df	p	Reconstructed Frequency
V	0	—	(6)						—
	1	—	(11)						—
Electricity	2	9	9						—
	3	5	5	1.7	0.961	1.00	2	> 0.75	—
	4	2	2						—
	5	0	1						—
	6	2	0						—
VI, VII	0	—	(99)						4,546
	1	—	(59)						2,727
Biology	2	16	18						818
	3	2	4	0.6	0.983	17.22	1	< 0.001	164
Medicine	4	1	1						24
	5	2	0						3
	6	0	0						0
	7	1	0						0
VIII	0	—	(1)						432
	1	—	(4)						1,080
Technology	2	6	5						1,350
	3	5	4						1,125
	4	1	2						703
	5	1	1	2.5	0.973	0.95	2	> 0.75	352
	6	0	0						146
	7	0	0						52
	8	0	0						16
	9	0	0						4
	10	0	0						1

Note: Correlation r is based on observed and expected frequencies only up to the terminal zero in the longest column.

hypothetically missing multiples, Table 2 shows that there should be over 6000 independent contributions, whereas the largest list to date contains a mere 264. Still, even if the additional multiples were located, the Poisson model would probably stand firm. Unless the new-found multiples drastically altered the *form* of the frequency distribution, the same parameter μ would obtain. Even more critically, such added multiples could very likely augment the model's fit.

2. The bias against low grade multiples implies that our estimated values of μ may be a little too large. That is, the bias operates so as to lower the proportion of low grade multiples to high grade multiples, thereby shifting the mean of the total distribution upwards. Hence, the resulting parameter estimates may be a bit more liberal towards the deterministic side than may actually hold true.

3. Even though the number of multiples could be substantially larger in all disciplines, the conclusion remains that inventions and discoveries are far from being inevitable. According to the model, a notable proportion of contributions fail to be made at all. Using the Poisson distribution we can estimate the reconstructed frequencies of total failures (see Table 3). Thus in astronomy, with μ = 1.1, some 33% of all attempts failed, leaving 238 discoveries undiscovered for the 478 achieved. So under the parameters of the model, discoveries in astronomy cannot be deemed truly inevitable even if astronomical discoveries are made by 1.1 astronomers, on the average. Naturally, as μ increases, the proportion of 'undiscoveries' declines. In the case of mathematics, where μ = 1.4, only 25% (or 108) discoveries were missed by all mathematicians. And in the case of technology, with the highest μ of all (=2.5), only 8% of the possible inventions failed to be made by at least one inventor. In concrete terms, only 432 inventions are lacking, while 4830 were achieved. Moreover, for technology, doublets are actually more common than singletons (25% versus 20%, respectively). Nonetheless, even in technology we can only assert a rather weak probabilistic species of 'determinism'. We will never know how the course of history might have been changed had the failures been successes, and some of the successes failures.

Price also observed that the Poisson model implies some complete failures, as well as multiple sucesses. However, he interpreted the failures as the repercussion of the multiple successes. The reverse is true. There is a perfect negative relation between the number of failures and the number of multiples. The real foundation of the failures is the parameter μ = $\underline{np}$. A small value of μ implies that the probabilities of success are low, so that the luck may often run out on a particular invention or discovery.

Conclusion

I do not by any means claim that the Poisson model provides a perfect account of the data, for it is a patent first approximation. Certainly both $\underline{p}$ and $\underline{n}$ should be allowed to vary across inventions without any constraint that their product be constant.[18] Nonetheless, unless my simplifying assumptions are extremely far from the truth, it is very likely that more refined models will lead to the same fundamental conclusion: namely, that the occurrence of independent discoveries and inventions probably cannot be taken as evidence for the inevitability of techno-scientific advance. This conclusion does not deny that the political and cultural milieu can influence the content and intensity of creativity in the sciences. This conclusion also does not contradict the proposition that some inventions or discoveries are necessarily antecedent to others, and therefore that the temporal ordering of contributions is partly determined (for example, the calculus may have to precede modern physics). All we need to maintain is that any specific invention or discovery will usually have a very low probability of appearing, and consequently that techno-scientific progress must be largely indeterminate.

Although the results may not endorse the social deterministic perspective, I must end by emphasizing that the so-called 'genius' theory is not supported either. Given that μ is around unity, and that the highest grade multiple is a nonet, the most likely parameters are something close to $\underline{n} = 10$ and $\underline{p} = 0.1$. Under these specifications, there are only 10 persons sufficiently competent to make an advance, but each has merely a one in ten chance of success. If we define a genius as a rare individual with an uncommonly high probability of success in his or her undertakings, then clearly each discovery requires too much 'luck' (or 'serendipity') to be worthy of the term 'genius'. So chance seems to have usurped the dominating position in explicating the appearance of multiples in the history of science and technology.

• *NOTES*

1. Bernard Barber, *Science and the Social Order* (Glencoe, Ill.: Free Press, 1952), 198-201; Leslie While, *The Science of Culture* (New York: Farrar, Strauss, 1949), 204-11.

2. W. F. Ogburn and D. Thomas, 'Are Inventions Inevitable?', *Political Science Quarterly*, Vol. 37 (1922), 83-93. See also William F. Ogburn, *Social Change* (New York: Delta, 1966), 90-102.

3. A.L. Kroeber, 'The Superorganic', *American Anthropologist*, Vol. 19 (1917), 163-214. See also Alfred L. Kroeber, *Configurations of Culture Growth* (Berkeley, Calif.: University of California Press, 1944), 12-13.

4. R. K. Merton, 'Singletons and Multiples in Scientific Discovery: A Chapter in the Sociology of Science', *Proceedings of the American Philosophical Society*, Vol. 105 (1961), 470-86.

5. Jacob Schmookler, *Invention and Economic Growth* (Cambridge, Mass.: Harvard University Press, 1966), 189-95. Schmookler advances another very crucial criticism — namely, that many claimed independent discoveries are in point of fact spurious. For instance, to ascribe the advent of the steamboat to several inventors errs in grouping diverse inventions under a single generic term, and in confusing independent discovery with successive evolutionary development. The upshot of this criticism is that the number of multiples may be *over*estimated. Accordingly, the results to follow are in all likelihood biased in favour of the social deterministic theory.

For a more detailed discussion of this second of Schmookler's criticisms, see Edward Constant, 'On the Diversity and Co-Evolution of Technological Multiples: Steam Turbines and Pelton Water Wheels', *Social Studies of Science*, Vol. 8 (1978), 183-210. Constant's paper should also be examined by any historian who wishes to assert that 'Constant and Simonton independently and simultaneously developed Schmookler's criticism of previous interpretations of multiples'. If the historian does not accept Constant's general argument, then at least the present Poisson model can explain this coincidence!

6. Derek de Solla Price, *Little Science, Big Science* (New York: Columbia University Press, 1963), 65-68.
7. E. C. Molina, *Poisson's Exponential Binomial Limit* (Princeton, NJ: Van Nostrand, 1942). See also George W. Snedecor and William G. Cochran, *Statistical Methods* (Ames, Iowa: University of Iowa Press, 1967, 6th edn.), 223-25.
8. Merton, op. cit. note 4, 477-82.
9. Ibid.
10. Ogburn and Thomas, op. cit. note 2.
11. Snedecor and Cochran, op. cit. note 7, 236-37.
12. Actually, the sum of separate Poisson distributions can yield another Poisson distribution. Yet this outcome obtains solely if the separate distributions are independent, an unlikely assumption in the current situation. See Snedecor and Cochran, op. cit. note 7, 225.
13. Ogburn and Thomas, op. cit. note 2.
14. Snedecor and Cochran, op. cit. note 7, 232.
15. Ludwig Darmstaedter, *Handbuch zur Geschichte der Naturwissenschaften und der Technik* (Berlin: Springer, 1908).
16. Pitirim A. Sorokin, *Social and Cultural Dynamics* (New York: American Book, 1937), 134-35.
17. Merton, op. cit. note 4.
18. One reviewer suggested that the value of μ be permitted to have a Gamma distribution, in which case a compound Poisson model results, yielding the negative binomial distribution. Unfortunately, the distribution also has two free parameters, which takes us back to the estimation problem posed at the paper's outset. Nevertheless, since the form of the negative binomial closely approximates that of the Poisson, it is doubtful that application of this alternative model will alter the main conclusions, even if it would achieve a superior fit. See Merek Fisz, *Probability Theory and Mathematical Statistics* (New York: Wiley, 3rd edn., 1963), 164-68. For a brief discussion of the 'robustness' of the Poisson distribution when the assumptions are only approximate, see Snedecor and Cochran, op. cit. note 7, 225.

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