IEOR 4404 Simulation **Fall 2025**

Homework 1

Instructor: Henry Lam

Due Date: *Sep 19, 2025*

Problem 1. A pair of fair dice is rolled. Let E denote the event that the sum of the dice is equal to 7
(a) Show that E is independent of the event that the first die lands on 4
(b) Show that E is independent of the event that the second die lands on 3 .

Problem 2. Let A, B, C be events such that $P(A) = 0.2, P(B) = 0.3, P(C) = 0.4$ Find the probability that at least one of the events A and B occurs if

(a) A and B are mutually exclusive;
(b) A and B are independent.

Find the probability that all of the events A, B, C occurs if

(c) A, B, C are independent;
(d) A, B, C are mutually exclusive.

Problem 3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. That is, $X = N_H - N_T$ where N_H and N_T denote the number of heads and the number of tails.

a) What are the possible values of X?
b) If the coin is assumed fair, for $n = 3$, what are the probabilities associated with the values that X can take on?
c) Using results from part b), write down the probability mass function of X for $n = 3$.
d) Following part c), write down the cumulative distribution function of X for $n = 3$.

Problem 4. A product is classified according to the number of defects it contains and the factory that produces it. Let X_1 and X_2 be the random variables that represent the number of defects per unit (taking on possible values of 0, 1, 2, or 3) and the factory number (taking on possible values 1 or 2), respectively. The entries in the table represent the joint possibility mass function of a randomly chosen product.

X_1 \ X_2	1	2
0	$\frac{1}{8}$	$\frac{1}{16}$
1	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{3}{16}$	$\frac{1}{8}$
3	$\frac{1}{8}$	$\frac{1}{4}$

a) Find the marginal probability mass functions of X_1 and X_2.
b) Find $E[X_1]$ and $E[X_2]$.
c) Find $Var(X_1), Var(X_2)$ and $Cov(X_1, X_2)$.
d)Find the conditional probability mass function of X_2 given $X_1 = 0$.

Problem 5. A total number of n children of different heights are placed in a line at random. You select the first child from the line, and walk with her/him along the line, until you encounter a child who is taller, or until you reach the end of the line. If you do encounter a taller child, you also have him/her to accompany you further along the line, until you encounter yet again a taller child or reach the end of the line, etc. Let the random variable X denote the number of children to be selected from the line. What is the expected value of X?

Problem 6. If the density function of X equals

$$f(x) = \begin{cases} ce^{-2x}, & \text{for } 0 \leq x < \infty. \\ 0, & \text{for } x < 0. \end{cases}$$

a) Find the value c.
b) What is $P(X > 2)$?

Problem 7. Suppose that the Rockwell hardness X and abrasion loss Y of a specimen (coded data) have a joint density given by

$$f(x, y) = \begin{cases} 3(x + y), & \text{if } x + y \leq 1, x \geq 0, y \geq 0. \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the marginal densities of X and Y .
b) Find $E[X]$ and $Var(X)$.
c) Find $Corr(X, Y)$.
d) Find the conditional marginal density of X given $Y = 1/2$.

Problem 8. The cumulative distribution function of the random variable X is given as

$$F(x) = \begin{cases} 0, & \text{if } x < 0. \\ x/2, & \text{if } 0 \leq x < 1. \\ 2/3, & \text{if } 1 \leq x < 2. \\ 11/12, & \text{if } 2 \leq x < 3 \\ 1, & \text{if } 3 \leq x \end{cases}$$

a) Plot the distribution function $F(\cdot)$.
b) What is $P(X > 1/2)$?
c) What is $P(2 < X \leq 4)$?
d) What is $P(X < 3)$?
e) What is $P(X = 1)$?

Problem 9. Let $X \sim \text{Geo}(p)$ for some p in (0,1); that is, X is a geometric random variable with the parameter denoting probability of success $= p$. Then, is X memoryless, i.e., for all positive integers m and n, and n, is

$$P(X > m + n \mid X > m) = P(X > n)?$$

Problem 10. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes? If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?