Probability HW1

Problem #1

a) $4+2+4+3+11+6+4+4+1+4+2+3+3+1+2+2+6+0+1+3+2$

$= 73$

$70\ 122 \approx \boxed{3.1818}$

b) 0, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 6, 6, 11

$= \boxed{13}$

c) 2 · 6 times

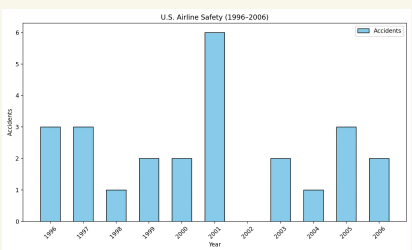
$= \boxed{2}$

d) $SD = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

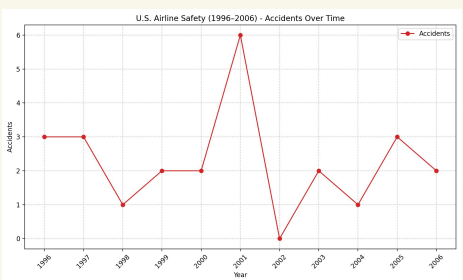
$\bar{x} = 3.318$

$SD = \sqrt{5.392} = \boxed{2.32}$

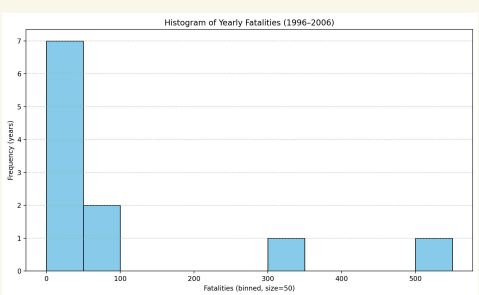
e) bar chart:



time series plot:



f) histogram



Problem #2

a) number of outcomes $= 2^4 = \boxed{16 \text{ possible}}$

b) outcome where 1 & 2 are working $= 4/16 = \{(1,1,0,0), (1,1,0,1), (1,1,1,0), (1,1,1,1)\}$

outcomes where 3 & 4 are working $= 4/16 = \{(0,0,1,1), (0,1,1,1), (1,0,1,1), (1,1,1,1)\}$

$\text{total} = \{(1,1,0,0), (1,1,0,1), (1,1,1,0), (1,1,1,1), (0,0,1,1), (0,1,1,1), (1,0,1,1)\} = 7$ outcomes

c)
$E = \{(0,0,0,0),(0,1,0,0),(0,0,0,1),(0,1,0,1)\} =$ 4 outcomes

Problem #3
if $E \subset F$, $E \cup (F \setminus E)$
$P(F) = P(E) + P(F \setminus E)$, $P(F \setminus E) \geq 0$
Therefore: $P(E) \leq P(F)$

Problem #4:
E happens and F doesn't $= E \cap F^c \Rightarrow (E \cap F^c) \cup (F \cap E^c)$
F happens and E doesn't $= F \cap E^c$
$P(E \cap F^c) = P(E) - P(E \cap F)$
$P(F \cap E^c) = P(F) - P(E \cap F)$
$P(E \cap F^c) \cup P(F \cap E^c) = [P(E) - P(E \cap F)] + [P(F) - P(E \cap F)]$
$= P(E) + P(F) - 2P(E \cap F)$

Problem #5
a) $P(4\text{th is a boy}) = \frac{5}{15} = \frac{1}{3}$
b) $P(12\text{th is a boy}) = \frac{5}{15} = \frac{1}{3}$ $P(12\text{th is a girl}) = \frac{10}{15} = \frac{2}{3}$
c) $\frac{14!}{15!} = \frac{1}{15}$
d) $\frac{5! \, 10!}{15!} = \frac{1}{3003}$
e) $\frac{4! \, 10!}{15!} = \frac{1}{15015}$

Problem #6
total outcomes $= 6^3 = 216$
Event = sum of two dice = one dice = $1+2+3+4+5$ outcomes
$P(\text{Event}) = \frac{15}{216} \approx 0.0694$

Problem #7
total outcomes $= 6^6 = 46{,}656$
A = all faces values appear = 6 outcomes B = one or more 6 appears
$P(A) = \frac{6!}{46{,}656} = \frac{720}{46656} \approx 0.0154$
$P(B) = 1 - P(\bar{B})$ $P(\bar{B}) = \frac{5^6}{6^6} = \frac{15625}{46656} \approx 0.3349$
$= 1 - 0.3349$
$= 0.6651$