

Homework 1

*Instructor: Henry Lam**Due: September 15, 2025*

Problem 1. The following table gives the number of commercial airline accidents and fatalities in the United States in the years from 1985 to 2006.

- Find the mean of the number of yearly airline accidents.
- Find the median of the number of yearly airline accidents.
- Find the mode of the number of yearly airline accidents.
- Find the standard deviation of the number of yearly airline accidents.
- Draw a bar chart and a time series plot for the number of accidents per year against year, from 1996 to 2006.
- Draw a histogram for the number of fatalities, from year 1996 to 2006. Use ranges of size 50 to create the histogram, i.e., 0-49, 50-99 and so on.

For the graphs above, you can either use any computer software or draw by hand.

U.S. Airline Safety, Scheduled Commercial Carriers, 1985–2006

Year	Departures (millions)	Acci- dents	Fatal- ities	Year	Departures (millions)	Acci- dents	Fatal- ities
1985	6.1	4	197	1996	7.9	3	342
1986	6.4	2	5	1997	9.9	3	3
1987	6.6	4	231	1998	10.5	1	1
1988	6.7	3	285	1999	10.9	2	12
1989	6.6	11	278	2000	11.1	2	89
1990	7.8	6	39	2001	10.6	6	531
1991	7.5	4	62	2002	10.3	0	0
1992	7.5	4	33	2003	10.2	2	22
1993	7.7	1	1	2004	10.8	1	13
1994	7.8	4	239	2005	10.9	3	22
1995	8.1	2	166	2006	11.2	2	50

Source: National Transportation Safety Board.

Problem 2. A system is composed of four components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector (x_1, x_2, x_3, x_4) where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

- How many outcomes are in the sample space of this experiment?
- Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working. Specify all the outcomes in the event that the system works.
- Let E be the event that components 1 and 3 are both failed. How many outcomes are contained in event E ?

Problem 3. Show that if $E \subset F$, then $\mathbb{P}(E) \leq \mathbb{P}(F)$.

Problem 4. Show that the probability of exactly one of the events E or F occurs is equal to

$$\mathbb{P}(E) + \mathbb{P}(F) - 2\mathbb{P}(E \cap F).$$

Problem 5. A group of 5 boys and 10 girls is lined up in random order—that is, each of the $15!$ permutations is assumed to be equally likely.

- a) What is the probability that the person in the $4th$ position is a boy?
- b) What about the person in the $12th$ position?
- c) What is the probability that a particular boy is in the $3rd$ position?
- d) What is the probability that all 5 boys are placed in the first 5 positions of the line?
- e) What is the probability that all 5 boys are placed in the first 5 positions of the line and a particular boy is in the 3rd position?

Problem 6. John, Pedro and Rosita each roll one fair die, independently of each other. What is the probability that the score of Rosita is equal to the sum of the scores of John and Pedro?

Problem 7. You throw a fair die six times in a row. What is the probability that all of the six faces values appear? What is the probability that one or more sixes appear?