Homework 3 – Subglacial conduit dynamics

Mathematical Modeling of Geological Processes – Fall 2015
Due: October 16, 2015
25 points

Summary

You will explore the dynamics of the growth and closure of subglacial conduits through analysis of a numerical model. This homework should be turned in as an ipython notebook that contains your code, relevant figures (chosen sparingly), and interpretation of your results.

Key questions to explore are:

1. What is the equilibrium conduit cross-sectional area for a recharge of 2 m^3/s? Can you confirm this analytically or through manual calculations?
2. If the system is started at this equilibrium state for $R = 2\,\mathrm{m}^3\,\mathrm{s}^{-1}$, how long does it take to adjust its area to a reduced recharge of 1 m^3/s or an increased recharge of 4 m^3/s?
3. How much does conduit diameter change over a diurnal cycle if recharge changes over that cycle according to $R = 2(2 + \cos(2\pi t/P)\,\mathrm{m}^3\,\mathrm{s}^{-1}$, where P is 24 hours?

Model

Subglacial conduits can be described mathematically using a reservoir and pipe equation,

$$\frac{dh}{dt} = \frac{R - Q}{A_\mathrm{R}}, \tag{1}$$

coupled to an equation for change in conduit cross-sectional area

$$\frac{dA_\mathrm{c}}{dt} = \frac{f\rho_\mathrm{w}}{8\rho_\mathrm{i}L_\mathrm{f}}\frac{P_\mathrm{wet}Q^3}{A_\mathrm{c}^3} - 2\left(\frac{1}{nB}\right)^n A_\mathrm{c}(P_\mathrm{i} - P_\mathrm{w})|P_\mathrm{i} - P_\mathrm{w}|^{(n-1)}, \tag{2}$$

where R is recharge into the system, Q is discharge in the subglacial conduit, A_R is the reservoir surface area, A_c is the conduit cross-sectional area, h is the water level in the reservoir, ρ_w and ρ_i are the densities of water and ice, respectively, P_wet is the wetted perimeter of the conduit, which is the length of the conduit perimeter that is wet, n is the ice flow law exponent, B is the Arrhenius parameter, and P_w and P_i are the water pressure and ice overburden pressure, respectively. The first equation accounts for conservation of mass within the flow system, and the second equation tracks changes in conduit cross section due to melt (first term on RHS) and creep closure (second term on RHS). You will also need several additional relationships, including:

$$P_\mathrm{w} = \rho_\mathrm{w} g h, \tag{3}$$

$$P_\mathrm{i} = \rho_\mathrm{i} g Z, \tag{4}$$

Parameter	**Value**	**Units**
ρ_{w}	1000	$\mathrm{kg\,m^{-3}}$
ρ_{i}	900	$\mathrm{kg\,m^{-3}}$
L_{f}	3.34×10^5	$\mathrm{J\,kg^{-1}}$
n	3	(unitless)
B	$5.8 * 10^7$	$\mathrm{N\,m^{-2}\,s^{1/n}}$
Z	1000	m
f	0.1	(unitless)
L	5000	m
A_{R}	100	$\mathrm{m^2}$

Table 1: Values of constants used in the model.

where Z is the ice thickness,

$$Q = A_{\mathrm{c}}\sqrt{\frac{2gh}{(1 + fL/D_{\mathrm{H}})}}, \tag{5}$$

and the definition of hydraulic diameter,

$$D_{\mathrm{H}} = \frac{4A_{\mathrm{c}}}{P_{\mathrm{wet}}}. \tag{6}$$

You can also assume that the conduits have a circular cross section and are totally water-filled. Values for constants are given in Table 1.