Understanding Cryptography – A Textbook for Students and Practitioners

by Christof Paar and Jan Pelzl

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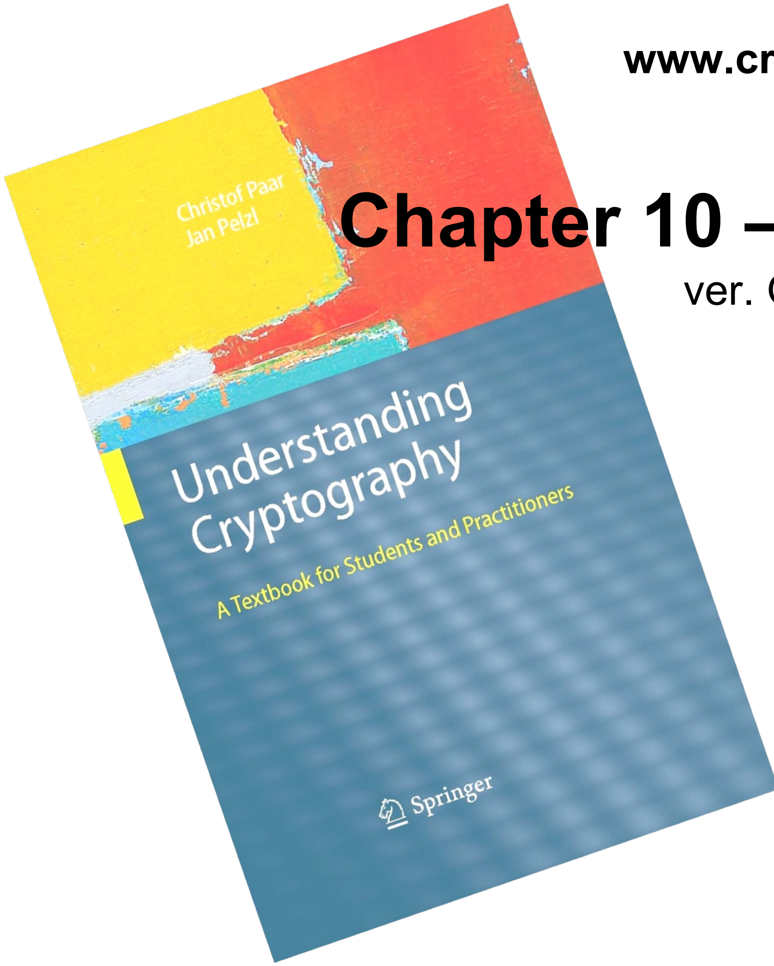

Chapter 10 – Digital Signatures

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These slides were prepared by Georg Becker, Christof Paar and Jan Pelzl

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Content of this Chapter

- The principle of digital signatures
- The RSA digital signature scheme
- The Digital Signature Algorithm (DSA)

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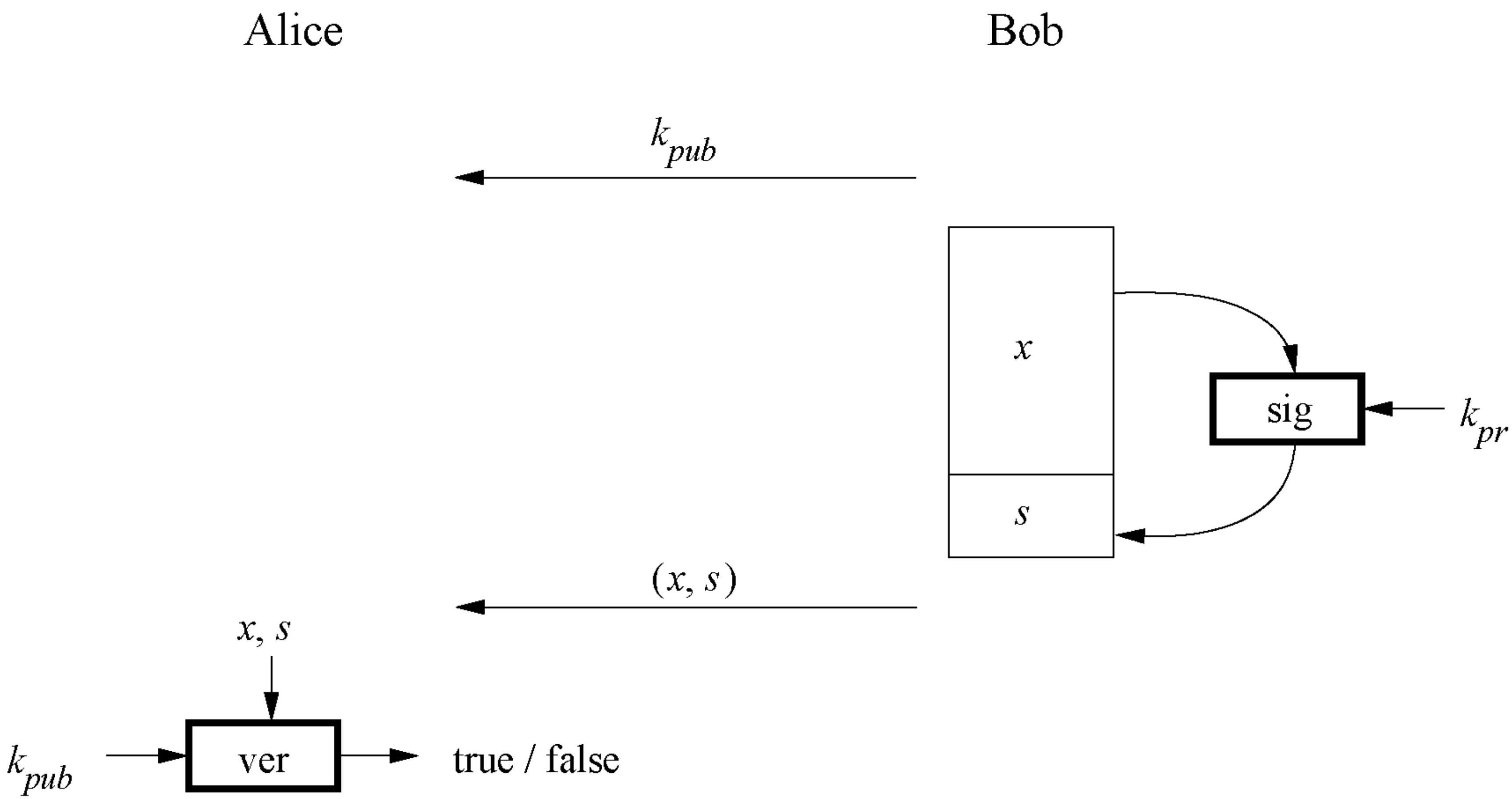
Motivation

- Alice orders a pink car from the car salesmen Bob
- After seeing the pink car, Alice states that she has never ordered it:
- How can Bob prove towards a judge that Alice has ordered a pink car? (And that he did not fabricate the order himself)

$\Rightarrow$ Symmetric cryptography fails because both Alice and Bob can be malicious

$\Rightarrow$ Can be achieved with public-key cryptography

Basic Principle of Digital Signatures



Main idea

- For a given message *x*, a digital signature is appended to the message (just like a conventional signature).
- Only the person with the private key should be able to generate the signature.
- The signature must change for every document.

⇒The signature is realized as a function with the message *x* and the private key as input.

⇒The public key and the message *x* are the inputs to the verification function.

Security Services

Digital signatures provide the following security services:

1. **Integrity:** Ensures that a message has not been modified in transit.
2. **Message Authentication:** Ensures that the sender of a message is authentic. An alternative term is data origin authentication.
3. **Non-repudiation:** Ensures that the sender of a message can not deny the creation of the message. (c.f. order of a pink car)

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Main idea of the RSA signature scheme

To generate the private and public key:

- Use the same key generation as RSA encryption.

To generate the signature:

- "encrypt" the message *x* with the private key

$$\boxed{s = sig_{K_{priv}}(x) = x^d \bmod n}$$

- Append *s* to message *x*

To verify the signature:

- "decrypt" the signature with the public key

$$\boxed{x' = ver_{K_{pub}}(s) = s^e \bmod n}$$

- If $x=x'$, the signature is valid

The RSA Signature Protocol

Alice		Bob
	$\xleftarrow{\quad K_{pub} \quad}$	$K_{pr} = d$ $K_{pub} = (n, e)$
		Compute signature: $s = sig_{k_{pr}}(x) \equiv x^d \bmod n$
	$\xleftarrow{\quad (x,s) \quad}$	

Verify signature:
$x' \equiv s^e \bmod n$
If $x' \equiv x \bmod n \rightarrow$ valid signature
If $x' \not\equiv x \bmod n \rightarrow$ invalid signature

Security and Performance of the RSA Signature Scheme

Security:

The same constrains as RSA encryption: *n* needs to be at least 1024 bits to provide a security level of 80 bit.

$\Rightarrow$ The signature, consisting of *s*, needs to be at least 1024 bits long

Performance:

The signing process is an exponentiation with the private key and the verification process an exponentiation with the public key *e*.

$\Rightarrow$ Signature verification is very efficient as a small number can be chosen for the public key.

Existential Forgery Attack against RSA Digital Signature

Alice	Oscar	Bob
		$K_{pr} = d$ $K_{pub} = (n, e)$
$\xleftarrow{(n,e)}$	$\xleftarrow{(n,e)}$	
	1. Choose signature: $s \in Z_n$ 2. Compute message: $x \equiv s^e \bmod n$	
$\xleftarrow{(x,s)}$		

Verification:

- compute $x' \equiv s^e \bmod n$
- Compare x and x':

  $x \equiv s^e \equiv x' \bmod n$

  Signature is valid!

Existential Forgery and Padding

- An attacker can generate valid message-signature pairs (x,s)
- But an attack can only choose the signature s and NOT the message x

⇒ Attacker cannot generate messages like “Transfer $1000 into Oscar‘s account”

Formatting the message x according to a *padding scheme* can be used to make sure that an attacker cannot generate valid (x,s) pairs.

(A messages x generated by an attacker during an Existential Forgery Attack will not coincide with the padding scheme. For more details see Chapter 10 in *Understanding Cryptography.)*

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Facts about the Digital Signature Algorithm (DSA)

- Federal US Government standard for digital signatures (DSS)
- Proposed by the National Institute of Standards and Technology (NIST)
- DSA is based on the Elgamal signature scheme
- Signature is only 320 bits long
- Signature verification is slower compared to RSA

The Digital Signature Algorithm (DSA)

Key generation of DSA:

1. Generate a prime p with $2^{1023} < p < 2^{1024}$
2. Find a prime divisor q of $p-1$ with $2^{159} < q < 2^{160}$
3. Find an integer α with $\mathrm{ord}(\alpha)=q$
4. Choose a random integer d with $0<d<q$
5. Compute $\beta \equiv \alpha^d \bmod p$

The keys are:

$k_{pub} = (p,q,\alpha,\beta)$

$k_{pr} = (d)$

The Digital Signature Algorithm (DSA)

DSA signature generation :

Given: message x, signature s, private key d and public key (p,q,α,β)

1. Choose an integer as random ephemeral key k_E with $0<k_E<q$
2. Compute $r \equiv (\alpha^{k_E} \bmod p) \bmod q$
3. Computes $s \equiv (\mathrm{SHA}(x)+d \cdot r)\, k_E^{-1} \bmod q$

The signature consists of (r,s)

SHA denotes the hash function SHA-1 which computes a 160-bit fingerprint of message x. (See Chapter 11 of *Understanding Cryptography* for more details)

The Digital Signature Algorithm (DSA)

DSA signature verification

Given: message x, signature s and public key (p,q,α,β)

1. Compute auxiliary value $w \equiv s^{-1} \bmod q$
2. Compute auxiliary value $u_1 \equiv w \cdot SHA(x) \bmod q$
3. Compute auxiliary value $u_2 \equiv w \cdot r \bmod q$
4. Compute $v \equiv (\alpha^{u_1} \cdot \beta^{u_2} \bmod p) \bmod q$

If $v \equiv r \bmod q \rightarrow$ signature is valid

If $v \not\equiv r \bmod q \rightarrow$ signature is invalid

Proof of DSA:

We show need to show that the signature (*r,s*) in fact satisfied the condition $r \equiv v \bmod q$:

$s \equiv (\mathrm{SHA}(x))+d \cdot r) \cdot k_E^{-1} \bmod q$

$\Leftrightarrow \quad k_E \equiv s^{-1}\,\mathrm{SHA}(x) + d \cdot s^{-1}\, r \bmod q$

$\Leftrightarrow \quad k_E \equiv u_1 + d \cdot u_2 \bmod q$

We can raise α to either side of the equation if we reduce modulo *p*:

$\Leftrightarrow \quad \alpha^{k_E} \bmod p \equiv \alpha^{u_1 + d \cdot u_2} \bmod p$

Since $\beta \equiv \alpha^d \bmod p$ we can write:

$\Leftrightarrow \quad \alpha^{k_E} \bmod p \equiv \alpha^{u_1} \beta^{u_2} \bmod p$

We now reduce both sides of the equation modulo q:

$\Leftrightarrow \quad (\alpha^{k_E} \bmod p) \bmod q \equiv (\alpha^{u_1} \beta^{u_2} \bmod p) \bmod q$

Since $r \equiv (\alpha^{k_E} \bmod p) \bmod q$ and $v \equiv (\alpha^{u_1} \beta^{u_2} \bmod p) \bmod q$, this expression is identical to:

$\Leftrightarrow \quad r \equiv v$

Example

Alice | Bob

Key generation:

1. choose p = 59 and q = 29
2. choose α = 3
3. choose private key d = 7
4. $\beta = \alpha^d = 3^7 \equiv 4 \bmod 59$

(p, q, α, β)=(59, 29, 3, 4) ←

Sign:

Compute has of message H(x)=26

1. Choose ephermal key k_E=10
2. $r = (3^{10} \bmod 59) \equiv 20 \bmod 29$
3. $s = ((26 + 7 \cdot 20) \cdot 3) \equiv 5 \bmod 29$

$(x,(r, s))$=(x,20, 5) ←

Verify:

$w \equiv 5^{-1} \equiv 6 \bmod 29$

$u_1 \equiv 6 \cdot 26 \equiv 11 \bmod 29$

$u_2 \equiv 6 \cdot 20 \equiv 4 \bmod 29$

$v = (3^{11} \cdot 4^4 \bmod 59) \bmod 29 = 20$

$v \equiv$ r mod 29 → valid signature

Security of DSA

To solve the discrete logarithm problem in *p* the powerful index calculus method can be applied. But this method cannot be applied to the discrete logarithm problem of the subgroup *q*. Therefore *q* can be smaller than *p*. For details see Chapter 10 and Chapter 8 of *Understanding Cryptography* .

p	q	hash output (min)	security levels
1024	160	160	80
2048	224	224	112
3072	256	256	128

Standardized parameter bit lengths and security levels for the DSA

Security of DSA

- Reuse of ephemeral key can lead to the disclosure of the signing key.
- Exercise: prove this.

- Real-world incident:
  - Sony Playstation uses the same constant as the ephemeral key in its digital signatures.
  - This was exploited in 2010 by a hacker to obtain the signing key:
  - https://www.bbc.com/news/technology-12116051

Elliptic Curve Digital Signature Algorithm (ECDSA)

- Based on Elliptic Curve Cryptography (ECC)
- Bit lengths in the range of 160-256 bits can be chosen to provide security equivalent to 1024-3072 bit RSA (80-128 bit symmetric security level)
- One signature consists of two points, hence the signature is twice the used bit length (i.e., 320-512 bits for 80-128 bit security level).
- The shorter bit length of ECDSA often result in shorter processing time

For more details see Section 10.5 in *Understanding Cryptography*

Lessons Learned

- Digital signatures provide message integrity, message authentication and non-repudiation.
- RSA is currently the most widely used digital signature algorithm.
- Competitors are the Digital Signature Standard (DSA) and the Elliptic Curve Digital Signature Standard (ECDSA).
- RSA verification can be done with short public keys *e*. Hence, in practice, RSA verification is usually faster than signing.
- DSA and ECDSA have shorter signatures than RSA
- In order to prevent certain attacks, RSA should be used with padding.
- The modulus of DSA and the RSA signature schemes should be at least 1024-bits long. For true long-term security, a modulus of length 3072 bits should be chosen. In contrast, ECDSA achieves the same security levels with bit lengths in the range 160–256 bits.