

Universidad de los Andes

PHYSICS DEPARTMENT

MEASUREMENT AND CHARACTERIZATION OF
GRANULATION PATTERNS IN THE IAG SOLAR FLUX
SPECTRUM

Quasi-final version

BSc Physics Final Project

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Abstract

Hello, this is my work:D

The present project analyses the three signatures of convection presentes on the solar spectrum. Measurement of the granulation patterns, characterization of the wavelength-dependence on many feautres of the star clears the concept of convection in the Sun. I hate the near infrared. I made a visualizer (im not doing that again)

Acknowledgements

Peace! Not crazy war!

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1. Introduction

The solar granulation patterns

For decades, the solar spectrum has served as the fundamental reference point for spectroscopic analysis. As our nearest star, the Sun enables detailed studies of stellar composition. However, advances in optical instrumentation have revealed previously undetected spectral details, providing new insights into fundamental solar properties. One key insight is the granulation patterns caused by convective motion, which are characterized by three distinct hydrodynamic signatures: Line broadening, line profile asymmetry, and the line-depth dependence of convective blueshift.

Inspired by David Gray's foundational research, this project aims to extract the granulation patterns from the IAG Solar Flux Atlas to calculate relative velocities and perform a detailed analysis based on the three signatures of convection.

1.1 The three signatures of convection

The stellar spectrum serves as astronomy's primary source of information, encoding details about a star's composition and velocity. However, in some stars this spectrum is modified due to fluid motions caused by density and temperature fluctuations in the outermost layer, where each spectral line exhibit a distinct velocity shift. These persistent convective motions generate a granular structure in the solar photosphere, a phenomenon supported by extensive research (see [8, 5, 9]).

The granulation in the Sun photosphere was first observed as a moving granular structure by Janssen in 1885. Later, in 1901, Plaskett associated this pattern with the convective cells observed at Bénard’s experiment [4], where fluids heated from below produces rising elements of hot gas convecting heat to the surface. This characteristic configuration produces three signatures of granulation in stellar spectra: Line broadening, line profile asymmetry, and line-depth dependence of convective blueshift [10]. The method for quantifying this relationship are known as the *granulation patterns*; which characterize the signatures mentioned above [9]. The physical origin of these patterns lies in the convective process on the solar photosphere itself.

When the Sun pushes material up through its outer layer, the spectrum exhibits a blueshift. As this material subsequently cools and falls back through the atmosphere, it produces a redshift but emits less light, making the blueshift dominant. This characteristic is called *convective blueshift*.

Neutral iron (Fe I) lines are optimal for calibrating this phenomenon due to their high abundance, minimal thermal broadening, limited isotopic variation and availability of accurate laboratory measurements of natural wavelengths. Furthermore, the even number of nucleons in the most abundant isotope resulting in no hyperfine structure shown. The use of a single chemical species offers the additional advantage of retrieving natural wavelengths from a single source, thereby eliminating potential discrepancies [11]. The project used the Nave list of Fe I lines, which presented the values of laboratory measurements as complete and accuracy for the purposes [12].

The study of granulation patterns has been significantly documented by David Gray, whose work has improved the precision of stellar radial velocity measurements (see [9] and references therein). This improvement stems primarily from his observation that granulation patterns in solar-type stars closely resemble the granulation pattern from the third signature of convection. The line depth-dependence of convective blueshift for different solar-type stars differing primarily by a scaling factor from the solar one. This is particularly significant given the considerable challenges of obtaining such precise measurements for other stars, which are often affected by spectral noise, stellar proper motions, and velocity uncertainties [9]. Further-

more, analyzing solar granulation patterns is crucial for validating photospheric hydrodynamic models [5], enabling improved calibration and testing of dynamic atmospheric models.

Following the motivation of treating the Sun as any other star, which does not have the angular resolution necessary to study the spatial granulation structure, the most accurate solar flux atlas is necessary.

1.2 The IAG solar flux atlas

In 2016, Reiners and collaborators published the unprecedented precision *Institut für Astrophysik und Geophysik* (IAG) solar flux atlas obtained with the FTS *Fourier Transform Spectrograph* at Göttingen, simultaneously reporting convective blueshifts for a sample of Fe I lines. This atlas provides highly precise and accurate data, with radial velocity uncertainties on the order of ± 10 m/s across the wavelength range from 4050 Å to 10 650 Å. In contrast to other FTS atlases, the entire visible wavelength range was observed simultaneously using only one spectrograph setting [13].

Despite the exceptional quality of the spectrum, the first derived granulation pattern for the third signature of convection appeared notably scattered and noisy. This was attributable to a rudimentary line position measurement methodology and a poor line selection which included numerous blended features, outdated wavelength references, and incomplete spectral coverage. Even if the atlas contains the dataset for the near infrared range, no treatment was implemented on this part of the spectrum. Consequently, the full potential of the IAG solar flux atlas for determining precise convective blueshifts remains unrealized, highlighting the need for the refined analytical methods applied in this project.

1.3 The IAG spatially resolved quiet Sun atlas

In 2023, Ellwarth and collaborators [6] published the *Institut für Astrophysik und Geophysik* (IAG) spatially resolved quiet sun atlas, also obtained with the FTS *Fourier Transform Spectrograph* at Göttingen. This atlas has the advantage of its coverage from the disk center ($\mu = 1.0$)

towards the solar limb ($\mu = 0$), where $\mu = \cos(\theta)$ with θ being the angle between the surface normal and the observer's position. This spatial resolution enables the study of how convective blueshifts vary across the solar disk due to changing projection angles, a dependency that cannot be observed in other stars.

The principal objective of this project is to characterize the solar granulation patterns by treating the Sun as any other star. To achieve this, the disk-integrated IAG solar flux atlas serves as our primary data source. However, a precise analysis of the relative velocities in this integrated light requires correcting for the center-to-limb variations that are uniquely quantified by the IAG spatially resolved quiet Sun atlas.

1.4 Motivation

In the first semester of 2025, Manuel Fuentes, a physics student at the Universidad de los Andes, implemented these improvements in a computational project focused on the visible spectral range. By developing enhanced measurement techniques and employing a carefully curated line list that fully covers the visible range with updated wavelength references, Fuentes achieved significantly sharper granulation patterns than previous analyses. This work demonstrated that proper line selection and modern wavelength standards can reliably extract convective signatures from high-quality solar spectra.

Under the guidance of Professor Benjamin, the present project study the visible range and extends the analysis into the near infrared range. This region contains spectral lines that originate from deeper photospheric layers. Although these lines are consequently weaker, they represent a rich source of information. This project also extends previous efforts by measuring the line profile asymmetry and line broadening, other spectral signs of granulation beyond convective blueshift; and exploring how these phenomena depend on the wavelength range.

By addressing these challenges, this project intended to produce a robust characterization of the solar granulation patterns and new insights into the photosphere's dynamics. These efforts were guided by the central research question: What are the direct spectroscopic consequences of solar convection?

2. Literature Review

Convective motion in the Sun

As previously mentioned, David Gray has significantly advanced the study of granulation patterns in the solar photosphere, with a particular focus on measuring relative velocities with high precision. This chapter explores the physical origins of the three signatures of convective motion in the Sun photosphere, and how this reveals the hydrodynamics on the outermost layer.

2.1 The solar interior and the solar outer atmosphere

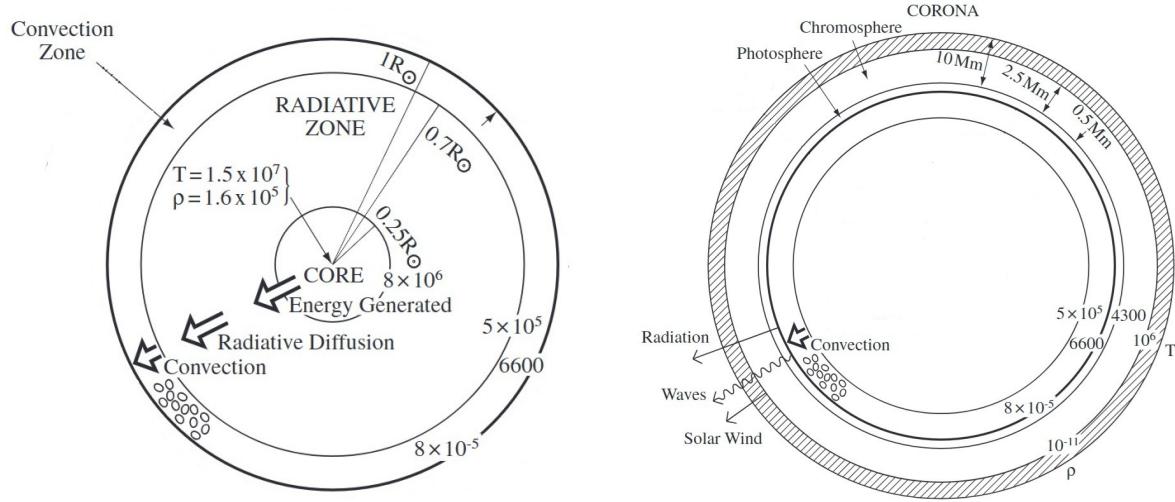
The Sun is classified as a yellow dwarf star of spectral type G2V. Its chemical structure is primarily composed of a large fraction of ionized hydrogen and a smaller proportion of helium. What makes the Sun unique in astronomical studies is its proximity to Earth, which allows for detailed observation unmatched for any other star. Structurally, the Sun is divided into two main regions: The solar interior and the solar outer atmosphere.

As illustrated in Figure 2.1a the overall structure of the solar interior is core, radiative and convective zone. From the core, He nuclei is built from H nuclei in the proton-proton chain as Equation (2.1) refers.



The proton-proton chain reaction in the core liberates approximately 26.7 MeV of energy in the form of high energy γ -rays, and 0.5 MeV of energy in the form of neutrinos. In this

zone, standard models estimate a temperature of 1.6×10^7 K and density to 1.6×10^5 kg/m³. Moving outward through the layers, both the density and temperature decrease significantly, as the energy is slowly transferred outwards by radiative diffusion [4]. This process progressively shifts the wavelength of the radiation from high energy γ -rays to the visible light that eventually escapes. The large temperature combined with the high density, allows the absorptions and remissions of photons that make this zone highly opaque and maintain the central material in a plasma state, functioning like a massive nuclear reactor.



(a) The interior structure of the Sun. The convection zone is responsible for the general movement that characterizes the granulation patterns.

(b) The outer structure of the Sun. The photosphere is the layer of the Sun where the convection cells overshoot from the convection zone.

Figure 2.1: The general structure of the Sun. Images modified from [1].

On the other hand, Figure 2.1b illustrates the overall structure of the solar outer atmosphere consisting of the photosphere, chromosphere and corona. In these layers, the density decreases rapidly with height above the solar surface. However, the temperature decreases to a minimum of approximately 4300 K in the upper photosphere before rising through the chromosphere and transition region to millions of degrees in the corona. From that point, the temperature falls slowly expanding outwards as the solar wind.

Nevertheless, the relevant layers for this project are the photosphere, a thin layer of plasma that emits most of the solar radiation; and the convection zone, in which all the convection process

take place. The radiation chain process from this target layers results on the emission of a continuous spectrum passes through the overlying chromosphere. Then specific wavelengths are absorbed for this layer, resulting in the characteristic Fraunhofer lines superimposed on the emitted spectrum, which allows observing the convection consequences [1].

2.2 The solar convection zone

In 1885 Janssen obtained the first clear photograph of photospheric granules (see Figure 2.2) providing the initial evidence and the starting point for numerous studies about granulation [2].

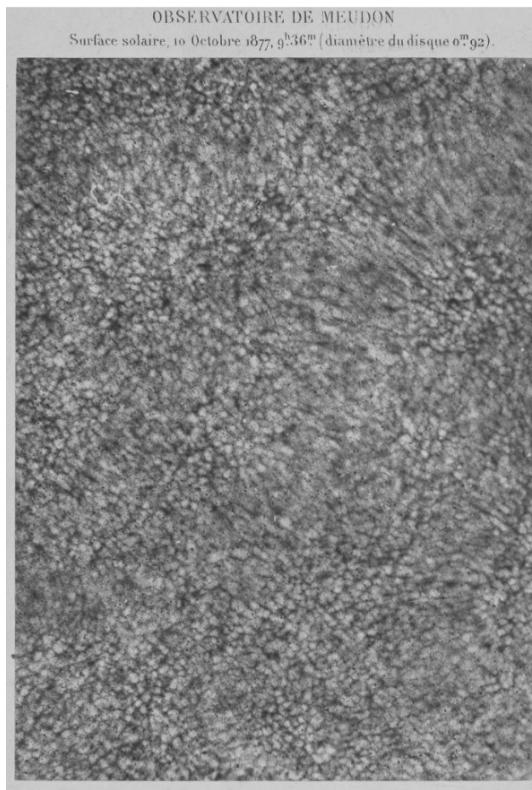


Figure 2.2: The first clear photograph of the photosphere where the granulation pattern is visible took by Janssen in 1885. Image taken from [2].

Following the history, in 1930, Unsöld theorized that the layers beneath the photosphere should be convective unstable [4]. This hypothesis was later supported by Plaskett when he related the observed granules to the convective cells studied in Bénard's laboratory experiments [14].

In this analogy, a fluid heated from below develops rising elements of hot gas that transport heat to the surface.

The elements of hot gas rising transporting heat are called *convective cells*, and the pattern generated by several cells on the photosphere is the *granulation* with each individual region referred to as a *granule*. Typical granules span approximately 700 km and have short lifetimes, lasting between five to ten minutes.

In the solar context, convection takes place in a highly compressible and stratified gas ubicated at approximately $0.86R_{\odot}$ and affected by a large temperature gradient. This physical regime leads to determine the conditions required for convection to occur and the resulting dynamics of the granules [4].

2.2.1 The Schwarzschild criterion

Consider an elementary parcel of material displaced in local hydrostatic equilibrium with its surroundings, characterized by radial profiles of pressure $P(r)$, density $\rho(r)$, and temperature $T(r)$. If the granule temperature is increased to a value T' , it will expand adiabatically to maintain pressure equilibrium, thereby decreasing the density relative to its surroundings (see Figure 2.3).

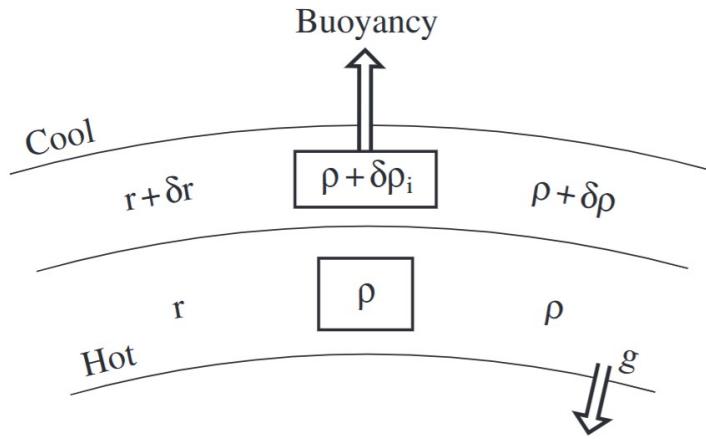


Figure 2.3: Diagram for the parcel of material displaced so slowly that remains in horizontal pressure. Image taken from [1].

This convective cell experiences a buoyancy force, causing it to rise. The buoyancy force persists until the granule's density matches that of its new surroundings after traveling a length l .

Let T'_n be the temperature of the rising element and T_n the temperature of its new surroundings.

The difference between the adiabatic gradient of the element and the radiative gradient of the surroundings governs the convection, as described by Equation (2.2)

$$T_n = T + \left(\frac{dT}{dr} \right)_R l \quad | \quad T'_n = T' + \left(\frac{dT}{dr} \right)_{ad} l \quad (2.2)$$

Where R refers for radiative temperature gradient and ad for the adiabatic temperature gradient. The onset of convection leads to the inequality (2.3), where two conditions can arise: The granulation is established when adiabatic gradient exceeds the radiative gradient; otherwise, the layer is stable and energy is transported by radiation.

$$-\left(\frac{dT}{dr} \right)_R > \left(\frac{dT}{dr} \right)_{ad} \quad (2.3)$$

If the granulation is established, the element continues to expand adiabatically as it rises, driven by buoyancy. Otherwise, if the layer is stable, the element will contract, becomes heavier than its surroundings and begins to move down toward its original position.

This onset of instability, when the vertical temperature gradient is too large, is known as the Schwarzschild criterion for convection. Conveniently this criterion is expressed in terms of the relation between T , P and γ (heat capacity ratio) for an adiabatic change (see Equation (2.4)).

$$-\left(\frac{dT}{dr} \right)_R > \frac{\gamma - 1}{\gamma} \left(\frac{T}{P} \right) \left(-\frac{dP}{dr} \right)_{ad} \quad (2.4)$$

In this form, the condition establish that convection can set when opacity rises rapidly, due to increasing the population of $n = 3$ level of hydrogen and γ lowered by ionization. The material and energy transported by this process ended on the low photosphere, where the granules exhibit different properties that will be examined in subsequent sections.

2.3 The Solar Photosphere

As mentioned in the previous section, the observation of this layer lead to the study of dynamics and reactions within Sun's outermost layer. From surface observations a distinct pattern of

granules with dynamic behavior is apparent, where individual cells continuously emerge and disappear (see Figure 2.4).

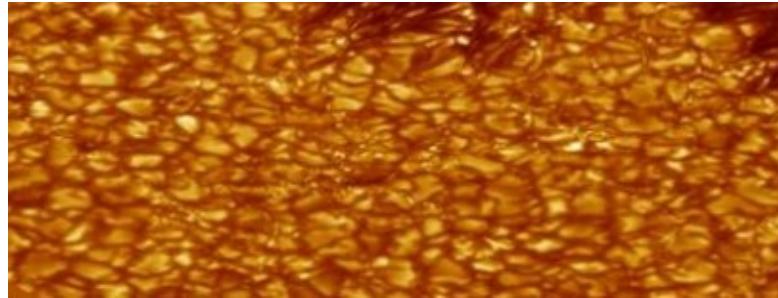


Figure 2.4: A view of granulation on the Sun's surface. Image taken from [3].

The bright areas of granules correspond to regions where hot gas rises through the solar atmosphere. As this gas releases energy in the form of photons at the photosphere, it cools and subsequently descends, creating the darker regions of intergranular lanes [15]. Furthermore, high-resolution observations reveal that these granules are in continual motion generating asymmetries in absorption line profiles (see Figure 2.5).

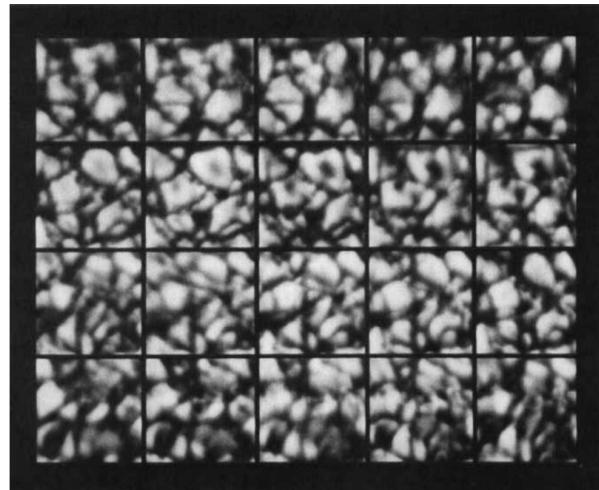


Figure 2.5: A time sequence showing granule evolution where the time intervals are about a minute. Image taken from [4].

2.3.1 Static photosphere: Limb darkening phenomenon

Because the temperature decreases outward through the photospheric layers, the observed intensity falls off towards the solar limb. Discovered by Halm in 1907 [5], this effect is known as *limb darkening phenomenon*, which make the disk intensity profile to appear more squared at increasing wavelength (see Figure 2.6).

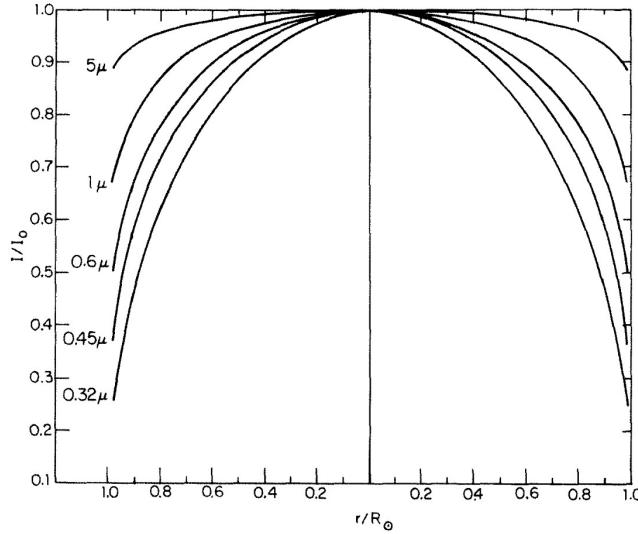


Figure 2.6: Squared profile for the disk intensity at increasing wavelengths, where 5μ refers to the infrared range and 0.32μ the violet range. Image taken from [4].

The analysis of this effect provides a direct technique for determining the temperature structure of the photosphere as a function of line depth.

For typical weaker lines, the convective blueshift diminishes toward the limb, with a net velocity change approximately of 400 m/s. As explained by Ellsworth et al. observations closer to the limb pass through the atmosphere at a shallower angle, resulting in longer optical paths through high atmospheric layers that allows the study of the layers where convective blueshift is less pronounced [6]. Because of this phenomena and following the objective of studying the line depth-dependence of convective blueshift, the center disk flux spectrum was taken as reference for analysis, where the limb darkening effect is negligible.

2.3.2 Dynamic photosphere: The C-curved profile bisector.

Analysis of changes, contrast and velocity field in the granulation structure have been inferred indirectly from observations of Fraunhofer line profile shapes [4]. The observations on absorption lines reveals that velocity of a rising granule decays less rapidly than its excess brightness, resulting in a characteristic C-shaped line profile bisector (see Figure 2.7).

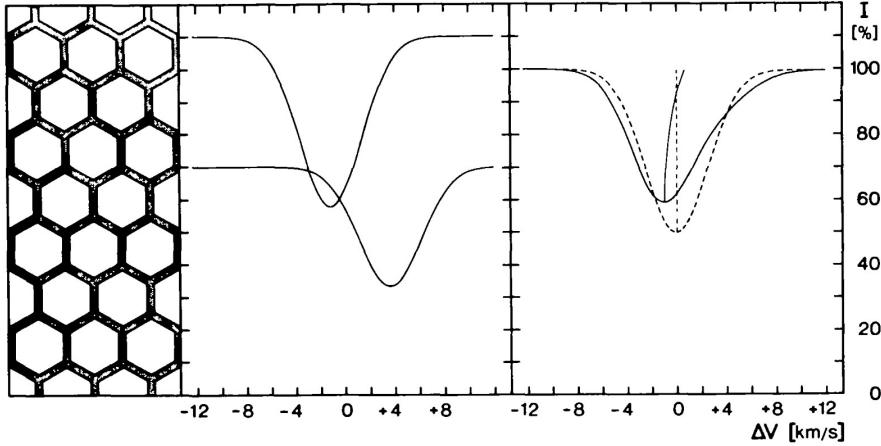


Figure 2.7: The C-shaped line profile bisector with the corresponding diagram of wavelength displacement due to convective blueshift. Image taken from [5].

The formation of the C-shaped line profile bisector occurs in three stages, corresponding to different heights in the photosphere.

First, the deepest part of the line profile is formed higher up, in a region of decelerated upflow, producing a smaller blueshift. Then, the mid-depth portion is formed in the brightest upflowing material, resulting in a blueshift. Finally, the line profile wings where the opacity is lowest, tend to be formed deepest in the cool material, producing a redshift.

This dynamic process induces characteristic asymmetries on the line profile bisector, which becomes the principal instrument to measure convection process from the solar atmosphere.

2.4 The three signatures of convection

The three signatures of convection in stars are described by Gray in his research (see [9, 7, 10] and references therein) as the principal characteristics to identify and treat the asymmetries due to convective motion.

2.4.1 First signature of convection: Line broadening

The absorption lines can be classified by decreasing intensity line profile as strongest and weakest lines [10].

In general, stronger absorption lines exhibit more pronounced line core curvature than weaker lines. However, the line core curvature is affected by different broadening effects due to Doppler, atomic and magnetic effects as we talk of the photosphere like a stratified hot gas with magnetic flux. **IN PROGRESSS**

This line core curvature C_c can be quantified by the second derivative of the line's intensity profile with respect to wavelength, evaluated at observed wavelength (see Equation (2.5)).

$$C_c = \lambda_{obs}^2 \left(\frac{d^2 p(\lambda_{obs})}{d \lambda_{obs}^2} \right) \quad (2.5)$$

In the near infrared range, the solar spectrum contains natural bands or groups of lines that are separated by regions of strong absorption from telluric elements. In other words, elements like O₂ and CO₂ that are absorbed by the atmosphere. This phenomenon modify the line core curvature of determining lines profiles, specially in the near infrared range.

IN PROGRESSS

2.4.2 Second signature of convection: Line profile asymmetry

As demonstrated by Nieminen [?], the asymmetry occurs because the c-curved profile bisector reflects velocity variations and a bisector slope (see Figure 2.8).

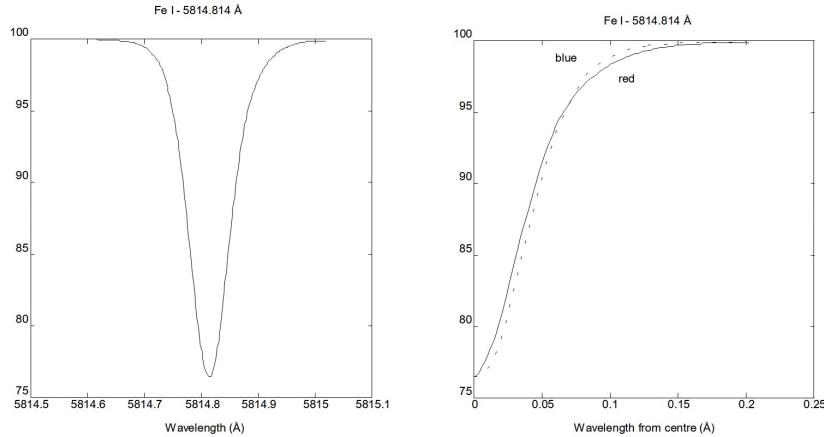


Figure 2.8: Asymmetries on an average absorption line. Can be observed the differences in intensity redshift profile. Image taken from [?].

According to Kirchhoff's laws, absorption line formation requires lower temperature conditions, which are found precisely in the Sun's outermost atmospheric layers [15]. These regions not only provide the appropriate temperatures for absorption but also exhibit comparatively higher opacity. Those spectral lines from Fe I are particularly valuable for solar granulation studies due to their high abundance, minimal thermal broadening, limited isotopic variation and availability of accurate laboratory measurements of natural wavelengths [?].

These asymmetries imprinted on the Fe I lines can be quantified by analyzing their bisectors. A convenient method for this measures involves using the third derivative of the line profile, which provides the slope of the lowest end of the bisector (see Equation (2.6)).

$$-\frac{c}{\lambda_{obs}} \left(\frac{1}{3C_c^2} \right) \left(\frac{d^3 p(\lambda_{obs})}{d\lambda_{obs}^3} \right) \quad (2.6)$$

The slope of the line profile is defined as zero when the line profile bisector is vertical and the line core is symmetric. The relation is derived in appendix B.

Since most of stellar observations are made with lower-resolution spectrographs and often lower signal-to-noise ratios, there is a need to study granulation through its signatures. Gray and Oostra identified a form of line profile asymmetry observable under these conditions, known as the *flux deficit*. This phenomenon can be interpreted as flux imbalances on the redshifted side of the line profile, caused by a net loss of radiative energy.

To quantify this asymmetry, the authors proposed calculating the flux deficit by taking the difference between the observed, altered profile and a the granulation curve. By treating the altered line profile as an approximate representation of the third signature of convection, one can observe the flux difference interpreted as radiation [7].

2.4.3 Third signature of convection: Line depth-dependent wavelength shifts

Many studies across the years have detected and observed the phenomenon of wavelength shifts against the line depth, or as it is called, convective redshift. This behavior was shown to be more present in the weaker lines which are related to infrared and violet range.

2.4.4 Solar granulation pattern

The solar granulation pattern is a plot of relative velocity against line depth, as shown Figure 2.9.

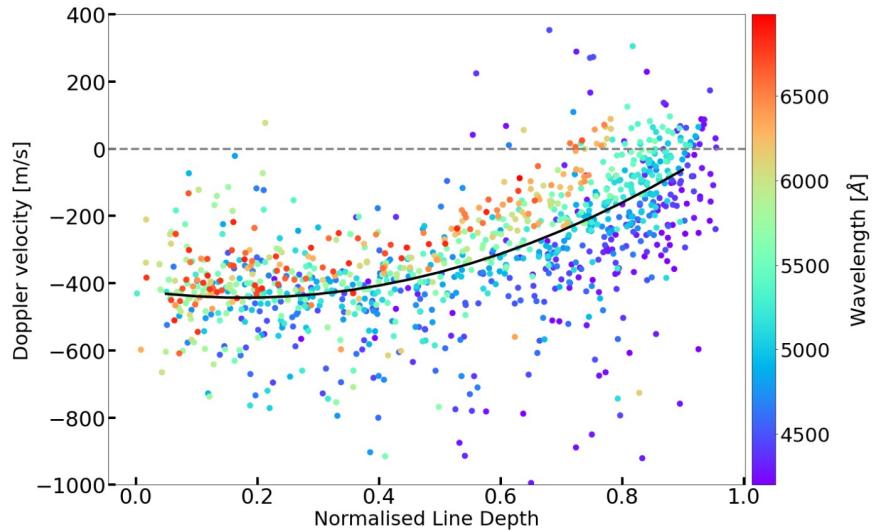


Figure 2.9: Granulation pattern for the IAG spatially resolved quiet sun atlas, is shown to have a strong blueshift in the weaker lines. Image taken from [6].

Notice in Figure 2.9 the wavelength-dependence os the pattern. The significance of the gran-

ulation pattern lies in its universality for solar-type stars; their patterns closely resemble the Sun's, differing primarily by a scaling factor (see [10]). Consequently, a detailed analysis contributes to the understanding and radiation of photospheric hydrodynamic models [5, 9]. However, a precise measurement of the granulation pattern requires a understanding of the convective blueshift phenomenon.

2.4.5 Convective Blueshift

When the Sun pushes material up through its outer layer, the spectrum exhibits a blueshift. As this material subsequently cools and falls back through the atmosphere, it produces a redshift, but emits less light, making the blueshift dominant.

That can be measured by the relativistic Doppler effect. This effect accounts for length contraction, as predicted by Einstein's theory of relativity. However, the measured negative redshift resulting from convective motions is known as *convective blueshifts* which are measured by the Equation (2.7).

$$v_r \approx c \left(\frac{\lambda_{obs} - \lambda_{rep}}{\lambda_{rep}} \right) \quad (2.7)$$

Since the strength of the convective distortions and shifts of spectral lines vary across the H-R diagram, we expect systematic errors in radial velocities [9].

2.4.6 Chromodependence characterization

Recent emphasis on measuring the granulation pattern has led to new interpretations of line depth-dependent wavelength shifts.

Gray and others authors have qualitatively explained the line-depth dependence on convective blueshift. Shallow lines come preferably from deep photospheric layers where convection is strong, so the convective blueshift is great. As long as deep lines come preferably from superficial layers, where gravity has slowed the convection and the blueshift is small. However, an explanation of why diagonal trend depends on color, or chromodependence along te line depth, has not been found.

In 2018, Gray and Oostra attempted to establish a standard curve determined by a third order

polynomial fit to the solar granulation pattern (see Figure 2.10).

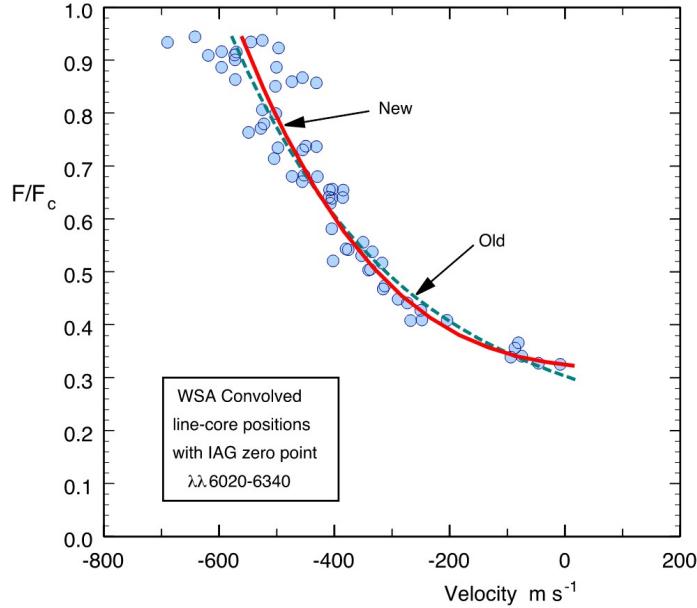


Figure 2.10: Standard curve proposed by Gray and Oostra on the spectral range the spectral range of 6020 Å to 6340 Å. Image taken from [7].

Nevertheless, the authors limited the spectral range of 6020 Å to 6340 Å, avoiding the chromodependence which becomes evident in a wider spectral range. This limitation motivates the present work to develop a strong characterization and give the methodology for dealing with this phenomenon.

A separate theoretical perspective, offered by Hamilton and Lester, attributes aspects of photospheric dynamics to rotation. The pronounced differential rotation with latitude observed seems to be the result of convective flows driven radially by the buoyancy force and deflected horizontally by the Coriolis force [4]. This rotation contributes to angular momentum through two forms: Meridional circulation and Reynolds stresses.

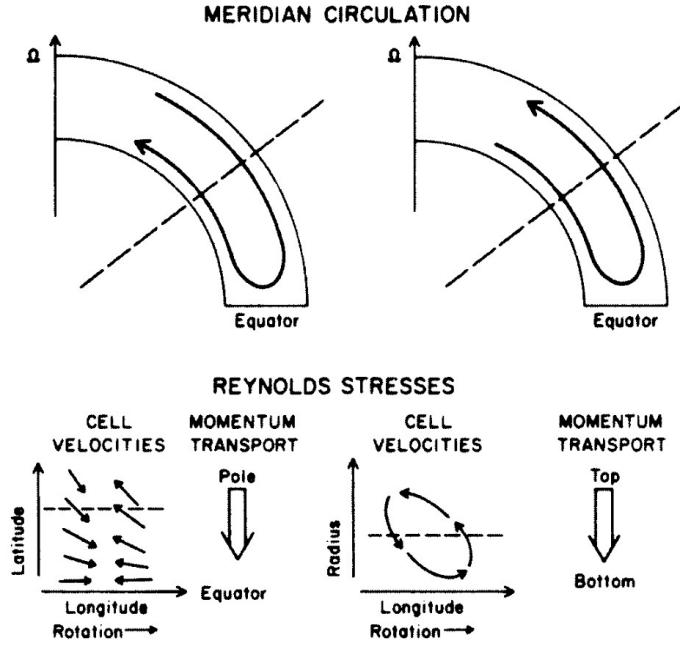


Figure 2.11: Contributions on angular momentum. Image taken from [4].

The first contribution involves axisymmetric meridional circulation (see Figure 2.11). In the absence of other transport mechanism, such a circulation would tend to spin up the poles and the interior, as the moving fluid conserves its angular momentum.

The second contribution, which tends to enforce solid body rotation, arises from Reynolds stresses associated with nonaxisymmetric convective motions. The mechanism can be understood by considering that, for equal velocities in the meridional plane, the flux of angular momentum per unit mass is greater toward the equator than away from it. This process depends critically on nonaxisymmetric convection, as it generates net fluxes of angular momentum in latitudinal and radial directions without a corresponding net mass flux. Neither buoyancy forces, which are strictly radial, nor pressure gradients, which must average to zero around the solar circumference, can directly influence the sun's axisymmetric rotation profile [4]. Therefore, the combined action of meridional circulation and Reynolds stresses is essential for shaping the observed differential rotation.

3. Methodology

The blend-free list of Fe I lines

Given the complexity introduced by convective motions on the spectrum, this project takes a computational approach leading, guided by Professor Benjamin and previous research. The analysis was conducted using the Python programming language. All data and code have been uploaded in a [GitHub repository](#), allowing anyone interested to reproduce the results and verify the authenticity of the conclusions presented.

This project follows the methodology established in previous studies (see [5] and references therein), utilizing a selected list of Fe I lines from Nave [12]. As previously discussed, these lines are ideal for the calibration process due to their minimal thermal broadening and reduced susceptibility to other atmospheric perturbations.

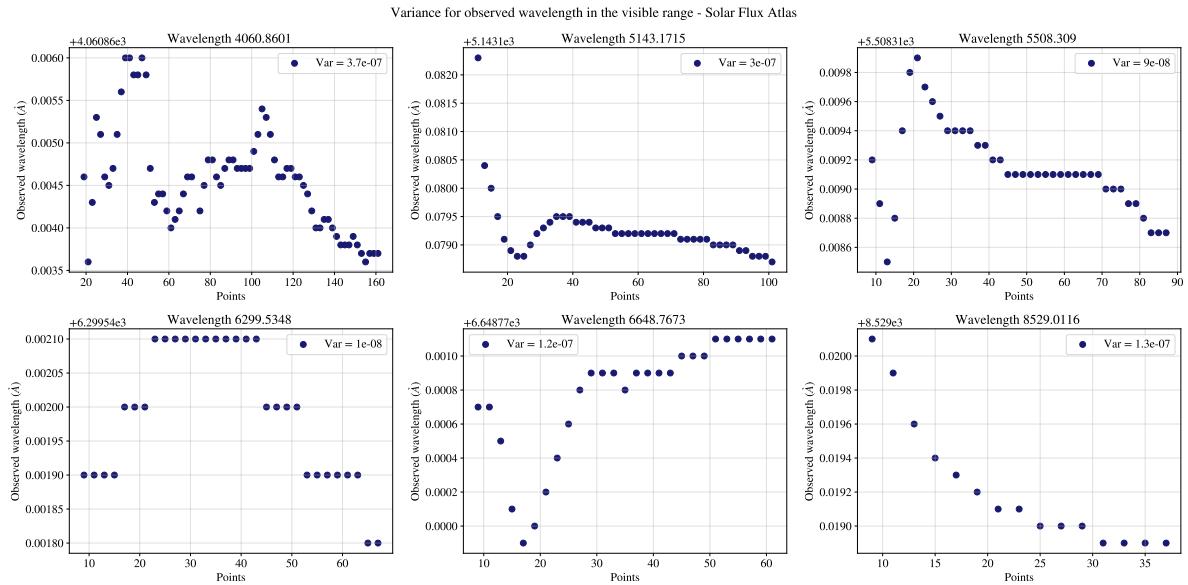
3.1 Computational approach

The computational approach focuses on identifying the solar granulation pattern by calculating relative velocities from Fe I lines.

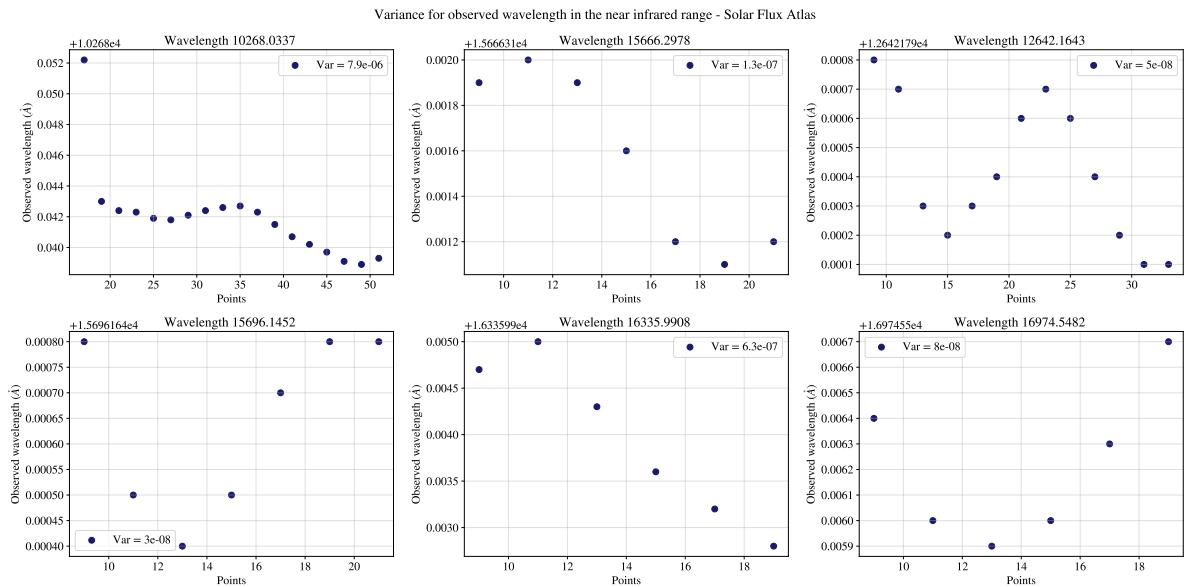
First, Fe I lines in the IAG Solar Flux Atlas and the IAG Spatially Resolved Quiet Sun Atlas using the blend-free list from Nave list of Fe I lines were identified [12]. For each identified line were selected bins of $0.1 \text{ m}\text{\AA}$ around the closest minimal point to the associated value from the Nave Fe I list. Beware, these points are not the observed wavelengths, just a reference to generate an observation window. Then, a fourth-order polynomial fit was adjusted due to the

wavelength window and found the minimal point, whose represent the observed wavelength. The efficacy of this approach for quantifying asymmetry via the bisector slope is supported by [?]. For an optimal fit was used a z-score standardization on each line core, which is explained in appendix A. The observed wavelength from the polynomial fit was used to calculate the relative velocity. Finally, the values for the line core curvature (see Equation (2.5)) and the line core bisector slope (see Equation (2.6)) were found.

Furthermore, the variance of the observed wavelength was analyzed by altering the number of points used in the fourth-order polynomial fit. Figure 3.1 shows that this parameter set performs well in the visible spectrum. However, its performance degrades in the infrared, where the number of data points defining the line core is reduced compared to a typical line in the visible range.



(a) Variance for several lines in the visible range for the Solar Flux Atlas.



(b) Variance for several lines in the near infrared range for the Solar Flux Atlas.

Figure 3.1: Variance of the observed wavelength altering the number of points on the fourth order polynomial fit

IN PROGRESS Then, the same analysis was performed to the blueshift. And we study the variance of the window of the polynomial fit. **IN PROGRESS**

HERE: Add the uncertainty analysis of the mean variance for the wavelength range that we use.

3.2 Blend-free Fe I line Nave list

The mentioned methodology implements the Nave list of laboratory-measured Fe I lines [12]. This list classifies lines with a quality grading (A, B, C, D), based on four wavenumber uncertainties ($\Delta\sigma$) and the corresponding wavelength uncertainties ($\Delta\lambda$). Where A denotes lines with error in wavenumbers less than 0.005 cm^{-1} ; grade B less than 0.01 cm^{-1} ; grade C less than 0.02 cm^{-1} ; and grade D greater than 0.02 cm^{-1} . All known blended lines and all lines measured only in grating spectra have been assigned the grade D [12].

However, not all the cataloged lines with grade A are clearly present in the solar spectrum, and within the near infrared range, many lines are severely blended. In collaboration with Professor Benjamin and Manuel Fuentes, we refined this initial list using different approaches to select the most reliable values.

3.2.1 Selection process for blend-free Fe I lines

The line selection process employed two approaches: Computational and visual. Initially, only quality A lines for the Nave list were selected. This group were subjected to four computational filters based on the properties of having a fourth-order polynomial fit for the line core.

The first filter takes into account the form of the C-curved line profile bisector. Lines whose c-curved bisectors exhibited excessive scatter, indicating a profile dominated by noise rather than a convective signature, were rejected.

The second filter selected lines whose curvature sign was consistent with an absorption line; this implies the coefficient of the fourth-order term needs to be positive.

The third filter ensures the selected lines represents an absorption curve by discarding closest points that more closely resemble slopes or continuum noise. The condition was to discard lines which difference between extreme points of the fit were less than half of the line depth. This describes an absorption curve, and not a slope or noise.

The final filter required the absolute difference between the observed wavelength and the measured laboratory wavelength to be less than 0.025 \AA . Larger discrepancies suggest misidentification or severe blending, rendering the calculated relative velocity unreliable. This threshold

is empirically supported by observations throughout the selection process and the statement for the mean velocity of the sun is 200 m/s to 600 m/s.

The final part of the selection process was a visual inspection to discard lines that were blended or absent from the solar spectrum. A custom visualizer (detailed in appendix C) was instrumental for this, allowing the simultaneously display of graphics related to dynamics aspects. This was particularly useful for selection lines in the near infrared range. The visual requirement focused on the geometry of the spectral line profile, its behavior on the three signatures plots.

The concluding part of the methodology was performed the plots which represents the three signatures of convection.

First of all, the granulation pattern (relative velocity), core curvature and core bisector against line depth was performed to all ranges in both atlases with the objective to observe the three signatures of convection. Subsequently, various parameters were plotted against line depth to specifically characterize the granulation.

MENTION THAT WE CHANGE THE LIMITS OF WAVELENGHT FROM REINERS TO STABLISH COPARISIONS WITH THE SPATIALLY NOW VISIBLE IS FROM 4000/7500 A AND NIR IS 7500/23000

4. Results and discussion

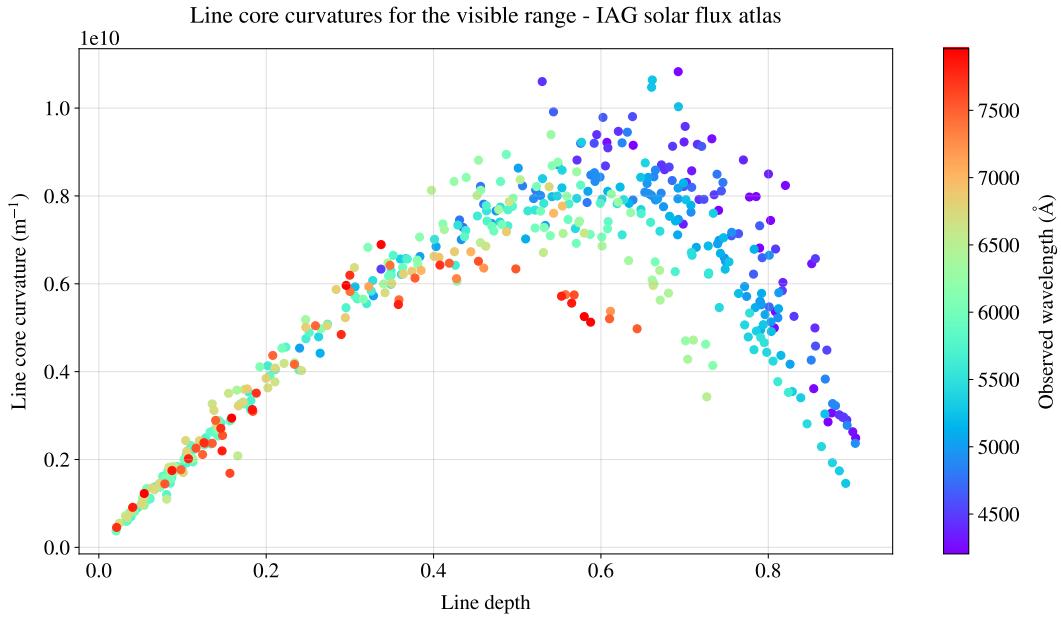
The three signatures of convection

Our results were separated into three sections following the signatures of convection. All of them analyses these signatures around the main phenomenon of wavelength-dependence.

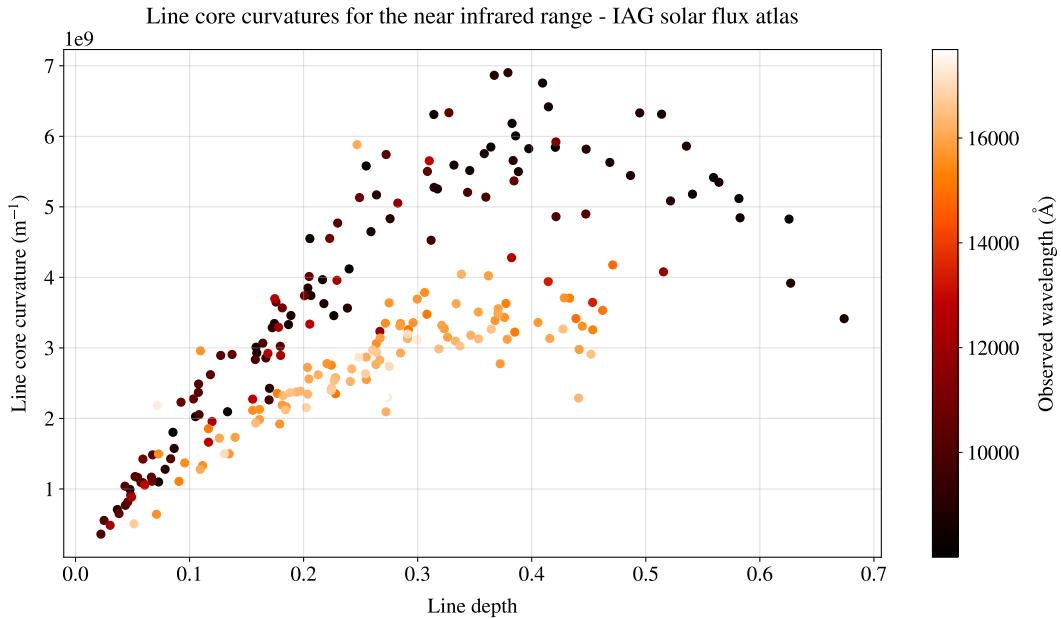
4.1 The first signature: Line broadening

4.1.1 Line depth-dependence on line core curvature

The line core curvatures were calculated following Equation (2.5), and plotted against line depth as shown Figure 4.1. In general, the behavior described in previous research was observed. For shallow lines, the line core curvature is proportional to depth as expected. Furthermore, for deep lines the line core curvature is reduced by saturation with strongly wavelength-dependence.



(a) Line core curvatures for the visible range. While shallow lines curvature is proportional to depth, deep lines curvature is reduced by saturation with is strongly wavelength-dependence.



(b) Line core curvatures for the near infrared range. The plot shows a conspicuous partition around $14\,000 \text{\AA}$

Figure 4.1: Line core curvatures for the IAG solar flux atlas separated in wavelength ranges.

As shown in Figure 4.1b the near infrared range exhibits a natural division at approximately

$14\,000\text{ \AA}$, a region dominated by telluric absorption lines from Earth's atmosphere. The division separates the J band, which refers to an atmospheric transmission window of 3000 \AA centered on $12\,500\text{ \AA}$; and the H band, which refers to an window of 3500 \AA centered on $16\,500\text{ \AA}$ [?]. Plotting the entire spectral range of the IAG solar flux atlas, a clear line depth-dependence of the line core curvature as a function of wavelength is observed.

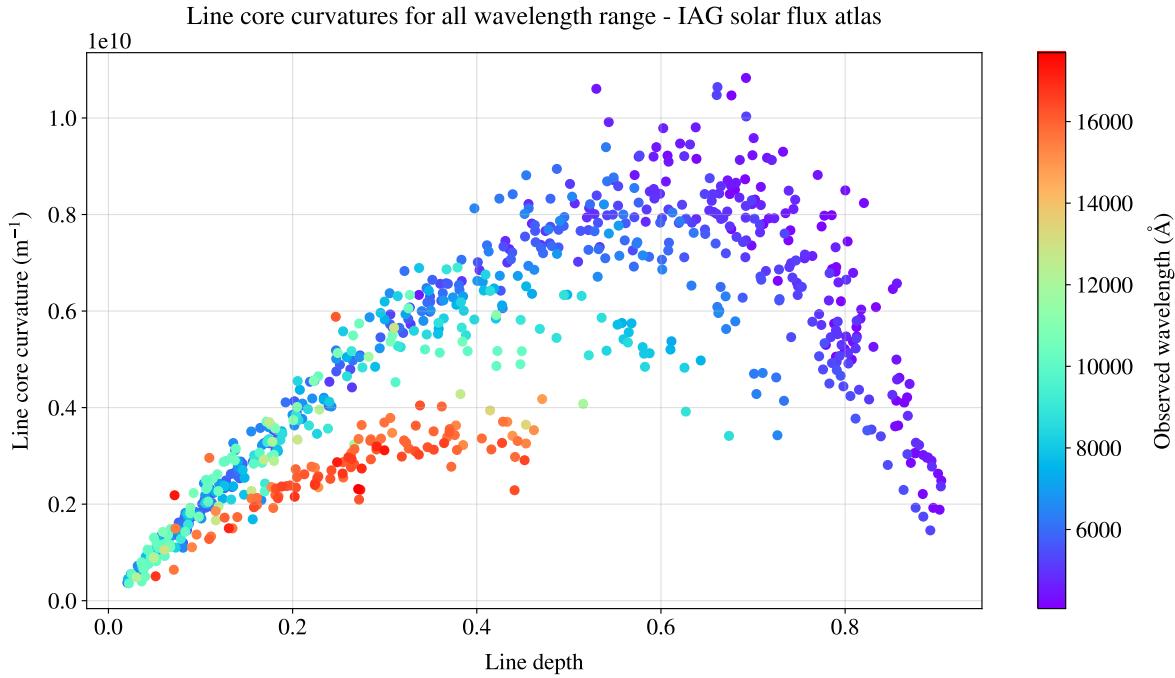


Figure 4.2: Line core curvatures in the IAG solar flux atlas. While the J band for the near infrared range adjust to the general shallow lines trend, the H band exhibit wavelength-dependence.

While the J band for the near infrared range adjust to the general shallow lines trend in Figure 4.2, the H band exhibit wavelength-dependence. Despite the line depth-dependent shifts along wavelength, the visible range of the IAG solar flux atlas exhibits a clear linear tendency for shallow lines (see Figure 4.2). Consequently, a linear fit was applied to the visible range for the IAG solar flux atlas in the range $0.0 - 0.1$ of line depth.

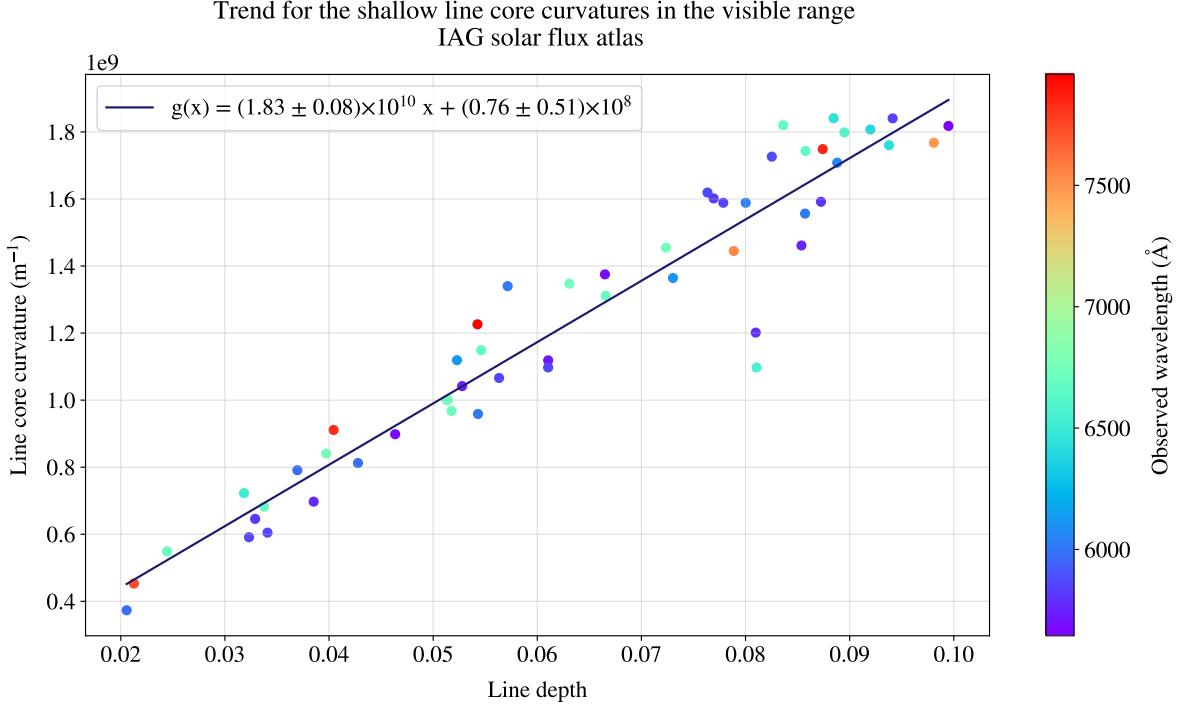


Figure 4.3: Linear fit applied to the range (0.0 – 0.1) of line depth in the visible range of the IAG solar flux atlas.

As shown in Figure 4.3, the slope of the relationship has a value of $(1.83 \pm 0.08) \times 10^{10}$. The absence of wavelength-dependence in this spectral range indicates that line core curvatures have a net dependence on velocities (thermal, convective and rotational), with negligible influence from atomic effects. To confirm this statement, the line core curvature was also plotted for the visible range using the IAG spatially resolved quiet sun atlas at the solar limb $\mu = 0$ as shown Figure 4.4. In other words, this analysis uses light from the center of the solar disk, which is not affected by rotational Doppler broadening, thereby isolating the local effects.

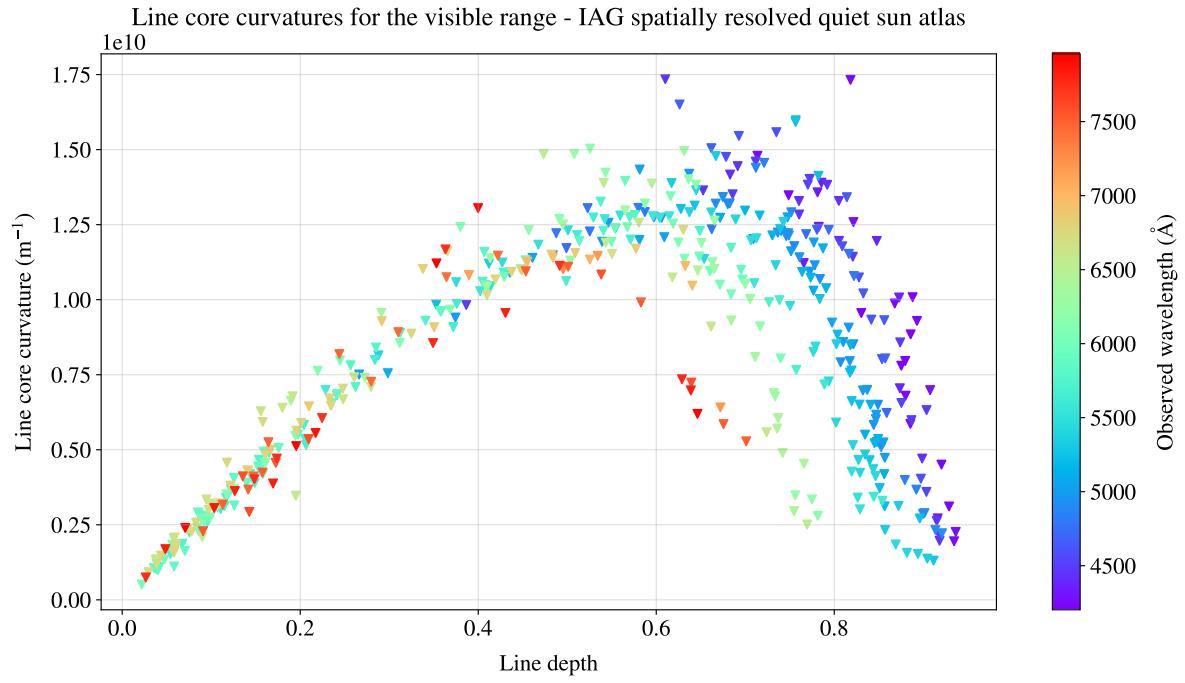


Figure 4.4: Line core curvatures for the visible range of the disk center data from the IAG spatially resolved quiet sun atlas.

A linear fit was applied taking the same range $0.0 - 0.1$ of line depth, the Figure 4.5 shows a value of $(2.94 \pm 0.16) \times 10^{10}$ for the slope.

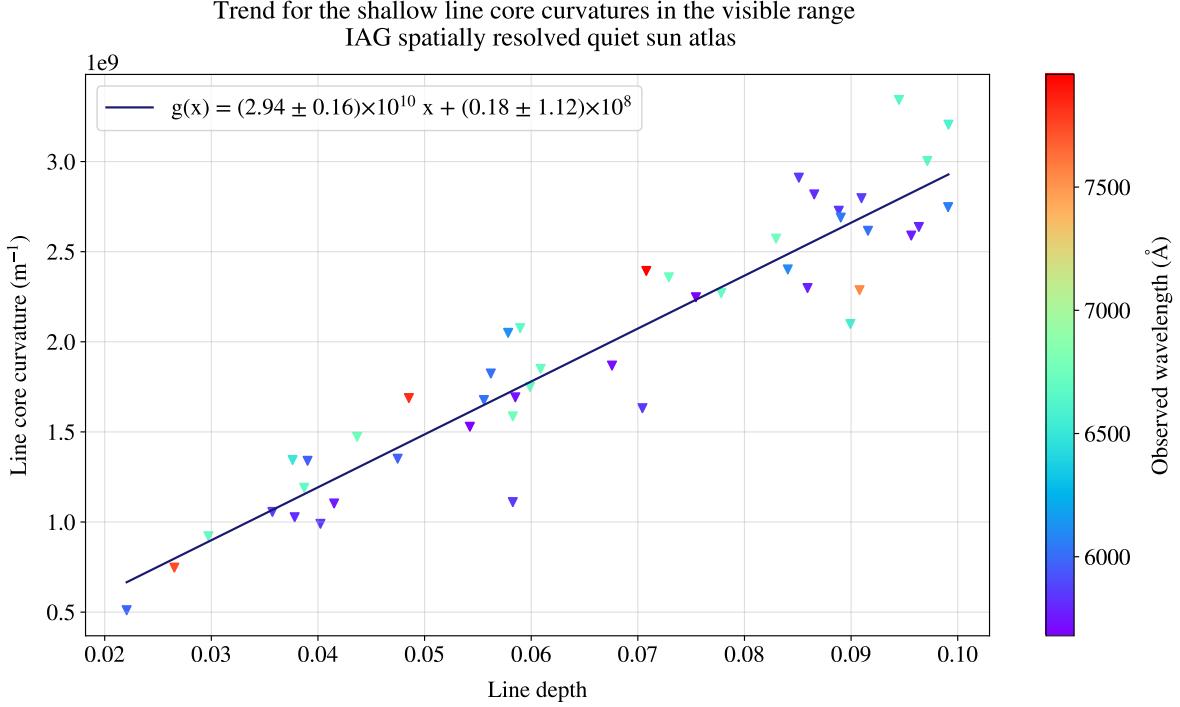


Figure 4.5: Linear fit applied to the range (0.0 – 0.1) of line depth in the visible range of the IAG spatially resolved quiet sun atlas.

The result of found a greater slope in the center-disk confirm that rotation is a negligible effect for the line broadening. This allows deduce the variance of convection speed, knowing the thermal velocity.

IN PROGRESS.. As mention in chapter 3, we can induce from here the σ_{conv} as we the other parameters described in equation () and confirm that rotation is negligible. Benja comment: Here you could discuss all that can be learned about line broadening. For example: The slope of the disk-center spectrum doesn't depend on rotation, meaning it depends only on thermal broadening and convection. The thermal is known, so the variance of the convection speeds can be deduced. The slope of the flux spectrum is lower, because it includes rotation. The rotation is also known (but more work to calculate); but the three contributions can be assessed. Deeper lines deviate from the initial straight line, and this part is wavelength-dependent. The cause (s) should be discussed: Saturation in the deepest lines, but maybe other broadening effects (Lorentzian) may become important, and damping coefficients are somewhat related to

wavelength by atomic structure. [IN PROGRESS..](#)

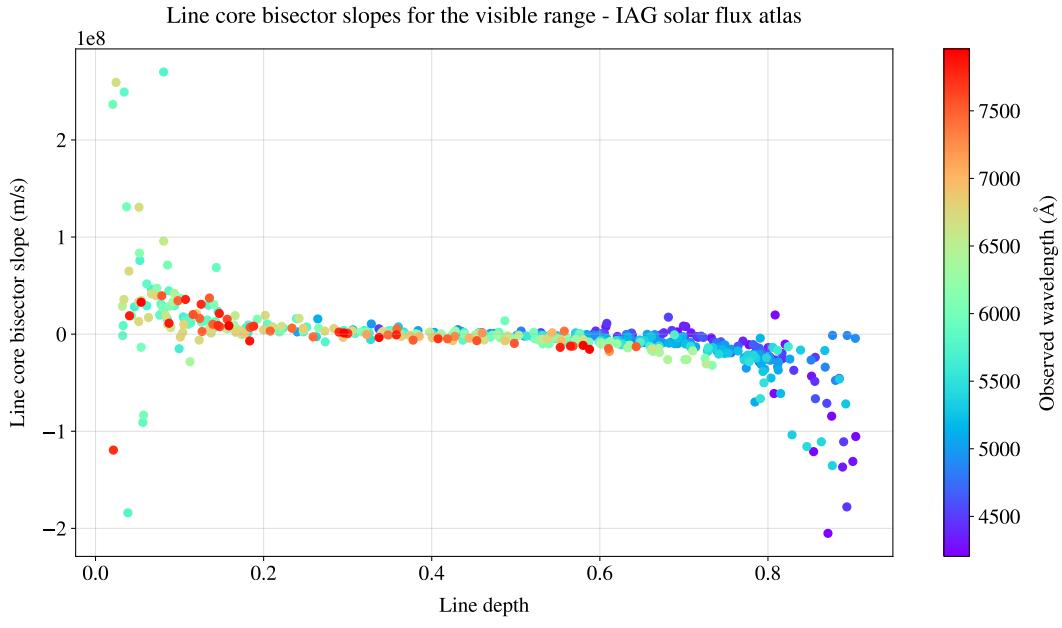
4.2 The second signature: Line profile asymmetry

4.2.1 The line core bisector slope

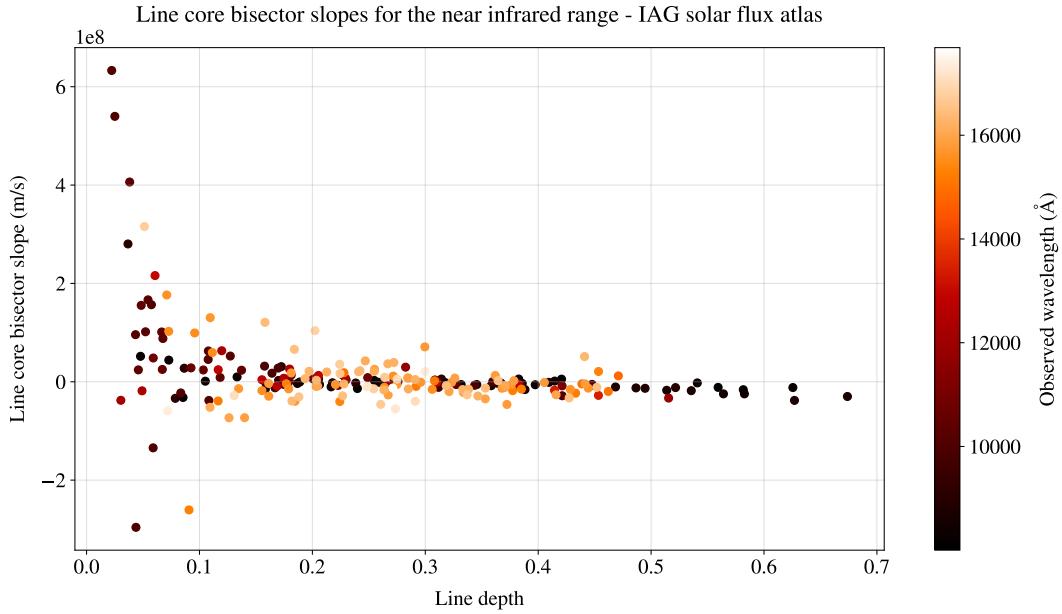
As mention before, convection is the only mechanism that creates asymmetric line profiles. Consequently, the slope of the line core is a indirect line profile asymmetry measure, expressed as a velocity.

The line core bisector slope was calculated following the equation (2.6) and plotted against line depth, as shown Figure 4.6.

The shallow lines, due to their small depth, do not exhibit a fully developed C-shaped bisector. Instead, their bisectors shows as a positive slope. In contrast, deep lines experiences the convective blueshift in its totality, exhibit a negative bisector slope. The line depths in the middle range represent lines profiles which are not greatly affected by convection.



(a) Line core bisector slopes for the visible range.



(b) Line core bisector slopes for the near infrared range.

Figure 4.6: Line core bisector slopes for the IAG solar flux atlas separated in wavelength ranges. The behavior of both plots is according to the C-curved shape of the line profile bisector affected by convection movement.

Searching for a comparision with the IAG spatially resolved quiet sun atlas, only the visible

range was analysed. Then a linear fit was applied to the line core bisector slope data in the line depth range of (0.3 – 0.6) to quantify this transition, as shown in Figure 4.7.

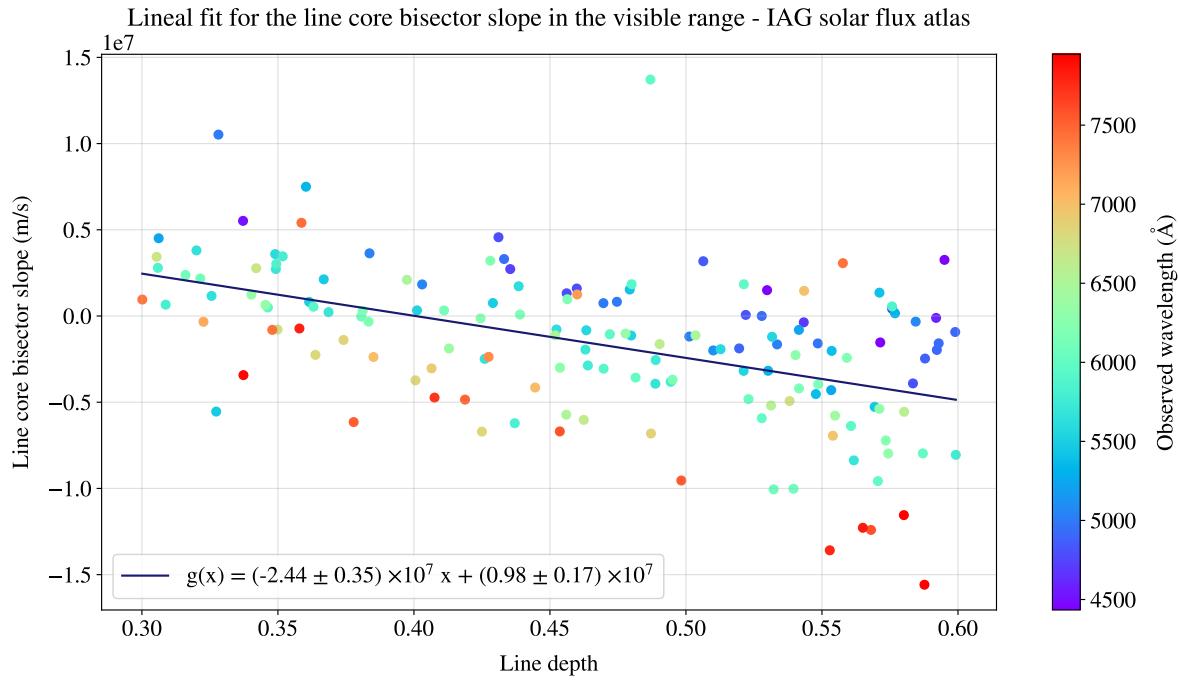


Figure 4.7: Linear fit adjusted to the range (0.3 – 0.6) of line core bisector slopes in the IAG solar flux atlas, where is evident an wavelength-dependence.

As there is an evident wavelength-dependence along the line core bisector slopes, a point of comparision was searched comparing with the disk center spectrum (see figure 4.8).

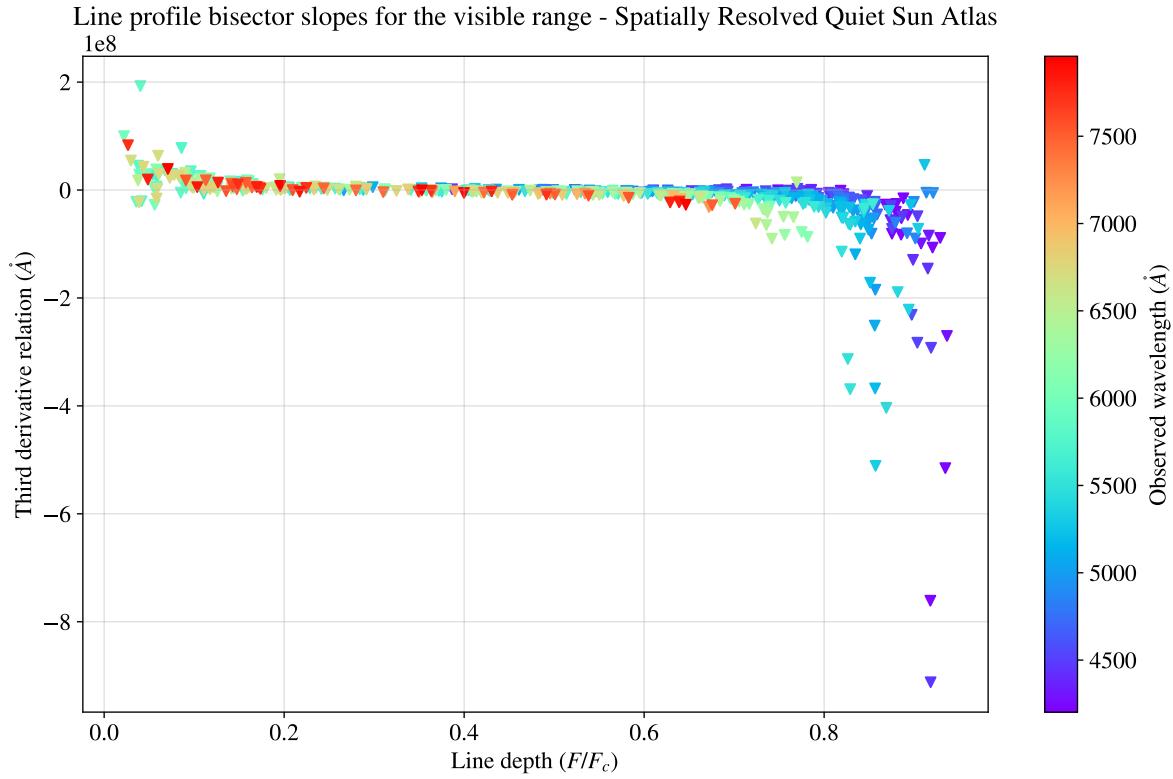


Figure 4.8: Line profile bisector slopes for the visible range in the IAG spatially resolved quiet sun atlas for $\mu = 0$.

The bisector slope derived from the disk center is smaller in magnitude compared to the integrated flux, despite both exhibiting the same characteristic trend (see Figure 4.9).

Lineal fit for the line core bisector slope in the visible range - IAG spatially resolved quiet sun atlas

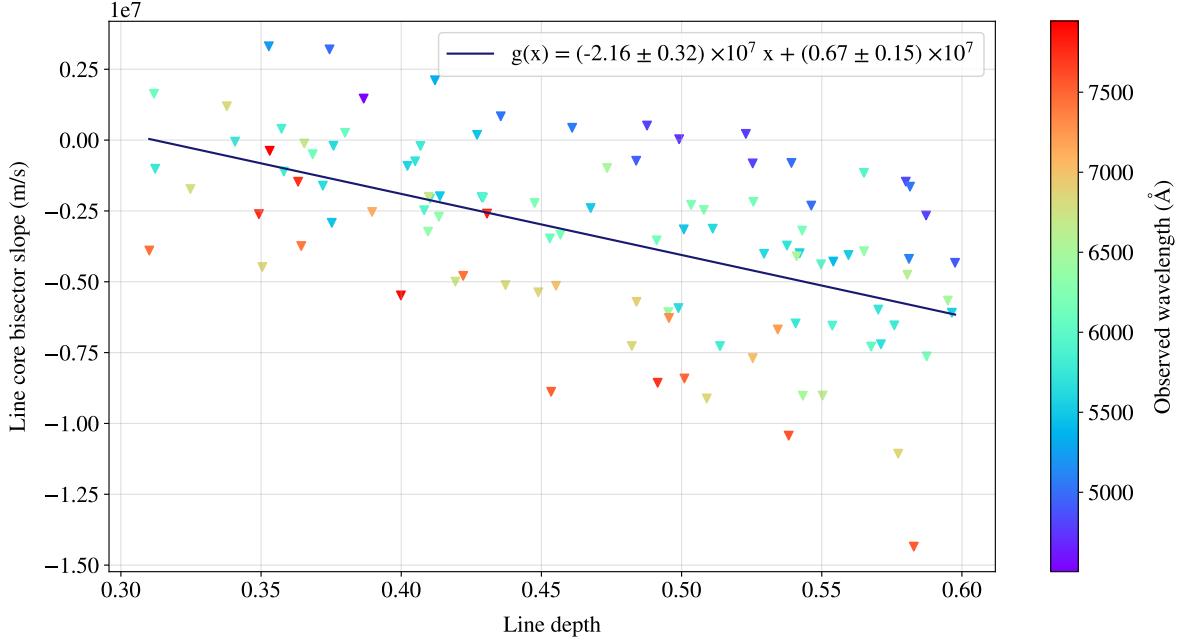


Figure 4.9: Linear fit adjusted to the range (0.3–0.6) of line core bisector slopes int disk-center spectrum, where is evident an wavelength-dependence.

The phenomenon of chromodependence is evident of Figures 4.7 and 4.9, where the line profile bisectors slopes are organised by decreasing wavelength. This behavior is clearer on the disk-center spectrum than the disk-integrated flux spectra.

If line depth is separate into bins of 0.1 and plot the line profile bisector slope against wavelength, shifts in the line bisector slope become apparent as shown in Figure 4.10.

Line Depth bins on the distribution of excitation potential for lowest energy level - IAG solar flux atlas

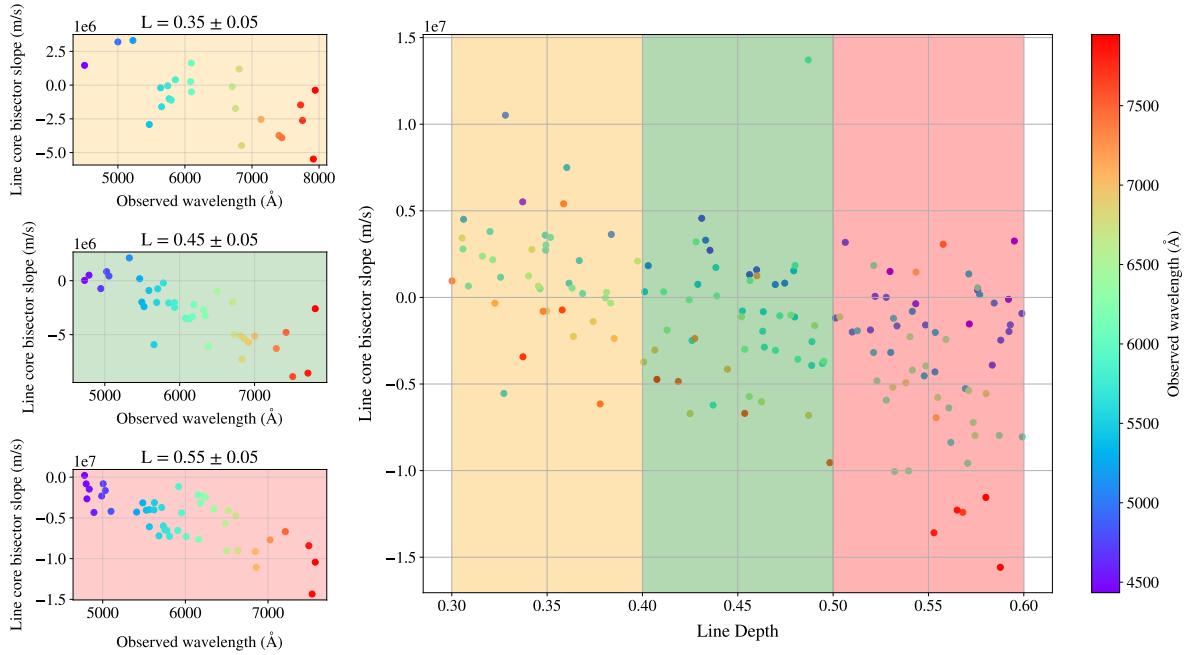


Figure 4.10: Line depth bins of 0.1, where wavelength-dependence of the slopes becomes apparent.

For the integrated flux spectrum was found that, for a fixed flux, the slopes decrease with wavelength (see Figure 4.11).

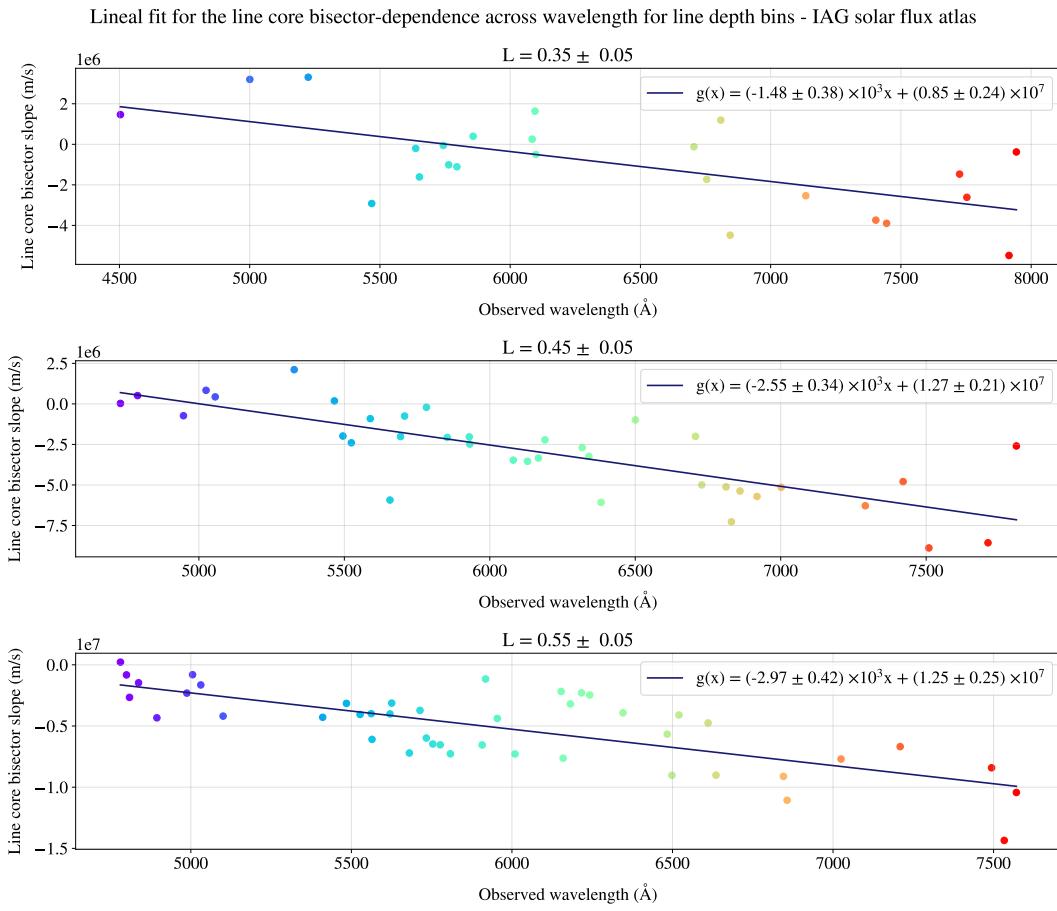


Figure 4.11: Individual plots of line core bisector slope shift across wavelength representing each bin of line depth for the IAG solar flux atlas.

The same analysis was performed on the disk-center flux and the same behavior was found (see Figure 4.12).

Lineal fit for the line core bisector-dependence across wavelength for line depth bins - IAG spatially resolved quiet sun atlas

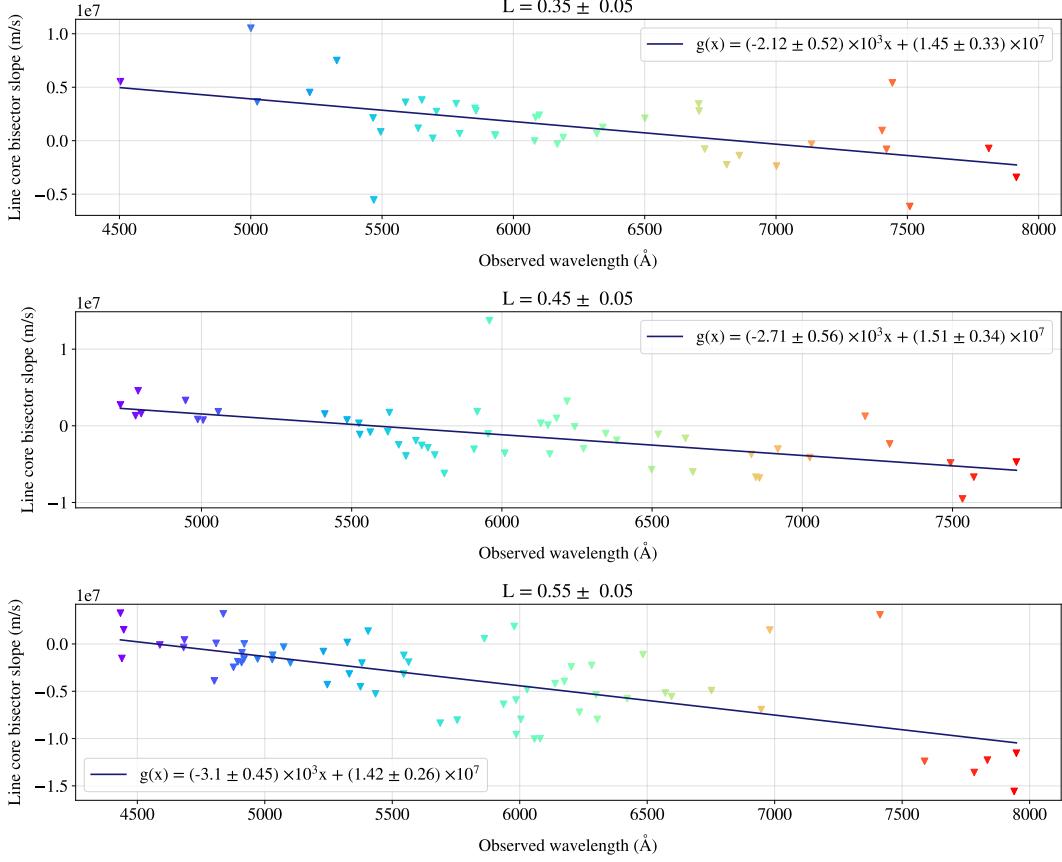


Figure 4.12: Individual plots of line bisector slope shift across wavelength representing each bin of line depth for the Spatially Resolved Quiet Sun Atlas.

IN PROGRESS.. So, for fixed flux we have chromodependence.. IN PROGRESS..

4.2.2 Flux deficit IN PROGRESS

Due to radiation, the redshift part of a line is displaced on the flux, inducing a rotation on the c-curved line profile bisector. This phenomenon is called flux deficit. Hamilton and Lester [8] noticed that the behavior of the third signature mimics the line profile bisectors. The mean of line profile bisectors on ranges of wavelength follows the behavior of granulation pattern, this last is discussed in the next section. Later, Gray and Oostra [7] show that the bisectors need to follow the granulation pattern as the form of we calculated velocities and bisectors.

For comparison with Gray and Oostra work, was taken the line 6254.2850 Å as shown in

Figure 4.13.

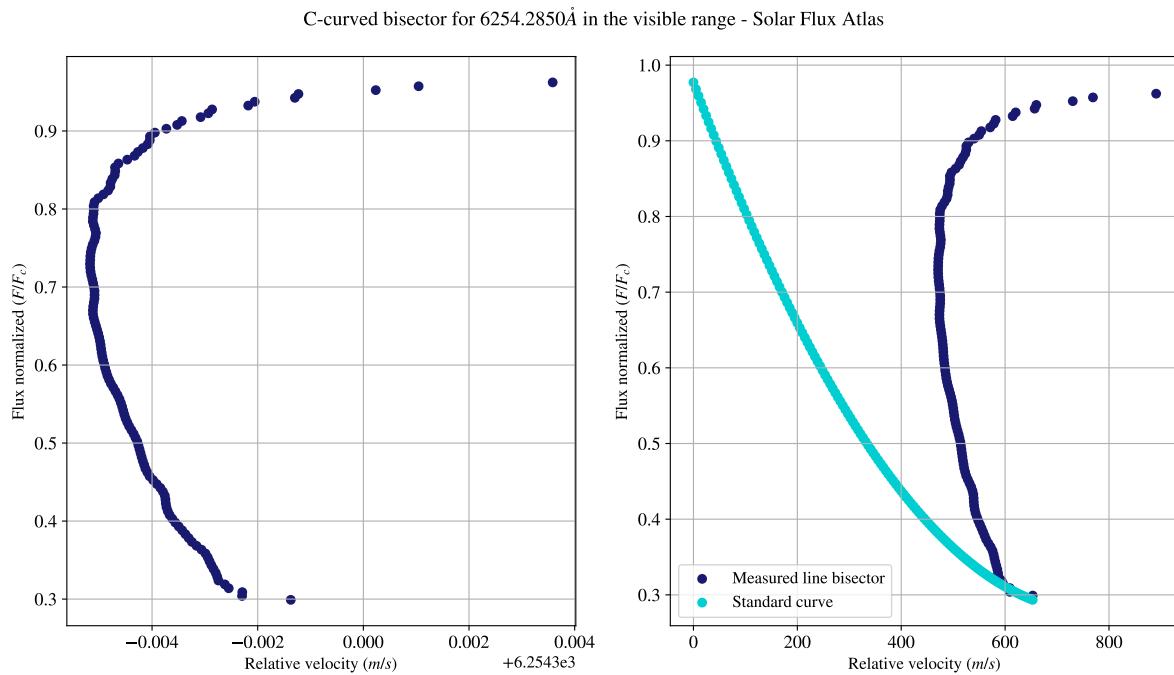


Figure 4.13: C-curved line profile bisector for the 6254.2850\AA and the standard curve for the green range from the third signature plot.

Following the same method, the figure 4.14 shows a flux imbalance described on the distribution and temperatures of the mean, the maximum and the RMS point of the bisector.

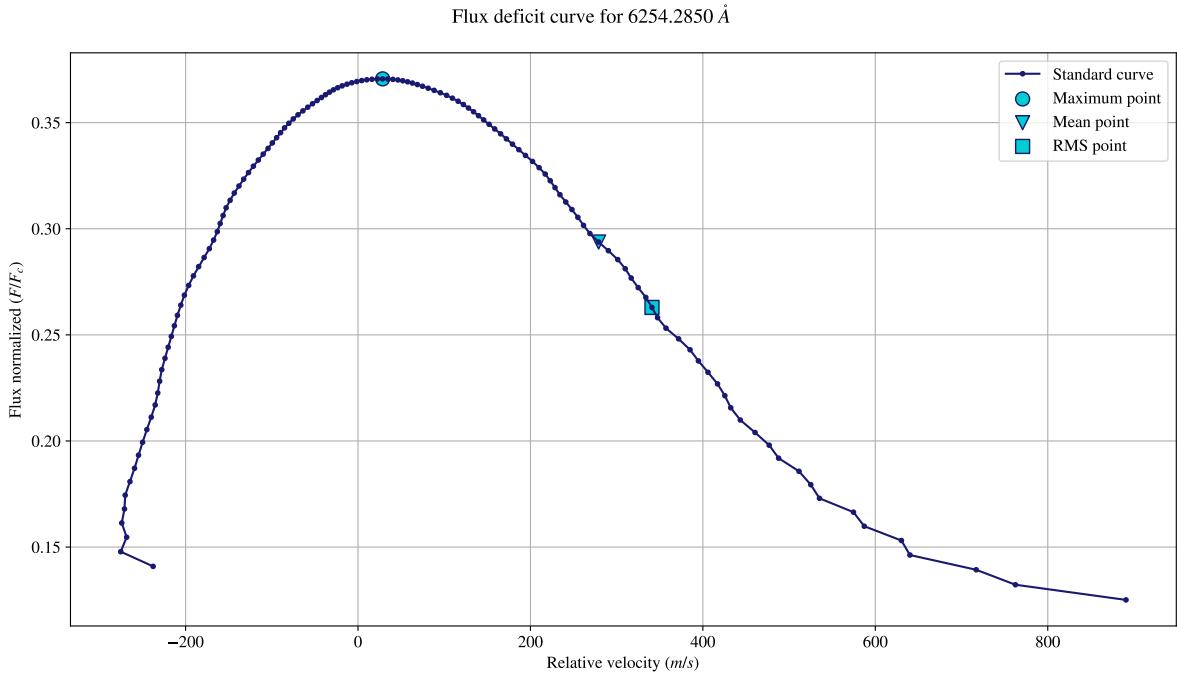


Figure 4.14: Flux deficit curve for the 6254.2850\AA and the temperatures of the mean, the maximum and the RMS point of the bisector.

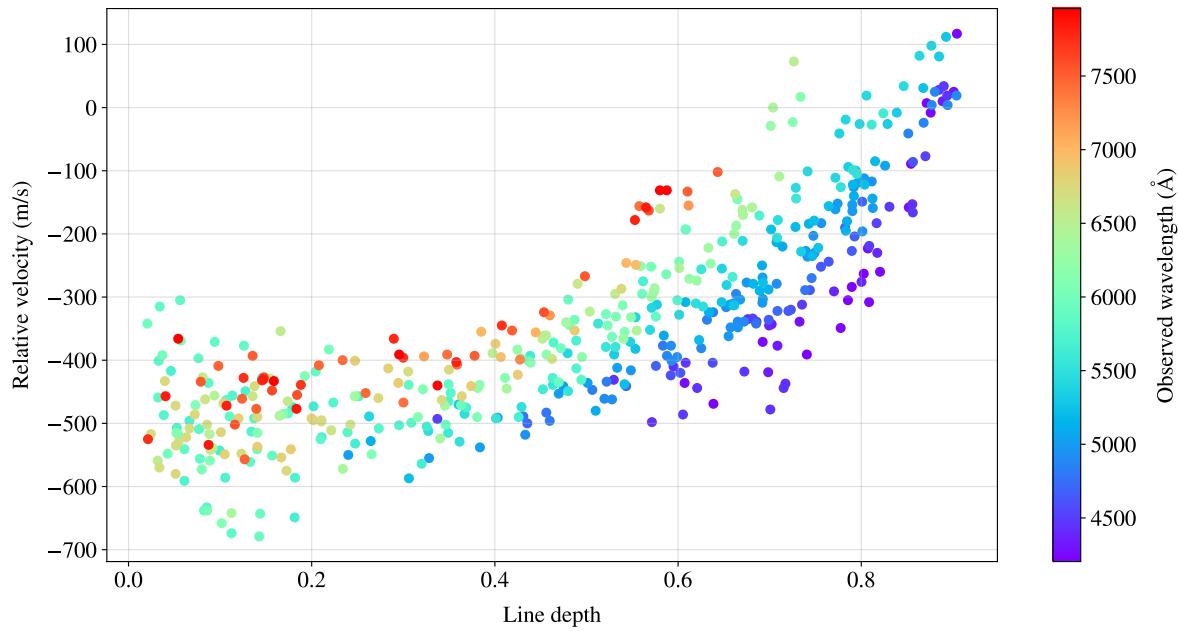
The idea is add the flux deficit using the color curves and just report the velocities with the percent of the line corresponds that velocity. As we found the standard curves for all the ranges, the same analysis was realized for a random line in the range of the respective standard curve (im working on that). From this maybe get a description of line asymmetry in the all visible range.

4.3 The third signature: Line depth-dependence on wavelength shifts

4.3.1 The granulation pattern

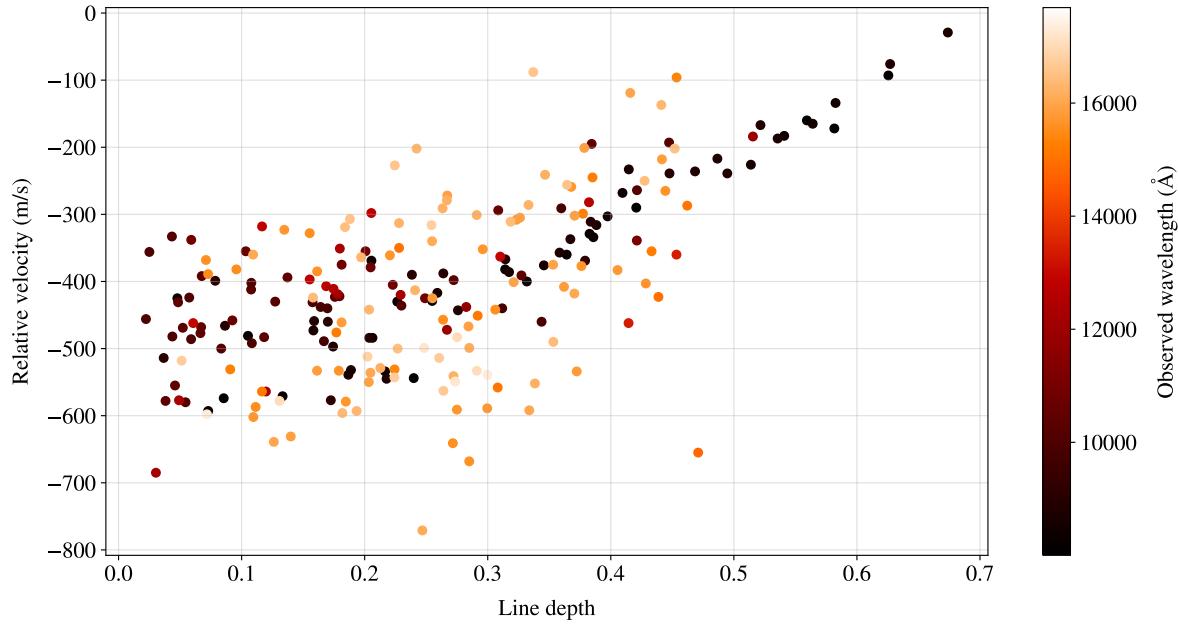
The granulation pattern is presented as the plot of relative velocity against line depth for the IAG solar flux atlas.

Granulation pattern for the visible range - IAG solar flux atlas



(a) Visible range for the IAG solar flux atlas.

Granulation pattern for the near infrared range - IAG solar flux atlas



(b) Near infrared range for the IAG solar flux atlas.

Figure 4.15: Granulation pattern obtained for both wavelength ranges in the IAG solar flux atlas. Note that the relative velocity is wavelength-dependent

As shown in Figure 4.15, the convective velocity depends on line depth, as described in literature. Moreover, this *granulation curve* is not unique or universal, but depends on the wavelength range. To further characterize this trend, we performed an analysis of line depth versus wavelength was performed at fixed velocities.

To measure these wavelength shifts, the analysis was restricted to the spectral range from 4300 \AA to 5600 \AA . One of the plots of Figure 4.16 illustrates why the restricted range is convenient: In longer wavelengths it presents more complex structures which make a linear fit useless. The wavelengths from both atlases were then sorted into velocity bins of 50 m/s.

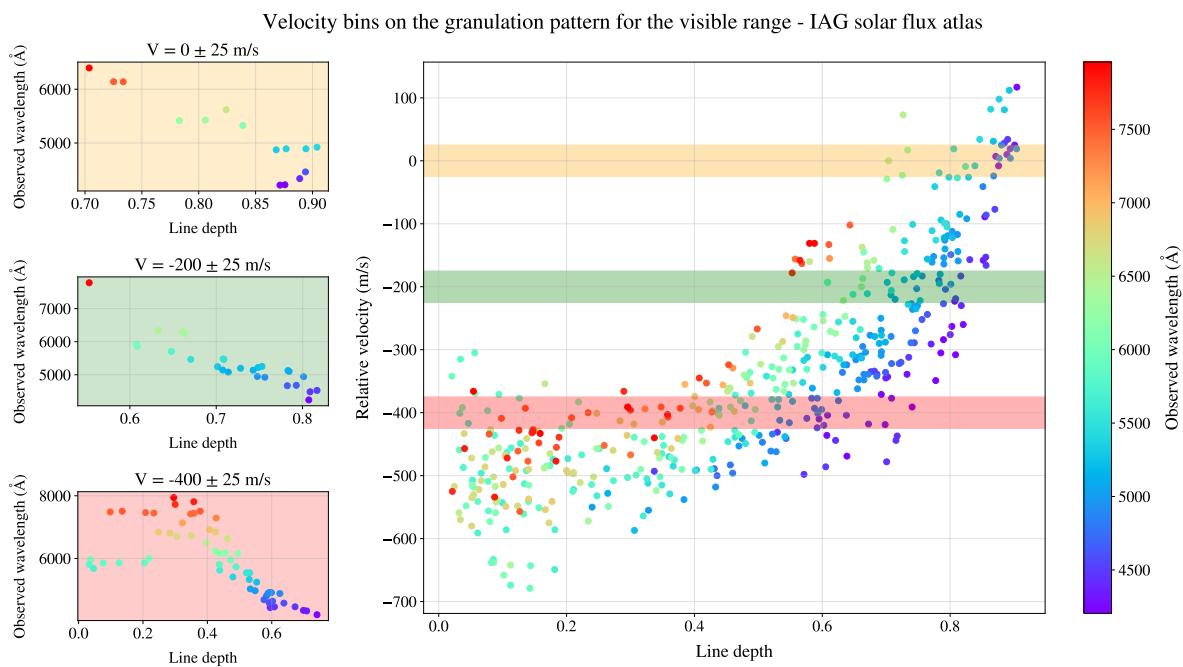


Figure 4.16: Velocity bins for the relation between wavelength and line depth.

For each velocity bin was plotted the wavelength against line depth to measure a frequency shift with a lineal fit (see Figure 4.17).

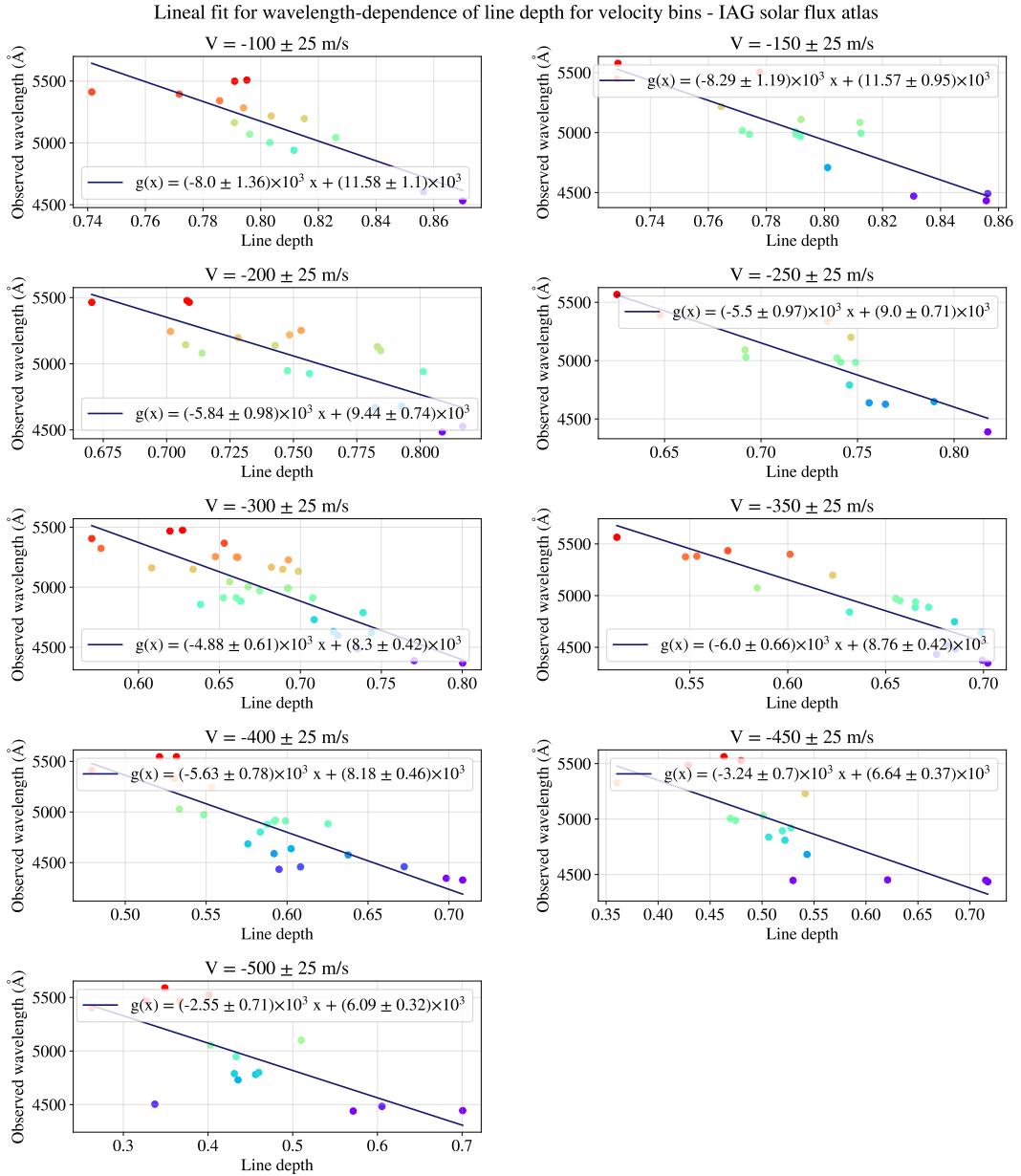


Figure 4.17: Individual plots of wavelength across line depth representing each bin of velocity for the IAG solar flux atlas.

The same analysis was performed to compare with the disk-center spectrum. The Figure 4.18 shows the results for the wavelength-dependence along line depth in the IAG spatially resolved quiet sun atlas.

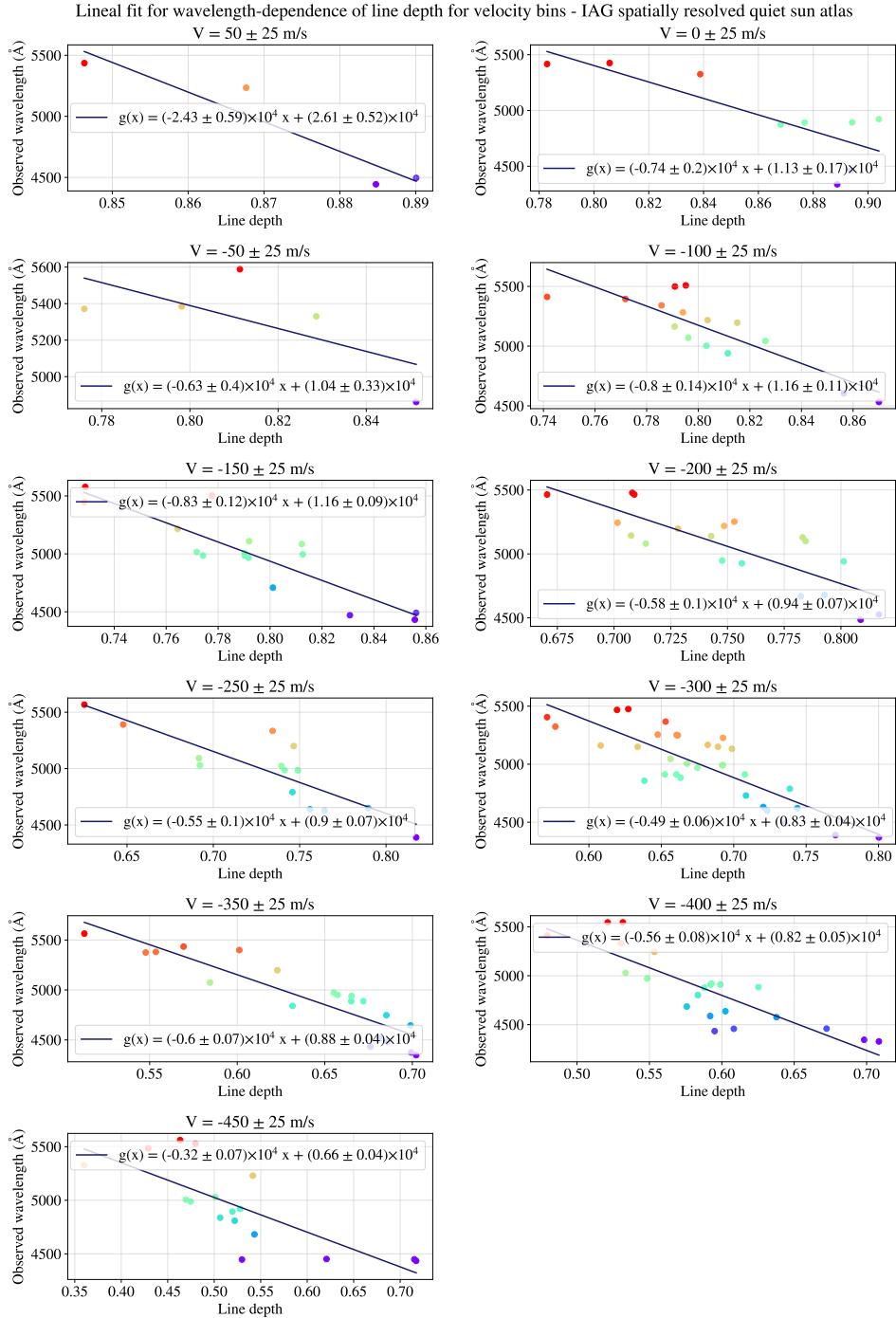


Figure 4.18: Individual plots of wavelength across line depth representing each bin of velocity for the IAG spatially resolved quiet sun atlas. The value for slopes on this spectrum are higher than the disk-integrated flux atlas.

IN PROGRESS The guiding hypothesis was: “If the velocity shift is present only in the disk-

integrated spectrum and absent at the disk-center, then solar rotation is the cause.” However, the velocity shift was observed in both spectral datasets and the parameters on the disk-center spectrum are greater than IAG solar flux atlas. This was initially unexpected, as rotational doppler broadening is negligible at the disk center.

Furthermore, the shift on every lineal fit expresses the granulation relation. [IN PROGRESS](#)

4.3.2 Line depth-dependence on excitation potential for the lowest energy level

As shown Figure 4.19 the relative velocity was plotted against excitation potential for lowest energy level. The distribution of wavelength along excitation potential is according to literature, showing that lower wavelength have less excitation potential on the lowest energy level than high wavelength.

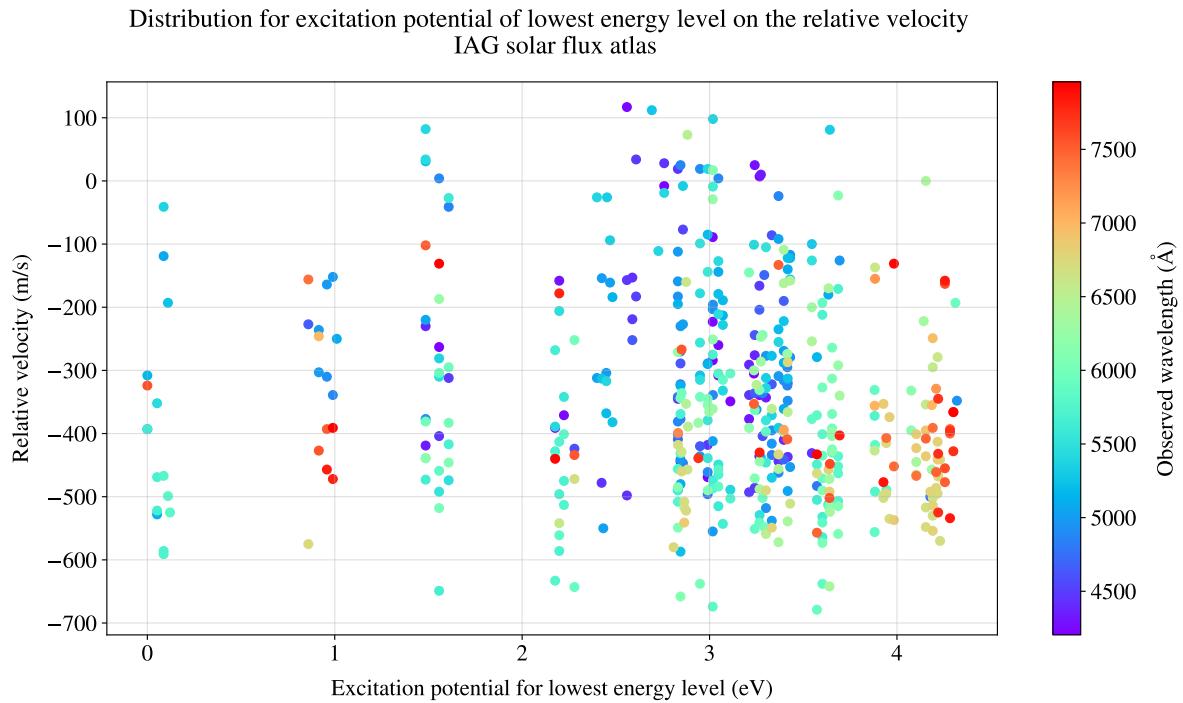


Figure 4.19: Relative velocity against excitation potential for lowest energy level in the visible range of the IAG solar flux atlas.

However, if relative velocity is separate on bins of 100 m/s and plot the excitation potential for

lowest energy level against line depth, shifts in the range (2.5 to 5.0) eV of excitation potential become apparent as shown Figure 4.20.

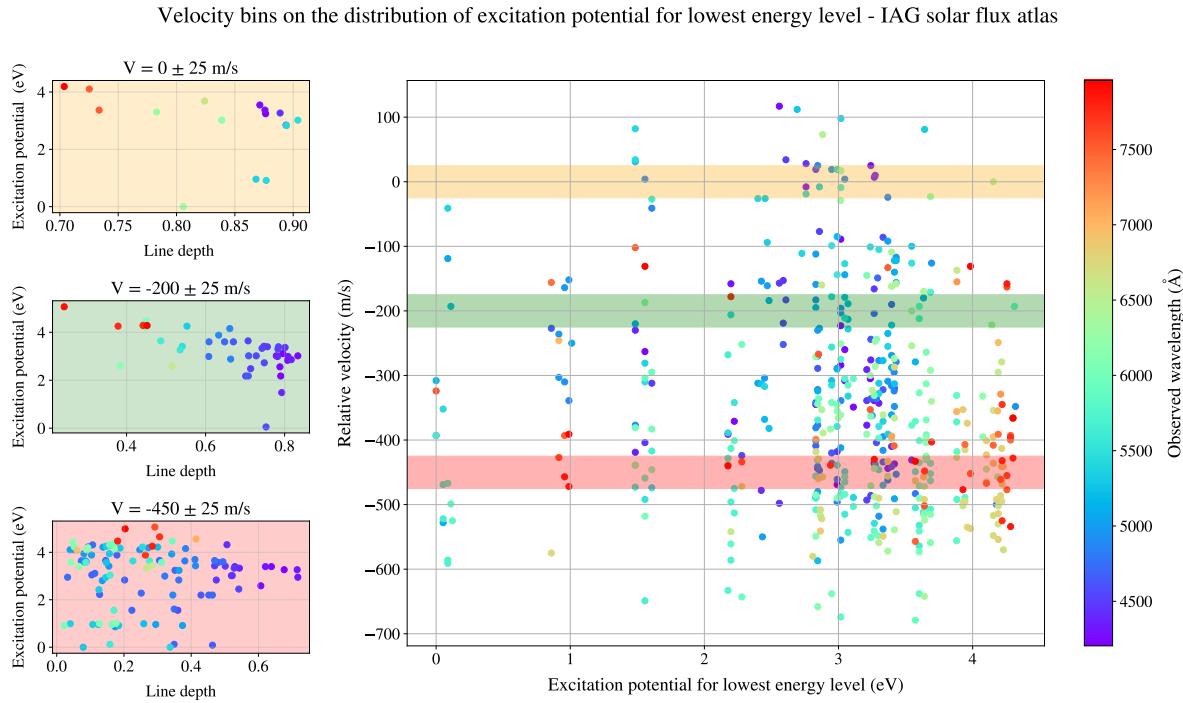


Figure 4.20: Velocity bins of 100 m/s across Figure 4.19, with this separation is visible the excitation potential shift in the range (2.5 to 5.0) eV across line depth.

Figure 4.21 explicitly shows the dependence on the highest values (2.5 to 5.0) eV for the excitation potential of lower energy levels across line depth, which can be modeled with a lineal fit.

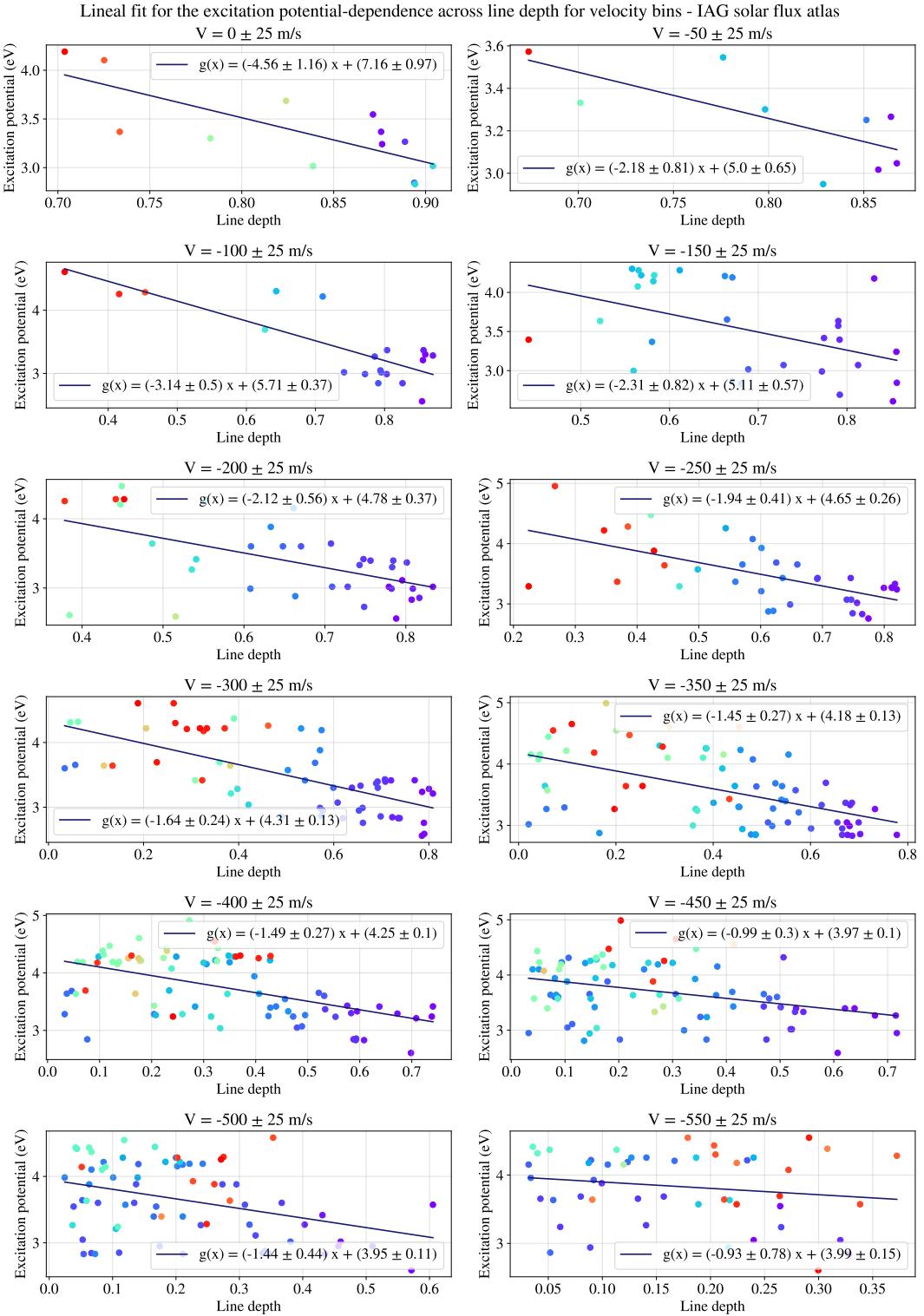


Figure 4.21: Individual plots for the range (2.5 to 5.0) eV of excitation potential of lower energy level across line depth, representing each bin of velocity for the visible range of IAG solar flux atlas. The Table 4.1 shows the coefficients for each applied lineal fit.

For the velocity range of -175 m/s to -525 m/s the calculated slopes are similar, as shown in Table 4.1. This indicates a consistent ratio on energy shifts in the energy across the velocity range.

Velocity bin (m/s)	Slop (eV)	σ_{Slope}	Shift (eV)	σ_{Shift}
0	-4.5638	1.1564	7.1636	0.9651
-50	-2.1781	0.8069	5.0005	0.6495
-100	-3.1355	0.4958	5.7114	0.3697
-150	-2.3102	0.8208	5.1098	0.5688
-200	-2.1203	0.5594	4.7782	0.3747
-250	-1.9361	0.4121	4.6533	0.2588
-300	-1.6399	0.2408	4.3146	0.1314
-350	-1.4534	0.2694	4.1778	0.1251
-400	-1.4863	0.2683	4.2510	0.1030
-450	-0.9916	0.2973	3.9734	0.0982
-500	-1.4429	0.4428	3.9506	0.1097
-550	-0.9308	0.7750	3.9919	0.1497

Table 4.1: Values for the lineal fit coefficients on the different velocity bins for the Figure 4.21.

The values σ_{Slope} and σ_{Shift} refers to the standard error of the respective lineal coefficient.

To corroborate the only dependence on excitation potential of lowest energy level, the same analysis was performed on the IAG spatially resolved quiet sun atlas at $\mu = 0$.

Distribution for excitation potential of lowest energy level on the relative velocity
IAG spatially resolved quiet sun atlas

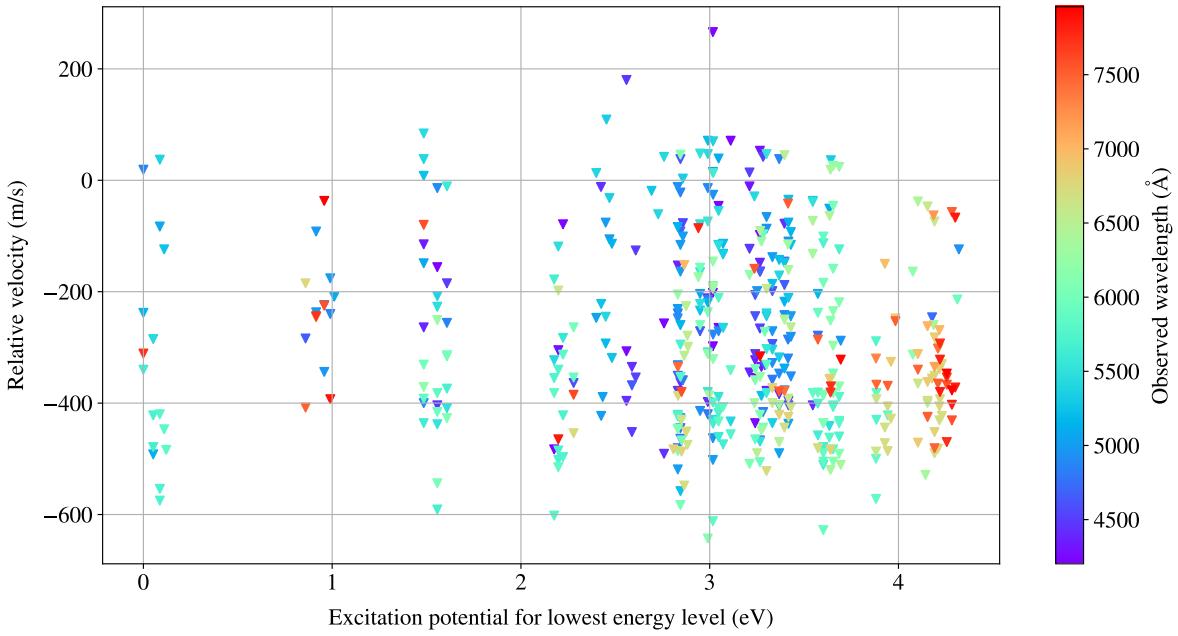


Figure 4.22: Relative velocity against excitation potential of lowest energy level in the visible range for the IAG spatially resolved quiet sun atlas.

Figure 4.22 shows the relative velocity against excitation potential of lowest energy level for disk center data.

Separating the relative velocity on bins of 100 m/s and taking the range (2.5 to 5.0) eV of excitation potential, the disk-center flux exhibits the same qualitative behavior as the IAG solar flux atlas (see Figure 4.23).

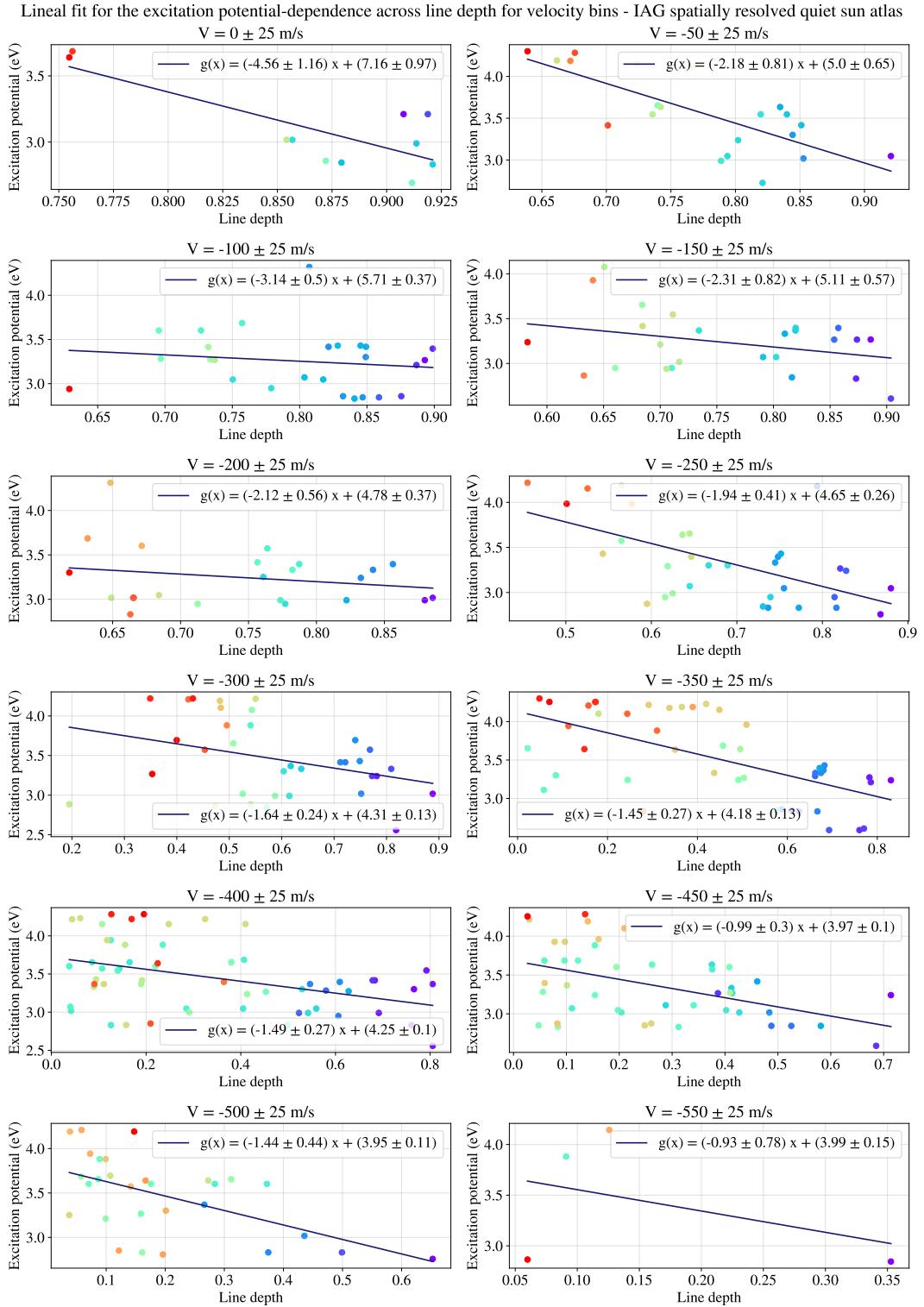


Figure 4.23: Individual plots for the range (2.5 to 5.0) eV of excitation potential of lowest energy level across line depth, representing each bin of velocity for the IAG spatially resolved quiet sun atlas.

However, there is a quantitative difference, the lineal fit coefficients are higher to the reported for the disk-integrated atlas (see Table 4.2).

Velocity bin (m/s)	Slop (eV)	σ_{Slope}	Shift (eV)	σ_{Shift}
0	-4.5638	1.1564	7.1636	0.9651
-50	-1.9274	1.1583	4.7903	0.9496
-100	-2.9149	2.0695	5.5099	1.6744
-150	-2.7907	1.2421	5.4525	0.9195
-200	-2.8771	0.9840	5.3471	0.7277
-250	-2.1774	0.7516	4.8238	0.5241
-300	-0.9802	0.3714	3.8374	0.2419
-350	-0.3246	0.3473	3.4399	0.1931
-400	-0.9455	0.3184	3.8607	0.1511
-450	-0.7942	0.3032	3.7832	0.1174
-500	-1.1728	0.4614	3.7206	0.1291
-550	-1.8835	1.0826	3.9324	0.1770

Table 4.2: Values for the lineal fit coefficients on the different velocity bins for the Figure 4.23.

The values σ_{Slope} and σ_{Shift} refers to the standard error of the respective lineal coefficient.

IN PROGRESS.. The rotation eliminates one of the chromodependence but this dont confirm anything **IN PROGRESS..**

4.3.3 Characterization of chromodepence on granulation pattern

The Figure 4.24 show the standard curve proposed for Gray and Oostra [7], establish on the range 6020 Å – 6340 Å of wavelength.

Gray and Oostra standard curve for the range $(6020 - 6340)\text{\AA}$

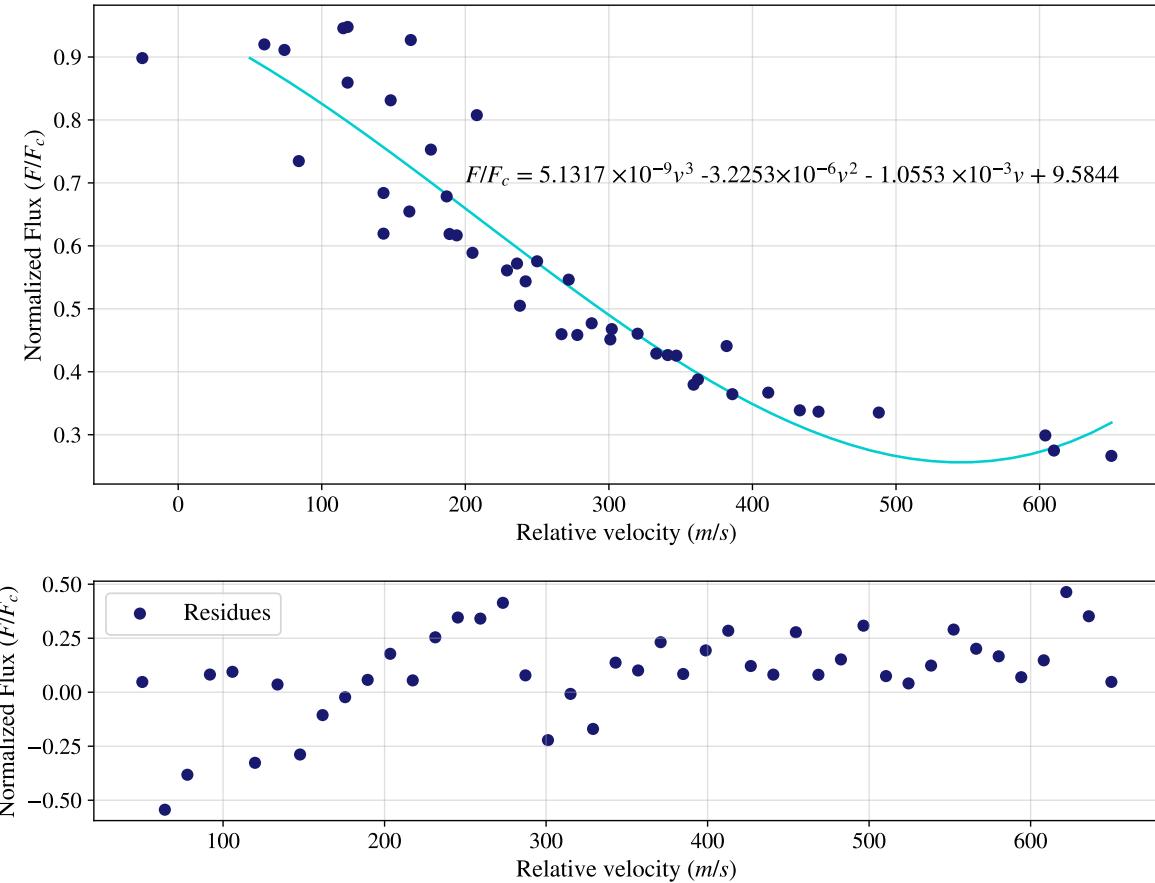


Figure 4.24: Standard curve given by Gray and Oostra and recalculated for comparison with the current work.

Since the standard curve does not account for the full wavelength range and the given third-order polynomial fit take just the low curve points, a different approach was adopted. The spectrum was separated into distinct color ranges, and a second-order polynomial was adjusted to each segment as shown Figure 4.25.

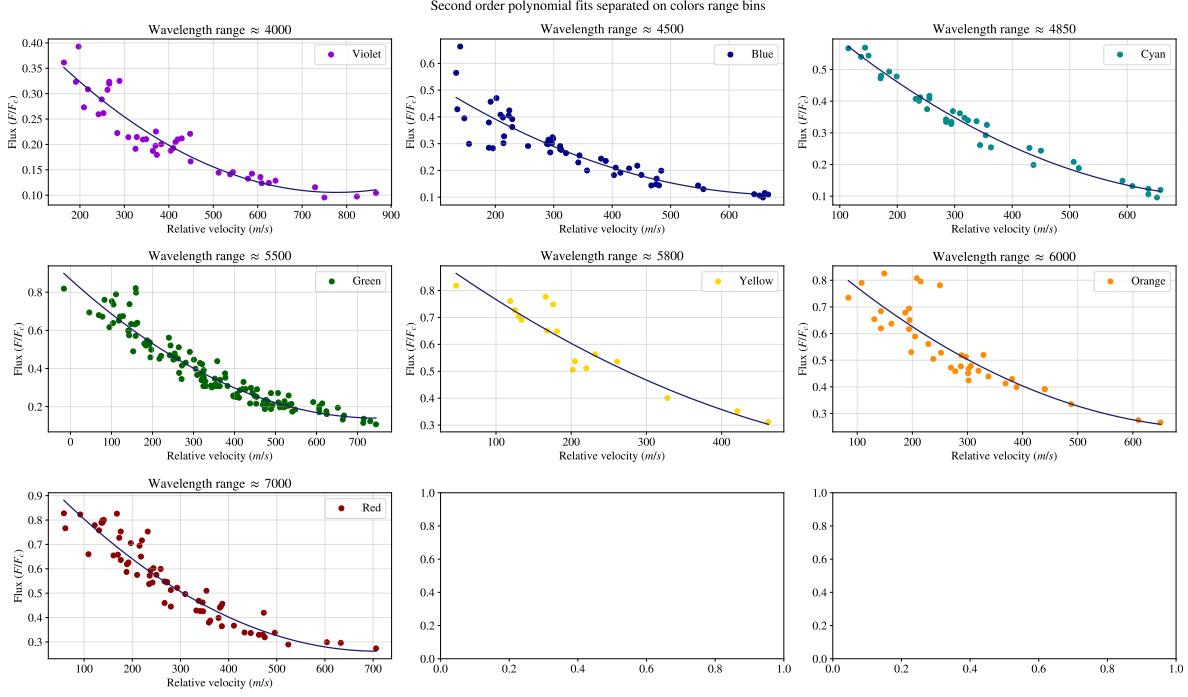


Figure 4.25: Different color curves adjusted to a specific range on granulation pattern for the visible range in the IAG solar flux atlas. The inclination on the curves is more pronounced in the violet than the red range.

The trend in the curves is more pronounced in the violet and red ranges. This can be interpreted as a shift and flattening of the standard curve from Gray and Oostra towards shorter wavelengths. Using the curve for the range $(4970 \text{ \AA} - 5700 \text{ \AA})$ as a reference (green color curve), distinct scaling coefficients were calculated for each color segment, as shown in Table 4.3.

Wavelength range (\AA)	Color coefficient
3800-4270	0.5359
4270-4760	0.8195
4760-4970	0.8348
4970-5700	1.000
5700-5810	1.0595
5810-6180	0.9834
6180-7800	1.1149

Table 4.3: Scaling factors α_c to the new standard curve shown in the Equation (4.1).

The Equation (4.1) reports a new, color dependent standard curve. Its coefficients (α_c) vary with the color range, generating the corresponding granulation pattern for each spectral segment listed in Table 4.3.

$$F/F_c = \alpha_c(1.2925 \times 10^{-6}v^2 - 0.0019v + 0.8671) \quad (4.1)$$

However, when the color coefficients α_c are plotted against wavelength, they exhibit a quadratic tendency that increases with longer wavelengths. This finding leads us to model the coefficient α_c as a function of wavelength, as shown in equation (4.2)

$$\circledast(\lambda) = -6.7598 \times 10^{-8}\lambda^2 + 9.1584 \times 10^{-4}\lambda - 2.0041 \quad (4.2)$$

Then, the characterization for the granulation pattern it can be described by equation 4.3

$$F/F_c(\lambda, v) = \circledast(\lambda)[1.2925 \times 10^{-6}v^2 - 0.0019v + 0.8671] \quad (4.3)$$

On the other hand, chromodependence was also identified in the near infrared range, although it is less pronounced than in the visible spectrum. There were defined specific wavelength ranges by excluding regions dominated by telluric absorption lines, and was found linear relationships for each range (see Figure 4.26). However, unlike the visible range these relationships do not exhibit a unified correlation.

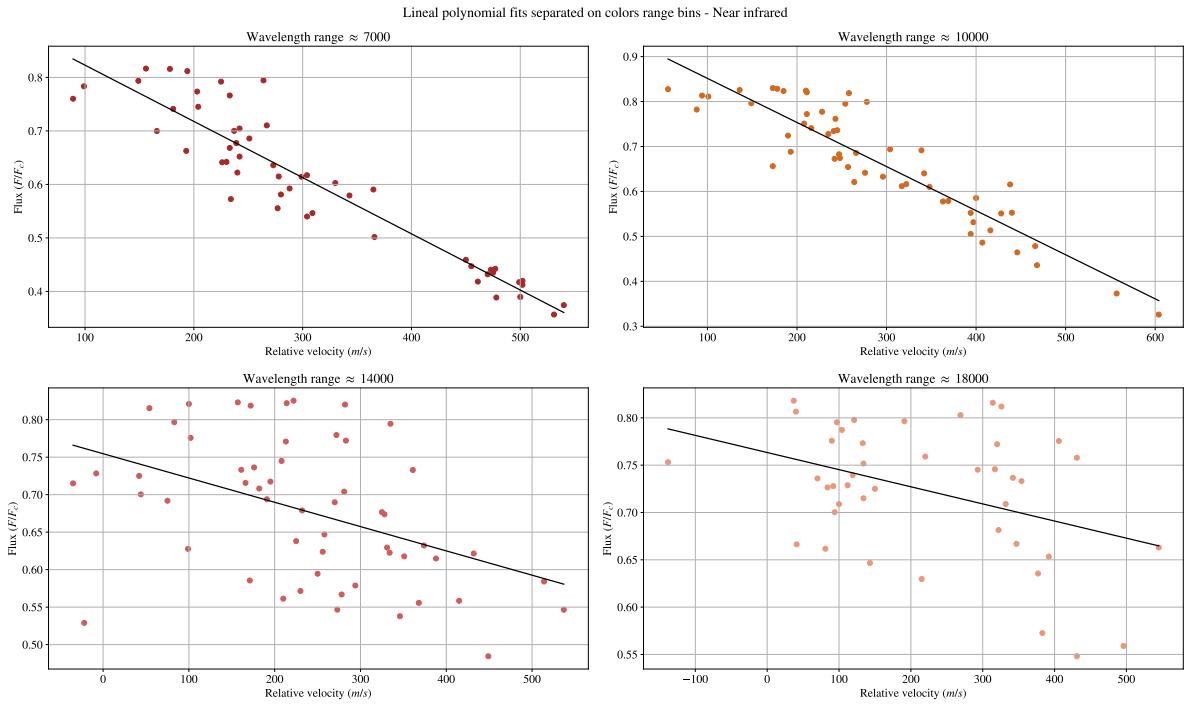


Figure 4.26: Different color curves fitted to a specific ranges on granulation pattern for the near infrared range in the Solar Flux Atlas.

The coefficients for each linear fit applied are shown in the table 4.4.

Wavelength range	Slope	Shift
7000-8500	-0.0010	0.9279
8500-11000	-0.0009	0.9496
11000-16000	-0.0003	0.7546
16000-23000	-0.0001	0.7633

Table 4.4: Coefficients for the first order polynomial fit in each wavelength range in the near infrared.

4.4 Higher quality graphs

To show the improvement on the quality in the third signature plots, the figure 4.27 compare the plot obtained by Ellwarth [6] and the granulation pattern obtained in this project.

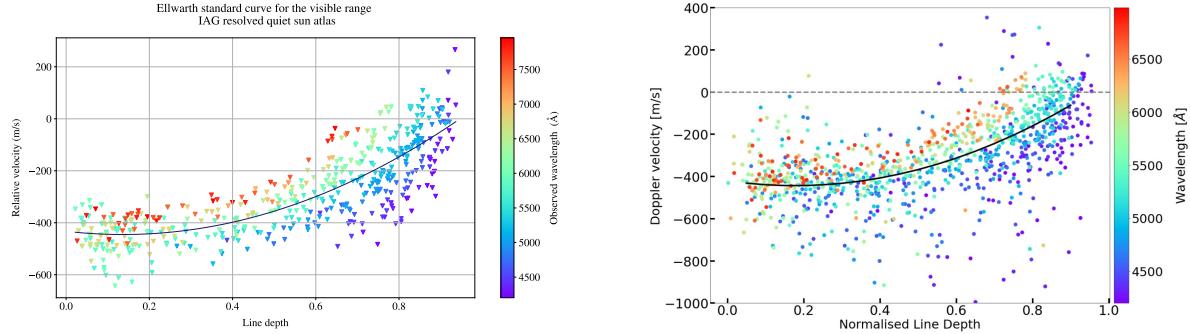


Figure 4.27: We realized the same graphic for the comparision with the Ellwarth article to show the less scattered points.

The less scattered points and the improvement on the chromodepence identification is evident.

IN PROGRESS Discuss all aspects about the standard granulation curve and the effects that broaden or disperse this curve: The curve depends strongly on wavelength and weakly on excitation potential. Discuss the hypothesis that the chromodependence is a consequence of rotation and why now believe that is not true. **IN PROGRESS**

5. Conclusions

Wavelength and line depth dependences

In conclusion, this project characterized the convective blueshift in solar absorption lines and its dependence on both line depth and wavelength range.

Specifically, was produced a refined, blend-free list of Fe I lines and derived solar granulation patterns with minimal scatter. Also provided a detailed description of how the granulation pattern varies with wavelength, establishing a new, color-dependent standard curve.

(In progress..)

A. Appendix

Z-score Standardization

The function `np.polyfit()`, during the process of calculating the fourth-order polynomial fit, presents an over estimation on the coefficients due to the large difference of magnitude order between axes. To deal with this difference, was applied a z-score standardization on the selected bins of wavelengths around the minimum reference point. This process helps to avoid the dominance of certain features over others due to differences in their scales [16].

For the standardization was applied Equation (A.1) on the selected bins for wavelength around the minimum reference point.

$$\lambda_{\text{scaled}} = \frac{\lambda_{\text{original}} - \mu(\lambda_{\text{original}})}{\sigma(\lambda_{\text{original}})} \quad (\text{A.1})$$

Where $\mu(\lambda_{\text{original}})$ refers to the mean and $\sigma(\lambda_{\text{original}})$ to the standard deviation of the wavelength range. In terms of calculating derivatives for the first and second signature of convection, the standardization on the variable required a re-scaled factor of conversion for obtain the original values. Based on the definition for the standardization, the derivatives have to follow Equation (A.2).

$$\frac{d}{d\lambda_{\text{original}}} = \frac{1}{\sigma(\lambda_{\text{original}})} \frac{d}{d\lambda_{\text{scaled}}} \quad (\text{A.2})$$

Taking the derivative of the expression (A.1) a factor related to the standard deviation appears. Therefore, the original values for derivatives evaluated in the observed wavelength are expressed in Equation (A.3)

$$\frac{d^2}{d\lambda_{\text{original}}^2} = \frac{1}{\sigma(\lambda_{\text{original}})^2} \frac{d^2}{d\lambda_{\text{scaled}}^2} \quad \frac{d^3}{d\lambda_{\text{original}}^3} = \frac{1}{\sigma(\lambda_{\text{original}})^3} \frac{d^3}{d\lambda_{\text{scaled}}^3} \quad (\text{A.3})$$

The treatment proposed solves the problem for the large difference of magnitude order between axes.

B. Appendix

The third derivative relation

The third derivative relation or called the line core bisector slope was deducted by Professor Benjamin and showed below.

Define the slope for the minimum point as Equation (B.1), where the points b, c, h are illustrated in Figure (B.1).

$$CBS = \lim_{h \rightarrow 0} \frac{\frac{b+c}{2} - a}{h} \quad (B.1)$$

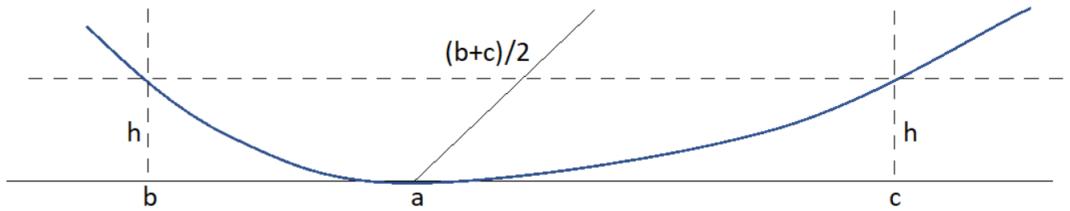


Figure B.1: Illustration for the points a, b, c and the height whose defines the line core bisector slope (see Equation (B.1)).

Note that in a symmetrical line the bisector is vertical and the slope, according to this definition, is zero. Now, the curve may be expanded around the point a by Taylor series. Notice this value is chosen at the minimum, so the first derivative evaluated in a is zero; and the difference between the function evaluated in points a and c is equal to the height.

$$2h = (c - a)^2 f''(a) + \frac{1}{3}(c - a)^3 f'''(a) \quad (B.2)$$

Taylor expansion series is represented by Equation (B.2). The minimal points were abbreviated as $c - a = x > 0$ and likewise $b - a = y < 0$ obtaining Equation (B.3).

$$2h = x^2 f''(a) + \frac{1}{3}x^3 f'''(a) \quad 2h = y^2 f''(a) + \frac{1}{3}y^3 f'''(a) \quad (\text{B.3})$$

Therefore, x and y are two equations roots, which are needed to trend like Equation (B.4) for the cubic polynomial equation.

$$CBS = \lim_{h \rightarrow 0} \frac{x}{y} = -1 \quad (\text{B.4})$$

If the polynomial third-order equation is written in the standard form, the coefficients becomes relation (B.5).

$$a = \frac{f'''}{3} \quad b = f'' \quad c = 0 \quad d = -2h \quad (\text{B.5})$$

A standard procedure is to change the variable to the difference of t with the ratio of second derivative against third derivative. Moreover, the variable change allows to express the Taylor series as a third-order polynomial fit, as shown in Equation (B.6).

$$t^3 + pt + q = 0 \quad \rightarrow \quad p = -3 \left(\frac{f''}{f'''} \right)^2 \quad q = 2 \frac{(f'')^3 - 3h(f''')^2}{(f''')^3} \quad (\text{B.6})$$

The Equation (B.6) have three solutions given Equation (B.10).

$$t_i = 2 \left(\frac{f''}{f'''} \right) \cos \left(\frac{1}{3} \arccos \left(\frac{3h(f''')^2}{(f'')^3} - 1 \right) - \frac{2\pi i}{3} \right) \quad i = 0, 1, 2 \quad (\text{B.7})$$

When h vanishes, there are two equal solutions $\cos(\theta) = 1/2$ and one differing $\cos(\theta) = -1$.

Taking a parameter ϵ as Equation (B.8), when h vanishes $\epsilon \ll 1$.

$$\epsilon = \frac{h(f''')^2}{(f'')^3} \ll 1 \quad (\text{B.8})$$

This limit impose a new limit on the trigonometric function as described Equation (B.9).

$$\arccos(3\epsilon - 1) = \pi - \delta \quad \delta \ll 1 \quad (\text{B.9})$$

Therefore, the solutions for the Taylor expansion (see Equation (B.10)).

$$t_{\pm} = 2 \frac{f''}{f'''} \left(\frac{1}{2} \cos \left(\frac{\sqrt{6\epsilon}}{3} \right) - / + \frac{\sqrt{3}}{2} \sin \left(\frac{\sqrt{6\epsilon}}{3} \right) \right) \quad \rightarrow \quad \delta = \sqrt{6\epsilon} \quad (\text{B.10})$$

For the line core bisector slope appears the condition (B.11).

$$x + y = t_+ + t_- - 2 \left(\frac{f''}{f'''} \right) = -\frac{2}{3} \left(\frac{f''}{f'''} \right) \epsilon = -2h \frac{f''}{3(f''')^2} \quad (\text{B.11})$$

Whence, finally, the core bisector slope is defined by Equation (B.12)

$$CBS = -\frac{f''}{3(f''')^2} \quad (\text{B.12})$$

In the code it was multiplied by the relation (c/λ) to see the behavior across line depth.

C. Appendix

Visualizer for outliers

For the process of Fe I line selection was created a visualizer, using the Tkinter python library, to aid in identifying blend lines or those outside the spectrum. In order to make this process less stressing a little tools bar was created to facilitate the label and deleting process is presented in Figure C.1.

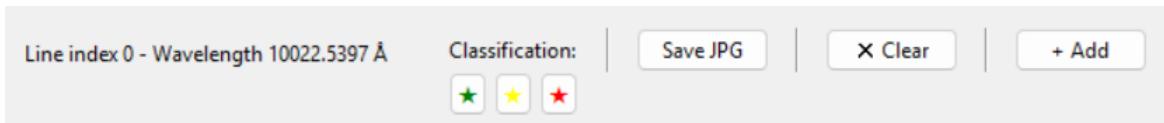
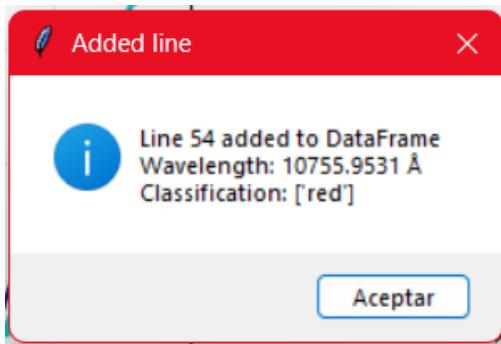


Figure C.1: Tools bar for the visualizer, contains a classification system of labels, the option to save images, and add to a Dataframe for dropping.

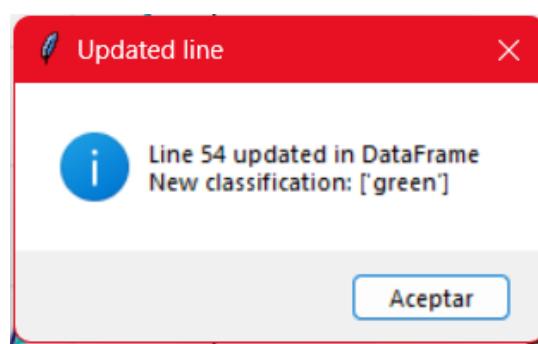
It features a custom classification system, shown as color stars. We present the guide using in the project: Red is for a discarded line, yellow indicates that the line needs further check, and green is for a line approved. If it is necessary to change the label, use the button "Clear" first and then give another label to the line. These buttons don't make changes to the Dataframe more than just a label for the user orientation. Even if the lines have or not the label for good ones, the code just deletes the lines which are added to the Dataframe of filtered lines.

To make a list of lines which are going to be deleted or filtered use the button "+ Add", which includes the line core information with the label given. If this process was successful, the message shown in Figure C.2a should appear. On the other hand, if the label of a line was changed when it was already saved as a discarded line, it can be updated. Give the other label

and use the button "+ Add", should appear the message from Figure C.2b.



(a) Message for an added line to the Dataframe.



(b) Message for an updated line to the Dataframe.

Figure C.2: The messages shows the index, wavelength and classification for the line added or updated.

When the visualizer is close, the code returns a Dataframe with the dropped lines (lines added) and filtered lines (rest of the lines). The option for "Save JPG" of the graph is available, saving the image in format JPG and showing the confirmation message with the index and wavelength of the line (see Figure C.3).

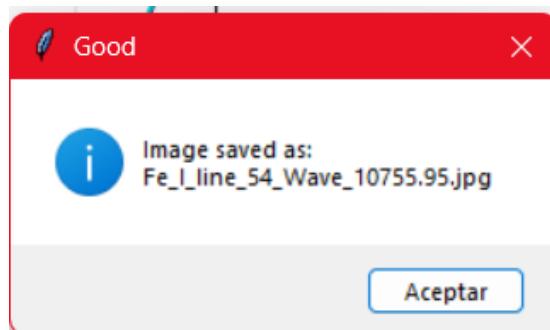


Figure C.3: Confirmation message for save the image in format JPG of the graph with the information of the line.

Furthermore, was created a navigation bar which help to locate a specific line or exploring the different graphs.

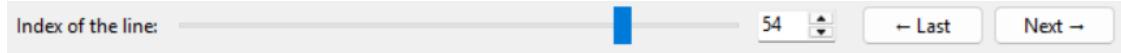


Figure C.4: Navigation bar for the visualizer, help to locate a specific line or exploring the different graphs using a slider, writing box, and buttons for moves.

This bar counts with a slider and a box to locate the line by using the index it appears in the original list (see Figure C.4). Also, counts with the buttons " \leftarrow Last" and " \rightarrow Next" to move between graphs.

Two versions of the visualizer were developed, each one applying different filters. The first filter displays the geometry of the line core and line profile as shown Figure C.5.

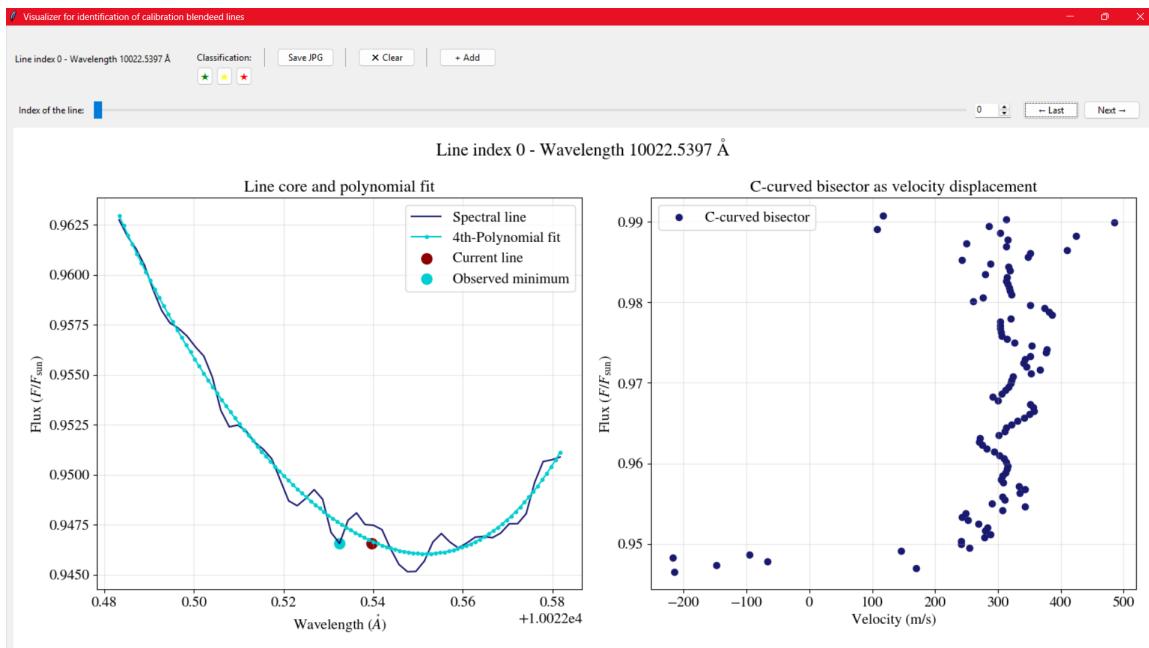


Figure C.5: General view for the results of the first filter. It shows the line core and the fourth-order polynomial fit with the line profile bisector.

This version shows the line core and the fourth-order polynomial fit, in parallel with the line profile bisector of each one in terms of velocity. The objective of the first filter is to reduce the number of lines for analyzing the three signatures of convection.

The second filter, which is build on the selected lines from the first filter, shows the three signatures of convection and the behavior of the selected line core in each one (see Figure C.6).

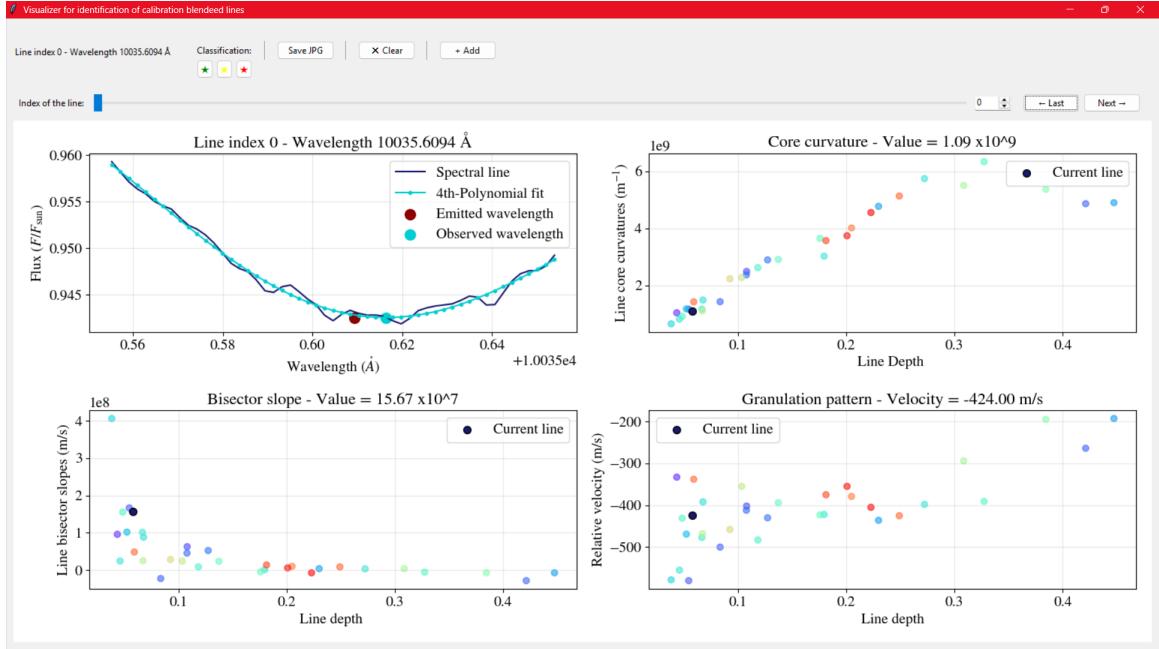


Figure C.6: General view for the results of the second filter. It shows the three signatures of convection with the line profile.

This version exhibits all the three signatures graphics including the line core with the fourth-order polynomial fit. In each graphic of derivatives the corresponding Fe I is resalted, this with the finally to select lines depending on his behavior.

The visualizer and the filters significantly reduced the time spent reviewing lines. For this reason, the code for this tool can be seen on [GitHub Repository](#), along with a test example provided below.

C.1 Test example

The main code for this project is contained in the Jupyter notebook file `Visualizer.ipynb`, and a test example dataset is provided in the excel file `Test_example`. On the first part of the code two DataFrames were created: One for the spectrum with columns `Wave` and `Flux`, representing wavelength in armstrongs and normalized flux, respectively; and another for the list of Fe I lines. Run the next cell code for upload the raw data from the excel file and create the Dataframe of the spectrum with columns `Wave` and `Flux`.

```

#Data from Vzier without treatment 10000 A- 11000 A
raw_data = pd.read_excel(r"C:\Users\clauw\Documents\
    Programming\Final-Project\Project\Visualizer\Data\
    Test_example.xlsx")

#Convert the wavelength in A cause its in cm^-1
raw_data['Wave A'] = (10**8)/raw_data['Wave (cm)']

#Invert the values for the index in python
raw_data = raw_data.sort_values('Wave A', ascending=True,
    na_position='first').reset_index(drop=True)

```

Feel free to modify this line to read the file type. The idea is the Dataframe results have the columns Wave (cm), nFlux and Wave A. There's no need for the flux to be normalized, because nothing in the code uses this condition. Then run the next cell code for create the Dataframe with calibration lines.

```

#List of Fe I lines
raw_lambda = pd.read_excel(r"C:\Users\clauw\Documents\
    Programming\Final-Project\Project\Visualizer\Nave\
    list example.xlsx")

raw_lambda['Nave list'] = (10**8)/raw_lambda['lambda']
raw_lambda = raw_lambda[raw_lambda['Nave list'].between
    (10000,11000,inclusive='right')] #Separate the range for
    use, in this case 10000-11000
raw_lambda = raw_lambda[raw_lambda['class']=='A'].
    reset_index(drop=True) #Just take the quality A lines

```

In this case, we use the Nave list of Fe I lines. A test example list is displayed on the folder Data.

For the first filter we use two different functions. The function `find_minimuns` was used to se-

lect the closest minimums to the Fe I lines, along with their associated flux values and Fe I line wavelengths. Beware, these Fe I line wavelengths serve as reference points for selecting bins around each line, rather than representing observed wavelengths. The closest lines to the Nave list values were selected as the minimum from the absolute value between the wavelengths. We discard any lines with distances between minimums and Fe I lines exceeding 0.1 Å. To use the function run the next code line.

```
closer_lines_first_filter = find_minimums(raw_data['Wave A',
    ], raw_data['nFlux'], raw_lambda['Nave list'])
```

The function `width_minimums` selecting the bins of 50 mÅ and 0.2 Å around the minimal point, respectively for the line core and the line profile. Each window corresponds to one index on the Dataframe of closer Fe I lines. Run the next cell to create the bins for the line core and the the line profile.

```
local_points_first_filter = width_minimums(
    closer_lines_first_filter, raw_data, 0.05) #The window for
    the observed wavelength
local_points_bis_first_filter = width_minimums(
    closer_lines_first_filter, raw_data, 0.2) #Window for the
    bisector with all the line
```

Then, the function First Filter finds the fourth-order polynomial fit and calculates the minimum point with the second derivative of the fit, that is the observed wavelength. This returns a Dataframe with the value of Fe I line associated, the polynomial fit, and the observed wavelength. In parallel, the line profile bisector is calculated using the midpoint method where equal points of flux are selected for comparison. Use the next line code for call the function.

```
Values_first_filter, Fit_first_filter, Local_first_filter,
Bisector_first_filter = First_Filter(
    local_points_first_filter, closer_lines_first_filter,
    local_points_bis_first_filter)
```

Furthermore, to ensure the typical absorption line shape, fourth-order polynomial fits with

non-positive second-order coefficients were discarded. Also an approximation was used to discard lines that do not belong to the spectrum. A computational form of a slope can be seen as the difference between the maximum and minimum point of the list of points. In terms of flux if it is seen the distance can't be more than a half of the absolute difference. This lets us discard pronounced slopes without affecting or filtering weaker lines.

To display the visualizer, uncomment the following line code in the cell corresponding to it.

```
#CHANGE THIS LINE - The order: local_points, values, fit,
bisectors
app = OutlierViewer(root, Local_first_filter,
Values_first_filter,Fit_first_filter,
Bisector_first_filter)
```

Note: Sometimes the first plot of the visualizer can be displaced due to the Tkinter library. To fix this, just move to another line and back to the wished one.

We recommend eliminating the lines that follow one of the conditions presented. First, the line profile bisector doesn't show a C-shaped bisector or it's too affected by the noise. Second, the polynomial fit and the line profile don't follow a common absorption line form. This can be interpreted as the position on other points to the fit. Third, the line core shows two minimums or a protuberance. These are blended lines. An example of this behavior is shown on Figure C.7 and an example of accepted lines is shown in Figure C.8.

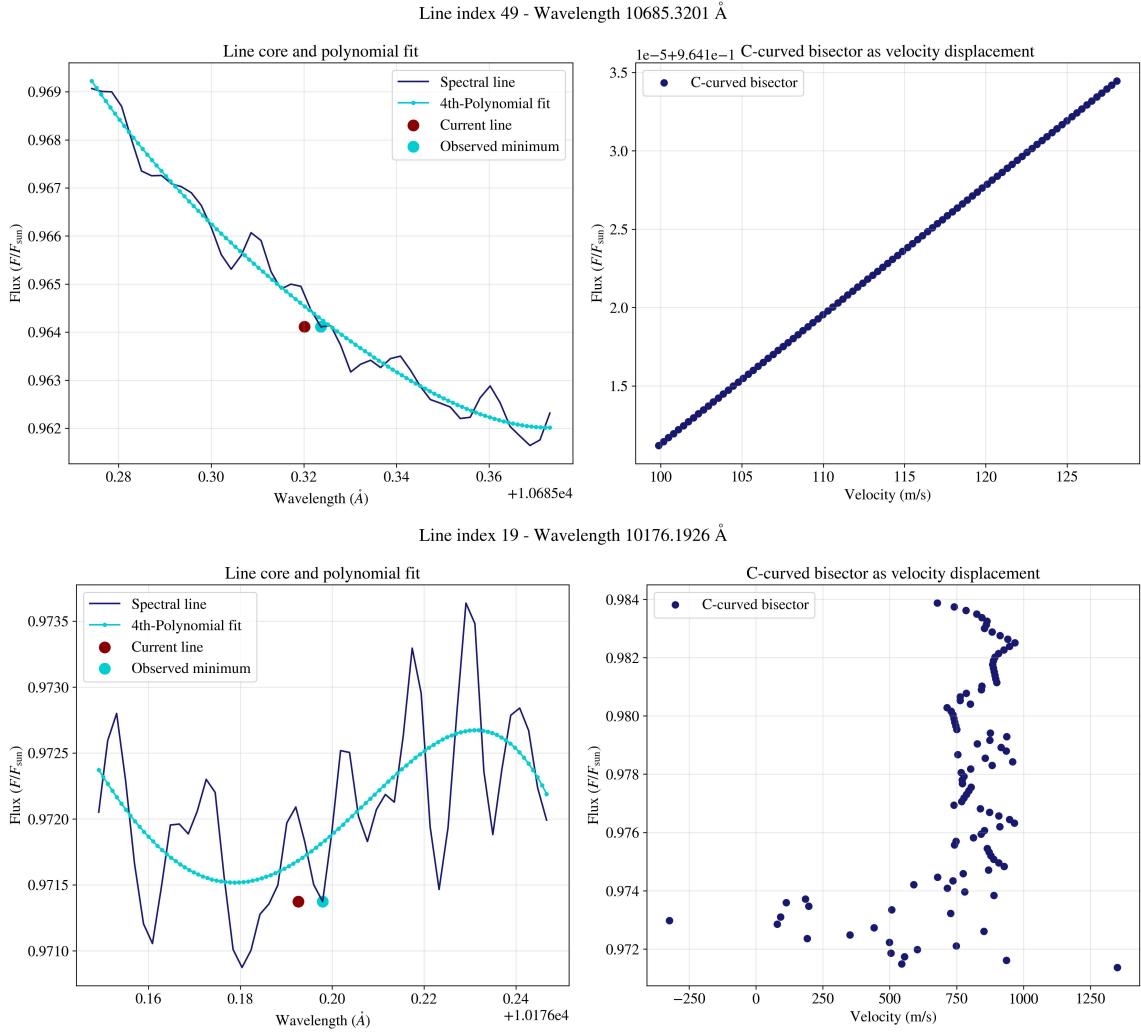


Figure C.7: Examples for discarded lines with the first filter of the visualizer. The line cores and line bisectors of the two plots shows one or more of the conditions presented.

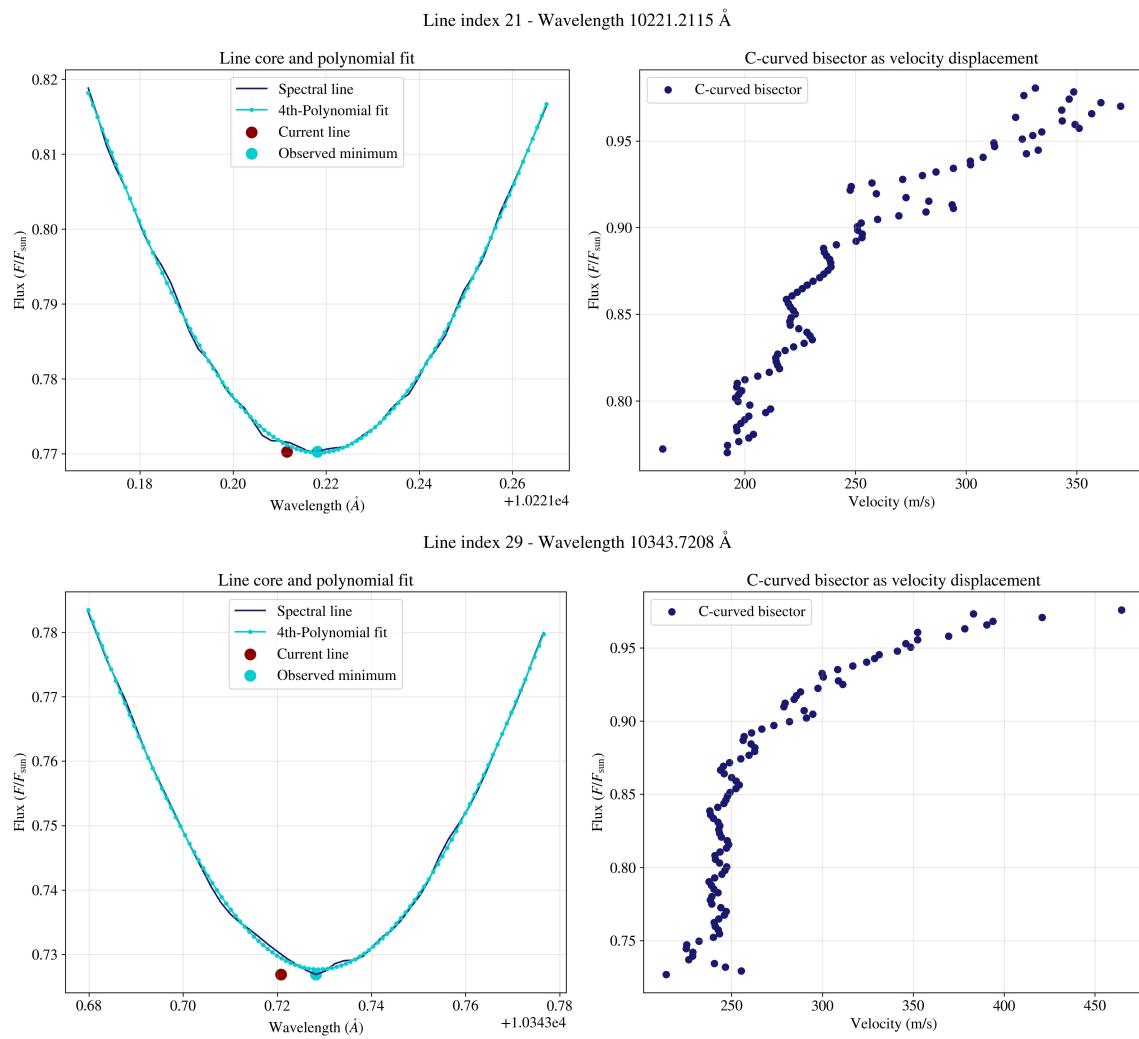


Figure C.8: Examples for accepted lines with the first filter of the visualizer.

Note: Make sure that the visualizer is closed after running another cell code.

To extract the Dataframe with the lines to discard or for further analysis run the next cell.

```
#Return the lines which are added to the dataframe of
#deleting lines
Filtered_lines_1 = app.return_lines()

#Save the selected lines to drop
Filtered_lines_1['Line'].to_excel('Dropped lines first')
```

```
filter.xlsx', index=False)
```

The next cell saves the filtered lines in an excel file.

```
index_to_drop_1 = Filtered_lines_1.index.values

#Drop the selected lines
New_values_1 = Values_first_filter.drop(index_to_drop_1).
    reset_index(drop=True)

#Save them in a excel file
New_values_1['Nave line'].to_excel('Nave list first filter.
    xlsx', index=False)
```

The second part of the code calculates the three signatures of convection for the filtered lines.

With this dataset, only run the cell corresponding to the new list of filtered lines for calibration.

```
#Lines from the excel file - This list have filtered values
raw_lambda = pd.read_excel(r"C:\Users\clauw\Documents\
    Programming\Final-Project\Project\Visualizer\Data\Nave
    list first filter.xlsx")
```

For the test example we provide a list for filtered lines in the file Nave list first filter.xlsx. Then, run the cell to redefine the closer lines and the corresponding windows of 50 mÅ.

```
closer_lines_second_filter = find_minima(raw_data['Wave A',
    ], raw_data['nFlux'], raw_lambda['Nave list'])

local_points_second_filter = width_minima(
    closer_lines_second_filter, raw_data, 0.05)
```

Then, run the next cell for applied the second filter and calculate the three signatures of convection.

```
Values_second_filter, Local_second_filter, Fit_second_filter =
```

```
Second_Filter(local_points_second_filter,  
closer_lines_second_filter['Nave list'])
```

This function calculate the observed wavelength based on the fourth-order polynomial fit. Then, uses the different relations related to the signatures of convection. To display the visualizer of the second filter, uncomment the following line code in the cell corresponding to it

```
#CHANGE THIS LINE - The order: local_points, values, fit  
app_2 = OutlierViewer(root, Local_second_filter,  
Values_second_filter,Fit_second_filter)
```

Note: Make sure that the visualizer is closed after running another cell code.

We recommend eliminating the lines that are affected by noise or don't follow the general behavior on the three signatures of convection. These are blended lines. An example of this behavior is shown on Figure C.9 and an example of accepted lines is shown in Figure C.10.

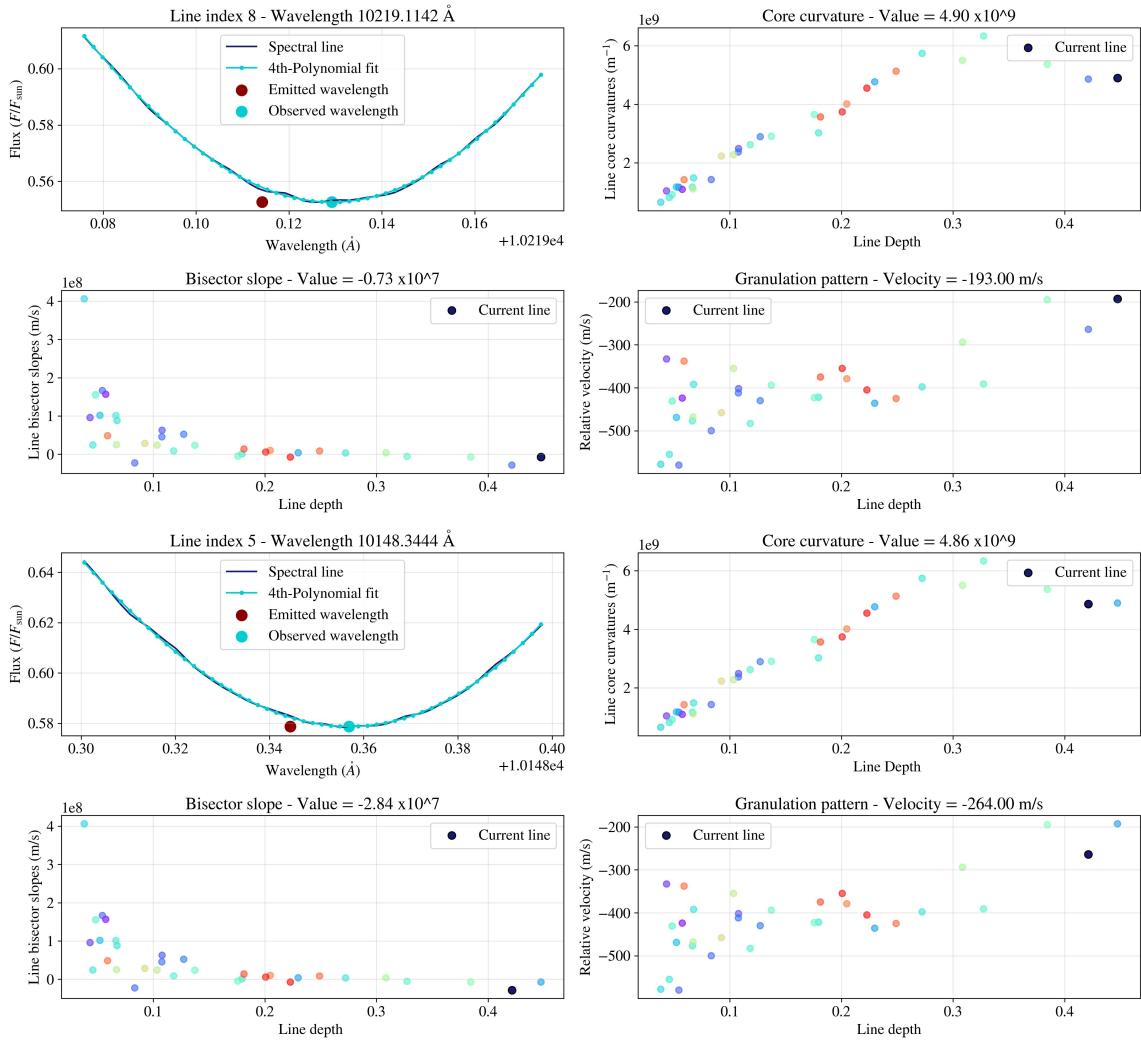


Figure C.9: Examples for discarded lines with the first filter of the visualizer.

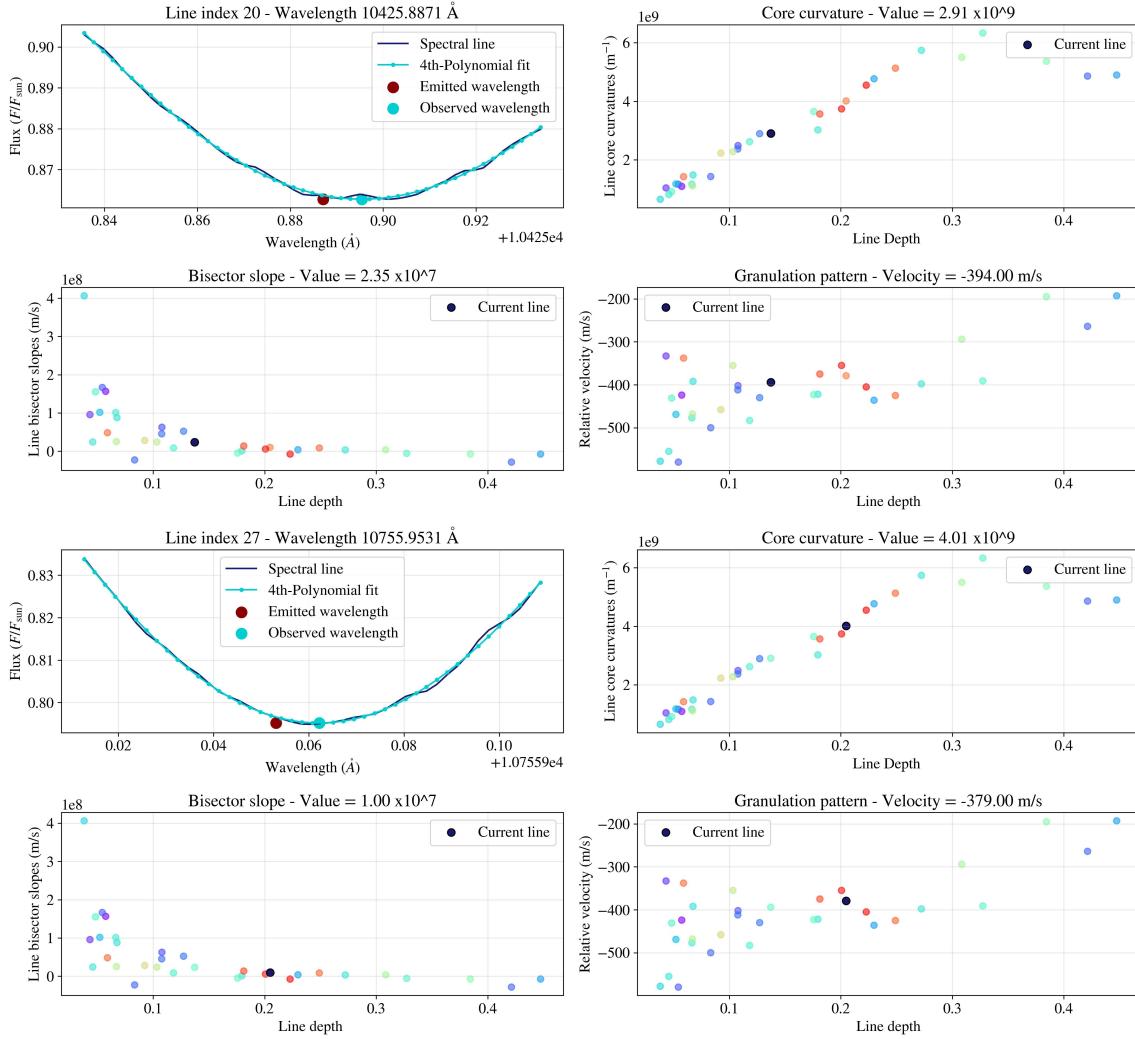


Figure C.10: Examples for accepted lines with the first filter of the visualizer.

To extract the Dataframe with the lines to discard or for further analysis run the next cell.

```
Filtered_lines_2 = app_2.return_lines()

#Save the selected lines to drop
Filtered_lines_2['Line'].to_excel('Dropped lines second
filter.xlsx', index=False)
```

The next cell saves the filtered lines in an excel file.

```
index_to_drop_2 = Filtered_lines_1.index.values
```

```
#Drop the selected lines
New_values_2 = Values_second_filter.drop(index_to_drop_2).
    reset_index(drop=True)

#Save them in a excel file
New_values_2.to_excel('Second_filter.xlsx', index=False)
```

After the process, the resulting excel is the definitive blend-free list of Fe I lines.

Bibliography

- [1] E. Priest, *Solar Magnetohydrodynamics*. Kluwer, 1982.
- [2] J. M. Malherbe, “Jules janssen: The birth of solar physics, the foundation of meudon observatory and the mont blanc adventure (1875-1895),” *HAL CNRS*, Aug. 2022.
- [3] S. A. Hamouda, F. Alfarjani, and F. Y. Elfituri, “Sunspots production and relation to other phenomena: A review,” *International Journal of Science and Research Methodology*, 06 2018.
- [4] P. Foukal, *Solar Astrophysics*. Cambridge research and instrumentation, 1990.
- [5] D. Dravins, L. Lindegren, and A. Nordlund, “Solar granulation - influence of convection on spectral line asymmetries and wavelength shifts,” *Astronomy and Astrophysics*, vol. 96, pp. 345–364, Mar. 1981.
- [6] M. Ellwarth, B. Ehmann, S. Schäfer, and A. Reiners, “Convective characteristics of fe i lines across the solar disc,” *Astronomy and Astrophysics*, vol. 680, p. A62, Dec. 2023.
- [7] D. F. Gray and B. Oostra, “The solar-flux third granulation signature,” *The Astrophysical Journal*, vol. 852, p. 42, Jan. 2018.
- [8] D. Hamilton and J. B. Lester, “A technique for the study of stellar convection: The visible solar flux spectrum,” *Publications of the Astronomical Society of the Pacific*, vol. 111, p. 1132, Sept. 1999.
- [9] D. Gray, “The third signature of stellar granulation,” *The Astrophysical Journal*, vol. 697, p. 1032, May 2009.

- [10] D. F. Gray and T. Pugh, “The third signature of granulation in bright-giant and supergiant stars,” *The Astronomical Journal*, vol. 143, p. 92, Mar. 2012.
- [11] B. Oostra and P. C. Vargas Muñoz, “The differential redshift of titanium lines in k stars,” *Revista mexicana de astronomía y astrofísica*, vol. 58, no. 2, pp. 173–180, 2022.
- [12] G. Nave, S. Johansson, R. C. M. Learner, A. P. Thorne, and J. W. Brault, “A new multiplet table for fe i,” *The Astrophysical Journal Supplement Series*, vol. 94, p. 221, Sept. 1994.
- [13] A. Reiners, N. Mrotzek, U. Lemke, J. Hinrichs, and K. Reinsch, “The iag solar flux atlas: Accurate wavelengths and absolute convective blueshift in standard solar spectra,” *Astronomy and Astrophysics*, vol. 587, p. A65, Mar. 2016.
- [14] H. H. Plaskett, “Solar granulation,” *Monthly Notices of the Royal Astronomical Society*, vol. 96, p. 402, Mar. 1936.
- [15] B. Carroll and D. Ostlie, *An Introduction to Modern Astrophysics*. Cambridge University Press, 2017.
- [16] J. P. Boyd, *Solving Transcendental Equations: The Chebyshev Polynomial Proxy and Other Numerical Rootfinders, Perturbation Series, and Oracles*. SIAM, Oct. 2014.
- [17] J. S. Aponte, *Medición de la Velocidad Convectiva en la Fotosfera Solar*. Bachelor’s thesis, Universidad de los Andes, 2017.