Assignment

Section 3.4: 2(fj), 8, 11, 14, 15; Section 4.1: 10; Section 4.2: 23, 29, 30; Section 4.3: 10, 11, 12, 15

Work

3.4

2.

8. Let W denote the subspace of R^5 consisting of all vectors having coordinates that sum to zero. The vectors

$$
\begin{aligned}
u_1 &= (2, -3, 4, -5, 2), & u_2 &= (-6, 9, -12, 15, -6), \\
u_3 &= (3, -2, 7, -9, 1), & u_4 &= (2, -8, 2, -2, 6), \\
u_5 &= (-1, 1, 2, 1, -3), & u_6 &= (0, -3, -18, 9, 12), \\
u_7 &= (1, 0, -2, 3, -2), & u_8 &= (2, -1, 1, -9, 7)
\end{aligned}
$$

generate W. Find a subset $\{u_1, u_2, \ldots, u_8\}$ that is a basis for W.

$$
\mathsf{R}^5 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} : x_1 + x_2 + x_3 + x_4 + x_5 = 0, x_1, \ldots, x_5 \in \mathbb{R} \right\} \tag{1}
$$

$$
\begin{pmatrix}
2 & -6 & 3 & 2 & -1 & 0 & 1 & 2 \\
-3 & 9 & -2 & -8 & 1 & -3 & 0 & -1 \\
4 & -12 & 7 & 2 & 2 & -18 & -2 & 1 \\
-5 & 15 & -9 & -2 & 1 & 9 & 3 & -9 \\
2 & -6 & 1 & 6 & -3 & 12 & -2 & 7
\end{pmatrix} \tag{2}
$$

$$
\rightsquigarrow \begin{pmatrix}
1 & -3 & 0 & 4 & 0 & 1 & 0 & -1 \\
 & & 1 & -2 & 0 & -2 & 0 & 1 \\
 & & & & 1 & -4 & 0 & -2 \\
 & & 0 & & & & 1 & -1 \\
 & & & & & & & 0
\end{pmatrix} \tag{3}
$$

It follows that $\{u_1, u_3, u_5, u_7\}$ is linearly independent by theorem 3.16. Therefore $\{u_1, u_3, u_5, u_7\}$ is a basis for W.

11.

14.

15.

4.1

10.

4.2

23.

29. Prove that if E is an elementary matrix, then $\det(E^t) = \det(E)$.

(a) **Types 1 & 2**

$$E^t = E \quad \text{(by HW.3.1.5)} \tag{4}$$
$$\Rightarrow \det(E^t) = \det(E) \tag{5}$$

(b) **Type 3**

E^t is an type 3 elementary matrix (by HW.3.1.5) $\det(E) = \det(I) = 1$ for any type elementary operation on I_n

$$\det(E^t) = \det(I) \text{ because } E^t \text{ is type 3} \tag{6}$$
$$\Rightarrow \det(E) = \det(E^t) = 1 \tag{7}$$

30. Let the rows of $A \in \mathsf{M}_{n\times n}(F)$. be $a_1, a_2, \ldots, a_n$ and let B be the matrix in which the rows are $a_n, a_{n-1}, \ldots, a_1$. Calculate $\det(B)$ in terms of $\det(A)$.

(a) **n is even**

In A, swap

$$a_{n-1} \text{ with } a_1 \tag{8}$$
$$a_{n-2} \text{ with } a_2 \tag{9}$$
$$\vdots$$
$$a_{n-\frac{n}{2}+1} \text{ with } a_{n-\frac{n}{2}} \tag{10}$$

From the fact that ${}^n/_2$ swaps were performed it follows from Theorem 4.6 that

$$\det(B) = (-1)^{\frac{n}{2}} \det(A) \tag{11}$$

(b) **n is odd** In A, swap

$$a_{n-1} \text{ with } a_1 \tag{12}$$
$$a_{n-2} \text{ with } a_2 \tag{13}$$
$$\vdots$$
$$a_{n-\frac{n+1}{2}+1} \text{ with } a_{n-\frac{n+1}{2}} \tag{14}$$

From the fact that $n - \frac{n+1}{2}$ swaps were performed it follows from Theorem 4.6 that

$$\det(B) = (-1)^{\frac{n-1}{2}} \det(A) \tag{15}$$

4.3

10.

11.

12.