Assignment

Section 4.4: 1, 5, 6; Section 5.1: 3(bc), 4(ceh), 7, 12, 14, 15, 19, 22

Work

4.4

1. Label the statements as true or false.

(a)	True	(g)	True
(b)	True	(h)	False
(c)	True	(i)	True
(d)	False	(j)	True
(e)	False	(k)	True
(f)	True		

5. Suppose that $M \in \mathsf{M}_{n\times n}(F)$ can be written in the form

$$M = \begin{pmatrix} A & B \\ O & I \end{pmatrix}$$

where A is a square matrix. Prove that $\det(M) = \det(A)$

Suppose $A \in \mathsf{M}_{k\times k}(F)$

Perform Type 3 operations such that the partition (A B) becomes an upper triangular matrix.

$$\Rightarrow M' = \begin{pmatrix} A' & B' \\ O & I \end{pmatrix} \tag{1}$$

M' is an upper triangular matrix so the determinant of M is the product of its diagonal terms.

$$\Rightarrow \det(A) = \prod_{i=1}^{n} M'_{ii} \tag{2}$$

$$M'_{ii} = 1 \quad \text{if } (k+1 \leq i \leq n) \tag{3}$$

$$\Rightarrow \prod_{i=1}^{n} M'_{ii} = \prod_{i=1}^{k} M'_{ii} \tag{4}$$

The first k diagonal terms of M' are the diagonal terms of A. It follows that

$$\prod_{i=1}^{k} M'_{ii} = \prod_{i=1}^{k} A'_{ii} \tag{5}$$

Because A' was obtained from A using Type 3 operations, and M' was obtained from M using Type 3 operations

$$\det(A') = \det(A) \tag{6}$$

$$\det(M') = \det(M) \tag{7}$$

$$\therefore \det(M) = \det(M') = \det(A') = \det(A) \tag{8}$$

6.

5.1

3.

4.

7.

12.

14.

15.

19.

22.