

## Assignment

Section 3.4: 2(fj), 8, 11, 14, 15; Section 4.1: 10; Section 4.2: 23, 29, 30; Section 4.3: 10, 11, 12, 15

## Work

### 3.4

2.

8. Let  $W$  denote the subspace of  $\mathbb{R}^5$  consisting of all vectors having coordinates that sum to zero. The vectors

$$\begin{aligned} u_1 &= (2, -3, 4, -5, 2), & u_2 &= (-6, 9, -12, 15, -6), \\ u_3 &= (3, -2, 7, -9, 1), & u_4 &= (2, -8, 2, -2, 6), \\ u_5 &= (-1, 1, 2, 1, -3), & u_6 &= (0, -3, -18, 9, 12), \\ u_7 &= (1, 0, -2, 3, -2), & u_8 &= (2, -1, 1, -9, 7) \end{aligned}$$

generate  $W$ . Find a subset  $\{u_1, u_2, \dots, u_8\}$  that is a basis for  $W$ .

$$\mathbb{R}^5 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} : x_1 + x_2 + x_3 + x_4 + x_5 = 0, x_1, \dots, x_5 \in \mathbb{R} \right\} \quad (1)$$

$$\begin{pmatrix} 2 & -6 & 3 & 2 & -1 & 0 & 1 & 2 \\ -3 & 9 & -2 & -8 & 1 & -3 & 0 & -1 \\ 4 & -12 & 7 & 2 & 2 & -18 & -2 & 1 \\ -5 & 15 & -9 & -2 & 1 & 9 & 3 & -9 \\ 2 & -6 & 1 & 6 & -3 & 12 & -2 & 7 \end{pmatrix} \quad (2)$$

$$\rightsquigarrow \begin{pmatrix} 1 & -3 & 0 & 4 & 0 & 1 & 0 & -1 \\ & & 1 & -2 & 0 & -2 & 0 & 1 \\ & & & & 1 & -4 & 0 & -2 \\ & 0 & & & & & 1 & -1 \\ & & & & & & & 0 \end{pmatrix} \quad (3)$$

It follows that  $\{u_1, u_3, u_5, u_7\}$  is linearly independent by theorem 3.16. Therefore  $\{u_1, u_3, u_5, u_7\}$  is a basis for  $W$ .

11.

14.

15.

## 4.1

10.

## 4.2

23.

29. Prove that if  $E$  is an elementary matrix, then  $\det(E^t) = \det(E)$ .

(a) **Types 1 & 2**

$$E^t = E \quad (\text{by HW.3.1.5}) \quad (4)$$

$$\Rightarrow \det(E^t) = \det(E) \quad (5)$$

(b) **Type 3**

$E^t$  is an type 3 elementary matrix (by HW.3.1.5)  $\det(E) = \det(I) = 1$  for any type elementary operation on  $I_n$

$$\det(E^t) = \det(I) \text{ because } E^t \text{ is type 3} \quad (6)$$

$$\Rightarrow \det(E) = \det(E^t) = 1 \quad (7)$$

## 4.3

30.

11.

12.