Assignment

3.1: 5, 12; 3.2: 5(beg), 6(adf), 14, 20; 3.3: 2(ad), 3(ad), 7(bd), 9, 10

Work

3.1

5. Prove that E is an elementary matrix if and only if E^t is.
   Claim: $E \rightsquigarrow E^t$

$$I_n = \begin{bmatrix} e_1 & e_2 & \cdots & e_i & \cdots & e_j & \cdots & e_n \end{bmatrix} \tag{1}$$

   (a) Claim: The interchange of any two rows i and j is equivalent to interchanging any two columns i and j
   By applying the interchange to E it follows that:

$$E = \begin{bmatrix} e_1 & e_2 & \cdots & e_j & \cdots & e_i & \cdots & e_n \end{bmatrix} \tag{2}$$
$$\Rightarrow E^t = \begin{bmatrix} e_1 & e_2 & \cdots & e_j & \cdots & e_i & \cdots & e_n \end{bmatrix} = E \tag{3}$$

   (b) Claim: Multiplying any row i with nonzero scalar c is equivalent to multiplying any column j with the same scalar c.
   By applying the scaling to E is follows that:

$$E = \begin{bmatrix} e_1 & e_2 & \cdots & ce_j & \cdots & e_n \end{bmatrix} \tag{4}$$
$$\Rightarrow E^t = \begin{bmatrix} e_1 & e_2 & \cdots & ce_i & \cdots & e_n \end{bmatrix} = E \tag{5}$$

   (c) Claim: Adding any scalar multiple of row i to row j is equivalent to adding any scalar multiple of column i it column j
   By applying the replacement to E it follows that:

$$E = \begin{bmatrix} e_1 & e_2 & \cdots & ce_i + e_j & \cdots & e_n \end{bmatrix} \tag{6}$$
$$\Rightarrow E^t = \begin{bmatrix} e_1 & e_2 & \cdots & ce_j + e_i & \cdots & e_n \end{bmatrix} \tag{7}$$

$$\therefore E^t \text{ is elementary} \tag{8}$$

12. Let A be an $m \times n$ matrix. Prove that there exists a sequence of elementary row operations of types 1 and 3 that transforms A into an upper triangular matrix.

   (a) For $m = 2$

      i. If $a_{11} = 0$ and $a_{21} \neq 0$ interchanging rows 1 and 2 creates an upper triangular matrix.

      ii. If $a_{11} \neq 0$ adding the row 1 scaled by a_{21}/a_{11} and subtracted from row 2 creates an upper triangular matrix.

(b) For $m = k$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{pmatrix} \tag{9}$$

i. If $m > n$

$$A \rightsquigarrow \begin{pmatrix} a_{11} & \cdots & \cdots & a_{1n} \\ & a_{22} & & \vdots \\ & & \ddots & \vdots \\ & & & a_{nn} \\ & \text{\huge 0} & & a_{n+1,n} \\ & & & \vdots \\ & & & a_{mn} \end{pmatrix} \tag{10}$$

ii. If $m < n$

$$A \rightsquigarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} & a_{1,m+1} \cdots & a_{1n} \\ & a_{22} & & \vdots & & \vdots \\ & \text{\huge 0} & \ddots & \vdots & & \vdots \\ & & & a_{mm} & a_{m,m+1} \cdots & a_{mn} \end{pmatrix} \tag{11}$$

(c) For $m = k + 1$

i. If $m > n$

ii. If $m < n$

3.2

5.

6.

14.

20.

3.3

2.

3.

7.

9.

10.