Data on the 155 motif equivalence classes (mECs) partitioning the 9608 graphs on n=5 Created by Carina Curto and Katherine Morrison (PIs) in collaboration with Caitlyn Parmelee, Sumita Garai, and Joshua Paik. August 16, 2020.

This is an annotated version of n5\_mECs\_README.txt, adding information about attractors. Dynamic attractors all occur in Groups I-III, which consist of 1053 graphs. Group IV has 8555 graphs, each one containing stable fixed points only (cliques).

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GROUP I: 39 classes of graphs with no proper core motifs
(these are either new n=5 cores or they are graphs that contain no core motifs)
[Total: 39 graphs, 3.7% "unstable core" classes, 0.4% of all n=5 graphs]
mEC 1: 5-cycle (core motif), Group I-A (graph 1)
mEC 2: 5-star (core motif), Group I-A (graph 2)
mEC 3: 5-star with 2-cliques (core motif), Group I-A (graph 3)
mEC 4: 4-cycle[1,3][2,4] (core motif), Group I-B (graph 4)
mEC 5: 4-cycle[1,3][2,4] plus edge out (core motif), Group I-B (graph 5)
mEC 6: 4-cycle[1,3][2,4] plus 2 edges out (core motif), Group I-B (graph 6)
mEC 7: clique union 4-cycle + pt (core motif), Group I-C (graph 7)
mEC 8: clique union 4-cycu + pt (core motif), Group I-C (graph 8)
mEC 9: clique union 4-ufd + pt (core motif), Group I-C (graph 9)
mEC 10: clique union 3-cycle + 2-clique (or fusion 3-cycle + pt) (core motif), Group I-C (graph 10)
mEC 11: pure cyclic union on pt, 2-clique, pt, pt (or extended 4-cycu) (core motif), Group I-D (graph 11)
mEC 12: extended 4-cycu down-skip edge node 2 (** no core motifs for eps = 0.51, delta = 1.76 **),
Group I-D (graph 12)
mEC 13: extended 4-ufd (core motif), Group I-D (graph 13)
mEC 14: extended 4-ufd down-skip edge node 2 (core motif), Group I-D (graph 14)
mEC 15: extended 4-ufd + down-skip edge node 3 (core motif), Group I-D (graph 15)
mEC 16: pure cyclic union pt, 3-clique, pt (core motif), Group I-E (graph 16)
mEC 17: cyclic union pt, 3-clique, pt w/ added back edge (core motif), Group I-E (graph 17)
mEC 18: cyclic union pt, 3-clique, pt w/ two added back edges (core motif), Group I-E (graph 18)
mEC 19: pure cyclic union pt, 3-cycle, pt (core motif), Group I-F (graph 19)
mEC 20: cyclic union pt, 3-cycle, pt w/ added back edge (core motif), Group I-F (graph 20)
mEC 21: cyclic union pt, 3-cycle, pt w/ dropped down edge (core motif), Group I-F (graph 21)
mEC 22: cyclic union pt, 3-cycle, pt w/ added back edge and dropped down edge (v1) (core motif),
Group I-F (graph 22)
mEC 23: cyclic union pt, 3-cycle, pt w/ added back edge and dropped down edge (v2) (core motif),
Group I-F (graph 23)
mEC 24: cyclic union pt, 3-cycle, pt w/ added back edge and dropped down edge (v3) ** 2 attractors **
(core motif), Group I-F (graph 24)
mEC 25: pure cyclic union pt, 2-clique, 2-clique (core motif), Group I-G (graph 25)
mEC 26: cyclic union pt, 2-clique, 2-clique w/ added back edge (core motif), Group I-G (graph 26)
mEC 27: cyclic union pt, 2-clique, 2-clique w/ two added back edges (v1) (core motif), Group I-G (graph 27)
mEC 28: cyclic union pt, 2-clique, 2-clique w/ two added back edges (v2) (core motif), Group I-G (graph 28)
mEC 29: cyclic union pt, 2-clique, 2-clique w/ three added back edges (v1) (core motif), Group I-G (graph 29)
mEC 30: cyclic union pt, 2-clique, 2-clique w/ three added back edges (v2) (core motif), Group I-G (graph 30)
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mEC 31: cyclic union pt, 2-clique, 2-clique w/ two added back edges (v3) (core motif), Group I-G (graph 31)
mEC 32: envelope attractor (v1) (** no core motifs for eps = 0.51, delta = 1.76 **), Group I-H (graph 32)
mEC 33: envelope attractor (v2) (** no core motifs for eps = 0.51, delta = 1.76 **), Group I-H (graph 33)
mEC 34: envelope attractor (v3) (core motif), Group I-H (graph 34)
mEC 35: envelope attractor (v4) (core motif), Group I-H (graph 35)
mEC 36: envelope attractor (v5) (core motif), Group I-H (graph 36)
mEC 37: double envelope attractor (v1) (core motif), Group I-I (graph 37)
mEC 38: double envelope attractor (v2) (core motif), Group I-I (graph 38)
mEC 39: weird attractor (core motif), Group I-J (graph 39)
GROUP II: 66 "unstable core" classes with at least one unstable 4-motif: 4-cycle, 4-cycu, 4-ufd
[Total: 289 graphs, 3% of all n=5 graphs]
** These 12 classes split into smaller attractor equivalence classes
mEC 40: 4-cycle [13 graphs] Group II (graphs [40:52])
**mEC 41: 4-cycle + sink [2 graphs] Group II (graphs [53:54])
mEC 42: 4-cycle + 2-clique [4 graphs] Group II (graphs [55:58])
mEC 43: 4-cycle + 3-cycle [2 graphs] Group II (graphs [59:60])
mEC 44: two 4-cycles [1 graph] Group II (graph 61)
**mEC 45: 4-cycu [57 graphs] Group II (graphs [62:118])
**mEC 46: 4-cycu + sink [7 graphs] Group II (graphs [119:125])
mEC 47: 4-cycu + 2-clique (v1) [4 graphs] Group II (graphs [126:129])
**mEC 48: 4-cycu + 2-clique (v2) [8 graphs] Group II (graphs [130:137])
**mEC 49: 4-cycu + 2-clique (v3) [7 graphs] Group II (graphs [138:144])
mEC 50: 4-cycu + 3-clique [2 graphs] Group II (graphs [145:146])
**mEC 51: 4-cycu + 3-cycle (v1) [2 graphs] Group II (graphs [147:148])
mEC 52: 4-cycu + 3-cycle (v2) [2 graphs] Group II (graphs [149:150])
mEC 53: two 4-cycus (v1) [1 graph] Group II (graph 151)
mEC 54: two 4-cycus (v2) [1 graph] Group II (graph 152)
mEC 55: two 4-cycus (v3) [1 graph] Group II (graph 153)
**mEC 56: 4-ufd [107 graphs] Group II (graphs [154:260])
**mEC 57: 4-ufd + sink [11 graphs] Group II (graphs [261:271])
**mEC 58: 4-ufd + 2-clique (v1) [8 graphs] Group II (graphs [272:279])
mEC 59: 4-ufd + 2-clique (v2) [4 graphs] Group II (graphs [280:283])
mEC 60: 4-ufd + 2-clique (v3) [8 graphs] Group II (graphs [284:291])
**mEC 61: 4-ufd + 2-clique (v4) [16 graphs] Group II (graphs [292:307])
mEC 62: 4-ufd + 3-clique (v1) [2 graphs] Group II (graphs [308:309])
mEC 63: 4-ufd + 3-clique (v2) [2 graphs] Group II (graphs [310:311])
**mEC 64: 4-ufd + 3-cycle (v1) [3 graphs] Group II (graphs [312:314])
mEC 65: 4-ufd + 3-cycle (v2) [3 graphs] Group II (graphs [315:317])
mEC 66: 4-ufd + fusion 3-cycle [1 graph] Group II (graph 318)
**mEC 67: 4-ufd + 4-cycu (v1) [2 graphs] Group II (graphs [319:320])
mEC 68: 4-ufd + 4-cycu (v2) [1 graph] Group II (graph 321)
mEC 69: 4-ufd + 4-cycu (v3) [1 graph] Group II (graph 322) – missing the 4-ufd attractor
mEC 70: two 4-ufds (v1) [1 graph] Group II (graph 323)
mEC 71: two 4-ufds (v2) [1 graph] Group II (graph 324)
mEC 72: two 4-ufds (v3) [1 graph] Group II (graph 325)
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mEC 73: two 4-ufds (v4) [1 graph] Group II (graph 326)
mEC 74: two 4-ufds (v5) [1 graph] Group II (graph 327)
mEC 75: two 4-ufds (v6) [1 graph] Group II (graph 328)
GROUP III: 28 "unstable core" classes whose only unstable core motifs are 3-cycle and fusion 3-cycle
[Total: 725 graphs, 69% of all "unstable core" graphs, 7.55% of all n=5 graphs]
** None of these classes split into smaller attractor equivalence classes
mEC 76: 3-cycle [270 graphs] Group III (graphs [329:598])
mEC 77: 3-cycle + sink [62 graphs] Group III (graphs [599:660])
mEC 78: 3-cycle + two sinks [4 graphs] Group III (graphs [661:664])
mEC 79: 3-cycle + 2-clique (disjoint) [52 graphs] Group III (graphs [665:716])
mEC 80: 3-cycle + 2-clique (overlap) [148 graphs] Group III (graphs [717:864])
mEC 81: 3-cycle + 2-clique + sink [14 graphs] Group III (graphs [865:878])
mEC 82: 3-cycle + two 2-cliques (v1) [3 graphs] Group III (graphs [879:881])
mEC 83: 3-cycle + two 2-cliques (v2) [14 graphs] Group III (graphs [882:895])
mEC 84: 3-cycle + two 2-cliques (v3) [12 graphs] Group III (graphs [896:907])
mEC 85: 3-cycle + three 2-cliques [2 graphs] Group III (graphs [908:909])
mEC 86: 3-cycle + 3-clique [7 graphs] Group III (graphs [910:916])
mEC 87: two 3-cycles (v1) [4 graphs] Group III (graphs [917:920])
mEC 88: two 3-cycles (v2, "butterfly") [41 graphs] Group III (graphs [921:961])
mEC 89: two 3-cycles + sink [5 graphs] Group III (graphs [962:966])
mEC 90: two 3-cycles + 2-clique (v1) [6 graphs] Group III (graphs [967:972])
mEC 91: two 3-cycles + 2-clique (v2) [3 graphs] Group III (graphs [973:975])
mEC 92: two 3-cycles + 2-clique (v3) [2 graphs] Group III (graphs [976:977])
mEC 93: two 3-cycles + two 2-cliques [2 graphs] Group III (graphs [978:979])
mEC 94: three 3-cycles (v1) [1 graph] Group III (graph 980)
mEC 95: three 3-cycles (v2) [1 graph] Group III (graph 981)
mEC 96: four 3-cycles [1 graph] Group III (graph 982)
mEC 97: fusion 3-cycle [49 graphs] Group III (graphs [983:1031])
mEC 98: fusion 3-cycle + sink [6 graphs] Group III (graphs [1032:1037])
mEC 99: fusion 3-cycle + 2-clique (v1) [10 graphs] Group III (graphs [1038:1047])
mEC 100: fusion 3-cycle + 2-clique (v2) [2 graphs] Group III (graphs [1048:1049])
mEC 101: fusion 3-cycle + 3-clique [2 graphs] Group III (graphs [1050:1051])
mEC 102: two fusion 3-cycles (v1) [1 graph] Group III (graph 1052)
mEC 103: two fusion 3-cycles (v2) [1 graph] Group III (graph 1053)
GROUP IV: 52 "stable core" classes that have no unstable core motifs
[Total: 8555 graphs, 89% of all n=5 graphs]
** None of these classes split into smaller attractor equivalence classes
mEC 104: 5-clique [1 graph] Group IV (graph 1054)
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mEC 105: 4-clique [20 graphs] Group IV (graphs [1055:1074]) mEC 106: 4-clique + sink [4 graphs] Group IV (graphs [1075:1078]) mEC 107: 4-clique + 2-clique [3 graphs] Group IV (graphs [1079:1081]) mEC 108: 4-clique + 3-clique [2 graphs] Group IV (graphs [1082:1083])

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mEC 109: two 4-cliques [1 graph] Group IV (graph 1084)
mEC 110: 3-clique [568 graphs] Group IV (graphs [1085:1652])
mEC 111: 3-clique + sink [127 graphs] Group IV (graphs [1653:1779])
mEC 112: 3-clique + two sinks [9 graphs] Group IV (graphs [1780:1788])
mEC 113: 3-clique + 2-clique (disjoint) [106 graphs] Group IV (graphs [1789:1894])
mEC 114: 3-clique + 2-clique (overlap) [202 graphs] Group IV (graphs [1895:2096])
mEC 115: 3-clique + 2-clique + sink [19 graphs] Group IV (graphs [2097:2115])
mEC 116: 3-clique + two 2-cliques (v1) [4 graphs] Group IV (graphs [2116:2119])
mEC 117: 3-clique + two 2-cliques (v2) [20 graphs] Group IV (graphs [2120:2139])
mEC 118: 3-clique + two 2-cliques (v3) [12 graphs] Group IV (graphs [2140:2151])
mEC 119: 3-clique + three 2-cliques [3 graphs] Group IV (graphs [2152:2154])
mEC 120: two 3-cliques (v1) [12 graphs] Group IV (graphs [2155:2166])
mEC 121: two 3-cliques (v2) [55 graphs] Group IV (graphs [2167:2221])
mEC 122: two 3-clique + sink [7 graphs] Group IV (graphs [2222:2228])
mEC 123: two 3-cliques + 2-clique (v1) [4 graphs] Group IV (graphs [2229:2232])
mEC 124: two 3-cliques + 2-clique (v2) [6 graphs] Group IV (graphs [2233:2238])
mEC 125: two 3-cliques + two 2-cliques [2 graphs] Group IV (graphs [2239:2240])
mEC 126: three 3-cliques (v1) [2 graphs] Group IV (graphs [2241:2242])
mEC 127: three 3-cliques (v2) [1 graph] Group IV (graph 2243)
mEC 128: four 3-cliques [1 graph] Group IV (graph 2244)
mEC 129: 2-clique [2787 graphs] Group IV (graphs [2245:5031])
mEC 130: 2-clique + sink [1024 graphs] Group IV (graphs [5032:6055])
mEC 131: 2-clique + two sinks [106 graphs] Group IV (graphs [6056:6161])
mEC 132: 2-clique + three sinks [6 graphs] Group IV (graphs [6162:6167])
mEC 133: two 2-cliques (disjoint) [879 graphs] Group IV (graphs [6168:7046])
mEC 134: two 2-cliques (overlap) [863 graphs] Group IV (graphs [7047:7909])
mEC 135: two 2-cliques (disjoint) + sink [85 graphs] Group IV (graphs [7910:7994])
mEC 136: two 2-cliques (overlap) + sink [181 graphs] Group IV (graphs [7995:8175])
mEC 137: two 2-cliques (overlap) + two sinks [11 graphs] Group IV (graphs [8176:8186])
mEC 138: three 2-cliques (v1) [152 graphs] Group IV (graphs [8187:8338])
mEC 139: three 2-cliques (v2) [146 graphs] Group IV (graphs [8339:8484])
mEC 140: three 2-cliques (v3) [32 graphs] Group IV (graphs [8485:8516])
mEC 141: three 2-cliques + sink (v1) [14 graphs] Group IV (graphs [8517:8530])
mEC 142: three 2-cliques + sink (v2) [5 graphs] Group IV (graphs [8531:8535])
mEC 143: four 2-cliques (v1) [16 graphs] Group IV (graphs [8536:8551])
mEC 144: four 2-cliques (v2) [6 graphs] Group IV (graphs [8552:8557])
mEC 145: four 2-cliques (v3) [1 graph] Group IV (graph 8558)
mEC 146: four 2-cliques (v4) [20 graphs] Group IV (graphs [8559:8578])
mEC 147: four 2-cliques + sink [3 graphs] Group IV (graphs [8579:8581])
mEC 148: five 2-cliques (v1) [3 graphs] Group IV (graphs [8582:8584])
mEC 149: five 2-cliques (v2) [1 graph] Group IV (graph 8585)
mEC 150: six 2-cliques [1 graph] Group IV (graph 8586)
mEC 151: sink [764 graphs] Group IV (graphs [8587:9350])
mEC 152: two sinks [221 graphs] Group IV (graphs [9351:9571])
mEC 153: three sinks [32 graphs] Group IV (graphs [9572:9603])
mEC 154: four sinks [4 graphs] Group IV (graphs [9604:9607])
mEC 155: five sinks [1 graph] Group IV (graph 9608)
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## mEC Notes:

- mEC 2: The Gaudi attractor appears as a 2nd attractor for eps = 0.095, delta = 0.11 and eps = 0.1, delta = 0.12.
- mEC 9: The attractor for this core motif has surprise symmetry.
- mEC 12: The attractor for this graph (which has \*no\* core motifs) has surprise symmetry. This class contains 1 graph that has parameter dependent core motifs (# 12).
- mEC 13: The attractor for this core motif has surprise symmetry.
- mEC 14: The attractor for this core motif has surprise symmetry.
- mEC 15: The attractor for this core motif has surprise symmetry.
- mEC 17: The attractor for this core motif has surprise symmetry.
- mEC 18: The attractor for this core motif has surprise symmetry.
- mEC 19: The attractor for this core motif has surprise symmetry.
- mEC 26: The attractor for this core motif has surprise symmetry.
- mEC 27: The attractor for this core motif has surprise symmetry.
- mEC 28: The attractor for this core motif has surprise symmetry. This graph is s\_i-equivalent to graph 27.
- mEC 29: The attractor for this core motif has surprise symmetry.
- mEC 30: The attractor for this core motif has surprise symmetry. This graph is s\_i-equivalent to graph 29.
- mEC 31: The attractor for this core motif has surprise symmetry.
- mEC 32: This class contains 1 graph that has parameter dependent core motifs (# 32).
- mEC 33: This class contains 1 graph that has parameter dependent core motifs (# 33).
- mEC 45: This class contains 5 graph(s) that have parameter dependent core motifs (# 86 93 97 113 117).
- mEC 46: This class contains 1 graph(s) that are missing an attractor (# 124). This class contains 1 graph that has parameter dependent core motifs (# 124).
- mEC 48: This class contains 2 graph(s) that are missing an attractor (# 133 136). This class contains 2 graph(s) that have parameter dependent core motifs (# 133 136).
- mEC 49: This class contains 4 graph(s) that are missing an attractor (# 139 142 143 144). This class contains 4 graph(s) that have parameter dependent core motifs (# 139 142 143 144).
- mEC 51: This class contains 1 graph(s) that are missing an attractor (# 148). This class contains 1 graph that has parameter dependent core motifs (# 148).
- mEC 57: This class contains 1 graph(s) that are missing an attractor (# 267).
- mEC 58: This class contains 2 graph(s) that are missing an attractor (# 276 279).
- mEC 61: This class contains 3 graph(s) that are missing an attractor (# 293 299 305).
- mEC 64: This class contains 1 graph(s) that are missing an attractor (# 314).
- mEC 67: This class contains 2 graph(s) that are missing an attractor (# 319 320). This class contains 2 graph(s) that have parameter dependent core motifs (# 319 320).
- mEC 69: This class contains 1 graph(s) that are missing an attractor (# 322). This class contains 1 graph that has parameter dependent core motifs (# 322).
- mEC 75: This class contains 1 graph(s) that are missing an attractor (# 328).
- mEC 94: The attractors are quasiperiodic for eps = 0.25, delta = 0.5.
- mEC 96: This is baby chaos for eps = 0.25, delta = 0.5.

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## **Attractor Notes:**

mECs containing at least one graph that has a core motif with \*no\* corresponding attractor (14 mECs): [46, 48, 49, 51, 57, 58, 61, 64, 66, 67, 68, 69, 74, 75]

Graphs that have a core motif with \*no\* corresponding attractor (19 graphs): [ 124, 133, 136, 139, 142, 143, 144, 148, 267, 276, 279, 293, 299, 305, 314, 319, 320, 322, 328 ]

mECs containing at least one graph with parameter-dependent core motifs (10 mECs): [12, 32, 33, 45, 46, 48, 49, 51, 67, 69]

Graphs that have parameter dependent core motifs (19 graphs): [12, 32, 33, 86, 93, 97, 113, 117, 124, 133, 136, 139, 142, 143, 144, 148, 319, 320, 322]

### **Attractor Equivalence Classes**

(Preliminary: from Katie's classification using Hausdorff distance to compare attractors within an mEC.)

For the CTLN parameters eps = .51, delta = 1.76 we identified 145 dynamic attractors by pooling together distinct attractors within each mEC.

### mEC 41 splits into 2 attECs:

41A. [53]

41B. [54] **Note:** this is just a difference in whether or not the peripheral neuron fires, BUT it is rather high firing of the peripheral node in graph 54, so I would call these different.

### mEC 45 splits into 4 attECs:

45A. [ 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 87, 88, 89, 90, ... 91, 95, 96, 98, 99, 101, 102, 103, 104, 105, 107, 108, 110, 111, 112, 115, 116, 118, 100, 106, 109 ]

45B. [86, 97, 117] Note: this attractor has very high firing of peripheral node

45C. [93, 113] Note: this attractor has serious symmetry breaking and high peripheral node firing

45D. [80, 92, 94, 114] Note: these attractors are similar to that of graph 113, but with lower peripheral nodes

# mEC 46 splits into 2 attECs:

46A. [ 119, 120, 121, 122, 123, 125 ]

46B. [ 124 ] Note: this split is real! – graph 124 is missing its 4-cycu attractor

#### mEC 48 splits into 3 attECs:

48A. [ 130, 131, 134, 135 ]

48B. [ 132, 137 ] Note: this split is real! — we have peripheral node firing AND symmetry breaking

48C. [ 133, 136 ] Note: this split is real! – both these graphs are missing the 4-cycu attractor

## mEC 49 splits into 2 attECs:

49A. [ 138, 140, 141 ]

49B. [139, 142, 143, 144] Note: this split is real! – all graphs in the 2nd class are missing the 4-cycu attractor

#### mEC 51 splits into 2 attECs:

51A. [147]

51B. [ 148 ] Note: this split is real! – graph 148 is missing its 4-cycu attractor

### mEC 56 splits into 4 attECs:

56A. [ 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 254, 255, 256, 257, 258, 259 ]

Note: These all have no or low firing peripheral node 5

56B. [250, 260] Note: these attractors have very high firing of peripheral node 5

56C. [ 183 ] Note: this split is real! - very different attractor for the 4-ufd

56D. [217, 253] Note: this split is real! - another very different attractor for the 4-ufd

## mEC 57 splits into 2 attECs:

57A. [ 261, 262, 263, 264, 265, 266, 268, 269, 270, 271 ]

57B. [267] Note: this split is real! – graph 267 is missing the 4-ufd attractor

## mEC 58 splits into 2 attECs:

58A. [ 272, 273, 274, 275, 277, 278 ]

58B. [276, 279] Note: this split is real! – both these graphs are missing the 4-ufd attractor

## mEC 61 splits into 3 attECs:

61A. [ 292, 294, 295, 296, 297, 300, 301, 302, 303, 306, 307, 304 ] **Note:** These all have no or low firing peripheral node 5

61B. [ 298 ] Note: this split is real! – very different attractor for 4-ufd

61C. [ 293, 299, 305 ] Note: this split is real! - all the graphs in this class are missing the attractor for their 4-ufd

### mEC 64 splits into 2 attECs:

64A. [312, 313]

64B. [314] Note: this split is real! – graph 314 is missing the attractor for its 4-ufd

#### mEC 67 splits into 2 attECs:

67A. [319]

67B. [ 320 ] Note: this split is real! - 319 is missing the 4-cycu attractor while 320 is missing the 4-ufd attractor